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SEQUENCE ESTIMATION TECHNIQUES FOR ENHANCED DIGITAL SUBSCRIBER LOOP TRANSMISSION CAPABILITY

by

Vilas Joshi

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment of the requirements for the degree of Doctor of Philosophy

Ottawa-Carleton Institute for Electrical Engineering Faculty of Engineering Department of Systems and Computer Engineering Carleton University June 1988 ©Vilas Joshi, 1988
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DIGITAL SUBSCRIBER LOOP TRANSMISSION CAPABILITY

submitted by Vilas Joshi,
in partial fulfilment of the requirements for the degree of
Doctor of Philosophy

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ABSTRACT

The Digital Subscriber Loop (DSL) is an important element of Integrated Services Digital Networks (ISDNs), providing integrated voice and data services to the customer over existing cables of the loop plant. This work evaluates the channel capacity performance bounds of the subscriber loop channel when the interfering noise is crosstalk and white noise. The mathematical model used to evaluate the channel capacity of the subscriber loop is also established.

Maximum Likelihood Sequence Estimation (MLSE) is one of the keys to approach the performance promised by the channel capacity results. From the subscriber loop channel responses, performance-related parameters were computed for ideal matched filter reception, MLSE and decision feedback equalizer (DFE). Using these parameters we have made a comparative evaluation of the MLSE and DFE receiver theoretical performance for DSL channels.

The results of a study into the echo canceller and DFE tap length requirements in different DSL system configurations are also presented.

We have proposed two reduced state sequence estimation (RSSE) algorithms (Decision Feedback Sequence Estimation and M-algorithm) as less complex alternatives to Viterbi Algorithm (VA). A major interference limiting the range of the DSL systems is near-end crosstalk (NEXT). We have established the stationary and cyclostationary NEXT noise models that will be used to evaluate the performance of DSL systems. Using these NEXT noise models the theoretical performance achiev-
able with a VA receiver is evaluated. An extensive simulation study was carried out to fully establish the performance of the proposed RSSE receivers compared with the performance of more conventional DSL systems using the DFE. We show that for DSL channels the proposed RSSE algorithms can reduce the complexity of VA while obtaining performance superior to that of the DFE. We demonstrate that in the presence of cyclostationary crosstalk a substantial increase in loop range is achievable with RSSE receivers, compared to conventional DFE receivers.

Finally, we present the results of a preliminary investigation into the application of channel coding along with soft decision Viterbi decoding for DSL transmission.
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Contents

ABSTRACT ................................................................. iii
ACKNOWLEDGEMENTS .................................................. v
CONTENTS ................................................................. vi
LIST OF FIGURES ......................................................... ix
LIST OF TABLES ........................................................... xv

1 INTRODUCTION .......................................................... 1
   1.1 Objectives ......................................................... 5
   1.2 Thesis Organization ................................................ 9

2 CHARACTERISTICS OF SUBSCRIBER LOOPS ....................... 11
   2.1 The Loop Plant ................................................... 11
   2.2 Attenuation ......................................................... 12
   2.3 Inductive Loading ................................................ 12
   2.4 Gauge Discontinuity .............................................. 14
   2.5 Bridged Taps ....................................................... 14
   2.6 Noise ................................................................. 15
   2.7 Crosstalk ........................................................... 16

3 SYSTEM DESIGN ......................................................... 24
   3.1 System Objectives ............................................... 26
   3.2 Evaluation Criteria .............................................. 27
7.4.1 The NEXT Noise Model ..................................... 127
7.5 The Performance of MLSE Systems With Cyclostationary Crosstalk 136
7.6 Simulation Set-Up ............................................. 152
7.7 Simulation Results (Stationary Crosstalk) ......................... 158
7.8 Simulation Results (Cyclostationary Crosstalk) .................... 161
7.9 Discussion ..................................................... 165

8 PERFORMANCE OF TRELLIS CODING ON DSL CHANNELS 169
8.1 Trellis Code .................................................. 170
8.2 System Model ................................................ 170
8.3 Simulation Set-Up .......................................... 174
8.4 Simulation Results ......................................... 174
8.5 Discussion .................................................. 177

9 CONCLUSIONS .................................................. 180
9.1 Concluding Remarks ........................................ 180
9.2 Suggestions for Future Work ................................ 183

REFERENCES .................................................... 186
A CABLE CHARACTERISTICS DATA ............................. 198
B IMPEDANCE BALANCING HYBRID ................................ 202
C TRANSFORMER DATA .......................................... 203
D ABCD PARAMETERS FOR BRIDGED TAPS ..................... 204
List of Figures

1.1 Typical Application of the S-Interface and U-Interface Transceivers on an ISDN Digital Subscriber Loop. The S-interface is in the customer premises, and the U-interface provides transport to the serving center or central office. [4] .............................................. 3

1.2 A Typical DSL Arrangement Showing Sources of Echo Due to Imperfect Isolation in the Hybrid and Open Ended Bridged Tap Reflection. Note: Dashed lines are echo signal path, solid lines are the intended direction of transmission .................................................. 6

2.1 Attenuation of Typical Telephone Cables at 55°F. Dashed lines indicate square root dependence on frequency. [15] .................................................. 13'

2.2 Frequency Characteristics of 26 AWG PIC Telephone Cable. [16] .................................................. 13

2.3 Signal and FEXT model. .................................................. 20

2.4 Signal and NEXT model. .................................................. 20

2.5 NEXT LOSS for pair-to-pair and 49 disturbers. [7] .................................................. 22

3.1 Basic Echo Canceller Arrangement For Two-Wire Full Duplex Transmission .................................................. 25

3.2 Power Spectra of Different Line Codes. .................................................. 31

3.3 WAL-2 Waveforms and Coding State Diagrams .................................................. 33

3.4 Manchester Waveforms and Coding State Diagram .................................................. 33

3.5 Bipolar Waveforms and Coding State Diagrams .................................................. 33

3.6 WACX Waveforms and Coding State Diagrams .................................................. 35

3.7 Encoding with WACX .................................................. 37
3.8 Power Spectrum of the MS43 Code. [51] ........................................ 40

4.1 Channel Model. ....................................................... 50
4.2 NEXT Limited Capacity. ............................................ 54
4.3 ELCL FEXT Limited Capacity (22 AWG). ....................... 55
4.4 White Noise Limited Capacity (22 AWG). ....................... 56
4.5 White Noise Limited Capacity (26 AWG). ....................... 57
4.6 White Noise Limited Bandwidth (22 AWG). ..................... 58
4.7 White Noise Limited Bandwidth (26 AWG). ..................... 59
4.8 NEXT + White Noise Limited Capacity (22 AWG). ............. 60
4.9 NEXT + White Noise Limited Capacity (26 AWG). ............. 61
4.10 NEXT + White Noise Limited Bandwidth (22 AWG). ........... 62
4.11 NEXT + White Noise Limited Bandwidth (26 AWG). ........... 63
4.12 Main Capacity Versus BW'. ..................................... 65

5.1 Performance of DFE and MLSE Receiver On Subscriber Loop Channel Alone. .................................................. 75
5.2 Performance of DFE and MLSE Receiver On Subscriber Loop Channel With Hybrid and Transformer. ......................... 76
5.3 Performance of DFE and MLSE Receiver On Subscriber Loop Channel With Hybrid, Transformer and Bipolar Pulse Shaping. ........................................... 77
5.4 The Variation in $R_0$ with Loop Length (160 kbaud). .......... 78
5.5 The Variation in $d^2_{MLSE}$ with Loop Length (160 kbaud). .... 79
5.6 The Variation in $d^2_{DFE}$ with Loop Length (160 kbaud). .... 80
5.7 The Variation in $R_0$ with Loop Length (80 kbaud). .......... 81
5.8 The Variation in $d^2_{MLSE}$ with Loop Length (80 kbaud). .... 82
5.9 The Variation in $d^2_{DFE}$ with Loop Length (80 kbaud). .... 83
5.10 The System Model With Noise-Whitening Filter. .......................... 86
5.11 The Sketch of a Typical $|H(f)|^2$ Function. .............................. 87
5.12 The Sketch of a Typical Autocorrelation Function $R_k$. ................. 87
5.13 NEXT Limited Performance of DFE and MLSE Receiver On Subscriber Loop Channel With Transformer. .............................. 89

6.1 Block Diagram of a General DSL System. ................................ 94
6.2 Fixed Front-End Equalizer Structure (Differentiator). .................. 95
6.3 Ratio of Residual Echo to Total Echo, Without Receiver Filter. (160 kbaud). .................................................. 97
6.4 Ratio of Residual Echo to Total Echo, With Differentiator. (160 kbaud). .................................................. 98
6.5 Ratio of Residual Echo to Total Echo, With and Without Differentiator. (80 kbaud). .................................................. 100
6.6 Ratio of Residual Echo to Total Echo, With Receiver LPF and Bipolar Pulse Shaping (160 kbaud). .................................................. 101
6.7 Ratio of Residual Echo to Total Echo, With Receiver LPF and Bipolar Pulse Shaping (80 kbaud). .................................................. 102
6.8 Plots of Signal to DFE Tap Power Ratio, Without Receiver Filter. (160 kbaud). .................................................. 104
6.9 Plots of Signal to DFE Tap Power Ratio With Differentiator (160 kbaud). .................................................. 105
6.10 Plots of Signal to DFE Tap Power Ratio, With and Without Differentiator. (160 kbaud). .................................................. 106
6.11 Plots of Signal to Residual ISI Power Ratio Without Receiver Filter. (160 kbaud). .................................................. 107
6.14 Plots of Signal to Residual ISI Power Ratio and Signal to DFE Tap Power Ratio Without Receiver Filter. (80 kbaud). .................................................. 110
6.15 Plots of Signal to Residual ISI Power Ratio and Signal to DFE Tap Power Ratio, With Differentiator. (80 kbaud). 111
6.16 Plots of Signal to Residual ISI Power Ratio and Signal to DFE Tap Power Ratio, Without Receiver Filter. (80 kbaud). 112
6.17 Plots of Signal to Residual ISI Power Ratio and Signal to DFE Tap Differentiator. (80 kbaud). 113
6.18 Plots of Signal to DFE Tap Power Ratio and Signal to Residual ISI Power Ratio, With Receiver LPF and Bipolar Pulse Shaping (160 kbaud). 114
6.19 Plots of Signal to DFE Tap Power Ratio and Signal to Residual ISI Power Ratio, With Receiver LPF and Bipolar Pulse Shaping (80 kbaud). 116

7.1 Conventional DSL Transceiver System Model Using DFE. 120
7.2 A M-Algorithm Search in An 8-State Trellis With M=4. 123
7.3 Stationary NEXT Noise Model. 129
7.4 Crosstalk Power Variation With Sampling Phase (Bipolar Coding, 160 kbaud). 132
7.5 Crosstalk Power Variation With Sampling Phase (Bipolar Coding, 80 kbaud). 133
7.6 Crosstalk Power Variation With Sampling Phase (No Coding, 160 kbaud). 134
7.7 Crosstalk Power Variation With Sampling Phase (No Coding, 80 kbaud). 135
7.8 Cyclostationary NEXT Noise Model. 137
7.9 The Variation in $d^2_{min}$ with Sampling Phase ($\tau/T$) for Case 1. (All Loop Length, L, are in Miles). 141
7.10 The Variation in $d^2_{min}$ with Sampling Phase ($\tau/T$) for Case 2. (All Loop Lengths, L, are in Miles). 142
7.11 The Plot of $r_{conventional}$ and $r_{adaptive}$ Against Phase Shift $\phi$ for Case 1. (Loop Length L=5.0 Miles). 145
7.12 The Plot of $r_{conventional}$ and $r_{adaptive}$ Against Phase Shift $\phi$ for Case 1. (Loop Length L=7.0 Miles). ........................................... 146

7.13 The Plot of $r_{conventional}$ and $r_{adaptive}$ Against Phase Shift $\phi$ for Case 1. (Loop Length L=8.0 Miles). ........................................... 147

7.14 The Plot of $r_{conventional}$ and $r_{adaptive}$ Against Phase Shift $\phi$ for Case 2. (Loop Length L=5.0 Miles). ........................................... 148

7.15 The Plot of $r_{conventional}$ and $r_{adaptive}$ Against Phase Shift $\phi$ for Case 2. (Loop Length L=7.0 Miles). ........................................... 149

7.16 The Plot of $r_{conventional}$ and $r_{adaptive}$ Against Phase Shift $\phi$ for Case 2. (Loop Length L=8.0 Miles). ........................................... 150

7.17 M-Algorithm and DFSE Transmitter and Receiver System Model With Stationary Crosstalk. ........................................... 154

7.18 Conventional Decision Feedback Transmitter and Receiver System Model With Stationary Crosstalk. ........................................... 155

7.19 M-Algorithm and DFSE Transmitter and Receiver System Model With Cyclostationary Crosstalk. ........................................... 156

7.20 Conventional Decision Feedback Transmitter and Receiver System Model With Cyclostationary Crosstalk. ........................................... 157

7.21 Maximum Reach of the DSL Systems for Binary Transmission at 160 kb/s (Stationary Crosstalk). ........................................... 159

7.22 Maximum Reach of the DSL Systems for Quaternary Transmission at 80 kbaud (Stationary Crosstalk). ........................................... 160

7.23 Maximum Reach of the DSL Systems for Binary Transmission at 160 kb/s (Cyclostationary Crosstalk). ........................................... 163

7.24 Maximum Reach of the DSL Systems for Quaternary Transmission at 80 kbaud (Cyclostationary Crosstalk). ........................................... 164

8.1 Rate 2/3 Convolution Code. ........................................... 171

8.2 Trellis Diagram for the Rate 2/3 Encoder Shown in Figure 8.1. 171

8.3 8-Level PAM Signal Set. ........................................... 172

8.4 The Transmitter and Receiver System Model With Trellis Coding and Cyclostationary Crosstalk. ........................................... 173
8.5 Maximum Reach of the DSL System With Trellis Code and Comparison With Conventional DFE Detection. (Cyclostationary Crosstalk). 176

A.1 Cable Characteristics for 19 AWG PIC Cable at 70°F. [106] 199
A.2 Cable Characteristics for 22 AWG PIC Cable at 70°F. [106] 200
A.3 Cable Characteristics for 26 AWG PIC Cable at 70°F. [106] 201

B.1 An Impedance-Balancing Hybrid Structure 202

D.1 ABCD Parameters 206
D.2 Use of ABCD Parameter Matrix 206
D.3 Line With a Bridged Tap 207
D.4 The Equivalent Line 208
List of Tables

3.1 2B1Q Conversion Table ................................................. 38
3.2 Translation Table for MS43 ............................................ 39
4.1 Modem Milestones. [70] .................................................. 47
4.2 Evolution of Data Communications. [74] ............................. 48
7.1 The DFSE and M-Algorithm Performance Gains in Miles Compared to DFE Receiver ......................................................... 165
Chapter 1

INTRODUCTION

The increasing demand for a variety of new data services like simultaneous transmission of digitized speech and data, database communication capabilities, teletext and remote utility reading etc., has prompted the trend towards integration of voice and data in telecommunication networks. The existing analogue telephone network is expected to evolve towards ISDNs (Integrated Services Digital Networks) [1], [2], [3], [4]. Whether the concept of ISDN becomes a reality will depend primarily on the success in the efficient and economical digitization of the present telecommunication systems. The network has already been seeded with digital facilities in the interoffice transmission network as well as in toll switching. More recently, local digital switching has become a reality. The portion of the network which has remained largely untouched by digital trends is the subscriber loop. The final link to the customer premises continues to be predominantly analogue. The technologies for digital transmission over existing subscriber loops are the keys to the end-to-end connectivity in the realization of ISDN. This is due to the fact that a large part of the investment of the existing telephone system is in the subscriber loops, which are twisted pairs of wires that connect a subscriber to the local telephone exchange.
The ISDN holds the promise of being able to provide a rich set of digital data services through a single technology, in common with the voice network. The ISDN concept consists of a uniform access system which is digital, high speed, multichannel, internationally standardized and with sophisticated signalling. In addition the network provides variable bandwidth services including circuit switching, packet switching, and end-to-end signalling. This is the basis for developing a total network system which is flexible, intelligent and therefore able to respond quickly to market opportunities with the appropriate features and services. This will lead to more economical equipment for both the carrier and the user thereby enlarging the telecommunications market.

Today CCITT standardization activities concerning ISDN are accelerating and recommendations for the basic access are almost complete. The ISDN technologies are undergoing trials at different scales, in various locations, with various customers and multiple vendors.

There are several interfaces that provide the ISDN basic access at the physical layer. The Figure 1 shows the physical interfaces for the basic access to ISDN [4]. The basic user interface standard recommended by CCITT suggests a 144 kb/s (2B+D) information rate for ISDN [3], [4], [5]. The proposed data rate of 144 kb/s includes provision for two voice/data channels at 64 kb/s each (B+B) and one data channel at 16 kb/s (D). The S-interface is for transmission within the customer premises (residential, business or from private branch exchange to the telephone equipment). The S-interface uses separate wire pairs for the two directions of transmission between the network termination (NT) and terminal equipment (TE). The S-interface line rate is 192 kb/s [5]. The extra 48 kb/s are used for framing, control and synchronization.

The U-interface is for the connection of central office to the customer premises. The U-interface describes the full duplex data signal on the two-wire subscriber
Figure 1.1: Typical Application of the S-Interface and U-Interface Transceivers on an ISDN Digital Subscriber Loop. The S-interface is in the customer premises, and the U-interface provides transport to the serving center or central office. [4]
loop between central office line termination (LT) and customer premises network termination (NT). The T1D1.3 working group of Exchange Carrier Standards Association has proposed a U-interface bit rate of 160 kb/s (144 kb/s + 16 kb/s used for housekeeping) [6]. The proposed U-interface line code is 2B1Q (2 binary, 1 quaternary). This is a 4-level pulse amplitude modulation (PAM) code without redundancy. Thus the proposed symbol rate at the U-interface is 80 kbaud.

Many different techniques for transmission over subscriber loop have been proposed, the major effort is to achieve compromise between circuit complexity and transmission performance. We are interested in subscriber loops for digital transmission application and we will denote such a facility as the Digital Subscriber Loop (DSL). The DSL is a technology that provides full duplex service on a single twisted metallic pair (subscriber loop) at a rate sufficient to support ISDN basic services and additional framing, timing recovery and operations functions. The physical termination of the DSL at the network (central office) end is LT and the termination at the user (subscriber) end is NT. The proposed system range of the DSLs is 3.5 miles or about 42 dB loss at 40 kHz, corresponding to 99% loop penetration [6].

To economically realize the end-to-end digital transmission several techniques have been investigated to utilize existing 2-wire twisted pairs as a means of transmitting full duplex data at 144 kb/s. Field trials and simulations have been carried out to compare the performance obtained by three promising methods: Frequency Division Multiplexing (FDM), Time Compression Multiplexing (TCM) and Echo Cancellation (EC) [4], [7], [8], [9]. Among them the Echo Cancellation (EC) technique has emerged as the most promising since it has greater range due to narrow bandwidth, less cable attenuation, less transmission delay and reduced crosstalk coupling. The EC method is gaining acceptance in most parts of the world as a method of choice; while Japan is pursuing the TCM method [4], [10]. The T1D1.3 Working Group has recommended the EC technique for DSL application [6].
Figure 2 illustrates a simplified DSL arrangement. We will denote the subscriber (customer) end of the DSL as the near-end and central office (CO) end as the far-end. The hybrid’s capability of isolation is dependent on the accuracy of the balance network to match the line. For practical implementation a single compromise balance network is employed that best matches the greatest number of loops. The subscriber loop is characterized by a number of imperfections such as cable loss, distortion, bridged taps, gauge changes, crosstalk and impulse noise. One of the major impairments in the DSL is echoes due to imperfectly matched hybrids and effects of bridged taps. Figure 2 shows some of the sources of echoes in the DSL. There are additional constraints imposed in the practical implementation of a scheme such as remote powering of the subscriber set from the local exchange, synchronous operation in which near-end clock is slaved to the far-end (CO) clock, and the most important one being that any DSL scheme should be implemented in VLSI due to the cost considerations.

The major building blocks of DSL transceiver are echo cancellers, equalizer, line coding, timing recovery and filtering. Various signal processing techniques have been suggested and implemented for different blocks of the DSL system [4], [7], [10], [12], [13]. As the DSL transmission rate evolved from 80 kb/s initially, to 160 kb/s at present, more and more stringent design requirements have been imposed on the DSL system designer.

1.1 Objectives

The demands for higher data rates in the transmission of digital information are continuously growing. With the advent of ISDN, even higher speed data transmission will become available and the demand is likely to grow. As the demands for facilities in the local telephone network grows the need to transmit at rates much higher than 160 kb/s and/or over loops much longer than 3.5 miles is likely to arise.
Figure 1.2: A Typical DSL Arrangement Showing Sources of Echo Due to Imperfect Isolation in the Hybrid and Open Ended Bridged Tap Reflection. Note: Dashed lines are echo signal path, solid lines are the intended direction of transmission.
In view of this possibility it is important to establish the channel capacity bounds of the subscriber loops. One of the objectives of this study is to establish these channel capacity upper bounds on subscriber loop performance. The channel capacity bounds will serve as theoretical limits rather than as definite goals for a DSL system designer. These channel capacity performance bounds will form a benchmark for evaluating various DSL system design on a common basis. Our channel capacity results clearly indicate that reliable communication at rates much higher than 160 kb/s and/or over loops much longer than 3.5 miles is theoretically possible.

Equalization in a DSL system is indispensable. Adaptive decision feedback equalizer (DFE) has advantages over linear equalizer with regards to VLSI implementation and noise enhancement. Forney [14] has shown that Maximum Likelihood Sequence Estimation (MLSE) implemented by Viterbi Algorithm (VA) is the optimum solution to countering intersymbol interference (ISI). Another objective of this thesis is to obtain comparison of the relative theoretical performance of these two equalization techniques: MLSE and DFE. The comparison between DFE receiver and MLSE receiver gives us an indication of the size of the gap in performance between DSL systems in use today and what could theoretically be achieved by much more complex receiver design. The theoretical performance plots of MLSE and DFE receiver also provide a convenient basis for comparing inherent performance limits for each receiver technique.

The complexity of the MLSE receiver implemented by the VA for channels having long impulse response is prohibitive. In the case of DSLs, the ISI is 10 to 20 taps long and the VA receiver is unacceptably complex. Receiver complexity considerations have resulted in reduced emphasis on MLSE techniques for combating ISI. One of the keys to approach the performance promised by channel capacity results is MLSE. So an approach is needed that will reduce the complexity of the VA receiver but still offer partial performance gains of the MLSE. In this study we have investigated two reduced state sequence estimation (RSSE) algorithms (Decision
Feedback Sequence Estimation, DFSE and M-algorithm) which are suboptimum but less complex alternative to VA.

