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UMI
An Experimental Investigation of the Aerodynamic Effects of Roughness Consisting of Arrays of Depressions in Surfaces

by

Martin John Baumann, B.Eng

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment of the requirements for the degree of Master of Engineering in Aerospace Engineering

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

Department of Mechanical and Aerospace Engineering Carleton University Ottawa, Ontario March 12, 1999

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acceptance of the thesis

An Experimental Investigation of the Aerodynamic Effects of
Roughness Consisting of Arrays of Depressions in Surfaces
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Martin John Baumann. B.Eng

in partial fulfilment of the requirements for
the degree of Master of Engineering

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ABSTRACT

Wind tunnel tests were carried out on flat plates containing regular and random arrays of depression surface roughness. Arrays of cylindrical depressions and hemispherical depressions were used as the regular roughness, while small dimples in vinyl floor coverings were used to investigate flow over a randomly roughened surface. Skin friction coefficients were determined using the velocity defect-law. The skin friction coefficients found using the defect-law were found to be in good agreement with skin friction coefficients calculated using the momentum integral method. The skin friction coefficients for the depression roughness were found to be less than skin friction coefficients of protrusion roughness of the corresponding geometry and spacing.

Computational work was also performed using a discrete element roughness model. Boundary layer development computations were done to identify an equivalent protrusion roughness height that would produce the same boundary layer behaviour as the actual depression roughness geometries. Skin friction coefficients generated by the computer code were matched to the experimental skin friction coefficients found from the defect-law method to identify this ‘equivalent height’. Equivalent protrusion heights were found to be about one order of magnitude less than the actual depression depths.
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LIST OF SYMBOLS

A,B  constants in the law-of-the-wall, Equations (2-6, 2-8)

A_F  projected frontal area of a protrusion roughness element (Figure 2.4a)

A_S  planar surface area which a roughness element occupies (Figure 2.4a)

A_w  total upstream-facing wetted surface area of a roughness element (p. 58)

A_XS  depression cross section area perpendicular to the flow

b  span of roughness element (Figure 2.4a)

C(λ)  roughness spacing parameter, Equations (2-8, 2-9)

C(λ_eq)  roughness spacing parameter found using equivalent height (pg. 56)

C_f  skin friction coefficient, \( \frac{\tau_w}{\frac{1}{2} \rho U^2} \)

d_{act}  actual full depth of depression roughness elements

H  boundary layer shape factor, \( \frac{\delta^*}{\theta} \)

k  height of protrusion roughness element (Figure 2.4a)
$k_e$ equivalent protrusion height of depression roughness element (Chapter 5)

$k_s$ standard sand height (p. 7)

$k^-$ roughness Reynolds number, $\frac{k u_*}{v}$

$L$ streamwise distance between 2-D roughness elements (Figure 2.3a)

$\frac{dP}{dx}$ freestream pressure gradient

$S$ streamwise length of a 2-D roughness element (Figure 2.3a)

$U, U_e, U_\infty$ freestream fluid velocity

$u$ local velocity inside the boundary layer

$u^-$ law-of-the-wall coordinate, $\frac{u}{u_*}$

$u_*$ wall friction velocity, $\sqrt{\frac{\tau_w}{\rho}}$

$\frac{\Delta u}{u_*}$ Log law-of-the-wall intercept shift due to roughness effects (Equation 2-8, 2-9)

$x$ streamwise distance measured from the leading edge of a flat plate

$y$ vertical distance from the surface of a flat plate

$y^-$ law-of-the-wall coordinate, $\frac{y u_*}{v}$
$Z_b$ spanwise aspect ratio, $\frac{k}{b}$ (p. 58)

$Z_r$ roughness element bluntness factor, $\frac{A_w}{A_F}$ (p. 58)

$\delta$ boundary-layer thickness at 0.99(U)

$\lambda_D$ Dvorak spacing parameter, $\frac{L}{S}$ (Figure 2.3a)

$\lambda_s$ Simpson spacing parameter, $\frac{A_s}{A_F}$ (Figure 2.4a)

$\lambda_{eff}$ Waigh spacing parameter (p. 58)

$\lambda_{eq}$ Simpson spacing parameter based on equivalent height (p. 56)

$\theta$ momentum-defect thickness of boundary layer

$v$ kinematic viscosity of the fluid

$\xi$ pressure-gradient parameter (Equation 2-5)

$\rho$ density of the fluid

$\tau_w$ wall shear stress
1.0 INTRODUCTION

In engineering practice, surfaces are often not smooth in an aerodynamical sense. Even if the given surface was indeed manufactured to be aerodynamically smooth, degeneration due to a number of factors (nature, usage, etc.) will often degrade the smoothness of the surface, with potentially large effects on the flow over said surface. Turbine blades, aircraft fuselage/wings, ship hulls, are all examples in which surface roughness can have a large effect on fluid dynamical performance. To address design and operational issues associated with surface roughness, it is clear that an accurate predictive model for behaviour of turbulent flow over rough surfaces is needed. The objective of this thesis is to contribute towards development of a model for aerodynamic effects of depression or ‘hole’ type roughness. The work was mainly experimental.

Extensive studies have been conducted by numerous authors to estimate and correlate effects of many different types of roughness. Early work by Nikuradse (1933) and others concentrated on the effects of a so-called ‘equivalent sand roughness’ on the well established logarithmic law-of-the-wall for smooth walls. Since that time, Prandtl (1934), Schlichting (1936), Hama (1955), Dvorak (1969) and a host of others have tried to refine and apply the approach outlined by Nikuradse to distributions of two and three dimensional roughness elements. The great majority of previous work has, however, dealt with protrusion type excrescences on the surface. These roughness types have ranged from regular arrays of transverse bars (two dimensional), and of hemispherical, conical, and other shapes (three dimensional), to randomly arranged stones. Little work has been done on flow development
and on determination of skin friction coefficients for depression type roughnesses.

This thesis presents experimental work on depression type roughness and discusses the results in relation to previous work on flow over rough surfaces.

Chapter two of this thesis presents a literature review focussed on the near-wall behaviour of flow over smooth and rough surfaces. It begins with a description of the logarithmic law-of-the-wall for turbulent flows over both smooth and rough walls (White 1991). Chapter two then presents a summary of current correlations between log-law parameters and roughness parameters and finally presents methods for determining the skin friction coefficient. Chapter three shows the apparatus and experimental procedures. Chapter four then presents the experimental work and results, followed by an examination and discussion of the experimental results. Chapter five outlines some computational work and compares computed results with those obtained experimentally. The concept of equivalent roughness height as it relates to depression type roughness will also be introduced and discussed in Chapter five. Chapter six presents conclusions and chapter seven shows recommendations for future work. All References, Figures and Appendices appear after Chapter seven.
2.0 LITERATURE REVIEW

2.1 Law-of-the-Wall for Smooth Walls

In engineering practice, most flows are turbulent. If the boundary layer is reasonably thick, roughness affects only near-wall flow in a direct sense. An important and well-established feature of near-wall turbulent flow over smooth and rough walls, is the logarithmic law-of-the-wall. This will now be discussed. The following presentation is based on White (1994) and White (1991).

It has been previously theorized by Prandtl (1926) and Von Kármán (1930), that the velocity profiles in turbulent boundary layer flow can be sub-divided into three distinct regions:

- Inner Layer: where velocity distribution is directly dependent on viscous shear imposed by the wall
- Outer Layer: where velocity is dependent on shear due to turbulent eddies
- Overlap Layer: where the inner and outer layers must smoothly coalesce

Prandtl (1926) argued that the inner layer behaviour is directly dependent only on local parameters and is not directly dependent on any free-stream properties; thus,

\[ u = f(\tau_w, \rho, \mu, y) \]  \hspace{1cm} (2-1)
CHAPTER TWO: LITERATURE REVIEW

Performing dimensional analysis on Eqn. 2-1 yields a dimensionless version of the inner law, Eqn. 2-2.

\[
\frac{u}{u_\tau} = F\left(\frac{yu_\tau}{v}\right); \quad \text{where} \quad u_\tau = \left(\frac{\tau_w}{\rho}\right)^{\frac{1}{2}} \tag{2-2}
\]

\(u_\tau\) is the wall friction velocity, so-called as it has units of velocity but it is not a velocity in the normal sense. It is convenient to express the parameters in the above equation in so-called law-of-the-wall coordinates. These coordinates are defined as follows, \(u^* = \frac{u}{u_\tau}\) and \(y^* = \frac{yu_\tau}{v}\).

Von Kármán (1930) argued that the velocity defect, \((U - u)\), in the outer layer is independent of viscosity but dependent on the free-stream parameters of velocity and pressure gradient and also on boundary layer thickness. The wall skin friction, \(\tau_w\), must also be included. Thus, Von Kármán assumed that:

\[
U - u = g(\tau_w, \rho, y, \delta, \frac{dP_e}{dx}) \tag{2-3}
\]

where \(g\) denotes a functional dependence.