The objective is to establish the performance of the DSL systems using these two RSSE algorithms. Another objective is to compare the performance of RSSE systems with more conventional DSL transceiver schemes using DFE.

The dominant source of noise in the subscriber loops is near-end crosstalk (NEXT). The NEXT noise on a subscriber loop twisted pair is interference, through capacitive coupling from signals on neighboring twisted pairs in the same cable. In a mature ISDN environment all the twisted pairs in cables terminating in a central office will be carrying basic ISDN signals with their clocking obtained from the same master clock in the central office. In this case NEXT interference between any two loops is cyclostationary. Moreover if all the transmitted ISDN bit streams at the central office are phase- as well as frequency-synchronized or if only a few close neighbors can be assumed to cause most of the NEXT interference to a given loop, then the overall NEXT interference will be mainly cyclostationary. In this case the time-varying ensemble statistics of the crosstalk must be taken into account.

We first investigated the theoretical performance obtainable from a MLSE receiver with stationary NEXT and cyclostationary NEXT noise models. Simulations were carried out to evaluate the performance of the two RSSE algorithms investigated (DFSE and M-algorithm) and conventional DSL systems. In this study we evaluate the performance of the RSSE and conventional DSL systems using both the stationary and cyclostationary crosstalk noise models.

We show that for DSL channels with colored noise (stationary and cyclostationary NEXT). The RSSE algorithms investigated offer superior performance compared to more conventional DFE receivers without incurring unacceptably large complexity of the VA. In particular, we show that the improvement can be especially significant in the presence of cyclostationary crosstalk, because of the freedom
that sequence estimation receivers afford in the choice of receiver sampling phase. We evaluate this advantage for VA receivers.

VLSI realizability is the ultimate deciding factor when implementing any DSL system, as only the VLSI technology can implement a complex DSL system at low cost. The complexity of the echo canceller and equalizer largely determines the complexity of the overall circuit. So it is necessary to investigate the length requirements of the echo canceller and DFE in different system configurations to achieve a certain satisfactory level of performance. We have evaluated the complexity requirements of echo canceller and DFE for various line codes commonly used in the DSL. The effect of various receiver filter designs on the complexity is also investigated.

We have also begun an investigation of the application of channel coding for DSL application. Specifically we have investigated trellis codes, along with soft decision Viterbi decoding. The aim is to establish the performance of bandwidth efficient trellis coded transmission over subscriber loops.

1.2 Thesis Organization

The thesis is organized in nine chapters. This subsection includes a brief description of the contents of each of these chapters.

Chapter 2 discusses the characteristics of the subscriber loops, which affect the digital transmission over loops. It also gives some of the statistics of the loop in North America.

Chapter 3 defines the system objectives and evaluation criteria. It also discusses the design issues of line coding, echo canceller, equalization and timing recovery.
Chapter 4 presents the channel capacity performance bounds of the subscriber loop channel when the interfering noise is crosstalk and white noise.

The comparative evaluation of the theoretical performance of MLSE receiver and DFE receiver is presented in Chapter 5.

The results of the investigation into the complexity requirements of the echo canceller and DFE for different DSL system configurations is presented in Chapter 6.

Chapter 7 reports the experimental study by computer simulation of two RSSE algorithms for DSL applications. The first part of the chapter describes the two RSSE algorithms; namely the DFSE and the M-algorithm, under investigation. It also describes the noise models (stationary and cyclostationary NEXT noise models) which will be used in simulations to evaluate the performance of DSL systems. The theoretical performance of sequence estimation on DSL with stationary and cyclostationary crosstalk noise is also reported in Chapter 7.

The second part of Chapter 7 describes the simulation philosophy and the simulation set-up. It then presents the simulation results and discussions.

Chapter 8 reports the results of the preliminary investigation into using trellis coding for transmission over subscriber loops.

Finally, the different conclusions and extensions for future work are presented in Chapter 9.
Chapter 2

Characteristics of Subscriber Loops

2.1 The Loop Plant

One of the basic questions that has to be answered in the evolution of the DSL system is, what is the capability of the existing loop plant to carry digital signals. The following loop characteristics determine the transmission capability of the loop [15]:

1. Attenuation
2. Inductive Loading
3. Gauge Discontinuities
4. Bridged Taps
5. Noise
6. Crosstalk

The subscriber loop characteristics and their effect on the DSL system design are discussed in the following sections.
2.2 Attenuation

The plot of attenuation (dB/mile) versus frequency of exchange cable is shown in Figure 2.1 for various gauges [15]. Figure 2.2 shows the insertion loss characteristics of a 26 gauge wire alone [16]. The loop acts like a low-pass filter with pass-band attenuation, attenuating and dispersing the pulse as loop length increases. It should be noted that the loss of a twisted pair line exhibits a transition in the 80 kHz to 500 kHz band; beyond about 300 kHz, \( z_0 \) tends to its resistive asymptotic value and the loss follows a \( \sqrt{f} \) law with frequency. However below this, both the attenuation constant, \( \alpha \), of the line and the insertion loss between resistive termination changes much more slowly with frequency. In fact \( \alpha \) varies within the transition region as approximately \( f^{0.2} \). This means that the loss penalty incurred with an increase in the line rate or code spectral content with frequency is not as severe as in a normal \( \sqrt{f} \) dominated system.

Consequently, this allows due regard to be paid in the choice of line code to such features as timing content, distortion, tolerance, ease of detection, low frequency content and somewhat reduces the need to prejudice these properties by the usual emphasis on spectral efficiency.

2.3 Inductive Loading

Approximately 23% of the existing loop plant, primarily those extending beyond 18 kft are loaded [15]. A single load coil introduces 10-40 dB of additional attenuation at frequencies of interest here. This severe limitation confines the DSL system application to nonloaded loops.
Figure 2.1: Attenuation of Typical Telephone Cables at 55°F. Dashed lines indicate square root dependence on frequency. [15]

Figure 2.2: Frequency Characteristics of 26 AWG PIC Telephone Cable. [16]
2.4 Gauge Discontinuity

Over 80% of the loop experiences at least one gauge change and 21% have four or more such discontinuities. The typical composition of an average U.S. loop of average length of 7748 feet is [17]:

- 4500 feet of No.26 AWG
- 2408 feet of No.24 AWG
- 798 feet of No.22 AWG
- 42 feet of No.19 AWG

Gauge changes cause two types of transmission impairments:

1) Because of the loss characteristics of each gauge is different, variation in gauge introduces different loss.

2) It causes reflections to the sending end.

2.5 Bridged Taps

Bridged taps introduce echoes that degrade the performance of the system. Another effect of bridged taps is to load the loop providing additional attenuation and ISI [17], [18], [4]. Approximately 80% of the existing loops have at least one bridged tap [15]. The effect of bridged tap is not necessarily related to the length of the loop it sits on, but more to the length of the bridged tap itself. That is, a bridged tap of a critical length and appropriate gauge placed anywhere along the line may severely degrade its performance. The expected worst case length of a bridged tap is an odd multiple of one quarter of wavelength.
2.6 Noise

(1) **Power Induction**: Electromagnetic coupling between nearby power lines and telephone cables results in significant longitudinal voltages at 60 Hz power frequency and its harmonic on the telephone pairs. Baseband data signals requiring good response at low frequencies may be seriously impaired by this interference. Use of a code which results in a power spectrum that approaches zero at low frequencies and permits use of a high pass filter can solve this problem [19].

(2) **Impulse Noise**: One of the major factors limiting the range of the DSL system is impulse noise. The impulse noise originates from relays in the analog telephone circuits and couples through crosstalk into loops carrying digital signals. Although modern digital telephone exchanges are relatively free of impulse noise sources, analog exchanges which employ relays will continue to operate in many networks for several more decades. Therefore it is necessary to ensure that DSL systems which share the same cable as the switched circuits from these analog exchanges can tolerate the expected levels of impulse noise [10].

Though measurements have been made in the voiceband, knowledge about impulse noise in the frequency range used by DSL systems is very limited [15], [16]. Major sources of impulse noise are dial pulsing transients, switching and high voltage ringing pulses in analog exchanges. The interfering impulses are short compared with the time between them and as a result, the receiving circuit resolves them as independent events. The available data indicates that the impulse noise power spectrum density is highest at low frequency [15], [19]. As the frequency increases this power declines. For any line code power spectral density which behaves in the inverse manner at low frequencies, low-frequency pre-emphasis at transmitter does not significantly alter signal power. Corresponding de-emphasis at the receiver reduces the effect of impulse noise.
If impulse noise performance is measured for existing subscriber loops, it would vary according to the number and calling rate of analog telephone circuits, and cable crosstalk characteristics. The different calibration techniques for impulse noise field measurements are given in references [10], [7], [20]. The analytic models to estimate error probability of DSL's in the presence of impulse noise is reported in [10], [21], [22]. The statistical characteristics of the impulse noise can be analyzed by using the measured data under various conditions. Using impulse noise characteristics it is possible to estimate the error rate performance of a DSL system due to impulse noise. Despite these latest efforts to characterize the impulse noise in DSL the information about impulse noise remains sketchy. As the digitization of telephone networks increases the impulse noise is less likely to be the limiting factor.

2.7 Crosstalk

Another major impairment to digital transmission in subscriber loops is crosstalk. In multipair cables, crosstalk among neighbours, at high transmission rates may be a dominant source of disturbance. The two types of crosstalk involved are near-end crosstalk (NEXT) and far-end crosstalk (FEXT).

The maximum transmit power of a DSL system is dictated by the compatibility requirements with other systems which share the same cable facilities [7], [19]. These would include program channel services, Digital Data Phone Services (DDS), or Dataroute as well as analog carrier (e.g. SLC-8) and digital carrier such as T1. The DSL system transmit level must be limited in order to ensure that the performance of the systems being interfered with remains acceptable. For interference into an analog loop carrier systems, crosstalk must be limited to ensure that its idle noise requirements are not exceeded. In the case of digital systems the crosstalk must not close the eye opening at the decision point to cause the BER requirement to be exceeded. The minimum-possible receiver power level is governed by the impairment
caused by other systems crosstalking into the DSL. Here the crosstalk into the DSL systems must be kept low enough to ensure that its BER requirement is maintained.

Once the maximum transmit level and minimum receive level are obtained, their difference (in dB) puts a bound on the range of the DSL system. However, the range can also be limited by self-induced crosstalk (i.e., one DSL crosstalking into other similar systems (both NEXT and FEXT). The result is the maximum signal loss that the DSL system can tolerate without the level becoming so low with respect to crosstalk that BER requirements are exceeded. This defines another system range. The minimum of these two ranges represents the actual system range unless the impulse noise puts even tighter limitations on system range. The system range in dB may be translated into distance or, of greater significance, the percentage of loop population, on which DSL systems may operate at certain BER.

The computations necessary to carry out the range evaluations limited by crosstalk are by no means simple. They involve utilizing the power spectra of the transmitted signals of the interfering system; processing them through the gain of the receiver circuit of the system being interfered with and determining the resulting crosstalk noise level and their effects. Furthermore, crosstalk losses are statistical in nature, and vary randomly from pair combination to pair combination in multipair cable. Assumption must be made with respect to the probability distribution of pair-to-pair crosstalk loss. Also the total crosstalk power induced in a pair is the result of coupling from all the disturbers on the other pairs in the cable. Hence the assumption must be made with respect to the number of disturbers present and the statistical multi-disturber crosstalk model [7], [19], [23], [24], [25], [26].

The pair-to-pair NEXT and FEXT models and corresponding mathematical formulations involved to calculate the pair-to-pair coupling loss is given below [24], [25].
(1) **FEXT**

Far-end crosstalk is the coupling between cable conductors of signals transmitted in the same direction. The average far-end coupling path between two conductors is described by equation (2.1):

\[
F^2(f) = K_F(\ell/\ell_0)(f/f_0)^2
\]

where \( K_F \) is a random variable defined as the loss between two cable conductors measured at the same reference length \( \ell_0 \) and reference frequency \( f_0 \). Equation (2.1) gives the received power of a transmitted tone of frequency \( f \) divided into the coupled power of the same signal received on an adjacent conductor of equal length \( \ell \). Since this ratio is based on the received power equation (2.1) is called Equal Level Coupling Loss (ELCL). In dB the loss is:

\[
ELCL = 10 \log K_F + 10 \log(\ell/\ell_0) + 20 \log(f/f_0)
\]

Equation (2.2) shows that FEXT coupling loss decreases at 6 dB/octave rate with frequency and decreases at 3 dB/octave rate with conductor length \( \ell \). The term \( 10 \log K_F \) is modelled very well as a normal variable with mean \( m_F \) and standard deviation \( \sigma_F \) over all possible pairs of conductors in the cable. Figure 2.3 shows two conductors in the same cable, with signals transmitted in the same directions, having response \( C(f) \) and the corresponding FEXT coupling model. With the aid of Figure 2.3 we can compute the power of a single system coupled into one other system. The mean FEXT noise power \( P_F \) coupled at the output of the receiver 2 is:

\[
P_F = \int_0^w \left| G(F) \right|^2 \left| C(f) \right|^2 \left| R(f) \right|^2 F^2(f) df
\]

(2.3)
\[ G(f) \mid^2 = \text{PSD of the disturbing system. (Transmitter 1)} \]

\[ C(f) = \text{Cable response.} \]

\[ R(f) = \text{Receiver gain of the disturbed system. (Receiver 2)} \]

\[ W = \text{System bandwidth.} \]

Pair-to-pair FEXT interfering noise in dB is:

\[ P_F = m_F + 10 \log_2 \left( \frac{\ell}{\ell_0} \right) + 10 \log \int_0^W |G(f)|^2 \mid |C(f)|^2 \mid R(f) \mid^2 \left( \frac{f}{f_0} \right)^2 \, df \tag{2.4} \]

(2) NEXT

The proximity of strong transmitted signals to weak received signals gives rise to NEXT. Near-end coupling loss (NCL) between two transmission paths in the cable is expressed by the power ratio in the equation:

\[ N^2(f) = K_N \left( \frac{f}{f_0} \right)^{3/2} \tag{2.5} \]

In decibels this reduces to:

\[ NCL = 10 \log K_N + 15 \log \left( \frac{f}{f_0} \right) \tag{2.6} \]

The value \(10 \log K_N\) is a random variable expressing the coupling loss between the selected conductors measured at frequency \(f_0\). Equation (2.6) shows that NEXT coupling loss decreases at 4.5 dB/octave rate with frequency. As with the FEXT coupling loss, the variable \(10 \log K_N\) has a normal distribution with mean \(m_N\) measured at reference frequency \(f_0\) and standard deviation \(\sigma_N\). The Figure 2.4 shows the two pairs transmitting in opposite directions within the same cable and corresponding NEXT coupling model. The mean NEXT noise power \(P_N\) coupled at the output of the receiver 2 is given by:
Figure 2.3: Signal and FEXT model.

Figure 2.4: Signal and NEXT model.
\[ P_N = \int_0^N |G(f)|^2 |R(f)|^2 N^2(f) df \] (2.7)

So pair-to-pair NEXT interfering noise in dB is:

\[ P_N = m_N + 10 \log \int_0^W |G(f)|^2 |R(f)|^2 (f/f_0)^{3/2} df \] (2.8)

With this mathematical formulation one can find the pair-to-pair NEXT and FEXT interference, first when the DSL is the disturbing system and second when the DSL is the disturbed system.

The NEXT is the dominant crosstalk coupling in DSL systems using Echo Cancellation Techniques [7], [10], [15], [26], [27]. When the two-directional pair groups are contained in the same cable, NEXT tends to be more damaging than FEXT.

The total crosstalk power induced in a pair is coupled from a large number of other pairs. To find the crosstalk coupling due to multiple disturber one has to characterize the power sum of pair-to-pair crosstalk losses. The crosstalk power sum is the total crosstalk interference which appears on a given pair as a result of coupling from all disturbers on other pairs in the same cable. In almost all the earlier literature on crosstalk, the probability distributions of both pair-to-pair crosstalk loss and crosstalk power sum are assumed to be normal on dB scale i.e., log normal on power scale [24], [25], [28]. The results by Lin [23] indicate that the gamma distribution is a more satisfactory approximation than the conventionally accepted normal distribution for modelling multipair crosstalk behaviour.

The NEXT loss model suggested by the T1D1.3 working group gives the 1% NEXT loss as a function of frequency with loss decreasing at 15 dB per decade of frequency and having 57 dB loss at 80 kHz. This NEXT loss is the power sum of the pair-to-pair NEXT loss due to 49 simultaneously active disturbers in the same
Figure 2.5: NEXT LOSS for pair-to-pair and 49 disturbers. [7]
binder group [6]. Figure 2.5 shows the pair-to-pair and 49 disturbers. 1% NEXT loss as a function of frequency [7].

In all the studies on crosstalk interference in DSL it is assumed that the crosstalk interference from digital system is wide sense stationary. Campbell et al. [29], [30], [31] have shown that it may be necessary to consider the cyclostationary nature of the digital signals when determining crosstalk interference between such signals in multipair cable. In digital systems the decisions are made on periodic samples of the received signal and when considering the interference between such systems the relationship between sampling instant and time-varying ensemble statistics must be taken into account. The DSL systems are presently engineered assuming randomly phased crosstalk disturbers. These systems may suffer a loss in margin if the disturbing system clocks are phase aligned. This is likely to happen in a synchronous network like DSL. This loss in margin occurs because of the possibility of the disturbed system sampling at instants when the crosstalk interference power is greater than the time-average power [29], [30], [31].

Typically more than 50% of the crosstalk power is coupled from fewer than 6 interfering pairs and if the system clocks of these pairs are phase aligned the crosstalk would be predominantly cyclostationary [32], [33]. One way of obviating this loss in margin is by appropriate discrete phase randomization of the DSL system clocks [30].

However there is a positive side to this cyclostationary nature of the crosstalk. Due to the peak-to-average variation of the NEXT interference there exists a range of decision points for which system sampling instants can correspond to smaller than average values of the NEXT interference. The results reported in Chapter 7 clearly show that NEXT cyclostationarity can be exploited by a sequence estimation receiver to substantially improve the maximum loop range.
Chapter 3

SYSTEM DESIGN

The proposed U-interface bit rate in ISDN is 160 kb/s. The U-interface transceiver design is complicated due to bridged taps, echoes, crosstalk, impulse noise and ISI. VLSI realizability of any DSL system is a must for cost effectiveness. The EC technique for DSL application is gaining favor in most standardization activities [6]. Figure 3.1 gives the block diagram of a DSL system using EC method. The method of transmission is synchronous, with near-end clock at the subscriber end being slaved to the stable master clock at the far-end (central office). The major building blocks of DSL systems are echo canceller, equalizer, line coding, and timing recovery circuit. The EC technique has been extensively studied for DSL application [4], [10], [11], [12]. The DSL system design considerations involve defining various system objectives and establishing criteria which can be used to evaluate different DSL systems. The objectives defined above will influence the design of line code, equalizer, echo canceller and timing recovery circuit.

The first part of this chapter discusses the system objectives and evaluation criteria. The second part addresses design issues of line coding, echo canceller, equalization and timing recovery.
Figure 3.1: Basic Echo Canceller Arrangement For Two-Wire Full Duplex Transmission.
3.1 System Objectives

Certain essential and desirable features of the DSL system are specified by the following objectives:

(1) Full duplex operation.

(2) Operation over two-wire local lines of conventional length (typically up to about 3.5 miles) and characteristics without repeater.

(3) Long term bit error rate better than 1 in $10^7$.

(4) Transmission system should be bit sequence independent i.e., there should be no restriction on the data pattern transmitted by the customer.

(5) Central battery powering. Remote powering of DSL system is required to guarantee basic communication services even during power failure. The power consumption of the DSL system must be sufficiently low so as to allow powering from the central battery over the line.

(6) It will be advantageous to have a power down mode when idling, but it should have very rapid wake up and alignment time to minimize call set up times (e.g., 200 msec. from off-hook to transmission of the first signal message).

(7) Simple installations, particularly at the subscriber end without sensitive setting up or alignment procedures and it should be cheap. It is desirable that no plant conditioning be required beyond the removal of load coils.

(8) There should be sufficient rejection of pulses, transients and ringing voltages to prevent errors in the digital channel.

(9) Master/slave operation, where the system clock at the subscriber end is slaved to the master clock at the central office end.
(10) Reasonable system penetration

(11) System compatibility, an ISDN DSL system should co-exist with different classes of existing system without significant interference to or from.

(12) VLSI realization of the DSL system is essential for solving main problems like power consumption, circuit compactness and cost effectiveness.

3.2 Evaluation Criteria

In order to evaluate any transmission system for its suitability, it is necessary to first establish the criteria upon which the evaluation will take place, i.e., to set forth what factors are most important in judging the worth of DSL systems. These factors can be enumerated as follows:

(1) System Range: This is the maximum nonrepeatered length of the DSL system. It is often represented as a loss range in dB which is related to cable length of different gauges via the appropriate loss per unit length of the particular cable. The loss range in dB proposed by the T1D1.3 working group is 42 dB loss at 40 kHz [6]. Evaluation of the range of a system is a most complex process. There are a number of sources of limitations which must be considered. The sources of limitations include:

1. Cable loss

2. Crosstalk

3. Compatibility with other transmission system which must coexist in the same cable

4. Impulsive noise due to central office switching
5. Eye-opening performance, due to such impairments as imperfect equalization, inadequate echo cancellation and jitter.

The limits on system range imposed by each of these factors must be evaluated. The minimum of these ranges represents the actual system range.

(2) System Penetration: The penetration of the loop plant represents the percentage of the loops or the percentage of customers which the system can serve. Penetration is closely related to system range. Given the range in dB, the penetration is no greater than the percentage of loops whose loss is within the range of the system. To determine penetration, actual loop statistics such as those given in the Bell loop survey [34] can be used. The system penetration proposed by the T1D1.3 working group is 99% coverage of North American non-loaded loop population.

(3) Cost, Complexity, Reliability and Power Dissipation: Though it may be difficult to define these factors on an absolute basis, the DSL systems can be compared based on relative performance with respect to these factors.

3.3 Design Techniques

The discussion so far has been at the system level, where we have defined the desirable features of the DSL system. We also established various criteria based on which the evaluation of any DSL scheme can be carried out. Based on the design objectives stated above for the DSL system we now present the discussion of those major design issues of line coding, equalization, timing recovery and echo cancellation which must exist in the design of a DSL system using the EC method.
3.3.1 Line Code

One of the critical choices in the design of DSL systems is the selection of line codes. The line code has a strong influence on the performance and complexity of the DSL transceiver. It is desirable for the line code to have the following properties [16], [35], [36]:

(1) As dc isolation between system and cable is necessary (provided by transformer) the line code should have zero dc component and low spectral density at lower frequencies to reduce the distortion caused by the channel at lower frequencies.

(2) The line code should be simple to encode and decode in order to give low complexity in the system.

(3) No cable dependent adjustment should be needed.