Performing dimensional analysis on Eqn. 2-3 yields the relation:

\[
\frac{U - u}{u_\tau} = G\left(\frac{y}{\delta}, \xi\right) \tag{2-4}
\]
Expanding the functional dependance from Eqn. 2-4 yields Eqn. 2-4a shown below.

\[
\frac{U-u}{u_\tau} = A[\ln(\frac{y}{\delta})] + D(\frac{y}{\delta}, \xi) \tag{2-4a}
\]

Equation 2-4a is referred to as the velocity defect law and an application of it in the present work is discussed later. The value of ‘A’ is determined experimentally and ‘D’ denotes functional dependance. The value of ‘A’ used herein is 5.62.

The parameter ‘\xi’ represents the effect of the local pressure gradient and the wall shear stress. It is defined as

\[
\xi = \frac{\delta}{\tau_w} \frac{dP_e}{dx} \tag{2-5}
\]

‘\xi’ is zero in zero-pressure gradient flows.

It would seem reasonable that both the near-wall relationship and the defect law both apply in an interface or overlap layer. Millikan (1937) argued that both of the above equations can only apply simultaneously if a logarithmic relationship prevails in the overlap layer. This relation is shown below:

\[
\frac{u}{u_\tau} = A(\ln^+ \gamma) + B \tag{2-6}
\]

Equation 2-6 is known as the logarithmic law-of-the-wall, or simply as the log law.
CHAPTER TWO: LITERATURE REVIEW

The values of 'A' and 'B' are determined empirically. The values used were 5.62 and 5.0 respectively, according to Kind and Lawrysyn (1991). Note that the value of 'A' in Eqn. 2-6 is the same 'A' value as in Eqn. 2-4a. It is understood that Eqn. 2-6 is only applicable to flows with zero or moderate pressure gradients. The reason for this limitation is that in severe adverse pressure gradients, bordering on separation conditions, pressure gradient effects dominate skin friction effects, invalidating some of the assumptions underlying Eqn. 2-1 and 2-2. The outer layer can also be considered to behave similarly to a wake as discussed by Coles (1956). In the case of very favourable pressure gradients, the boundary layer can re-laminarize thereby negating any of the turbulent boundary layer theory outlined herein. The differences in pressure gradient effects can be seen in Fig. 2-1 (White, 1991).

2.2 Roughness Effects on the Law of the Wall

One would expect that the addition of roughness to a flow system would have substantial effects on the law of the wall. It has been found, notably by Clauser (1954) and Schlichting (1960), that addition of roughness elements results in a downward shift of the log law region but the log law is still followed. The slope of the logarithmic line does not vary, there is simply a downward shift in its intercept. This downward shift is a result of the increase of wall shear stress, and thus \( u_\tau \), caused by the roughness elements and is illustrated in Fig. 2-2 (White 1991).

It was proposed by Nikuradse (1933) and Prandtl and Schlichting (1934) among
CHAPTER TWO: LITERATURE REVIEW

others that a parameter can simply be added to the smooth-wall log law to represent the intercept shift due to roughness effects. The parameter is arbitrarily denoted as \( \frac{\Delta u}{u_w} \). They proposed that this parameter is a function of roughness Reynolds number, \( k' \) or \( \frac{k_u}{\nu} \), where ‘k’ is described as the mean roughness height. However Nikuradse (1933) and Prandtl and Schlichting (1934) noted that additional variables, such as other components of roughness element geometry, as well as element spacing and pattern of placement, must also have an effect. This leads to the preliminary statement that:

\[
\frac{\Delta u}{u_w} = f\left(\frac{k_u}{\nu}\right) + C(\lambda)
\]  

(2-7)

where \( C(\lambda) \) would account for the aforementioned roughness variables other than ‘k’.

The above relation would be a useful tool if the arbitrary function relating the roughness Reynolds number to the log-law shift could be determined. In his experiments, Nikuradse (1933) dealt with roughness on the inner surface of circular pipes. At the time, he used closely spaced sand grains as his roughness medium. In his work he used an effective height, \( k_r \), which is defined as “the size of mesh of the coarser of the two sieves through which sand was sifted” (Prandtl, 1960), which in effect corresponds to the maximum sand grain size. From his experiments, Prandtl and Schlichting (1934) found there existed three roughness regimes that varied directly with the roughness Reynolds's number.
CHAPTER TWO: LITERATURE REVIEW

The three regimes are:

\[
\frac{k_s u_\tau}{v} < 4
\]

hydraulically smooth; roughness does not protrude through the viscous sublayer and has no perceptible effect on the flow.

\[
4 < \frac{k_s u_\tau}{v} < 60
\]

transitional roughness regime; roughness extends somewhat outside the viscous sublayer. Roughness effect is due to both viscous effects and pressure or form drag on the roughness elements.

\[
\frac{k_s u_\tau}{v} > 60
\]

fully rough; roughness projects well outside the viscous sublayer which results in skin friction being dependent mainly on pressure or form drag on the roughness elements. Viscosity is unimportant.