(4) Frequency spectrum with small bandwidth and situated at lower frequencies to minimize the influence of crosstalk, thermal noise and impulse noise.

(5) Sufficient timing information for clock recovery at the receiver.

(6) Error monitoring.

(7) Code should be bit sequence independent.

(8) A linear relationship between coded and uncoded systems.

(9) Ease of equalization.

(10) Finite error propagation.

It can be seen that many of these requirements are conflicting and also not all of these requirements are equally important; e.g., although biphase and WAL-2 code fulfill most of these requirements very well, both of these codes effectively
fulfill most of these requirements very well, both of these codes effectively double the bandwidth requirement which results in increased line loss and increased crosstalk. A particular advantage of balanced binary code is that no AGC is required for different cable loss. The multilevel codes by reducing the baud rate reduce the crosstalk interference. The choice of a line code is always a compromise between conflicting objectives.

Here we discuss some of the traditional choices of line codes for DSL application such as WAL-2, biphase, AMI, 2B1Q, MS43 and a new line code called WACX which has been recently proposed for DSL application [37]. Figure 3.2 gives the plots of the power spectral density of some of these traditional codes.

(1) WAL-2

Power spectrum of WAL-2 code is given by [38]:

\[ |G(f)|^2 = \left( \frac{\sin \pi f T}{\pi f T} \right)^2 \]

Figure 3.3 gives the WAL-2 waveform and coding state diagram. WAL-2 is a two level code. It has more energy in the high frequency range which increases crosstalk and line loss. But self-equalizing properties reduce the code's sensitivity to cable distortion in a wide range of line lengths and can result in an open eye at the receiver with little or no line equalization. The effective echo impulse response is much shorter (8 to 16 baud intervals). The code is perfectly balanced on a symbol basis and low frequency energy is very low; facilitating the use of line coupling transformers with fairly high low frequency cutoff. It has sufficient timing information which is bit sequence independent. It does not require accurate agc.

(2) Manchester Biphase:

The Manchester code has the power spectrum given by [38]:

30
(a) Power spectra of M WAL2 (unbroken line) WAL2 (broken line) and biphase (dotted line)

(b) Power spectra of Bipolar code

Figure 3.2: Power Spectra of Different Line Codes.
\[ |G(f)|^2 = \left( \frac{\sin^2 \pi f T}{\pi f T} \right)^2 \]

Figure 3.4 gives the Manchester code waveforms and coding state diagram [39]. It is a two level code. The spectral peak of Manchester code is shifted below 1/T. It has zero dc component. The code is perfectly balanced. Manchester code exhibits excellent properties as far as the following items are concerned:

(a) Timing extraction.

(b) Transparency to information content (bit sequence independent).

(c) Error monitoring.

(d) Low spectral density at low frequencies.

(e) Low sensitivity to line distortion.

(f) Short echo impulse response.

(g) Low complexity for a code and decode circuitry.

Although it requires line equalization and requires twice the bandwidth compared to bipolar, a fixed equalizer (one or two taps) can cover the wide range of line lengths. It has better crosstalk performance compared to WAL-2. It does not require AGC.

(3) Bipolar:

The power spectrum of bipolar or AMI code is given by [38]:

\[ |G(f)|^2 = \left( \frac{\sin^2 \pi f T}{\pi f T} \right)^2 \]
Figure 3.3: WAL-2 Waveforms and Coding State Diagrams.

Figure 3.4: Manchester Waveforms and Coding State Diagram.

Figure 3.5: Bipolar Waveforms and Coding State Diagrams.
Figure 3.5 gives the bipolar waveform and coding state diagram. It is a three level code [39], [40]. Besides simplicity of code translation, this code has the advantage of small low frequency content. The small low-frequency power implies reduced tails of the echo signal and reduced length of the echo canceller. Error monitoring can be achieved by checking code violations. Primary advantages are lack of dc component, the lack of error propagation and ease of implementation. It has its spectral peak at half the bit rate resulting in much better crosstalk performance compared to WAL-2 and Manchester code. The major disadvantage of bipolar is its lack of bit sequence independence causing potential timing recovery problems. This problem can be overcome using scramblers. The other alternative is to use codes like HDB3 and B6ZS [40].

The bipolar code has a 3 dB penalty on a mean power basis compared to the ideal NRZ based system. If the system is peak power limited, the penalty is 6 dB. Due to the increase in the number of levels, the signal requires an accurate agc to detect the three levels correctly.

(4) WACX Code:

This new line code can be defined as the combination of WAL-1 and WAL-2 waveforms. Levy and Surie first proposed the use of this code for DSL application [37]. In that paper they used the name WAL COMPLEX (WACX) for this code. Figure 8.6(a) gives the code waveforms. It also illustrates why this code is defined as a combination of WAL-1 and WAL-2. Figure 3.6(b) gives the coding state diagram and Figure 3.6(c) gives the basic waveform used to represent one and zero. The power spectrum of the WACX code is given by:

\[ |G(f)|^2 = \left( \frac{\sin \pi f T \cdot \sin \pi f \frac{T}{2}}{\pi f \frac{T}{2}} \right)^2 \]
Figure 3.6: WACX Waveforms and Coding State Diagrams.
The power spectrum of WACX code is the same as that of bipolar code with 50% duty cycle. The code has a bandwidth equal to the bit rate. It is a two level code and is well balanced. Figure 3.7 shows the encoding using WACX code. It consists of transmitting two interleaved sequences of odd and even signal elements \( s_0(t) \) and \( s_e(t) \) respectively. Each sequence is periodic at the rate \( 1/2T \). The odd bit sequence \( s_0(t) \) is staggered, so that the center of the signal elements of one series are placed at instances of time where all the elements of another series are null, so there is no interference between elements of two series. This method of sending two interleaved sequences of signal elements where periodic signalling is replaced by staggered signalling was originally conceived by Bennett and Feldman [41]. At the receiver the received signal will be sampled at \( 0, T/2, 2T, 5T/2, 4T, 9T/2, \) etc. This will necessitate two equalizers and two echo cancellers at the receiver, one cancelling at odd sampling instances and one cancelling at even sampling instances. Each equalizer and echo canceller will be operating every \( 2T \) seconds.

As the power spectrum is similar to the bipolar power spectrum, its crosstalk performance will be similar to bipolar code. Unlike the bipolar code it is two level code, so it does not require an agc; also it is bit sequence independent. It has sufficient timing information. The paper by Levy and Surie also discusses the bit error rate performance of this code for a given receiver structure and describes a timing recovery method for DSL systems using this type of encoding.

The suitability of this code to transmit over DSL is yet to be fully established.

(5) 2B1Q Code:

The T1D1.3 working group has recommended 2B1Q (2 binary, 1 quaternary) line code as a standard for U-interface. It is a 4-level PAM code with zero redundancy. The proposed U-interface bit rate is 160 kbit/s and corresponding symbol rate with 2B1Q code is 80 Kbaud. The conversion table of this code is given in
Figure 3.7: Encoding with WACX.
Table 3.1. It is a very simple code and has excellent performance in the presence of crosstalk [6], [32].

Table 3.1

<table>
<thead>
<tr>
<th>Transmitted Bits</th>
<th>Quaternary Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>+3</td>
</tr>
<tr>
<td>11</td>
<td>+1</td>
</tr>
<tr>
<td>01</td>
<td>-1</td>
</tr>
<tr>
<td>00</td>
<td>-3</td>
</tr>
</tbody>
</table>

(6) MS43 Code:

Block codes are widely used in digital transmission. The MS43 block code has been proposed for the U-interface [4], [40]. The U-interface was nationally standardized in West Germany on the basis of MS43 line code [51], [58]. MS43 is a more sophisticated line code in the class of 4B3T block code, in which 4 bit binary blocks are mapped onto 3 ternary transmitted digits. For a U-interface bit rate of 160 Kbit/s the symbol rate in case of MS43 line code is 120 Kbaud. The translation table for MS43 line code is shown in Table 3.2. There are only four possible total disparity states at the end of a word, -2, -1, 0 or 1. (If positive pulses are given weight +1 and negative pulses -1, the disparity is the sum of these weights for a sequence).
Table 3.2
Translation Table for MS43. [40]

<table>
<thead>
<tr>
<th>Binary</th>
<th>Ternary transmitted when total disparity is:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2</td>
</tr>
<tr>
<td>0 0 0 0</td>
<td>+ + +</td>
</tr>
<tr>
<td>0 0 0 1</td>
<td>+ + 0</td>
</tr>
<tr>
<td>0 0 1 0</td>
<td>+ 0 +</td>
</tr>
<tr>
<td>0 0 1 1</td>
<td>0 - +</td>
</tr>
<tr>
<td>0 1 0 0</td>
<td>0 + +</td>
</tr>
<tr>
<td>0 1 0 1</td>
<td>- 0 +</td>
</tr>
<tr>
<td>0 1 1 0</td>
<td>- + 0</td>
</tr>
<tr>
<td>0 1 1 1</td>
<td>- + +</td>
</tr>
<tr>
<td>1 0 0 0</td>
<td>+ - +</td>
</tr>
<tr>
<td>1 0 0 1</td>
<td>0 0 +</td>
</tr>
<tr>
<td>1 0 1 0</td>
<td>0 + 0</td>
</tr>
<tr>
<td>1 0 1 1</td>
<td>0 - -</td>
</tr>
<tr>
<td>1 1 0 0</td>
<td>+ 0 0</td>
</tr>
<tr>
<td>1 1 0 1</td>
<td>+ 0 -</td>
</tr>
<tr>
<td>1 1 1 0</td>
<td>+ - 0</td>
</tr>
<tr>
<td>1 1 1 1</td>
<td>+ + -</td>
</tr>
</tbody>
</table>

Figure 3.8 shows the power spectrum of the MS43 code [51]. The advantage of this code is that it has a low-frequency, dc-free performance spectrum. The allocation shown in Table 3.2 improves the low frequency content of MS43 code compared with 4B3T code, giving it better transmission properties [40]. With this low-frequency spectrum the transmission system benefits from low line attenuation, low crosstalk distortion and high possible signal levels without causing EMI problems.

Error monitoring and alignment can be performed by monitoring the bounds on the total disparity (-2, -1, 0, 1) [40]. The disadvantage of the block codes is that their implementation is more complex than other codes like 2B1Q, AMI, etc.
Figure 3.8: Power Spectrum of the MS43 Code. [51]
3.3.2 Echo Canceller

The design objective for echo canceller can be obtained as given below.

In practice less than 10 dB transhybrid loss can be expected through the hybrid with a single compromise balance network. The longest loop may attenuate the transmitted signal by as much as 40-50 dB. Thus, in this worst case condition the near-end echo signal may be 30-40 dB higher in level than the far-end signal assuming equivalent transmission levels at both ends. If SNR of 20 dB or more is required for design BER the echo canceller may be required to attenuate the residual echo signal entering the receiver by as much as 60-70 dB relative to the original level. The echo canceller design is obviously critical if this degree of performance is required. Because of the level of cancellation required the echo canceller design is sensitive to the small nonlinearities in the channel, data converters and nonlinearity due to the saturation of transformers [42]. The echo canceller performance has been shown to be sensitive to jitter arising from timing recovery system [43].

The time duration of the echo response of a subscriber loop depends upon loop length and location and length of the bridged taps. The echo canceller should have sufficient length to cancel the longest echo response. Because the echo canceller is the most complex portion of the DSL system, there is high priority on minimizing the sampling rate used. The penalty for operating the echo canceller on samples taken at K times the baud rate is that the complexity of echo canceller is proportional to K.

There are various adaptive filter structures for implementing echo cancellers and the different adaptation algorithms which can be used [11]-[13], [35], [44]-[50]. One of the most popular adaptive filter structures used for echo cancellers is adaptive transversal filters with sequential implementation and operating according to the stochastic iteration (LMS) algorithm. Another popular approach is to implement
echo canceller by the look-up table structure. A combination of N binary input data are used as the address for random access memory (RAM). The content of this address is the compensation signal. Several DSL systems using the EC technique have been implemented and tested successfully [46]-[50], [13].

The need to implement DSL systems using VLSI will decide which filter structure and adaptation algorithm is selected for echo canceller. Many echo cancellers at present perform cancellation in the analog domain to avoid non-linearities in A/D converter. However with the rapid advances in CMOS technology, fully digitized implementation seems to be a better approach [51], [52], [58], [48].

3.3.3 Equalization

The dispersion of the signal pulse due to the $\sqrt{f}$ attenuation and distortion due to the presence of bridged taps make adaptive equalization essential, specially when operating over long loops. Linear equalizers which have been used extensively in data communication applications have two drawbacks in VLSI implementation of DSL systems. Multiplication, an essential operation of the linear equalizer, takes a lot of silicon area and/or computation time. Another important drawback of the linear equalizer is the noise enhancement when there are spectral nulls in the input signal caused by bridged taps.

Adaptive equalization without multiplication can be achieved with a decision feedback equalizer (DFE) [53], [54]. Because of the inherent nonlinearity of the DFE, the noise enhancement problem is reduced. A major drawback of DFE is that it is only capable of removing ISI of the trailing type that is within its span. All other disturbances including precursors ISI therefore have to be suppressed as well as possible in advance. To achieve this a front-end filter is needed before DFE. This filter must allow simple (hence preferably non-adaptive) implementation and yet be active over a broad range of loop lengths. Our measurements of various subscriber
loop impulse responses indicates that the telephone channel behaves quite closely as a minimum phase channel. This indicates that the major ISI component is of trailing type, which can be canceled using DFE. But for very long loop some form of front-end equalizer will be required to cancel precursor ISI [4], [32], [47], [49], [55]-[58].

A linear $\sqrt{f}$ equalizer implemented using switched capacitor technology has been used as a front end equalizer to compensate for the loop loss of up to 50 dB [49], [55], [56], [57]. The $\sqrt{f}$ equalizer is followed by the DFE to compensate the postcursor ISI caused by the bridged taps.

In case of digital implementation a forward linear equalizer with 1 or 2 taps will be needed to cancel the precursor ISI. The DFE can compensate for both loop loss and bridged taps [4], [32], [58].

3.3.4 Timing Recovery

Timing recovery is one of the most critical functions in DSL systems. The subscriber or near-end clock is slaved to the central office master clock in the DSL system.

The design goals for timing recovery systems are:

1. Minimum timing jitter
2. Insensitivity to the effects of bridged taps

There are two broad classes of timing recovery techniques namely continuous time and discrete time techniques. The continuous time method includes the Spectral Line [59], Threshold Crossing [60], Sampled-Derivative [60] and the Maximum Likelihood Estimation [60] method. The discrete time class includes such methods as the Wave Difference Method (WDM) [56], [61] and Baud-Rate Sampling [62].
As no known continuous-time technique can meet the required specification when implemented in VLSI technology, only sampled data (discrete-time) technique need be considered for timing recovery circuits. The continuous time schemes can be 'digitized' by performing sampling at a high enough rate to allow a complete reconstruction of the signal and would perform in a functionally equivalent way. However, the penalty for oversampling by a factor of K is that the complexity of the echo canceller is proportional to K. As echo canceller is the most complex portion of the DSL system, there is a high priority to minimize the sampling rate used. The wave difference method (WDM) also requires the sampling rate equal to at least twice the baud rate.

The baud rate sampling technique has recently invoked wide-spread interest due to the need to keep the sampling rate of the DSL system at a minimum. Various baud rate timing recovery schemes which have been proposed in recent publications is an indication of the trend and the progress being made towards implementing DSL systems using baud rate timing recovery techniques [51], [52], [58], [63]. The baud rate sampling technique for timing recovery offers potential advantages of combining the DFE with echo canceller [64]. The adaptive reference echo canceller (AREC) scheme can be implemented without the need for a separate reference former [11]. The DFE itself can be used as a reference former. Thus the same error signal can be used for echo canceller and DFE adaptation giving the advantages of AREC-like fast convergence without additional complexity.

Special frame formats which include a known synchronization burst e.g. 11-bit Barker code [51], [58] can simplify the timing recovery problem [49]. Another technique is the Echo Canceller Burst Mode (ECBM). ECBM technique has three periods of time in which only the useful-far-end received signal or only the echo signal or both are present [47]. During the periods of time when the echo signal is absent, it is possible to extract the timing information from the far-end signal. This can also result in considerable reduction in the complexity of echo canceller
and timing recovery circuit. The performance of an adaptive echo canceller at
the subscriber end of a DSL has been shown to be sensitive to jitter arising from
the timing recovery subsystem [43]. At the central office end, the transmitter and
echo canceller sampler clock may be obtained from the network standard clock
with negligible jitter. It is at the subscriber end where major degradation in echo
canceller performance due to timing jitter occurs.

It has been shown that a narrow bandwidth PLL is needed to extract a stable
clock with extremely small jitter from the received signal [65], [66]. This requires
an analog PLL. The present trend towards implementing DSL systems with almost
exclusively digital techniques makes it desirable to minimize the amount of analog
circuitry in the DSL system. A digital PLL (DPLL) is easier to implement but
to obtain low enough clock jitter DPLL requires a very fast clock. The DPLL
with slower clock will introduce discrete phase jumps which causes unacceptable
degradation in echo canceller performance. One approach to compensate for the
effect of phase jumps is to design the frame format so that there are intervals of
time where echo canceller accuracy is less critical, and force the phase slips to occur
at the beginning of those times e.g. ECBM technique, special codes like Barker
code [4], [47], [49], [51], [58].

One approach to compensate for phase jumps without special frame format is
to store tap sets of tap coefficients in the echo canceller one for each of two relative
transmit-receive phases [67]. Another method is to use echo canceller incorporating
jitter compensation algorithm. This echo canceller shows less sensitivity to jitter
and also facilitates faster timing adaptation [68], [69].

The desired echo cancellation and timing recovery requirements seems to be
achievable with the currently foreseeable signal processing techniques. The major
performance limiting impairments in DSL are distortion and noise. In this thesis
we have emphasized the treatment of those impairments.
Chapter 4

CHANNEL CAPACITY OF THE SUBSCRIBER LOOP

The U-interface bit rate in ISDN has evolved from 80 kb/s to the present 160 kb/s. This transmission must be reliable for loops up to about 3.5 miles long, in the presence of impairments of crosstalk noise, bridged taps, residual echo, jitter and impulse noise. However as the demands for communication facilities in the local telephone network grows, the need to transmit at rates higher than 160 kb/s and/or over loops longer than 3.5 miles is likely to arise, within the framework of ISDN and possibly also outside it. It is interesting to look at the evolution of high speed voice band modems in this context. The evolution of high speed modems to ever higher bit rates using successively more complicated modulation schemes is summarized in Table 4.1 [70]. Recently Codex Corp. has announced a 19.2 kb/s modem. How far can this evolution go? The maximum theoretically achievable rate at which one can transmit is given by the channel capacity upper bounds [71], [72]. The channel capacity of the bandlimited telephone channel was estimated as of the order of 23.5 kb/s [73]. It is evident that by using powerful coding-decoding techniques it is possible to achieve rates close to channel capacity.
<table>
<thead>
<tr>
<th>Year</th>
<th>Model</th>
<th>Speed</th>
<th>Bandwidth</th>
<th>Constellation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962</td>
<td>Bell 201</td>
<td>2400</td>
<td>1200 Hz</td>
<td>4-phase</td>
<td>fixed eq.</td>
</tr>
<tr>
<td>1967</td>
<td>Milgo 4400/48</td>
<td>4800</td>
<td>1600 Hz</td>
<td>8-phase</td>
<td>manual eq.</td>
</tr>
<tr>
<td>1971</td>
<td>Codex 9600 C</td>
<td>9600</td>
<td>2400 Hz</td>
<td>16-QAM</td>
<td>adaptive eq.</td>
</tr>
<tr>
<td>1980</td>
<td>Paradyne MP14400</td>
<td>14,400</td>
<td>2400 Hz</td>
<td>64-QAM</td>
<td>rectangular</td>
</tr>
<tr>
<td>1981</td>
<td>Codex/ESE SP14.4</td>
<td>14,400</td>
<td>2400 Hz</td>
<td>64-QAM</td>
<td>hexagonal</td>
</tr>
<tr>
<td>1984</td>
<td></td>
<td>14,400</td>
<td>2400 Hz</td>
<td>128-QAM</td>
<td>8-state trellis</td>
</tr>
</tbody>
</table>

The steadily growing demands for data communication has prompted the development of high speed data transmission systems for the subscriber loop. With the advent of ISDN, even higher speed data transmission will become available and demand, as in the case of voice-band modem, is likely to grow. Table 4.2 summarizes the evolution of data communications in the U.S. [74]. In view of this possibility it is important to establish the channel capacity upper bounds on the subscriber loop performance. The channel capacity of the subscriber loop can be used as a measure of performance to evaluate the existing and future practical DSL system. One can thus judge the capability of a particular transmission scheme and determine how close one is to the ideal scheme. It will also indicate as to whether there is any significant performance gain to be achieved using more complex and sophisticated coding-decoding technique than the ones presently employed in DSL.
Table 4.2: Evolution of Data Communications. [74]

<table>
<thead>
<tr>
<th>Services</th>
<th>Speed</th>
<th>Products</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Line</td>
<td>1.2, 2.4, 4.8, 9.6 kbps or limited by modems</td>
<td>Offered by the Bell System</td>
<td>Early 1970's</td>
</tr>
<tr>
<td>Digital Line</td>
<td>56 kbps</td>
<td>DDS</td>
<td>1974</td>
</tr>
<tr>
<td>Packet Switched Data</td>
<td>10 kbps</td>
<td>TYMNET</td>
<td>1969</td>
</tr>
<tr>
<td></td>
<td>50 kbps</td>
<td>TELENET</td>
<td>1975</td>
</tr>
<tr>
<td></td>
<td>56 kbps</td>
<td>ACCUNET</td>
<td>1983</td>
</tr>
<tr>
<td>Circuit Switched Data</td>
<td>56 kbps</td>
<td>DMS-100 CSDDS</td>
<td>1981</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1AESS&lt;sup&gt;1&lt;/sup&gt; CSDC&lt;sup&gt;2&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>Data Over Voice</td>
<td>10 kbps</td>
<td>DMS-100 Electronic Business Set</td>
<td>1983</td>
</tr>
<tr>
<td>Public Packet Switching</td>
<td>4.8 kbps multiplexed up to 56 kbps</td>
<td>DATAPATH&lt;sup&gt;3&lt;/sup&gt; LADT&lt;sup&gt;4&lt;/sup&gt;</td>
<td>1984</td>
</tr>
<tr>
<td>Private Packet Network</td>
<td>200 kbps</td>
<td>Tandem Computer</td>
<td>1984</td>
</tr>
<tr>
<td>Computer to PBX</td>
<td>56 kbps</td>
<td>CPI&lt;sup&gt;5&lt;/sup&gt;</td>
<td>1985</td>
</tr>
<tr>
<td></td>
<td>64 kbps</td>
<td>DMI&lt;sup&gt;6&lt;/sup&gt;</td>
<td>1986</td>
</tr>
<tr>
<td>ISDN</td>
<td>64 kbps</td>
<td>4ESS&lt;sup&gt;1,3&lt;/sup&gt;</td>
<td>1987</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5ESS&lt;sup&gt;1&lt;/sup&gt;</td>
<td></td>
</tr>
<tr>
<td>Wideband Data</td>
<td>384 kbps, 1.5 mbps</td>
<td>4ESS&lt;sup&gt;1,3&lt;/sup&gt;</td>
<td>1987</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5ESS&lt;sup&gt;1&lt;/sup&gt;</td>
<td></td>
</tr>
</tbody>
</table>

1 Trademark of AT&T
2 Circuit Switched Digital Service, later referred to as Public Switched Digital Service (PSDS)
3 Registered trademark of Northern Telecom
4 Local Area Data Transport
5 Computer to PBX Interface by Northern Telecom
6 Digital Multiplex Interface by AT&T
4.1 Mathematical Model for Channel Capacity Calculation

It has been shown that when a transmission line having dispersion and Gaussian noise is used as a communication channel, the channel capacity is achieved by optimizing the transmitted signal spectrum \[72, 75, 76\]. If the total transmitted signal power is limited there is an upper bound to the channel capacity, and the maximum channel capacity can be approached only with signals having a certain frequency spectrum \[75\]. The Figure 4.1 shows a channel with dispersion and Gaussian noise.

\[ P(f) = \text{Power Spectral Density of the Transmitted Signal} \]

\[ N^2(f) = \text{Gaussian Noise Power Spectral Density} \]

\[ C(f) = \text{Subscriber Loop Channel Response} \]

The total transmitted signal power is:

\[ P = \int_0^{BW} P(f)df \tag{4.1} \]

The maximum channel capacity for constant power \(P\) occurs when \[72, 75\]

\[ P(f) = K - \frac{N^2(f)}{|C(f)|^2} \quad \text{if } \frac{N^2(f)}{|C(f)|^2} \leq K \]

\[ = 0 \quad \text{if } \frac{N^2(f)}{|C(f)|^2} > K \tag{4.2} \]

where \(K\) is a constant chosen to give the desired value of the total signal power \(P\). The bandwidth \(BW\) is the region in which \(P(f)\) is nonzero. The maximum upper bound on the channel capacity is given by \[72\]:

49
Figure 4.1: Channel Model.
\[ C = \int_{0}^{BW} \log \left( K \frac{|C(f)|^2}{N^2(f)} \right) df \] (4.3)

Given the noise PSD \( N^2(f) \) and the transmitted power \( P \), the channel capacity upper bound \( C \) for a given subscriber loop channel with response \( C(f) \) can be obtained.

### 4.2 Channel Capacity Results

The dominant sources of noise in the subscriber loop are crosstalk, white noise and impulse noise [7]. As the digitization of telephone network grows, the impulse noise is less likely to be the dominant factor. The crosstalk noise can be near-end crosstalk (NEXT) or far-end crosstalk (FEXT). White noise may be a reasonable model for the effects of residual echo, receiver noise, impulse noise and other impairments such as jitter. The channel capacity upper bounds of the subscriber loop limited by NEXT, FEXT, white noise and combinations of these is evaluated using equations (4.1) to (4.3). (The assumption is made here that NEXT and FEXT are stationary and Gaussian). The noise power spectral density \( N^2(f) \) in each of these cases is:

1. NEXT alone

\[ N^2(f) = P(f) \ast X_N(f) \]

where,

\[ X_N(f) = \text{NEXT transfer function} \]

\[ = K_N f^{1.5} \]
\[ K_N = \text{cable dependent constant} \]

\[ = 0.88 \times 10^{-13} \]

which represents 99% worst case NEXT loss of 57 dB at 80 kHz for 49 disturbing pairs [7], [23].

(2) FEXT alone

\[ N^2(f) = P(f) \ast X_P(f) \ast |C(f)|^2 \]

where

\[ X_P(f) = \text{FEXT transfer function} \]

\[ = K_P f^2 L \]

\[ L = \text{Length of the coupling path} \]

\[ K_P = \text{Cable dependent constant} \]

\[ = 2.6 \times 10^{-16} \]

which represents 99% worst case FEXT loss of 33 dB at 3.15 MHz and 1 Kft (304.8 m), for 49 disturbing pairs [7], [23].
(3) White Noise alone

\[ N^2(f) = N_0 \]

(4) NEXT + White Noise

\[ N^2(f) = P(f) \ast X_N(f) + N_0 \]

The subscriber loop channel response \( C(f) \) for different line lengths is obtained using typical measured cable data for 22 AWG and 26 AWG wire [17], [18]. The characteristics of the cable types used (e.g., 22 gauge PIC cable at 70° and 26 gauge PIC cable at 70°F) are given in Appendix A. The transmitted power \( P \) in all cases is 0.01 watts.

The NEXT limited capacity is plotted in Figure 4.2 against the subscriber loop length in miles. There are two capacity plots, one for 22 AWG wire and one for 26 AWG wire. The bandwidth of the transmitted power spectrum is \( BW = \infty \), for the NEXT limited case. The equal level FEXT limited capacity is plotted against loop length in miles in Figure 4.3 for 22 AWG wire. The bandwidth of the transmitted power spectrum, \( BW = \infty \) in FEXT case also. The white noise limited capacity for two typical values of \( N_0 \) is plotted in Figures 4.4 and 4.5. The bandwidth, \( BW \), of the transmitted power spectrum is plotted in Figures 4.6 and 4.7, for the white noise limited case.

The channel capacity limited by the combination of white noise and NEXT is plotted in Figures 4.8 and 4.9 for two typical values of \( N_0 \).

The corresponding bandwidth, \( BW \), of the transmitted power spectrum is plotted in Figures 4.10 and 4.11.
Figure 4.2: NEXT Limited Capacity.
Figure 4.3: ELCL FEXT Limited Capacity (22 AWG).
Figure 4.4: White Noise Limited Capacity (22 AWG).
Figure 4.5: White Noise Limited Capacity (26 AWG).
Figure 4.6: White Noise Limited Bandwidth (22 AWG).
Figure 4.7: White Noise Limited Bandwidth (26 AWG).
Figure 4.8: NEXT + White Noise Limited Capacity (22 AWG).
Figure 4.9: NEXT + White Noise Limited Capacity (26 AWG).
Figure 4.10: NEXT + White Noise Limited Bandwidth (22 AWG).
Figure 4.11: NEXT + White Noise Limited Bandwidth (26 AWG).

\[ A : N_0 = 5.0 \times 10^{-15} \]
\[ B : N_0 = 5.0 \times 10^{-13} \]
To obtain more insight into the channel capacity results the channel capacity is divided into two parts.

Total Capacity = Main Capacity + Tail Capacity

\[
\text{Main Capacity} = \int_{0}^{BW'} \log \left( K \frac{|C(f)|^2}{N_0^2(f)} \right) df
\]  \hspace{1cm} (4.4)

\[
\text{Tail Capacity} = \int_{BW'}^{BW''} \log \left( K \frac{|C(f)|^2}{N_0^2(f)} \right) df
\]  \hspace{1cm} (4.5)

The plots of Main Capacity against \(BW''\) were obtained for different loop length by varying \(BW'\). Only NEXT interference is considered. A representative plot of Main Capacity against \(BW''\) is presented in Figure 4.12. Plots for other loop length are similar in shape and differ only in magnitude.

4.3 Summary

\(N_0 = 5.0E-15\) is the typical white noise figure found in the subscriber loops. As seen from the channel capacity plots the main interference limiting the capacity of the subscriber loop is NEXT [32]. The equal level FEXT has negligible effect in limiting the channel capacity of the subscriber loops. The white noise typically has a minor effect on the channel capacity performance of the subscriber loop. The plots of bandwidth against loop length indicate that as the loop length (attenuation) increases the bandwidth of the ideal transmitted signal decreases. The Main Capacity Versus \(BW'\) plot in Figure 4.12 indicates that as the \(BW'\) increases the Tail Capacity contribution decreases and Main Capacity approaches Total Capacity. In the end case when \(BW = BW'\), Main Capacity = Total Capacity. It is clear from Figure 4.12 that there is a knee portion in the plots of Main Capacity against \(BW''\) and any additional increase in \(BW'\) beyond the knee portion only gives diminishing
Figure 4.12: Main Capacity Versus BW’.
returns. This indicates that after a certain point any signalling which increases the bandwidth of the transmitted signal may not be efficient and some other form of bandwidth efficient signalling e.g., multilevel signalling may be considered. The channel capacity plots are valuable aids to a system designer while evaluating different DSL systems. The maximum theoretical rate at which one can transmit over a loop of given length, can be easily obtained from the channel capacity plots e.g., from Figure 4.9 it can be seen that over the loops 3.5 miles long (26 AWG) limited by NEXT and white noise we should be able to (theoretically at least) transmit at rates up to 500 kb/s. Conversely for a given transmission rate one can easily obtain maximum theoretical reach of the DSL. Using the channel capacity plots e.g. from Figure 4.9 it can be seen that at 160 kb/s the theoretical reach of the ideal 26 AWG DSL system, limited by NEXT and white noise, is 6 miles.

The channel capacity bounds can be used for comparison purposes with more practical but suboptimal DSL systems, to determine how much more the performance of the practical system can be improved by further design. The channel capacity bounds will serve as theoretical limits rather than as definite goals for a DSL system designer, which should be approached only as far as practical constraints allow. The channel capacity performance bounds will form a benchmark for evaluating various DSL system designs on a common basis.

It seems clear from the channel capacity plots that the reliable communication at bit rates significantly higher than 160 kb/s and/or over loops significantly longer than 3.5 miles is theoretically possible. Coding, e.g., trellis coding, and maximum-likelihood sequence estimation are the techniques for approaching the performance promised by these channel capacity results.
Chapter 5

Comparison of DFE and MLSE Receiver Performance on DSL Channels

Adaptive equalization is a time-honoured solution to the problem of data transmission over dispersive channels, e.g., subscriber loops. Linear equalizers are commonly used in practice due to their simple implementation. Linear equalizers can incur significant noise enhancement penalty when used to counteract the intersymbol interference on channels with severe distortion, e.g., spectral nulls. For such channels nonlinear equalization techniques such as decision feedback equalization (DFE) and maximum likelihood sequence estimation (MLSE) offer superior performance [77].

Forney [14] has shown that the MLSE implemented using Viterbi Algorithm (VA) is ultimate solution to the problem of data transmission over dispersive channels with Gaussian noise. The performance achieved by the MLSE receiver can be shown to be effectively as good as could be attained by any receiver structure. MLSE is one of the keys to approach the performance promised by channel capacity results presented in Chapter 4.
Even though VA is conceptually easy to implement the complexity of the VA, for channels with long impulse response, e.g., subscriber loop, is prohibitive [77]. Complexity considerations make application of MLSE to DSL impossible. DSL systems presently use DFE to combat ISI. The comparison between DFE receivers and MLSE receivers gives us an indication of the size of the gap in performance between DSL systems in use today and what could theoretically be achieved by much more complex receiver design.

One of our goals is to make a comparative evaluation of the MLSE receiver and DFE receiver performance for DSL channels [78], [79], [80].

5.1 Performance Calculation for DFE and MLSE

The received signal in DSL channels is of the form

\[ r(t) = \sum_n a_n h(t - nT) + \nu(t) \]  \hspace{1cm} (5.1)

where \( \{a_n\} \) are data symbols, \( h(t) \) is the overall channel impulse response and includes the subscriber loop channel response \( c(t) \) and any other filtering used in DSL, e.g., hybrid, transformer, pulse shaping, etc. The \( \nu(t) \) represents the additive noise.

If only a single isolated data pulse is transmitted and channel noise is white Gaussian, the optimum receiver is matched filter followed by a decision quantizer, and the resulting probability of error per symbol is

\[ P_e = Q(d_{MF}/2\sigma) \]  \hspace{1cm} (5.2)

where
\[ d_{MF}^2 = \left[ \int_{-\infty}^{\infty} |h(t)|^2 \, dt \right] \cdot E(a_n^2) \]  \hspace{1cm} (5.3)

is the squared matched filter output and \( \sigma^2 \) is the noise variance at the matched filter output. \( Q(x) \) is the probability that a zero-mean unit-variance Gaussian random variable exceeds \( x \).

The \( d_{MF}^2 \) is the energy of an isolated pulse. The \( d_{MF} \) parameter also has counterparts for ideal MLSE and DFE (infinite number of taps, no error propagation) reception in white Gaussian noise. That is, the symbol error probability for those systems can be approximated by equation (5.2) (apart from the constants multiplying the expression), with \( d_{MF} \) replaced by \( d_{MLSE} \) and \( d_{DFE} \) respectively, which are functions of \( h(t) \) [14], [78], [79].

It can be shown that \( d_{MF} \geq d_{MLSE} \geq d_{DFE} \) with their corresponding error probabilities being in inverse order. The quantities \( d_{MLSE}^2 / d_{MF}^2 \) and \( d_{DFE}^2 / d_{MF}^2 \) are measures of the effective decrease in the "signal-to-noise" (S/N) ratio (relative to the detection of an isolated pulse) resulting from ISI. The determination of the quantity \( d_{MLSE}^2 \) (which is related to the minimum distance) is therefore very important as \( d_{MLSE}^2 \) is a measure of potential performance which can be obtained using receivers of arbitrary complexity if the noise is Gaussian and white. We use ratios \( d_{MLSE}^2 / d_{MF}^2 \) and \( d_{DFE}^2 / d_{MF}^2 \) as a measure of comparing the relative asymptotic (large SNR) performance of MLSE and DFE receivers, given the DSL channel response [79], [80]. The \( d_{MLSE} \) and \( d_{DFE} \) for a given DSL channel response are estimated using a method of upper bound and lower bound described in [78], [79]. The procedure is described below.

For an ideal MLSE receiver, the parameter \( d_{MLSE}^2 \) is defined as

\[ d_{MLSE}^2 = \left[ e_0 e_{1}^{\min} \cdots \sum_{m_1 \geq 0}^{N} \sum_{m_2 \geq 0}^{N} \epsilon_{m_1 \epsilon_{m_2}} \left( \sum_i h(iT - m_1 T) h(iT - m_2 T) \right) \right] \]  \hspace{1cm} (5.4)
where minimization is over all error sequences \((\epsilon_0 \cdots \epsilon_N)\), \(N \to \infty\) and the \(\epsilon_i\) are the possible differences between data symbols, i.e., \(\epsilon_n = a_n - \hat{a}_n\). The exact value of \(d^2_{MLSE}\) can be determined numerically by the direct minimization of equation (5.4); by letting \(N \to \infty\) while exhaustively minimizing over error sequences. This gives a sequence of upper bounds on the \(d^2_{MLSE}\) which approach \(d^2_{MLSE}\) monotonically. The difficulty with this method is that the number of error sequences which must be checked grows as \(3^N\) (if the data is binary) and the computation effort becomes unreasonable. This difficulty is solved by deriving a sequence of lower bounds on \(d^2_{MLSE}\) which also approach \(d^2_{MLSE}\) monotonically. The process is halted at a value of \(N\) when the upper bound and lower bounds are close enough to give \(d^2_{MLSE}\) within the desired accuracy [79].

The lower bound for error sequence of length \(N + 1\) symbols is given by [79]

\[
d^2_{LB} = \left[ \epsilon_0, \epsilon_1^{\text{min}}, \ldots, \epsilon_N \sum_{n=0}^{N} \sum_{m=0}^{n} \epsilon_m C_{n-m} \right]^2
\]

(5.5)

where the \(\{C_n\}\) are obtained recursively as given below [79].

The autocorrelation function of the pulse sequence is given by

\[
R_k = \int_{-\infty}^{\infty} |H(f)|^2 e^{j2\pi f kT} df
\]

(5.6)

\[
= \int_{-\infty}^{\infty} h(t)h(t + kT) dt
\]

(5.7)

\[
= \int_{-1/2T}^{1/2T} R(f)e^{j2\pi f kT} df
\]

(5.8)

where
\[ R(f) = T \sum_{k=-\infty}^{\infty} R_k e^{-j2\pi f kT} \quad (5.9) \]

defining
\[ \frac{1}{2} \log_e \frac{R(f)}{T} = \sum \rho_k e^{-j2\pi f kT} \quad (5.10) \]

We have
\[ \rho_k = T \int_{-1/2T}^{1/2T} \frac{1}{2} \log_e \frac{R(f)}{T} e^{j2\pi f kT} df \quad (5.11) \]

and the \( \{C_n\} \) are given by
\[ C_0 = \exp(\rho_0) \quad (5.12) \]

\[ C_n = \frac{2}{n} \sum_{m=0}^{n-1} (n - m) \rho_{n-m} C_m \quad n \geq 1 \quad (5.13) \]

The parameter \( d_{DFE}^2 \) for the DFE receiver is given by
\[ d_{DFE}^2 = C_0^2 \quad (5.14) \]

also,
\[ d_{MP}^2 = R_0 \quad (5.15) \]

Using the formulas given above, for every \( N \), the minimization of (5.5) is carried out to obtain a lower bound on \( d_{MLSE}^2 \). This minimizing sequence \( \{\epsilon_0, \ldots, \epsilon_N\} \) is substituted into equation (5.4) to obtain the upper bound on \( d_{MLSE}^2 \). When the upper bounds and lower bounds are sufficiently close the process is terminated. Once the \( d_{MLSE}^2, d_{DFE}^2 \) and \( R_0 \) are obtained the ratios \( d_{MLSE}^2/R_0 \) and \( d_{DFE}^2/R_0 \) are calculated to compare the performance of MLSE and DFE receivers.
5.2 The Performance of DFE and MLSE on DSL With White Noise

The performance of DFE and MLSE receiver was evaluated for six different cases. The interfering noise considered was additive white Gaussian noise. The subscriber loop channel response \( c(t) \), was obtained using cable data for 22 AWG wire.

Case 1: Symbol rate, \( 1/T = 160 \) kbaud, and binary transmission. The overall response \( h(t) \) includes the subscriber loop channel response \( c(t) \) alone.

Case 2: Symbol rate, \( 1/T = 80 \) kbaud and 4-level transmission. The overall response \( h(t) \) includes the subscriber loop channel response \( c(t) \) alone.

Case 3: Symbol rate, \( 1/T = 160 \) kbaud and binary transmission. The overall response \( h(t) \), includes the subscriber loop channel response \( c(t) \), hybrid (Appendix B) and transformer (Appendix C).

Case 4: Symbol rate, \( 1/T = 80 \) kbaud and 4-level transmission. The overall response \( h(t) \), includes the subscriber loop channel response \( c(t) \), hybrid (Appendix B), transformer (Appendix C).

Case 5: Symbol rate, \( 1/T = 160 \) kbaud and binary transmission. The overall response \( h(t) \), includes the subscriber loop channel response \( c(t) \), hybrid (Appendix B), transformer (Appendix C) and bipolar pulse shaping.

Case 6: Symbol rate, \( 1/T = 80 \) kbaud and 4-level transmission. The overall response \( h(t) \), includes subscriber loop channel response \( c(t) \), hybrid (Appendix B), transformer (Appendix C) and bipolar pulse shaping.

In the case of binary transmission

\[ a_n = \{1, 0\} \]
\[ \epsilon_0 = \{1\} \text{ assuming "1" was transmitted} \]

\[ \epsilon_n = \{1, 0, -1\} n \geq 1 \]

and \[ d^2_{MLSE} = \left[ \epsilon_0, \epsilon_1^{\min}, \ldots, \sum_{m_1 \geq 0} \sum_{m_2 \geq 0} \epsilon_{m_1} \epsilon_{m_2} \left( \sum_i h(iT - m_1 T)h(iT - m_2 T) \right) \right] \] \hfill (5.16)

\[ d^2_{LB} = \left[ \epsilon_0, \epsilon_1^{\min}, \ldots, \epsilon_N \sum_{n=0}^N \left| \sum_{m=0}^n \epsilon_m C_{n-m} \right|^2 \right] \] \hfill (5.17)

In case of 4-level transmission

\[ a_n = \{3, 1, -1, -3\} \]

\[ \epsilon_0 = \{2, 4, 6\} \text{ assuming "3" was transmitted} \]

\[ \epsilon_n = \{6, 4, 2, 0, -2, -4, -6\} \quad n \geq 1. \]

and \[ d^2_{MLSE} = \left[ \epsilon_0 \epsilon_1^{\min}, \ldots, \frac{1}{4} \sum_{m_1 \geq 0} \sum_{m_2 \geq 0} \epsilon_{m_1} \epsilon_{m_2} \sum_i h(iT - m_1 T)h(iT - m_2 T) \right] \] \hfill (5.18)

\[ d^2_{LB} = \left[ \epsilon_0 \epsilon_1 \ldots, \epsilon_N \frac{1}{4} \sum_{n=0}^N \left| \sum_{m=0}^n \epsilon_m C_{n-m} \right|^2 \right] \] \hfill (5.19)

Using the method of upper and lower bounds described above the performance of DFE and MLSE receiver was evaluated for six different cases listed earlier. The S/N ratio penalty in dB (relative to the detection of an isolated pulse) for MLSE receiver \( (d^2_{MLSE}/R_0) \) and DFE receiver \( (d^2_{DFE}/R_0) \) is calculated for different subscriber loop length. The results of the calculation are plotted in Figures 5.1, 5.2

73
and 5.3. The Figure 5.1 shows the plots of $S/N$ ratio penalty against loop length for case 1 and case 2. The Figure 5.2 shows the plots of $S/N$ ratio penalty against loop length for cases 3 and 4. The Figure 5.3 shows the plots of $S/N$ ratio penalty against loop length for cases 5 and 6.

### 5.2.1 Discussion

As seen from Figures 5.1, 5.2 and 5.3, both binary and 4-level transmission with bipolar pulse shaping, cases 5 and 6 (Figure 5.3), offers the least $S/N$ ratio penalty relative to the detection of an isolated pulse for a given loop length compared to loops without pulse shaping, cases 1, 2, 3, and 4 (Figures 5.1 and 5.2). In all these cases the ratio of minimum distance ($d^2_{MLSE}$ and $d^2_{DFE}$) to $R_0$ is plotted and absolute value of $R_0$ varies considerably for each case.

Figures 5.4 - 5.9 show the absolute variation in $R_0$, $d^2_{MLSE}$ and $d^2_{DFE}$ with loop length. Figure 5.4 shows the variation in $R_0$ for 160 kbaud transmission. The Figure 5.5 shows the variation in $d^2_{MLSE}$ for 160 kbaud transmission. Figure 5.6 shows the variation in $d^2_{DFE}$ for 160 kbaud transmission. The Figures 5.7, 5.8, and 5.9 show the respective plots for 80 kbaud transmission. As seen from Figures 5.4 and 5.7 the absolute value of $R_0$ is considerably worse for channels with bipolar pulse shaping (Case 5 and Case 6) as compared to the value of $R_0$ for channel alone (Case 1 and Case 2). The same is true of $d^2_{MLSE}$ and $d^2_{DFE}$ (Figures 5.5, 5.6, 5.8 and 5.9). The plots of $R_0$, $d^2_{MLSE}$ and $d^2_{DFE}$ for channels with transformer (Case 3 and Case 4) and the plots $R_0$, $d^2_{MLSE}$ and $d^2_{DFE}$ for channels with bipolar pulse shaping (Case 5 and Case 6) are close.

As most of the energy in the subscriber loop channel is concentrated at low frequencies a low frequency cut-off (transformer or bipolar pulse shaping) causes significant degradation in $R_0$, $d^2_{MLSE}$ and $d^2_{DFE}$ as seen from Figures 5.4 - 5.9. This implies that absolute value of $S/N$ ratio is considerably worse for channels with
Figure 5.1: Performance of DFE and MLSE Receiver On Subscriber Loop Channel Alone.
Figure 5.2: Performance of DFE and MLSE Receiver On Subscriber Loop Channel With Hybrid and Transformer.
Figure 5.3: Performance of DFE and MLSE Receiver On Subscriber Loop Channel With Hybrid, Transformer and Bipolar Pulse Shaping.
Figure 5.4: The Variation in $R_0$ with Loop Length (160 kbaud).
Figure 5.5: The Variation in $d_{MLSE}^2$ with Loop Length (160 baud).
Figure 5.6: The Variation in $d_{DPE}^2$ with Loop Length (160 kbaud).
Figure 5.7: The Variation in $R_0$ with Loop Length (80 kbaud).
Figure 5.8: The Variation in $d_{MLSE}^2$ with Loop Length (80 kbaud).
Figure 5.9: The Variation in $d_{DFE}^2$ with Loop Length (80 kbaud).
bipolar pulse shaping (Case 5 and Case 6).

From Figures 5.1 and 5.2, it is clear that even MLSE has substantial $S/N$ ratio penalty on long loops, e.g., for binary transmission, Figure 5.1 (case 1) indicates that the $S/N$ ratio penalty is 10 dB for 6.0 mile long loop. The Figures 5.1 and 5.2 indicate that for both DFE detection and MLSE, the binary transmission (160 kbaud), cases 1 and 3, has significantly higher $S/N$ ratio penalty compared with 4-level transmission (80 kbaud) cases 2 and 4 e.g., the Figure 5.1 indicates that for loop 6.0 mile long, the binary transmission $S/N$ ratio penalty is about 5.0 dB higher than that for 4-level transmission (both DFE detection and MLSE). The Figure 5.2 indicates that for loop 6.0 mile long, the binary transmission $S/N$ ratio penalty is about 3.0 dB higher than that for 4-level transmission (both DFE detection and MLSE).

For loops up to 6.0 miles long, Figures 5.1, 5.2 and 5.3 indicate that a receiver based on DFE detection may achieve nearly the same theoretical performance (within about 1.5 dB) as a much more complex MLSE receiver. These plots also indicate that for loops much longer than 6.0 miles, the performance of the receiver based on the DFE detection could be considerably worse than that of MLSE receiver.

5.3 The Performance of DFE and MLSE On DSL With NEXT

In the Section 5.2 the performance of DFE and MLSE receiver was evaluated when the interfering noise considered was white Gaussian noise. One of the major sources of noise in DSL is near-end crosstalk (NEXT). In this section we evaluate the theoretical performance of MLSE and DFE receiver when the interfering noise is NEXT. The analysis presented in the previous section can be easily extended to the colored
Gaussian noise (NEXT) case by adding a noise whitening filter at the receiver [78]. The Figure 5.10 shows the corresponding block diagram of the transmitter-receiver structure.

\[ r(t) = \sum_n a_n h(t - nT) + \nu(t) \]

where \( \nu(t) \) is white Gaussian noise

If

\[ N^2(f) = \text{Colored noise spectral density} \]

In the case of NEXT

\[ N^2(f) = K_N f^{1.5} |G(f)|^2 \quad (5.20) \]

\[ W(f) = \frac{1}{\sqrt{N^2(f)}} \quad (5.21) \]

\[ |H(f)|^2 = \frac{|C(f)|^2 |G(f)|^2}{N^2(f)} \]

\[ = \frac{|C(f)|^2 |G(f)|^2}{K_N f^{1.5} |G(f)|^2} \quad (5.22) \]

\[ |H(f)|^2 = \frac{|C(f)|^2}{K_N f^{1.5}} \quad (5.23) \]

The Figure 5.11 shows a rough sketch of the typical \( |H(f)|^2 \) function for DSL channel (assuming \( |H(f)|^2 = 0 \) at \( f = 0 \)). The corresponding sketch of the autocorrelation function \( R_k \) given by equation (5.6) is shown in Figure 5.12.

85
Figure 5.10: The System Model With Noise-Whitening Filter.
Figure 5.11: The Sketch of a Typical $|H(f)|^2$ Function.

Figure 5.12: The Sketch of a Typical Autocorrelation Function $R_k$. 
For typical subscriber loop lengths, the autocorrelation function $R_k$ was evaluated using equations (5.6) and (5.23). As seen from the rough sketch in Figure 5.6 and confirmed by the computer results, it was found that $R_k$ for DSL channel with NEXT decreases at very slow rate, implying very severe ISI at the output of noise whitening filter. The computations required to evaluate the upper and lower bounds on $d_{MLSE}^2$ are prohibitive in this case due to the long tail of the autocorrelation function $R_k$. To alleviate this problem and get a better understanding of the performance of DFE and MLSE receiver with NEXT, the overall response $H(f)$ is obtained by combining subscriber loop channel response $C(f)$ and the transformer transfer function $T(f)$ (Appendix C).

In this case

$$|H(f)|^2 = \frac{|C(f)|^2 |T(f)|^2}{K_N f^{1.5}}$$

(5.24)

Even though such a conditioned response does not truly represent the NEXT system model shown in Figure 5.10, the practical implementation of noise whitening filter would provide a response which is closer to that given in equation (5.24) than the one given in equation (5.23). The equation (5.24) was used to evaluate the performance of DFE and MLSE receivers using the method of upper and lower bounds described in earlier sections. The performance was evaluated for two cases.

**Case 1**: Symbol rate $1/T = 160$ kbaud, binary transmission.

**Case 2**: Symbol rate $1/T = 80$ kbaud, 4-level transmission.

The results of the calculation are plotted in Figure 5.13.
Figure 5.13: NEXT Limited Performance of DFE and MLSE Receiver On Subscriber Loop Channel With Transformer.
5.3.1 Discussion

From Figure 5.13 it is clear that, as in white noise case, the MLSE receiver with NEXT, has substantial S/N ratio penalty relative to the detection of an isolated pulse, e.g., for a 4.0 mile long loop, binary transmission, the S/N ratio penalty is 11.0 dB. Also, Figure 5.7 indicates that for both, the DFE detection and MLSE, binary transmission (160 kbaud) has significantly higher S/N ratio penalty than that for 4-level transmission (80 kbaud), e.g., for 4.0 mile long loop, the binary transmission S/N ratio penalty is about 5.0 dB higher than the 4-level transmission S/N ratio penalty (for both, DFE and MLSE). It is also clear from Figure 5.7 that for loops up to 4.0 miles long, the DFE detection performance is only about 1.0 dB inferior to the MLSE receiver.

It is evident for Figures 5.1, 5.2, 5.3 and 5.13 that the performance characteristics of MLSE and DFE receivers, with white Gaussian noise and with NEXT, are similar in shape. Comparing Figures 5.2 and 5.13 it is evident that DFE and MLSE receiver with NEXT suffer much higher S/N ratio penalty than with white Gaussian noise, e.g., for 4.0 mile long loop, 4-level transmission, S/N ratio penalty with NEXT is about 6.0 dB higher than that with white noise.

5.4 Conclusions

The performance plots for DFE and MLSE receiver favor 4-level transmission (80 kbaud) compared to binary transmission (160 kbaud), both receiver techniques with 4-level transmission suffer considerably less S/N ratio penalty relative to the binary transmission penalty. The DFE and MLSE receivers with NEXT suffer much higher S/N ratio penalty than with white Gaussian noise.
It is evident from Figures 5.1, 5.2, 5.3 and 5.13 that in the case of both white Gaussian noise and NEXT case a DFE receiver may achieve nearly the same theoretical performance (within about 1.5 dB, for loops up to 6.0 miles) as a much more complex MLSE receiver. The relatively small difference in performance between MLSE and DFE receivers for stationary noise was borne out by the simulation results reported in Chapter 7 (Stationary Crosstalk). These plots also indicate that MLSE receivers could offer significant performance gains compared with DFE receivers, especially on long loops (much longer than 6.0 miles) where $S/N$ ratio is low and the effect of decision errors on DFE could be severe.

The analysis presented earlier and the plots shown in Figures 5.1, 5.2, 5.3 and 5.7 for DFE receiver do not take into account the effect of decision errors on the performance of DFE receiver. Thus the plots for DFE receiver do not represent the true performance of the practical DFE receiver. While these curves do not take into account imperfections such as finite numbers of tap coefficients, decision errors etc., they do provide a convenient basis for comparison of inherent performance limits for each receiver technique.
Chapter 6

DSL Complexity Considerations

Two major impairments in DSL are ISI and echo due to imperfect hybrid balance. As ISI is mostly of trailing type, DFE is a natural part of the DSL receiver. The complexity requirements of the echo canceller and DFE largely determine the complexity of the overall system. Hence it is imperative to keep the length of the echo canceller and DFE to a minimum without compromising their performance. So it is necessary to investigate the length requirements of the echo canceller and DFE in different DSL system configurations to achieve a certain satisfactory level of performance. The complexity requirements of echo canceller and DFE for various line codes commonly used in DSL are evaluated. In addition, the effect of various receiver filter designs on complexity is also considered.

6.1 The DSL System

The main features of the echo canceller DSL system relevant to the investigation into complexity are shown in Figure 6.1. The input to the front-end receiver filter, \( x(t) \), consists of echo signal, noise and far end signal \( s_o(t) \). As mentioned in Chapter
3, a fixed front-end equalizer is needed to reduce the precursor ISI which cannot be cancelled by the DFE, and to restrict the noise entering the system. We have studied the complexity requirements for two different types of receiver filter, one is a differentiator and the other is a lowpass filter designed to reduce the NEXT noise [81]. Figure 6.2 shows the differentiator structure in time domain. The differentiator acts as a fixed one tap pre-equalizer. The delay element in differentiator is $T/8$. The receiver filter transfer function $R(f)$ in the case of differentiator is

$$R(f) = 1 - \eta z^{-1} \quad (6.1)$$

and in the case of lowpass filter [81]

$$R(f) = \frac{1 + jk(f/f_0)}{1 + jd(f/f_0) - (f/f_0)^2}$$

$$f_0 = 0.3 \times \frac{1}{f}$$

$$k = 6.0$$

$$d = 1.7$$

The output of the receiver filter $y(t)$ can be modelled as

$$y(t) = \sum_{k=-\infty}^{\infty} a_k h(t - kT) + \sum_{k=-\infty}^{\infty} b_k e(t - kT) + \nu(t) \quad (6.3)$$

where

$a_k =$ far-end data symbols

$b_k =$ near-end data symbols

$h(t) =$ overall impulse response of the transmission path including coding, hybrid (Appendix B), transformer (Appendix C) and receiver filter.
Figure 6.1: Block Diagram of a General DSL System

Noise + Echo Signal
Figure 6.2: Fixed Front-End Equalizer Structure (Differentiator).
\( e(t) \) = impulse response of the echo path including coding, hybrid (Appendix B), transformer (Appendix C), and receiver filter.

\( \nu(t) = \) additive noise.

The echo canceller produces a replica of the echo signal \( e(t) \) to cancel the echo. The DFE removes the ISI from far-end signal by producing a replica of \( h(t) \).

## 6.2 Performance Measures

Using the theory given in Appendix D a program was developed to generate the impulse response for transmission and echo paths of the desired line configuration. This program simulates subscriber loops that are composed of sections of cable varying in gauge, temperature, length and including the effects of bridged taps, the transformer (Appendix B), the hybrid (Appendix C) and its balancing network parameter.

To determine the length requirements of echo canceller, we calculated from the echo impulse response, \( e(t) \), under different system configurations, the ratio of residual echo (uncancelled echo) to total echo on power basis. To achieve 60-70 dB echo cancellation, the residual echo power should be down by 60-70 dB compared to the total echo power. By varying the number of the echo canceller taps variations in the ratio are observed. From this, one can get the minimum number of echo canceller taps required for a given performance criteria in a particular system configuration.

The ratio of residual echo power to total echo power is calculated for different loops covering the wide variations of loop conditions encountered in actual practice. We have used five different codes: WAL-2, Manchester, MWAL-2, Bipolar and WACX to compare them in terms of their echo canceller length requirements. Figure 6.3 shows the results for echo path without any receiver filter. Figure 6.4 shows
Figure 6.3: Ratio of Residual Echo to Total Echo, Without Receiver Filter. (160 kbaud).
Figure 6.4: Ratio of Residual Echo to Total Echo, With Differentiator. (160 kbaud).
the results for the echo path which includes a differentiator as a receiver filter. The differentiator parameter used is \( \eta = 1.0 \). The transmission rate of 160 kbaud is assumed in both the cases. The Figure 6.5 shows the corresponding plots for 80 kbaud transmission rate. The receiver filter considered in this case is also a differentiator (\( \eta = 1.0 \)). All the loops (channels) used in Figures 6.3, 6.4 and 6.5 have 26 AWG cable for the main line section and 19 AWG cable for the bridged taps. The Figure 6.6 shows the plots of residual echo-to-total echo for the case when the receiver filter used is a low pass filter given in equation (6.2); only bipolar coding is considered. The transmission rate is 160 kbaud. The Figure 6.7 shows the corresponding plots for 80 kbaud transmission rate. All the loops shown in Figures 6.6 and 6.7 have 26 AWG cable for the main line section and 19 AWG cable for the bridged taps.

Using these plots we can obtain the minimum number of echo canceller taps required to achieve desired echo cancellation for different line codes. We can also compare the effects of receiver filter on the length requirements of echo canceller for different line codes.

We observe from these figures that without any receiver filter the MWAL or WAL-2 line code will require the least number of echo canceller taps compared to other codes for the same echo canceller performance. For DSL with receiver filter (differentiator), we note from Figures 6.4 and 6.5 that the Manchester code will require the least number of echo canceller taps compared to other codes to achieve the same amount of echo cancellation (both at 160 kbaud and 80 kbaud transmission rate).

Similar studies were carried out to determine the DFE tap length requirements for these codes. For the transmit channel response, \( h(t) \), the ratio of signal power at the correct sampling instance to the sum of the power in feedback taps is calculated. We have assumed baud-rate sampling in these results. The correct sampling
Figure 6.5: Ratio of Residual Echo to Total Echo, With and Without Differentiator. (80 kbaud).
Figure 6.6: Ratio of Residual Echo to Total Echo, With Receiver LPF and Bipolar Pulse Shaping (180 kband).
Figure 6.7: Ratio of Residual Echo to Total Echo, With Receiver LPF and Bipolar Pulse Shaping (80 kbaud).
instance is assumed to be at the time when the far-end signal component \( h(t) \), of the received signal \( y(t) \) reached its maximum. Another performance measure used is the ratio of signal power to the residual ISI (uncancelled ISI neglecting the precursor) power. The ratio of signal to residual ISI can be loosely considered as signal to noise ratio in the DFE receiver. For \( 10^{-7} \) BER requirement in DSL the signal to residual ISI ratio should be greater than 20 dB. Given the requirement that signal to residual ISI ratio should be greater than 20 dB, the minimum number of DFE taps required for a particular line code can be found from the plot of signal to residual ISI ratio. The variations in both the ratios is observed by varying the number of DFE taps. For channels with and without receiver filter (differentiator, \( \eta = 1.0 \)), the ratio of signal power to power in DFE taps is plotted in Figures 6.8, 6.9 and 6.10. The corresponding ratio of signal power to residual ISI power is plotted in Figures 6.11, 6.12 and 6.13. The transmission rate in the above figures is 160 kbaud. The line codes compared are WAL-2, MWAL, Manchester, Bipolar and WACX.

Similar plots were obtained when the transmission rate considered is 80 kbaud. For channels with and without receiver filter (differentiator, \( \eta = 1.0 \)), the ratio of signal power to power in DFE taps and ratio of signal power to residual ISI power is plotted in Figures 6.14, 6.15, 6.16 and 6.17. All the loops used in Figures 6.8 to 6.17 have 26 AWG cable for the main line section and 19 AWG cable for the bridged taps. The Figure 6.18 shows the plots of signal to DFE tap power ratio and signal to residual ISI power ratio for channels with bipolar coding and low pass filter given in equation (6.2). The transmission rate is 160 kbaud. In almost all the literature on DSL it is assumed that precursor ISI in subscriber loops is negligible and a fixed front end equalizer is sufficient to cancel the precursor ISI, but over longer loops the precursor ISI may not be insignificant. In Figure 6.18, for different loop lengths the ratio of signal power to precursor ISI power is also plotted. The corresponding plots for 80 kbaud transmission rate are shown in Figure 6.19. The
Figure 6.8: Plots of Signal to DFE Tap Power Ratio, Without Receiver Filter. (160 kbaud).
Figure 6.9: Plots of Signal to DFE Tap Power Ratio With Differentiator (160 kbaud).
Figure 6.10: Plots of Signal to DFE Tap Power Ratio, With and Without Differentiator. (160 kbaud).
Figure 6.11: Plots of Signal to Residual ISI Power Ratio Without Receiver Filter. (160 kbaud).
Figure 6.12: Plots of Signal to Residual ISI Power Ratio, With Differentiator. (160 kbaud).
Figure 6.13: Plots of Signal to Residual ISI Power Ratio With and Without Differentiator. (160 kbaud).
Figure 6.14: Plots of Signal to Residual ISI Power Ratio and Signal to DFE Tap Power Ratio Without Receiver Filter. (80 kbaud)
Figure 6.15: Plots of Signal to Residual ISI Power Ratio and Signal to DFE Tap Power Ratio, With Differentiator. (80 kbaud).
Figure 6.16: Plots of Signal to Residual ISI Power Ratio and Signal to DFE Tap Power Ratio, Without Receiver Filter. (80 kbaud).

112
Figure 6.17: Plots of Signal to Residual ISI Power Ratio and Signal to DFE Tap Differentiator. (80 kbaud).
Figure 6.18: Plots of Signal to DFE Tap Power Ratio and Signal to Residual ISI Power Ratio, With Receiver LPF and Bipolar Pulse Shaping (160 kbaud).
loops used in Figures 6.18 and 6.19 have 22 AWG cable for main line section and 19 AWG cable for bridged taps.

It can be seen from Figures 6.11, 6.12, 6.14 and 6.15 that WAL-2 and MWAL codes, even without receiver filter (differentiator), would give satisfactory performance with 4 DFE taps on the longest loops (both at 160 kbaud and 80 kbaud). This is to be expected due to the self equalizing properties of the WAL-2 and MWAL code. Figures 6.11, 6.12, 6.16, 6.17, 6.18 and 6.19 indicate that with fixed front end equalizer (differentiator or lpf), which shortens the tail of the ISI, 10 to 12 taps DFE would give satisfactory performance for bipolar and Manchester codes. From Figures 6.18 and 6.19 it is clear that over long loops the precursor ISI contribution is significant. This indicates that over long loops the precursor ISI could be a major source of interference if left uncancelled.

The plots for echo canceller and DFE tap length requirements are a valuable aid to the DSL system designer. Using these plots one can obtain an idea of the DFE and echo canceller tap length requirements for a given system configuration. These plots also provide a basis for comparing different systems in terms of their complexity requirements.
Figure 6.19: Plots of Signal to DFE Tap Power Ratio and Signal to Residual ISI Power Ratio, With Receiver LPF and Bipolar Pulse Shaping (80 kbaud).
Chapter 7

REDUCED STATE SEQUENCE ESTIMATION TECHNIQUES FOR DSL APPLICATION

The proposed U-interface bit rate in ISDN is 160 kb/s with a symbol rate of 80 kbaud. A DSL system employing PAM signalling with 4 amplitude levels (the 2B1Q line code) yields favourable bandwidth properties and crosstalk immunity [52]. However this results in severe ISI. The DFE is used in DSL systems to cancel the postcursor ISI. Unlike a linear equalizer a DFE does not cause noise enhancement and hence results in better performance. For channels with severe ISI, the maximum likelihood sequence estimation (MLSE) technique implemented by the Viterbi Algorithm (VA) has been shown to be the optimum receiver [14]. One of the keys to achieve the performance promised by channel capacity results (Chapter 4) is MLSE using VA.

The MLSE can provide significant improvement in performance compared to a DFE. For a K-symbol alphabet and ISI of L symbol long, the number of states in VA are $K^L$. The amount of processing per baud is proportional to $K^L$. Thus the complexity of the VA grows exponentially with L. Even though the VA is conceptually
simple to implement by virtue of its recursive nature, it is too complex for channels with severe ISI. In the case of fairly long subscriber loops, L is in the range of 10 to 20 taps or more and the complexity of the VA receiver for DSL application is prohibitive. Receiver complexity considerations have resulted in reduced emphasis on MLSE techniques for combating ISI.

An approach is needed that will reduce the complexity of the VA receiver but still offer partial performance gains of the MLSE. Most of the earlier work was directed towards shaping the channel into some desired one having a short impulse response, using a linear equalizer as a prefilter and then using the VA on this shortened channel [82], [83], [84]. However a linear equalizer enhances noise and if the desired impulse response is designed to be considerably different from the original channel due to complexity considerations, the noise enhancement will result in poor performance. Hill and Lee [85] proposed the use of a DFE as a prefilter to limit the complexity of the VA. DFE is no more complicated than a linear equalizer and does not cause noise enhancement. However the effects of error propagation in the DFE could degrade its performance considerably.

Another approach to reduce the complexity of the VA is to modify the MLSE itself. Vermeulen & Hellman [86] and Foschini [87] devised reduced-state MLSE techniques, where the VA keeps a smaller number of survivors than a complete MLSE. These reduced-state MLSE techniques were shown to be asymptotically optimum at high signal-to-noise ratios. But these techniques tend to be restricted in application to the specific trellises for which they were designed.

We have investigated two reduced state sequence estimation (RSSE) algorithms which are suboptimum but less complex alternatives to the VA. The two algorithms investigated are Decision Feedback Sequence Estimation (DFSE) and the M-algorithm. The DFSE algorithm was suggested by Eyuboglu and Qureshi [88] as an alternative way to reduce the complexity of the VA. Recently the use of the M-
algorithm has been suggested for decoding convolution codes [89]. Here we propose the M-algorithm as a RSSE algorithm to combat ISI. The two RSSE algorithms, DFSE and M-algorithm, discussed here have the following desirable characteristics:

1. The asymptotic (high SNR) detection performance can approach that of the VA.

2. A fixed number of processing operations per baud interval, just like the VA. This number is substantially lower than that for the VA.

3. Unlike the DFE these RSSE algorithms can utilize the energy in the precursor ISI.

The performance of DSL systems using these two RSSE algorithms will be compared with the performance of a more conventional DSL system using a DFE.

7.1 Decision Feedback Sequence Estimation

Figure 7.1 shows a conventional DSL system using a DFE (for simplicity the echo canceller is not shown).

The received signal, sampled at symbol rate \(1/T\), \(y_n\) is given by

\[
y_n = \sum_{j=0}^{L} a_{n-j} h_j + \nu_n
\]  

(7.1)

where \(a_n\)'s are the transmitted data symbols. The \(h_j\)'s are the samples of the overall impulse response. Additive noise samples are represented by \(\nu_n\)'s. They are assumed to be uncorrelated with the data.

The metric calculation in the VA is given by
Figure 7.1: Conventional DSL Transceiver System Model using DFE
\[ \mu_{n-L}(a_n) = \mu_{n-L-1}(a_{n-1}) - \left( y_n - \sum_{j=0}^{L} a_{n-j}h_j \right)^2 \] (7.2)

where \( a_{n-1} \) and \( a_n \) are possible successive states of the channel output. This choice of metric implicitly assumes that the additive noise samples are uncorrelated and Gaussian.

Rewriting equation (7.2) we have

\[ \mu_{n-L}(a_n) = \mu_{n-L-1}(a_{n-1}) - \left( y_n - \sum_{j=0}^{L'} a_{n-j}h_j - \sum_{j=L'+1}^{L} a_{n-j}h_j \right)^2 \] (7.3)

For a VA with truncated memory \( L' \) (\( L' < L \)) the term \( \sum_{j=L'+1}^{L} a_{n-j}h_j \) represents the residual ISI.

In the case of the DFE-MLSE structure proposed by Hill and Lee [85], the residual ISI is cancelled using a parallel DFE. The drawback of such a scheme is the error propagation in DFE. In the case of DFSE the estimate of the residual ISI sum given by \( \sum_{j=L'+1}^{L} a_{n-j}h_j \) for state \([a_n, \ldots, a_{n-L'}]\) is obtained from the path history associated with that state. Unlike the DFE-MLSE scheme the feedback mechanism in the case of DFSE is incorporated into the sequence estimator. This reduces the effect of error propagation. The number of states used in DFSE is reduced to \( KL' \) compared to \( KL \) in VA. For \( L' << L \) the complexity of DFSE is reduced considerably. In the case of DFSE the trellis merges much earlier than VA. This increases the probability of discarding the correct path. By a judicious choice of \( L' \), one may be able to obtain a desirable compromise between performance and complexity.
7.2 M-Algorithm

The M-algorithm has been widely used in speech and image encoding [90], [91], [92], [93]. The use of the M-algorithm has been suggested by Lin [89] and Aulin [94] for decoding convolutional codes. Here we are concerned with DSL systems and uncoded data transmission. The M-algorithm can simplify the detection complexity compared to the VA and still retain the high-SNR performance guaranteed by the VA. Here we propose the use of the M-algorithm to combat ISI in a DSL; offering a good performance/complexity tradeoff. The M-algorithm is a procedure for searching through a trellis. The aim is to perform a search through the trellis with M-states where $M << K^L$.

The brief description of the M-algorithm is as follows. Initially starting from the root node, we extend the trellis till we have M-states out of $K^L$ states. Once this initialization is completed, these M-states and corresponding M-paths (and associated path metrics) are passed on to the M-algorithm block. The working of the M-algorithm block is best illustrated by an example. The Figure 7.2 shows an 8-state trellis for K=2, L=3 and M=4 in this example. At a given time $nT$, where $T$ is the symbol period, we have 4 states (M=4), $S_0$, $S_1$, $S_3$ and $S_6$ and corresponding path metrics $\mu_0(n)$, $\mu_1(n)$, $\mu_3(n)$ and $\mu_6(n)$ respectively. There are 2 branches (K=2) out of each state. To extend the paths to the time $(n+1)T$, the cumulative metrics for new 8 paths are calculated. As shown in Figure 7.2 these 8 new paths reach states $S_0$, $S_1$, $S_3$, $S_4$, $S_5$, $S_7$. Two paths merge into the same states, $S_0$ and $S_4$, respectively. The M-algorithm at this stage essentially involves two sorts. The first sort checks for the states where there is more than one path merging. For each of these states the path with the best metric is retained and the other one is discarded. As seen in Figure 7.2, two states, $S_0$ and $S_4$ have more than one path merging in them. Out of two paths merging into states $S_0$ and $S_4$, one path with the best metric is retained for each state and the other one is discarded.
Figure 7.2: A M-Algorithm Search in An 8-State Trellis With $M=4$. 
This completes the first sort. The second sort involves retaining 4 states from the six different states which represent six surviving paths. The criteria used in this sort is to retain the 4 states with the best path metrics and discard the remaining two. This completes the second sort. Finally the information bit leading to those selected 4 states are stored in the corresponding surviving path memories. The maximum length of the path memories is D symbols into the past. D is also known as decoding delay. The algorithm then releases as output the oldest information bits from the path memory corresponding to the path with the best path metric among 4 retained paths. Thus the detected information bits are delayed by time DT. The algorithm purges the oldest symbols of all the survivors after releasing the output symbol. Now the algorithm is back at the situation when we started at time nT. The same procedure is repeated in extending paths to the time (n + 2)T.

One drawback of the M-algorithm is that sometimes the correct trellis path may be lost from the stored paths [85], [94]. This can happen if the transmitted path is not among the M survivors at some stage in the decoding process. At this stage one of two things can happen. Either the algorithm will reacquire the correct path within a short time or the receiver becomes lost. The algorithm in this case will wander aimlessly through the trellis indefinitely and only the channel noise can make the algorithm get back the transmitted path; this is a purely random phenomenon. The problem of correct path loss can be solved by transmitting data in blocks. In practice block transmission is likely to be used in DSL application for synchronization purposes. Another alternative is to build a recovery scheme to solve this correct path loss problem [89].

The earlier work by Lin [89] indicates that the correct path loss problem in the case of the M-algorithm decoding convolution and trellis codes results in the decoder progressing aimlessly through the trellis for an indefinite time. The results by Aulin [94] and Sheshadri [95] indicate that in the case of GPM and ISI, the M-algorithm does not ordinarily suffer from this problem, i.e., even if the correct
path is not one of the M-survivors, the algorithm retrieves the correct path within a short time. Our simulation results seem to indicate that on DSL channels (channels with ISI) the algorithm reacquires the correct path after a short time.

7.3 Performance Analysis

In the case of the M-algorithm, if the correct state is among the M-survivors, the detection performance will be the same as that of the VA. The performance of the M-algorithm will differ if the correct state is not among the M-survivors. Aulin [94] has given a method for analyzing the asymptotic (high SNR) detection performance of the M-algorithm in the presence of AWGN. A mathematical expression for the probability of an error event at high SNR is also derived. The evaluation of the error probability in this case involves a brute force search for the minimum distance and except for some simple cases, computation of error probability is impossible in a reasonable time [94]. Sheshadri [95] has given a lower bound to the bit error probability of the M-algorithm. Even this bound in case of M-algorithm is not guaranteed [95]. Still one can use this bound to compare the performance of the M-algorithm against the error performance of an optimum (VA) receiver. Also it can tell how large should M be so that its performance is close to that of the optimum receiver.

Exact error probability analysis of the DFSE is complicated because of decision feedback. Eyuboglu and Qureshi [88] have derived a bound for error probability of DFSE at high SNR. The bounds on error probability in the case of M-algorithm and DFSE are essentially of the form [88], [94], [95]

\[ P_e = C Q(D'_{\text{min}}/2\sigma) \]  

(7.4)

where \( C \) is a constant and \( D'_{\text{min}} \) is truncated minimum distance of the RSSE algo-
In the case of the M-algorithm, if \( M = K^L \), \( D'_{\text{min}} = D_{\text{min}} \) where \( D_{\text{min}} \) is the minimum distance of MLSE and in the case of DFSE \( D'_{\text{min}} = D_{\text{min}} \) if \( L' = L \).

This truncated minimum distance and the associated error events depend on the channel impulse response. Evaluation of error probability involves calculating error events for each channel and corresponding truncated minimum distance.

All these previous results on error probability analysis assume Gaussian noise. The dominant noise in DSL is coloured noise (NEXT). As explained in Chapter 5, computations required to evaluate \( D_{\text{min}} \) in case of DSL channels with NEXT noise are prohibitive due to the long tail of the autocorrelation function.

We have used simulations to evaluate the error performance of M-algorithm and DFSE based receivers. These simulations provide tighter results at the lower SNR's, where the bounds are weaker. Simulations also allowed investigation of detection dynamics. We have carried out extensive simulation study to fully establish the performance of two RSSE algorithms described earlier and compared it with the performance of more conventional DSL systems using the DFE.

### 7.4 System Model

We have described above two RSSE techniques which are less complex alternatives to the VA. Extensive simulations were carried out to evaluate the performance of the DFSE and M-algorithms for DSL application. The objectives of the simulations were twofold, one was to establish the performance limits of the DSL system using DFSE and M-algorithms and the second objective was to compare the performance of DFSE and M-algorithm systems with more conventional DSL transceiver schemes using DFE. The measure of performance used is the maximum reach (transmission range) of the corresponding system. The major interference limiting the transmission range (reach) of the DSL systems is NEXT. Here we first establish the NEXT
noise model which will be used in simulations to evaluate the performance of DSL systems.

7.4.1 The NEXT Noise Model

The transfer function of the NEXT coupling loss (NCL) suggested by the T1D1.3 working group is given by [6]

\[ N^2(f) = K_N (f/f_0)^{3/2} \]  \hspace{1cm} (7.5)

In dB

\[ NCL = m_N + 15 \log(f/f_0) \]  \hspace{1cm} (7.6)

\[ m_N = 57 dB \text{ at } 80 \text{ KHz} \]

As described in Section 2.6.3 this NEXT coupling loss is the power sum of the pair-to-pair NEXT loss due to 49 disturbers. The NEXT coupling loss model is valid under the following conditions.

1. It describes NEXT coupling loss which has only 1% cumulative probability of occurrence, i.e., only 1% of loops have less coupling loss.

2. The disturbing transmitters use the same binder group of the cable as the disturbed system.

3. All the 49 disturbing transmitters are simultaneously active.
The near-end crosstalk interference from 49 disturbers coupled at the output of the receiver filter of the disturbed system (Figure 7.1) can be simulated by a noise filter. This crosstalk noise filter can be conceptually divided into three sections: one that is shaped like the PSD of the disturbing system; one representing a model of NEXT coupling loss for 49 disturbers and one representing the receiver filter of the disturbed system. The crosstalk noise samples, $\nu_n$, (equation (7.1)) at the output of receiver filter can be simulated by applying a Gaussian random white noise source to the crosstalk noise filter input. This NEXT noise model suggested by the T1D1.3 working group assumes that the 49 disturbing clocks are randomly phased [6] and that the disturbance to any loop results from roughly equal contributions from many disturbing pairs. Figure 7.3 shows the NEXT noise model used in the simulations.

The crosstalk interference model described above is based on the average power spectral density of the digital signals. The crosstalk interference is assumed to be wide sense stationary (WSS) and only time-averaged properties of the signal ensemble are used in analysis. There may be some justification for using time averaged signal properties in the case of analog systems, where the subjective effect of interference may be insensitive to the cyclostationary nature of the interference. In the case of digital systems, the decisions are made on periodic samples of the received signal and the relationship between sampling instance and time-varying ensemble statistics must be taken into account while considering the crosstalk interference between such systems. The cyclostationary nature of the synchronous digital signal results in a periodic time-varying ensemble mean and variance [29], [30], [31]. The variation along the cable of the crosstalk coupling between pairs is a random process and the data sequences on the lines are independent random processes. This results in random crosstalk interference with cyclostationary statistics which depend on the phases of the transmitter clocks in the disturbing systems. As the interference in the disturbed system is relevant only at sampling instances, this cyclostationary
Figure 7.3: Stationary NEXT Noise Model.
nature of the crosstalk must be taken into account.

J.C. Campbell et al. [29], [39] and Smith [31] have shown that the incorrect assumption that crosstalk interference from digital systems is WSS and not cyclo-stationary will lead to some reduction in system margin. The results also show that the change from a nonsynchronous to synchronous network will lead to a reduction in system margin. This loss in margin occurs because of the possibility of the disturbed system sampling at instants when the crosstalk interference power is greater than the time-averaged power. They have identified the various network cases and investigated the appropriate design criteria for each case using a cyclostationary crosstalk noise model.

The ISDN is a synchronous network and DSL systems are presently engineered assuming randomly phased crosstalk disturbers (stationary crosstalk) [6]. In view of the above discussion it is important to investigate the effect of cyclostationarity of the crosstalk on the DSL system margin. The NEXT coupling loss model in DSL assumes 49 simultaneously active disturbers. The system clocks at the exchange end are derived from the same master clock. In one possible system architecture phase aligning of all the 49 disturbers at the exchange end is a possibility. Also as mentioned in Chapter 2, out of 49 disturbers 5 to 6 or fewer interfering pairs are the major crosstalk contributors [32], [33]. Here we assume a mature basic ISDN environment in which all the twisted pairs in cables terminating in central office are carrying basic ISDN signals with their clocking obtained from the same master clock in the central office. In this case the NEXT interference between any two loops is cyclostationary. The fewer the number of dominant disturbers, the more likely it is that the crosstalk is predominantly cyclostationary and that time-varying ensemble statistics of the crosstalk must be taken into account. Note that other prominent sources of cyclostationary additive noise in DSL systems include any residual echo signal after echo cancellation [96] and FEXT.
The deliberate phase randomization of the system clocks at the exchange end would reduce the cyclostationarity of the crosstalk and avoid the loss in margin. However there is a positive side to the cyclostationarity of the crosstalk. Due to the peak-to-average variation of the NEXT interference there exists a range of decision points for which the system sampling instants can correspond to smaller than average values of NEXT interference. In the case of cyclostationary crosstalk, the sampling phase of the disturbed system clock can be adjusted to sample the received signal at instants when the crosstalk interference power is smaller than the time-average power.

We have investigated the case where the system clocks of all the 49 disturbers are phase aligned. To evaluate the variation of the crosstalk interference with sampling phase, the time domain response of the crosstalk noise filter shown in Figure 7.3 was obtained for a different system structures. Using these responses the variation in crosstalk power with sampling phase was measured. Baud rate sampling was assumed. Over a baud period, the sampling phase was varied in steps of T/8. The receiver filter transfer function $R(f)$ in Figure 7.3 was the same as the one given by equation (6.2).

$$R(f) = \frac{1 + jk(f/f_0)}{1 + jd(f/f_0) - (f/f_0)^2}$$  \hspace{1cm} (7.7)

Figure 7.4 shows the plot of crosstalk power against sampling phase delay for bipolar coding and 160 kbaud transmission. The Figure 7.5 shows the plot for bipolar coding and 80 kbaud transmission. The Figure 7.6 shows the plot for no coding and 160 kbaud transmission. The Figure 7.7 shows the plot for no coding and 80 kbaud transmission.

From Figures 7.4-7.7 it is clear that in the case of the DSL systems considered, the peak-to-average variation of NEXT interference is in the range of 5 to 6 dB. It is evident from Figures 7.4-7.7 that in the case of cyclostationary crosstalk, proper
Figure 7.4: Crosstalk Power Variation With Sampling Phase (Bipolar Coding, 160 kbaud).
Figure 7.5: Crosstalk Power Variation With Sampling Phase (Bipolar Coding, 80 kbaud).
Figure 7.6: Crosstalk Power Variation With Sampling Phase (No Coding, 160 kbaud).
Figure 7.7: Crosstalk Power Variation With Sampling Phase (No Coding, 80 kbaud).
choice of sampling instant at receiver can result in reduced crosstalk interference. The cyclostationarity of the crosstalk results in a class of decision points for which NEXT interference can be made smaller than its time averaged value. The possibility to exploit the cyclostationary nature of the crosstalk (even by deliberately phase aligning the system clocks) to increase the system margin clearly exists. This is not possible in the case of stationary crosstalk. The cyclostationary NEXT noise model used in the simulations is shown in Figure 7.8. This model assumes that all the 49 disturbing system clocks are phase aligned or that strong crosstalk interference is mainly caused by only one or two close neighbours.

We have evaluated the performance of DSL systems using both the stationary and cyclostationary crosstalk noise models shown in Figure 7.3 and Figure 7.8, respectively.

### 7.5 The Performance of MLSE Systems With Cyclostationary Crosstalk

As seen from Figures 7.4 to 7.7 the sampling phase of the receiver has a significant influence on the cyclostationary crosstalk interference. It is important to investigate the effect of sampling phase on the performance of MLSE systems in the presence of cyclostationary crosstalk.

For a stationary white Gaussian noise channel model, the per-symbol error probability of a MLSE receiver is governed by the minimum distance among all the possible channel output sequences [14]. If the noise is cyclostationary, and the receiver samples at the baud rate, with the sampling phase \( \tau \), the sampled output of the channel is

\[
y_{nT+\tau} = \sum_{j=0}^{L} a_{n-j} h(jT + \tau) + \nu(nT + \tau) \tag{7.8}
\]
Figure 7.8: Cyclostationary NEXT Noise Model.
where \( \{a_n\} \) are the data symbols (assumed independent), \( h(t) \) is the end-to-end channel impulse response and \( \nu(nT + \tau) \) is sampled noise.

If the noise samples at \( T \)-second intervals are assumed independent and Gaussian, the symbol error probability is proportional to [14]

\[
Q \left( \frac{d_{\text{min}}(\tau)}{2\sigma(\tau)} \right)
\]

where

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2} dt
\]

and \( \sigma^2(\tau) \) is the variance of \( \nu(nT + \tau) \).

For an ideal MLSE receiver the parameter \( d_{\text{min}}^2 \) is defined as (Chapter 5)

\[
d_{\text{min}}^2(\tau) = \left[ \epsilon_0^\ast \epsilon_1^\text{min} \sum_{m_1 \geq 0} \sum_{m_2 \geq 0} \epsilon_{m_1} \epsilon_{m_2} \sum_i (h((i - m_1)T + \tau)h((i - m_2)T + \tau)) \right]
\]

(7.10)

In equation (7.10) the \( \{\epsilon_i\} \) are possible data symbol error values, and the minimum is taken overall the error patterns for which \( \epsilon_0 \neq 0 \).

The NEXT noise is not strictly Gaussian, nor are its baud-rate samples necessarily uncorrelated. However it may be approximately Gaussian since it consists of heavily filtered data symbols. The independence may be more questionable but our aim here is to achieve an intuitive understanding rather than precise performance results.

For a sampling phase \( \tau \), we have

\[
h(kT + \tau) = T \int_{-1/2T}^{1/2T} \hat{H}(f, \tau) e^{jk2\pi f T} df
\]

(7.11)

138
where,

\[
\hat{H}(f, \tau) = \sum_{k=-\infty}^{\infty} H(f - \frac{k}{T}) e^{j2\pi(f - \frac{k}{T})\tau} (7.12)
\]

and \(H(f)\) is the transfer function (Fourier transform of \(h(t)\)), and correspondingly

\[
\hat{H}(f, \tau) = \sum_{k} h(kT + \tau) e^{-j2\pi kT} (7.13)
\]

defining,

\[
E(f) = \sum_{m} \epsilon_{m} e^{-j2\pi fmT} (7.14)
\]

\[
\epsilon_{m} = T \int_{-1/2T}^{1/2T} E(f) e^{j2\pi fmT} dt (7.15)
\]

Using equations (7.11), (7.13), (7.14) and (7.15) we can rewrite the equation (7.10) for \(d_{\text{min}}^2\) as

\[
d_{\text{min}}^2 = \min \left[ T \int_{-1/2T}^{1/2T} |\hat{H}(f, \tau)|^2 |E(f)|^2 df \right] (7.16)
\]

From equation (7.16) it is quite clear that performance of MLSE system (\(d_{\text{min}}^2\)) will depend on the sampling phase \(\tau\). The results of the analysis carried out in Chapter 5 indicates that the error sequence which minimizes equation (7.10) on subscriber loops is \((1, -1, 0, 0, \cdots)\). We have evaluated the variation of \(d_{\text{min}}^2\) with sampling phase for this error sequence, i.e.,

\[
E(f) = 1 - e^{-j2\pi fT} (7.17)
\]

139
The variation of $d_{\text{min}}^2$ was evaluated for two different cases.

Case 1: Symbol rate $1/T=80$ kbaud and 4-level transmission. The overall response $h(t)$ includes the subscriber loop channel response, hybrid (Appendix B), transformer (Appendix C), Receiver filter.

Case 2: Symbol rate $1/T=80$ kbaud and 4-level transmission. The overall response $h(t)$ includes the subscriber loop channel response, hybrid (Appendix B) transformer (Appendix C) Receiver filter and bipolar pulse shaping.

The subscriber loop channel response was obtained using cable data for 22 AWG wire. The receiver filter transfer function was the same as the one given by equation (6.2). The variation in $d_{\text{min}}^2$ was evaluated by varying $\tau$ from 0 to $T$ in steps of $T/8$. The results of the calculation are plotted in Figures 7.9 and 7.10. Figure 7.9 shows the plot of $d_{\text{min}}^2$ (in dB) against sampling phase ($\tau/T$) for Case 1. The plots show variation of $d_{\text{min}}^2$ with sampling phase for different loop lengths in miles. Figure 7.10 shows the plots of $d_{\text{min}}^2$ (in dB) against sampling phase ($\tau/T$) for Case 2.

As seen from these plots for short loops on account of the $H(f)$ fold over, there is a large variation in $d_{\text{min}}^2$ with sampling phase. However as the length of the loops increases the fold over decreases and there is less variation in $d_{\text{min}}^2$ with sampling phase $\tau$.

In the case of NEXT, the disturbed and disturbing systems transmit in different directions and receive their clock information from different geographical locations. This introduces an unknown phase shift between these two clocks. Let $\phi$ be the relative phase between far-end signal and crosstalk (phase difference between disturbed and disturbing system clocks).

In order to evaluate the performance gains that can be achieved by exploiting
Figure 7.9: The Variation in $d_{\text{min}}^2$ with Sampling Phase ($\tau/T$) for Case 1. (All Loop Length, $L$, are in Miles.)
Figure 7.10: The Variation in $d_{\text{min}}^2$ with Sampling Phase ($\tau/T$) for Case 2. (All Loop Lengths, $L$, are in Miles.)
the cyclostationary nature of the crosstalk, let us define a measure of performance \( r \) as

\[
r(\phi) = \frac{d^2_{\text{min}}(\tau)}{\sigma^2(\phi + \tau)}
\]  

(7.18)

where \( \sigma^2(\phi + \tau) \) is the crosstalk noise power (stationary or cyclostationary). The crosstalk power \( \sigma^2(\phi + \tau) \) in the case of stationary crosstalk would not be affected by the sampling phase \( \tau \) and phase shift \( \phi \).

In the case of MLSE systems which do not exploit the cyclostationary nature of the crosstalk by adjusting the sampling phase \( \tau \), the sampling phase \( \tau \) is fixed irrespective of the phase shift \( \phi \) of the crosstalk interference. We have denoted such systems as conventional MLSE systems. The sampling phase \( \tau \) chosen is the one which gives system's maximum \( d^2_{\text{min}}(\tau) \), irrespective of phase shift \( \phi \), and is fixed. In this case, the ratio \( r(\phi) \) is given by

\[
r(\phi)_{\text{conventional}} = \frac{d^2_{\text{min}}(\tau^*)}{\sigma^2(\phi + \tau^*)}
\]  

(7.19)

where

\[
\max_{\tau} d^2_{\text{min}}(\tau) = d^2_{\text{min}}(\tau^*)
\]  

(7.20)

We have plotted the variation of crosstalk power with phase shift \( \phi \) in Figures 7.4 to 7.7. Using these figures along with Figures 7.9 and 7.10, the ratio \( r_{\text{conventional}} \) was obtained for different phase shift \( \phi \).

The MLSE systems which exploit the cyclostationary nature of the crosstalk, adjust the disturbed system clock sampling phase \( \tau \) such that the crosstalk power at the sampling instant is smaller than its time-averaged value. We have denoted
such systems as adaptive MLSE systems. In order to evaluate the performance of such a system the ratio $r(\phi)$ is obtained for different phase shifts $\phi$. The phase shift $\phi$ is varied in steps of $T/8$. For a given phase shift $\phi$, the ratio $r(\phi)$ is given as

$$r(\phi)_{\text{adaptive}} = \max_r \left[ \frac{d^{2}_{\text{min}}(r)}{\sigma^2(\phi + r)} \right]$$ (7.21)

The ratio $r(\phi)$ is obtained by varying $r$ in steps of $T/8$. For each sampling phase $r$, $d^{2}_{\text{min}}$ is obtained from Figures 7.9 and 7.10 and the corresponding $\sigma^2$ (crosstalk power) is obtained from Figures 7.4 to 7.7. Thus for every phase shift $\phi$, we obtained eight values of $r(\phi)$ (for eight different sampling phases $r$). Out of these eight sampling phases the adaptive MLSE system will select that sampling phase which maximizes the ratio $r(\phi)_{\text{adaptive}}$. Thus for every phase shift $\phi$, the adaptive MLSE system will adjust the sampling instant to maximize the ratio $r(\phi)$. The plot of $r(\phi)_{\text{adaptive}}$ was thus obtained by choosing this optimum ratio for each phase shift $\phi$ and varying $\phi$ in steps of $T/8$.

Figures 7.11, 7.12 and 7.13 show the plots of $r_{\text{conventional}}$ and $r_{\text{adaptive}}$ against phase shift $\phi$ for Case 1. Figures 7.14, 7.15 and 7.16 show the plots for Case 2. These figures show the plots for loop length = 5.0, 7.0 and 8.0 miles. All the loops are 22 AWG.

As seen from Figures 7.11 to 7.16 it is clear that peak-to-average variation in $r_{\text{conventional}}$ is of the order of 5-6 dB (for all loop lengths). It is also clear from these figures that for any phase shift $\phi$, $r_{\text{adaptive}} \geq \text{maximum (} r_{\text{conventional}} \text{)}$ and except for a small range of phase shift $\phi$'s, $r_{\text{adaptive}}$ is considerably greater than $r_{\text{conventional}}$. For the worst case of $\phi$, the difference of 10 to 12 dB are seen. In the case of conventional MLSE systems the performance in the presence of cyclostationary crosstalk will depend on the phase shift $\phi$. As this phase shift $\phi$ is random, the performance of such systems cannot be guaranteed. The plots for $r_{\text{conventional}}$ clearly indicate that in the case of cyclostationary crosstalk the MLSE systems which do not
Figure 7.11: The Plot of $r_{\text{conventional}}$ and $r_{\text{adaptive}}$ Against Phase Shift $\phi$ for Case 1. (Loop Length $L=5.0$ Miles).
Figure 7.12: The Plot of $r_{\text{conventional}}$ and $r_{\text{adaptive}}$ Against Phase Shift $\phi$ for Case 1. (Loop Length $L=7.0$ Miles).
Figure 7.13: The Plot of $r_{\text{conventional}}$ and $r_{\text{adaptive}}$ Against Phase Shift $\phi$ for Case 1. (Loop Length $L=8.0$ Miles).
Figure 7.14: The Plot of $r_{\text{conventional}}$ and $r_{\text{adaptive}}$ Against Phase Shift $\phi$ for Case 2. (Loop Length $L=5.0$ Miles).
Figure 7.15: The Plot of $r_{\text{conventional}}$ and $r_{\text{adaptive}}$ Against Phase Shift $\phi$ for Case 2. (Loop Length $L=7.0$ Miles).
Figure 7.16: The Plot of $r_{\text{conventional}}$ and $r_{\text{adaptive}}$ Against Phase Shift $\phi$ for Case 2. (Loop Length $L=8.0$ Miles).
adjust the sampling phase of the received signal can suffer considerable degradation in performance for unfavourable values of $\phi$.

In the case of stationary crosstalk the phase shift $\phi$ and sampling phase $\tau$ do not have any influence on the crosstalk power. Over long loops $d_{\text{min}}^2$ does not vary significantly with sampling phase $\tau$ and correspondingly the sampling phase $\tau$ has little influence on the ratio $r$(time-averaged value) in stationary crosstalk case. The plots for $r_{\text{conventional}}$ and $r_{\text{adaptive}}$ indicate that freedom in choosing the sampling phase with cyclostationary crosstalk can result in considerably greater $r_{\text{adaptive}}$ than time-averaged $r$ with stationary crosstalk. (Peak-to-average variation in $r_{\text{conventional}}$ is about 5-6 dB). Thus Figures 7.11 to 7.16 indicate that possibility to exploit the cyclostationary nature of the crosstalk clearly exists.

For loops 5.0 miles long the peak-to-average variation in $r_{\text{adaptive}}$ is of the order of 2.0 dB. For loops 8.0 miles long the variation in $r_{\text{adaptive}}$ is of the order of 1.2 dB. This variation is considerably smaller than the variation in $r_{\text{conventional}}$. It is evident from Figures 7.11 to 7.16 that as the length of the loops increase the variation in $r_{\text{adaptive}}$ decreases, such is not the case with $r_{\text{conventional}}$. This implies that over long loops the adaptive MLSE system performance will not be affected by the random phase shift $\phi$ between disturbed and disturbing system clocks.

These plots show that if the crosstalk in subscriber loop is cyclostationary the conventional MLSE systems can suffer considerable degradation in system margin. The adaptive MLSE systems on the other hand, can not only avoid this loss in system margin but can exploit the cyclostationary nature of the crosstalk to obtain better system performance.
7.6 Simulation Set-Up

Simulations were carried out to establish the maximum reach of the DSL system using DFSE, M-algorithm and DFE. To obtain the maximum reach of the DSL systems the BER performance of the DSL systems was measured by transmitting 5000 symbols over various subscriber loop channels. The maximum reach of the system was defined as, that channel for which zero errors were obtained after transmitting 5000 symbols and any additional increase in channel length caused error rate to degrade significantly. This definition is meaningful, given the system error probability’s high rate of change with received signal level.

The following is a list of characteristics and parameters that were employed in the simulations.

1. Bit Rate = 160 kbit/s
2. Symbol Rate = 80 kbaud (4-level transmission) or 160 kbaud (2-level transmission)
3. Bipolar Line Coding, 100% duty cycle
4. Subscriber Loops: 22 AWG twisted pair PIC cable at 70°F
5. Bridged Taps: (a) 0.5 miles long; (b) 19 AWG PIC cable at 70°F (for maximum reflection); (c) open ended
6. Receiver Filter: Transfer Function [81]

\[ R(f) = \frac{1 + jk(f/f_0)}{1 + jd(f/f_0) - (f/f_0)^2} \]

\[ f_0 = 0.3 \times \frac{1}{T} \]

\[ k = 6.0 \]

\[ d = 1.7 \]
7. The number of states in the DFSE was varied from 4 to 16.

8. The length of the channel response used in the metric calculation was 10 taps.

9. The M in the M-algorithm was varied from 8 to 32.

10. The decoding delay used in both, DFSE and M-algorithm is 40 symbols.

11. The number of taps in the DFE was 10.

The DSL channel response $C(f)$ includes hybrid (Appendix B) and transformer (Appendix C).

The block diagrams of the DSL systems with stationary crosstalk interference are shown in Figures 7.17 and 7.18. The Figure 7.17 shows the transmitter and receiver structure of the DSL system using DFSE and M-algorithm. The Figure 7.18 shows the corresponding transmitter and receiver structure of the DSL system using DFE.

The block diagram of the DSL system with cyclostationary crosstalk interference is shown in Figures 7.19 and 7.20. The Figure 7.19 shows the transmitter and receiver structure of the DSL system using the DFSE and M-algorithms. The Figure 7.20 shows the corresponding transmitter and receiver structure of the DSL system using a DFE. The DSL systems shown in Figures 7.17 to 7.20 were simulated to find the maximum reach of the DSL system using DFSE, M-algorithm and DFE. The reach was found for both binary transmission at 160 kbaud and 4-level transmission at 80 kbaud in each case. Initially a training sequence of 800 symbols is used for channel estimation. The BER performance was measured by transmitting 5000 symbols. The adaptation algorithm used to update the DFE tap coefficients was the LMS algorithm.
Figure 7.17: M-Algorithm and DFSE Transmitter and Receiver System Model With Stationary Crosstalk.
Figure 7.18: Conventional Decision Feedback Transmitter and Receiver System Model With Stationary Crosstalk.
Figure 7.19: M-Algorithm and DFSE Transmitter and Receiver System Model With Cyclostationary Crosstalk.
Figure 7.20: Conventional Decision Feedback Transmitter and Receiver System Model With Cyclostationary Crosstalk.
7.7 Simulation Results (Stationary Crosstalk)

In the case of stationary crosstalk noise the sampling phase of the received signal has no influence on the crosstalk noise. The best sampling phase of the received signal was used to measure the BER performance. On short loops precursor ISI is negligible and the best sampling phase corresponds to the instants where the height of the received signal is maximum. On longer loops precursor ISI is significant. As the DFE is not capable of cancelling precursor ISI, the precursor ISI in the case of a DFE is minimized by choosing a suboptimum sampling phase. This results in sampling of the received signal at an instant where the height of the received pulse is not maximum. This decrease in signal power causes degradation of signal-to-noise ratio. In the case of DFSE and M-algorithm, the estimated channel response includes one precursor tap. This allows the DFSE and M-algorithm to use best sampling phase even on long loops. As both of these algorithms utilize the energy in one precursor tap, unlike the DFE there is no degradation in their performance due to precursor ISI. One precursor ISI tap was found to be sufficient to cancel the dominant precursor ISI.

The results of the simulations are plotted in Figures 7.21 and 7.22. Figure 7.21 shows the maximum reach of the DSL system for binary transmission at 160 kbit/s. Figure 7.22 shows the maximum reach of the DSL system for quaternary (4-level PAM, 2B1Q) transmission at 80 kbaud. Figures 7.21 and 7.22 also give the composition of the maximum reach channels (all loop lengths are in miles).

As seen from Figure 7.22, the reach of the DFSE and M-algorithm DSL systems for binary signalling is the same. The performance gain achieved using M-algorithm and DFSE systems over conventional DFE systems in this case is 0.2 miles or equivalently about 1.6 dB on 22 AWG loop.

Similarly Figure 7.22 indicates that the reach of the DFSE and M-algorithm
Sys 1: DFE detection, binary transmission, (160 kb/s)
Sys 2: 16-state DFSE, binary transmission, (160 kb/s)
Sys 3: 16-state M-algorithm, binary transmission, (160 kb/s)

Figure 7.21: Maximum Reach of the DSL Systems for Binary Transmission at 160 kb/s (Stationary Crosstalk).
Figure 7.22: Maximum Reach of the DSL Systems for Quaternary Transmission at 80 kbaud (Stationary Crosstalk).
systems for quaternary transmission is the same. The performance gain achieved using the M-algorithm and DFSE over DFE is 0.3 miles or equivalently about 2.0 dB on 22 AWG loop.

7.8 Simulation Results (Cyclostationary Crosstalk)

The cyclostationary crosstalk noise model shown in Figure 7.8 assumes that the 49 disturbing system clocks are phase aligned. As the disturbing and disturbed systems transmit in different directions and receive their clock information from different geographical locations, there is an unknown phase shift between two clocks. In order to simulate this condition we have assumed that the disturbing system clocks are phase aligned with a phase shift \( \phi \). During the simulations this phase shift \( \phi \) is varied from 0 to \( T \) in steps of \( T/8 \).

As explained earlier due to the cyclostationary nature of the crosstalk interference, the sampling phase of the received signal has a significant influence on the DSL system performance. The proper choice of the sampling phase can result in smaller crosstalk interference due to the cyclostationarity of the crosstalk. One of the aims of the simulations is to evaluate whether the cyclostationarity of the crosstalk can be exploited to obtain better performance.

The philosophy in choosing the best sampling phase is to choose a sampling phase that minimizes the crosstalk interference and maximizes the height of the received signal, i.e., the best sampling phase corresponds to the instant at which the signal-to-noise ratio is maximized. The second factor which affects the choice of the sampling phase is the precursor ISI. Over long loops precursor ISI is significant. In the case of the DFE this has significant influence on the choice of sampling phase as DFE systems do not have mechanisms to cancel precursor ISI. Both these requirements of minimum precursor ISI and maximum SNR are sometimes conflict-
ing. The sampling phase in the case of DFE receiver is a compromise. In the case of DSL systems with DFSE and M-algorithm, the estimated channel response includes one precursor tap. This reduces if not eliminates the requirement of keeping the precursor ISI to a minimum while choosing the best sampling phase for these systems.

At the start of the simulations, for each of the 8 possible crosstalk interference phases, the optimum sampling phase as explained above is first obtained. The BER performance for a given channel is then evaluated by transmitting 5000 symbols. The reach of the DSL systems is evaluated for each of the crosstalk interference phases. The smallest reach among them is defined as the maximum reach of that DSL system. The simulations were carried out to evaluate this reach for each of the DSL systems shown in Figures 7.19 and 7.20.

The results of the simulations are plotted in Figures 7.23 and 7.24. The maximum reach of the DSL system for binary transmission at 160 kbaud is plotted in Figure 7.23. Figure 7.24 shows the maximum reach of the DSL system for 4-level (2B1Q) transmission at 80 kbaud.

As seen from Figure 7.23 the performance gain achieved using M-algorithm and DFSE systems over the conventional DFE system for binary transmission is 1.1 miles and 0.6 miles respectively on 22 AWG loop. The DSL systems using the M-algorithm outperforms both DFSE and DFE system for binary transmission.

Similarly, Figure 7.24 indicates that the reach of the DFSE and M-algorithm systems for quaternary transmission is the same. The performance gain achieved using DFSE and M-algorithm over conventional DFE scheme is 1.8 miles or equivalently about 10.5 dB on a 22 AWG loop.
Sys 1: DFE detection, binary transmission, (160 kb/s)
Sys 2: 8-state DFSE, binary transmission, (160 kb/s)
Sys 3: 8-state M-algorithm, binary transmission, (160 kb/s)

Figure 7.23: Maximum Reach of the DSL Systems for Binary Transmission at 160 kb/s (Cyclostationary Crosstalk).
Sys 1: DFE detection, 4-level transmission, (80 kbaud)
Sys 2: 4-state DFSE, 4-level transmission, (80 kbaud)
Sys 3: 8-state M-algorithm, 4-level transmission, (80 kbaud)

Figure 7.24: Maximum Reach of the DSL Systems for Quaternary Transmission at 80 kbaud (Cyclostationary Crosstalk).
Table 7.1: The DFSE and M-Algorithm Performance Gains in Miles Compared to DFE Receiver

<table>
<thead>
<tr>
<th></th>
<th>DFSE</th>
<th>M-Algorithm</th>
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<tbody>
<tr>
<td>Binary Transmission</td>
<td>0.2 miles</td>
<td>0.2 miles</td>
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<tr>
<td>Stationary Crosstalk</td>
<td></td>
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<tr>
<td>Quaternary Transmission</td>
<td>0.3 miles</td>
<td>0.3 miles</td>
</tr>
<tr>
<td>Stationary Crosstalk</td>
<td></td>
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</tr>
<tr>
<td>Binary Transmission</td>
<td>0.6 miles</td>
<td>1.1 miles</td>
</tr>
<tr>
<td>Cyclostationary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crosstalk</td>
<td>1.8 miles</td>
<td>1.8 miles</td>
</tr>
<tr>
<td>Quaternary Transmission</td>
<td></td>
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<tr>
<td>Cyclostationary</td>
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<tr>
<td>Crosstalk</td>
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### 7.9 Discussion

The Table 7.1 gives the performance gains (in miles) achieved using M-algorithm and DFSE compared to conventional DFE transceiver for different system conditions. As seen from Table 7.1, the DFSE and M-algorithm systems outperform more conventional DFE receivers for both, binary and quaternary transmission. We have used two crosstalk noise models, stationary and cyclostationary crosstalk noise, while evaluating the performance of different DSL systems. The DSL systems using the DFSE and M-algorithm achieve better performance compared to the DFE receiver when interfering crosstalk noise is either stationary or cyclostationary. Also, it is clear from these results that the DSL systems with cyclostationary crosstalk interference perform considerably better than DSL systems with stationary crosstalk interference. As discussed earlier our results indicate that the cyclostationary nature of the crosstalk interference can be exploited to enhance the performance of DSL systems.
It is evident from Table 7.1 that in the case of DSL systems using the M-algorithm and DFSE, the performance can be significantly improved by sampling the received signal at instants where cyclostationary crosstalk is smaller than its time-averaged value. In the case of stationary crosstalk the choice of sampling phase has no influence on the crosstalk interference. The M-algorithm and DFSE easily exploit the cyclostationary nature of the crosstalk due to their ability to utilize the energy in the channel precursor taps. This gives considerable freedom in choosing the best sampling phase. In the case of M-algorithm and DFSE the best sampling phase can be chosen to maximize SNR. Such liberty is not possible in the case of the DFE receiver. As the DFE receivers studied do not have any mechanism to cancel precursor ISI, any choice of sampling phase has to keep the precursor ISI to a minimum. This results in suboptimum sampling phase and corresponding degradation in performance as seen in Figures 7.21-7.24. The DFE receiver also suffers from the effects of error propagation.

The crosstalk interference in DSL is likely to be cyclostationary, especially if most interference comes from only a few close neighbours, or if all the transmitter clocks at the central office are phase aligned. The DSL systems engineered under the assumption that crosstalk is stationary are likely to suffer a reduction in system margin. On the other hand our studies indicate that the very nature of the cyclostationary crosstalk can be exploited to obtain better system margin.

From Figures 7.21-7.24 it is also clear that DSL systems using quaternary (2B1Q) signalling outperforms the DSL systems using binary signalling for all the three types of receiver structures studied. The DSL systems using quaternary signalling at 80 kbaud have a longer reach than the DSL systems using binary signalling at 160 kbaud.

The simulation results clearly indicate that M-algorithm and DFSE are indeed promising approaches to reduce the complexity of the VA for DSL application.
The VA for binary signalling would have required about $2^{12}$ states whereas the M-algorithm and DFSE systems studied have only about 16 states reducing complexity considerably. Similarly the VA for quaternary signalling would have required about $4^{10}$ states whereas M-algorithm and DFSE systems studied have about 16 states only, thereby reducing complexity considerably. Our studies show that a substantial increase in the reach of the DSL system can be achieved using the two proposed Reduced State Sequence Estimation (RSSE) techniques. By varying the number of states in the M-algorithm and DFSE we can obtain a good tradeoff between complexity and performance.

The noise considered in the simulation was NEXT alone, as NEXT is the dominant source of noise in DSL. It was modelled as non-white Gaussian noise although the M-algorithm and DFSE path metrics implicitly assumed white Gaussian noise. As mentioned in Chapter 5, Section 5.3, a receiver implementation that takes into account (with a whitening filter) would have a very long impulse response. The performance of such RSSE and DFE receivers might degrade significantly [77], [85], [95]. Also a noise-limiting receiver filter with 3 dB bandwidth of 40 KHz ($1/2T=40$ KHz) would somewhat attenuate the high frequency NEXT and NEXT noise would be "whitened" to some extent. Our simulation results show that for DSL channels with coloured noise, the M-algorithm and DFSE receivers (without any whitening filter) offer superior performance compared to more conventional DFE receivers, without incurring the unacceptably large complexity of the VA.

The effects of error propagation are less severe in the DFSE compared to DFE as feedback mechanism is incorporated in the sequence estimator. The M-algorithm is also not very susceptible to error propagation as long as the correct state is not lost. Thus the M-algorithm and DFSE offer better performance than conventional systems which use DFE to truncate the impulse response of the system [85].

Our results clearly show that the proposed RSSE algorithms can reduce the
complexity of MLSE while obtaining performance superior to that of the DFE. More important, they can permit a very substantial increase in system reach if the NEXT noise is cyclostationary. Since the channel capacity results of Chapter 4 indicated a tradeoff between reach and maximum bit rate, we can also infer that a significant increase in bit rate could be achieved by exploiting the cyclostationarity of NEXT in this way.
Chapter 8

PERFORMANCE OF TRELLIS CODING ON DSL CHANNELS

Combining channel coding and modulation by using a trellis encoder with a multilevel signalling and maximum likelihood (ML) Viterbi receiver, allows several dB's to be gained over conventional modulation schemes without sacrificing data rate or bandwidth [97]. Ungerboeck [97] investigated the design of multilevel trellis codes to achieve improved system performance. He has shown that, coding gains of 3-6 dB relative to an equivalent uncoded system could be achieved using a trellis structure with 4-64 states, in the presence of additive Gaussian noise [97]. Ungerboeck's work has given rise to a considerable research effort in the field of bandwidth efficient channel coding for digital transmission over telephone and satellite channels [73], [98-105].

The channel capacity results presented in Chapter 4 indicate that multilevel signalling along with coding is a way to achieve better system performance. We have begun an investigation of the application of channel coding for DSL transmission. Specifically we have considered the trellis codes, along with soft decision Viterbi decoding. The aim was to establish the performance of bandwidth efficient trellis coded transmission over subscriber loops.
8.1 Trellis Code

The trellis code used to evaluate the performance of a DSL system was given by Thapar [98]. It is rate 2/3 convolutional code with constraint length = 2. The corresponding convolution encoder is shown in Figure 8.1. The resulting trellis diagram is shown in Figure 8.2. The encoder output (bit triplets) is mapped on to 8-level PAM channel symbols. The bit-to-symbol mapping is based on the method of set partitioning given by Ungerboeck [97]. The corresponding 8-level PAM signal constellation is shown in Figure 8.3.

This particular choice of trellis code was somewhat arbitrary. Once the feasibility of using trellis coding for transmission over subscriber loops is established, the studies into the design of trellis codes for DSL application can be undertaken.

8.2 System Model

In order that the increased complexity of the coded system be justified, it must be shown that improved performance can be obtained on subscriber loop channels. In this study we have considered the situation where the encoded data sequences are transmitted over subscriber loop channels. The receiver performs maximum likelihood sequence decoding (VA) over unquantized received signal. The DSL has severe ISI. The equalizer (DFE) is used to cancel the ISI, in order to reduce the complexity of the VA receiver. The VA receiver is matched to the encoder only.

The simulations were undertaken to evaluate the performance of the coded DSL system. Figure 8.4 shows the transmitter and receiver structure of the DSL system using trellis coding and Viterbi detection. The coded DSL system used rate 2/3 trellis code shown in Figure 8.1 and VA for decoding. As the ISI in DSL is of trailing type, it is cancelled using DFE. The information rate of the coded DSL
Figure 8.1: Rate 2/3 Convolution Code.

Figure 8.2: Trellis Diagram for the Rate 2/3 Encoder Shown in Figure 8.1.
Figure 8.3: 8-Level PAM Signal Set.
Figure 8.4: Transmitter and Receiver System Model With Trellis Coding and Cyclostationary Crosstalk
system is 2/T bits (1/T=80 kbaud). The interference considered is cyclostationary NEXT alone. The cyclostationary NEXT noise model is the same as that given in Section 7.4.1, Figure 7.8. We have evaluated the performance of coded DSL systems using this cyclostationary crosstalk noise model.

The performance of the coded DSL system is compared with the performance binary (2-level) and quaternary 4-level uncoded DSL system. The measure of performance used is the maximum reach (transmission range) of the corresponding system.

8.3 Simulation Set-Up

The DSL system shown in Figure 8.4 was simulated to find the maximum reach of the DSL system using trellis coding. The simulation set-up is the same as that described in Chapter 7, Section 7.6. The information rate of the coded system is 160 kbit/s and the corresponding symbol rate is 80 kbaud (8-level transmission). The BER performance was measured by transmitting 5000 symbols over various loops. Decoding delay used in VA decoder is 20 symbols.

8.4 Simulation Results

The crosstalk noise model considered in the simulations is cyclostationary crosstalk noise model as shown in Figure 8.4. The simulation procedure is the same as that given in Section 7.8. As explained in Section 7.8, due to the cyclostationary nature of the crosstalk, the sampling phase of the received signal has significant influence on the DSL system performance. At the start of the simulations, for each of the crosstalk interference phases, the optimum sampling phase as explained in Section 7.8 is first obtained. The BER performance for a given channel is then evaluated
by transmitting 5000 symbols. The reach of the coded DSL system is evaluated for each of the eight crosstalk interference phases. The smallest reach among them is defined as the maximum reach of the coded DSL system.

It was found during the simulations that for DSL systems with trellis coding, the major problem is the error propagation in DFE. In DFE the feedback process is based on equalizer (quantizer) decisions and not VA decoder decisions. The decoded symbols from the VA decoder cannot be used in the feedback process due to the decoding delay. The error propagation prevents intended cancellation of ISI. This incomplete feedback equalization gives rise to residual interference at the input to the VA decoder. This residual interference was found to degrade the performance of the VA decoder.

The simulations were also carried out to find the maximum reach of the coded system when the ISI is correctly cancelled. This is achieved during simulations by feeding the correct transmitted symbols directly into DFE instead of tentative decisions from quantizer. The maximum reach thus obtained represents the ideal performance of the coded system.

The results of the simulations are plotted in Figure 8.5. It also gives the composition of the maximum reach channels (all loop lengths are in miles). The additional reach obtained by the correct cancellation of ISI (ideal reach) in the case of coded system, is shown in Figure 8.5 by the shaded region. In the last chapter we have already presented the maximum reach of the conventional uncoded system with DFE. The Figure 7.12 in the last chapter shows the block diagram of the uncoded system with DFE and cyclostationary crosstalk. The maximum reach of the conventional uncoded DSL system for binary transmission at 160 kbaud is plotted in Figure 7.15 (Sys1). The corresponding DSL system reach for 4-level transmission at 80 kbaud is plotted in Figure 7.16 (Sys1). In order to compare the performance of the coded system with an uncoded systems, the reach of the
Sys 1: Coded system, V.A. detection, 8-level transmission (80 kbaud)
Sys 2: DFE detection, 4-level transmission (80 kbaud)
Sys 3: DFE detection, binary transmission (160 kb/s)

Figure 8.5: Maximum Reach of the DSL System With Trellis Code and Comparison With Conventional DFE Detection. (Cyclostationary Crosstalk)
uncoded system from Figures 7.15 and 7.16 are also plotted in Figure 8.5.

8.5 Discussion

As seen from Figure 8.5 the maximum reach of the coded system is 4.0 miles and maximum reach of the 80 kbaud uncoded system is 5.2 miles. This is a somewhat surprising result, as one would have expected coded system to perform better than the uncoded system. Our simulation studies indicate that the performance of a coded system is ruined due to the error propagation in DFE.

The errors in tentative decisions by quantizer results in residual interference at the input to VA decoder. As seen from Figure 8.5, this error propagation in DFE degrades the performance of the coded system considerably. It is clear from Figure 8.5 that, a coded system can offer better performance than an uncoded system if the ISI is correctly cancelled. The ideal reach of the coded system is 5.5 miles compared to 5.2 miles of the uncoded system.

The noise considered in the simulations was cyclostationary crosstalk. The transceiver system model shown in Figure 8.4 is unable to exploit the cyclostationary nature of the crosstalk due to the presence of the DFE receiver used to cancel ISI (seen with ideal feedback DFE decisions). As mentioned in Chapter 7, Section 7.9, the DFE receiver is not capable of exploiting the cyclostationary nature of the crosstalk. This limits the performance gain that can be achieved, since cyclostationarity of the crosstalk cannot be exploited by the coded system.

One way to improve the performance of the coded system is to feed more than one equalized samples per baud to the decoder [103]. But this would require fractionally spaced DFE and corresponding fractionally spaced echo canceller. As DFE and echo canceller determine the complexity requirements of the DSL system, there is high priority on minimizing the sampling rate used. The penalty for operating
echo canceller/DFE on samples taken at K times the baud rate is that the complexity of the echo canceller/DFE is proportional to K [4]. The increased sampling rate for fractionally spaced DFE/echo-canceller results in additional complexity. Also so far as DFE is present the error propagation effect cannot be eliminated.

Another approach is to use a VA receiver whose metric is based on the complete knowledge of the channel as well as of the encoder. The system shown in Figure 8.4 has a separate quantizer and DFE to cancel the ISI. The VA receiver was matched to the encoder only. This system does not use the memory due to ISI to perform VA decoding. The VA receiver can be made to include both the code and the channel structure in the decoding process [101], [104], [105]. Such an extended-state VA receiver can be employed to optimally decode the message sequences in the presence of ISI. In this case the VA receiver decisions are based on the log-likelihood ratios of the sequences at the receiver filter output. The extended-state VA receiver allows one to exploit the memory in ISI. This approach eliminates the need for separate DFE and quantizer.

The complexity of such a MLSE receiver, implemented using the VA, is proportional to the number of states in the combined encoder and channel model [101], [105]. As seen in Chapter 7, complexity of the VA for DSL application (even to combat ISI alone) is prohibitive. Different solutions are therefore called for, trading some loss of optimality for reduced complexity. The Reduced State Sequence Estimation (RSSE) techniques, DFSE and M-algorithm, described in Chapter 7 can be used to reduce the complexity of this extended-state VA receiver (combined encoder and channel). The M-algorithm or DFSE receiver in this case would use the knowledge of both, the channel and the encoder, in the decoding process. Thus it is still possible to exploit the channel memory but with considerably-reduced complexity compared to a full extended-state VA. By judicious choice of the number of states in M-algorithm or DFSE, one may be able to obtain a desirable compromise between complexity and performance. As mentioned in Chapter 7, our studies indicate that
M-algorithm and DFSE receivers are considerably less susceptible to the effects of error propagation.

The feasibility study to evaluate the performance of trellis coded transmission over subscriber loop was actually done before the RSSE techniques investigation. The results shown in Figure 8.5 indicated that effects of error propagation, inherent to DFE, degrade the performance of VA decoder. The preliminary investigation into using an extended-state VA receiver to decode trellis code in the presence of ISI indicated that complexity of such a receiver is prohibitive. This lead us to investigate RSSE techniques for DSL application, which can reduce the complexity of the MLSE while retaining much of its performance gains. The results shown in Figure 8.5 are reported after the RSSE technique results (Chapter 7) due to the ease of presentation. In chronological order this work was done before the RSSE investigation.

Comprehensive studies will be required to establish the performance capabilities of these RSSE techniques (M-algorithm and DFSE) to decode trellis codes on channels with ISI (e.g., DSL channel).
Chapter 9

CONCLUSIONS

9.1 Concluding Remarks

The steadily growing demands for data/voice communication has prompted the development of high speed data transmission systems for subscriber loops. One of the research topics addressed in this thesis is the evaluation of the channel capacity upper bounds on the subscriber loop performance. The channel capacity bounds are evaluated when the interference is crosstalk and white noise. First the mathematical model used to evaluate the channel capacity of the subscriber loop is established. The channel capacity results plotted in Figures 4.2 - 4.12 clearly indicate that reliable communication at rates significantly higher than 160 kb/s and/or over loops much longer than 3.5 miles is theoretically possible. The channel capacity of the subscriber loop can be used as a measure of performance to evaluate existing and future practical DSL systems. The channel capacity results allow one to judge the capability of a particular transmission scheme and indicate whether there is any significant performance gain to be achieved using more complex and sophisticated coding-decoding techniques. Our channel capacity results indicate that multilevel signalling along with coding is one of the ways to achieve better system performance.
The performance achieved by the MLSE receiver implemented using VA can be shown to be effectively as good as could be attained by any receiver structure. DSL systems presently use DFE to combat ISI on subscriber-loops. In this work we have analyzed and compared the performance of MLSE and DFE receivers on subscriber loop channels in the presence of white Gaussian noise. The performance of MLSE and DFE receivers for a given DSL channel was estimated using the method of upper bounds and lower bounds. We have also extended the analysis to evaluate the performance of MLSE and DFE receivers in the presence of NEXT (coloured Gaussian noise). One of the important conclusions is that even MLSE has substantial S/N ratio penalty on long loops. The results also indicate that binary transmission (160 kbaud) has significantly higher S/N ratio penalty compared to quaternary transmission (80 kbaud). The results plotted in Figures 5.1, 5.2, 5.3 and 5.13 indicate that a DFE receiver may achieve nearly the same theoretical performance as a much more complex MLSE receiver in the presence of white noise or stationary NEXT.

The complexity requirements of the echo canceller and DFE largely determine the complexity of the DSL system. One of the contributions of this thesis is the evaluation of the complexity requirements of the echo canceller and DFE for different line codes commonly used in DSL systems. The effects of the front end receiver filter design on complexity is also evaluated. The plots for echo canceller and DFE tap length requirements provide a convenient basis for comparing different system designs in terms of their complexity requirements. The plots of the signal power to precursor ISI power indicate that over long loops (longer than 4 miles, 22 AWG) the precursor ISI could be a major source of interference if left uncancelled.

The MLSE can provide significant improvement in performance compared to a DFE. Although a VA receiver is implementable by virtue of its recursive nature, it is unacceptably complex for DSL channels. A simple and practical receiver is needed that might limit the complexity while retaining much of the performance advantage.
of the MLSE. One of the problems addressed in this thesis is the investigation of such reduced state sequence estimation (RSSE) algorithms. In this study we have proposed the Decision Feedback Sequence Estimation (DFSE) and M-algorithm as RSSE alternatives to reduce the complexity of the VA for DSL applications.

The dominant source of noise in subscriber loops is NEXT. The crosstalk interference in DSL is likely to be cyclostationary, especially if most of the interference comes from only a few close neighbours, or if all the transmitter clocks at the central office are phase aligned. DSL systems are presently being engineered assuming randomly phased crosstalk disturbers (stationary crosstalk). This will likely lead to some reduction in system margin. One of the research topics addressed in this work is the effect of cyclostationarity of the crosstalk on the DSL system margin. We have shown that in the case of cyclostationary crosstalk, the sampling phase of the disturbed system clock can be adjusted to sample the received signal at instants when the crosstalk interference power is smaller than its time-averaged power. The investigation into the performance of cyclostationary crosstalk indicates that adaptive MLSE systems can exploit the cyclostationary nature of the crosstalk to obtain better system margin.

A simulation study was carried out to establish the performance of the DSL system using DFSE, M-algorithm and DFE in the presence of stationary and cyclostationary NEXT. The simulation results indicate that the DFSE and M-algorithm systems outperform more conventional DFE receivers for both binary and quaternary transmission. We also show that RSSE systems with cyclostationary crosstalk interference perform considerably better than the DFE receivers. It is evident from the simulation results that in the case of DFSE and M-algorithm systems, the performance can be significantly improved by sampling the received signal at instants where the cyclostationary crosstalk is smaller than its time-averaged value. The M-algorithm and DFSE easily exploit the cyclostationary nature of the crosstalk due to their ability to utilize the energy in the channel precursor taps. This gives
considerable freedom in the choice of sampling phase. The results also show that as the DFE receivers studied do not have any mechanism to cancel the precursor ISI, DFE receivers are not capable of exploiting the cyclostationary nature of the crosstalk. It is also clear from the simulation results that DSL systems using quaternary signalling outperform the DSL systems using binary signalling for all three types of receiver structure studied. The simulation results clearly indicate that DFSE and M-algorithm are indeed promising approaches to reduce the complexity of VA for DSL application while offering superior performance compared to DFE receivers.

In this work we have also investigated the application of bandwidth efficient trellis coding along with soft decision Viterbi decoding for DSL transmission. The results of the preliminary investigation show that error propagation in the DFE degrades the performance of the coded system considerably. Also the results indicate that coded systems can not exploit the cyclostationarity of the crosstalk due to the presence of the DFE receiver used to cancel ISI.

9.2 Suggestions for Future Work

Below are outlined suggestions for future study.

1. The channel capacity plots of the subscriber loops indicate that a tradeoff between the reach and bit rate is possible. We have investigated RSSE techniques with a goal of increasing the reach of the DSL system. The proposed U-interface bit rate of 160 kb/s was assumed throughout this study. One of the areas open to future research is the evaluation of the DSL system capability to transmit at rates significantly higher than 160 kb/s, e.g., evaluating the performance capability of a DSL system that can connect more than one U-interface to the same cable. This will require a DSL system with a bit rate
of 320 kb/s or more.

2. Our studies have indicated that in the case of RSSE techniques freedom in choosing the sampling phase with cyclostationary crosstalk can result in considerably better system margin. This will require a timing recovery technique that will adjust the disturbed system clock sampling phase such that the crosstalk power at the sampling instant is smaller than its time-averaged value. Further studies are required to investigate techniques for deriving the optimum receiver sampling phase.

3. The results of our investigation into the application of channel coding for DSL transmission indicates that the performance of the coded system is ruined due to the error propagation in DFE. One way is to use a VA receiver whose metric is based on complete knowledge of the channel as well as encoder. The complexity of such an extended-state VA receiver is prohibitive. The RSSE techniques (e.g., DFSE and M-algorithm) can be used to reduce the complexity of this extended-state VA receiver. The RSSE receiver in this case would use the knowledge of both, the channel and encoder, in the decoding process. This will allow the receiver to exploit the channel memory but with considerably reduced complexity compared to a full extended-state VA. Further work is needed to establish the performance capabilities of such RSSE techniques. This will also require studies into the design of trellis codes for DSL application.

4. The VLSI realizability of any DSL system is a must for cost effectiveness. The implementation aspect will greatly influence the choice of DSL system architecture. We have established the performance capabilities of the RSSE algorithms for DSL transmission. One of the key issues that must be addressed in the future work is the feasibility of the VLSI implementation of the RSSE techniques for DSL application. The question of the choice of processing structures that are appropriate in VLSI circuit technology for realizing DSL
receivers based on the RSSE algorithms must be answered.

5. If the major interference in other types of digital communication systems is cyclostationary, the possibility to exploit the cyclostationarity of the interference should be investigated.
REFERENCES


Appendix A

CABLE CHARACTERISTICS DATA

This appendix contains the three data sheets from [106] that provide the characteristics by frequency for:

1. 19 AWG PIC cable at 70°F
2. 22 AWG PIC cable at 70°F
3. 26 AWG PIC cable at 70°F

This information is used along with the theory presented in Appendix D to generate the frequency responses and impulse responses for composite, (i.e., more than one cable type) subscriber loops with bridged taps of different gauges and lengths.
### Cable Characteristics 1Hz to 5MHz

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Figure A.1: Cable Characteristics for 19 AWG PIC Cable at 70°F [106].

---

19 GAUGE PIC CABLE AT 70 DEGREES F.
### Cable Characteristics 1Hz to 5MHz

#### 22 GAUGE PIC CABLE AT 70° DEGREES F.

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<th>G (nS/foot)</th>
<th>C (pF/foot)</th>
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Figure A.2: Cable Characteristics for 22 AWG PIC Cable at 70°F [106].

200
## Cable Characteristics 1Hz to 5MHz

### 26 GAUGE PIC CABLE AT 70° DEGREES F.

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Figure A.3: Cable Characteristics for 26 AWG PIC Cable at 70°F [106].

201
Appendix B

IMPEDEANCE BALANCING HYBRID

This appendix shows the impedance balancing hybrid used in this study [107].

![Diagram of impedance balancing hybrid]

Figure B.1: An Impedance-Balancing Hybrid Structure.
Appendix C
TRANSFORMER DATA

This appendix gives the transfer function of the transformer used in this study.

Transformer Transfer Function:

\[ T(f) = \frac{jf}{(a + jf)}, \quad a = 5000 \]
Appendix D

ABCD PARAMETERS FOR BRIDGED TAPS

Subscriber loops in the telephone loop plant may be composed of sections with different gauges and may contain bridged taps. In order to evaluate the frequency response of such lines, a line model is required.

A transmission line may be modelled using ABCD or hybrid parameters. See Figure D.1 where

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\]

\[A = \cos h\gamma \ell \quad R = \text{Resistance}\]
\[B = Z_0 \sin h\gamma \ell \quad L = \text{Inductance}\]
\[C = \frac{\sin h\gamma \ell}{Z_0} \quad G = \text{Conductance}\]
\[D = \cos h\gamma \ell \quad C = \text{Capacitance}\]

\[\gamma = \text{Propagation Constant} = \sqrt{(R + j2\pi fL)(G + j2\pi fC)}\]
\[ Z_0 = \text{Characteristic Impedance} = \sqrt{\frac{R + j2\pi fL}{G + j2\pi fC}} \]

\[ \ell = \text{Length of the Line} \]

Note: A, B, C, and D are complex quantities, accurate tables γ of versus frequency and \( Z_0 \) versus frequency for various gauges, temperatures and cable types have been published. See Appendix A, [18].

Lines made of a series connection of sections with various parameters may be characterized by matrix multiplication of the individual sections ABCD matrices, giving a composite ABCD matrix. To generate the characteristics for a line in the opposite direction the A and D terms are interchanged.

A line configured as in Figure D.2 has a Voltage Gain at a particular frequency given by:

\[ H = \frac{1}{Z_s \left( C + \frac{D}{Z_l} \right) + A + \frac{B}{Z_l}} \]

Consider Figure D.3, depicting a transmission line containing a single bridged tap. The three sections of this line may be characterized by their individual ABCD matrices. Here the loop sees the bridge tap as a parallel connection. An equivalent series connection ABCD matrix may be formed from the original bridged tap ABCD parameters, say A', B', C' and D'.

The conversion is given by:
In matrix form:

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
= \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\]

Figure D.1: ABCD Parameters.

Figure D.2: Use of ABCD Parameter Matrix.
Figure D.3: Line With a Bridged Tap.

\[ A = 1 \]

\[ B = 0 \]

\[ C = Y_0 \tan \beta \Gamma \]

where

\[ Y_0 = \text{admittance} = \sqrt{\frac{C^* D^*}{A^* B^*}} = \frac{1}{Z_0} \]
\[ \Gamma = \text{propagation constant} \]

\[ \log_e \frac{1}{2} \left( A' + D' + \frac{B}{Z_0} + Z_0 C' \right) \]  \[([17])\]

That is the ABCD matrix for a bridged tap line after conversion is:

\[
\begin{bmatrix}
1 & 0 \\
Y_0 \tan h\Gamma & 1
\end{bmatrix}
\]

The equivalent line for Figure D.3 is shown in Figure D.4.

![Figure D.4: The Equivalent Line.](image)

where

\[
\begin{bmatrix}
A_E & B_E \\
B_E & D_E
\end{bmatrix} = \begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
C C & 1
\end{bmatrix} \begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\]
where \( CC = \frac{C_3 D_3}{A_3 B_3} \tan h \left[ \text{complex log} \left( \frac{1}{2} \left( A_3 + D_3 \frac{B_3}{Z_0} + Z_0 C_3 \right) \right) \right] \)

where \( Z_0 = \sqrt{\frac{C_3 D_3}{A_3 B_3}} \)

Matrix reduction in this manner may be carried out for more complex lines, and the next ABCD parameters for the entire line obtained.

The frequency response of a composite cable is determined by evaluating the above matrices at each frequency.
END
07.07.89
FIN
M-algorithm and DFSE receivers are considerably less susceptible to the effects of error propagation.

The feasibility study to evaluate the performance of trellis coded transmission over subscriber loop was actually done before the RSSE techniques investigation. The results shown in Figure 8.5 indicated that effects of error propagation, inherent to DFE, degrade the performance of VA decoder. The preliminary investigation into using an extended-state VA receiver to decode trellis code in the presence of ISI indicated that complexity of such a receiver is prohibitive. This lead us to investigate RSSE techniques for DSL application, which can reduce the complexity of the MLSE while retaining much of its performance gains. The results shown in Figure 8.5 are reported after the RSSE technique results (Chapter 7) due to the ease of presentation. In chronological order this work was done before the RSSE investigation.

Comprehensive studies will be required to establish the performance capabilities of these RSSE techniques (M-algorithm and DFSE) to decode trellis codes on channels with ISI (e.g., DSL channel).