It should be noted that the above limits are by no means universally applicable to all roughness types. These limits only hold for closely-spaced sand grain roughness referred to in the literature as ‘standard sand roughness’. For example, Chen and Robertson (1974) among others, found that for sparsely distributed hemispherical roughness elements the flow did not become fully rough until \( k^+ \) achieved a value of 200 or more.

Having discussed the idea of roughness regimes, attention must again be drawn to
Eqn. 2-7. Noting that Eqn. 2-7 must be consistent with the standard log-law equation, Eqn. 2-6, and that in the fully rough regime viscosity must disappear from the relation for $\frac{\Delta u}{u_\tau}$, the $\frac{\Delta u}{u_\tau}$ relation then takes the form, for fully rough flows,

$$\frac{\Delta u}{u_\tau} = A(\ln \frac{k u_\tau}{v}) + C(\lambda)$$ (2-8)

where 'A' is the 'A' constant of Eqn. 2-6, and $C(\lambda)$ is a parameter that depends on spacing of the roughness elements and perhaps other parameters of the roughness. When Eqn. 2-8 is introduced into Eqn. 2-6, to represent roughness effects, we obtain for fully rough flow:

$$\frac{\Delta u}{u_\tau} = A(\ln \frac{v}{k}) + C(\lambda)$$ (2-9)

Note that the kinematic viscosity, $v$, does not appear in eqn 2-9; this is consistent with expectations for fully rough conditions.

2.3 Roughness Parameter $C(\lambda)$, and Spacing Parameter $\lambda$

As mentioned, Prandtl and Schlichting (1934) deduced that the roughness parameter, $C(\lambda)$, varied with roughness element variables, namely element shape and spacing between roughness elements. Many authors then tried to formulate correlations between the roughness parameter and the other variables. The correlation devised by Dvorak (1969) is one that is still widely used. He used a spacing parameter, $\lambda$, as his device to link roughness
geometry to the log-law roughness parameter, 'C'. Dvorak defined his spacing parameter as:

\[ \lambda_D = \frac{L}{S} \]  

(2-10)

where 'L' is the streamwise distance between roughness elements and 'S' is the streamwise length of the roughness elements themselves. These parameters are graphically shown in Figure 2.3a. It is obvious that this spacing parameter is only suitable for two dimensional roughness configurations. Using this spacing parameter, Dvorak formulated a correlation between the roughness parameter \( C(\lambda) \), and his spacing parameter \( \lambda_D \). His results are broken down into two groups, for densely and sparsely packed groups of elements.

His critical value for \( \lambda_D \) separating the groups is 4.7.

For \( \lambda_D < 4.7 \) (dense packing):

\[ C(\lambda) = 28.19 \log_{10}\lambda_D - 17.35 \]  

(2-11a)

For \( \lambda_D > 4.7 \) (sparse packing):

\[ C(\lambda) = 6.56 \log_{10}\lambda_D + 5.95 \]  

(2-11b)

These two equations are shown graphically in Figure 2.3b along with various experimental data.

Later, Simpson (1973) developed a three dimensional spacing factor that was based on the frontal area of a given roughness element, \( A_F \), and on the planar surface area which the element occupies, \( A_S \). These two areas are shown graphically in Figure 2.4a. Simpsons'
spacing parameter is:

\[ \lambda_s = \frac{A_S}{A_F} \quad (2-12) \]

Simpson’s modification of the spacing parameter does take into account three dimensional roughness but does not alter the Dvorak correlations. The correlation between the roughness parameter and Simpson’s spacing parameter is shown in Figure 2.4b with various experimental data. As seen in Figure 2.4b, the data does not correspond directly to the curve fit and there is substantial scatter.

2.4 Skin Friction Coefficient and Skin Friction Determination Methods

It is well known that the skin friction coefficient, \( C_f \), is simply a non-dimensional form of the wall shear stress. It is defined as:

\[ C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} \quad (2-13) \]

As discussed previously, wall shear stress can be shown in another form, that is, in terms of the wall friction velocity, \( u_w \). That is:

\[ \frac{C_f}{2} = \frac{\tau_w}{\rho U^2} = \left( \frac{u_w}{U} \right)^2 \quad (2-14) \]
CHAPTER TWO: LITERATURE REVIEW

Clauser (1954) used this result in proposing a way to determine \( C_f \) using the log law-of-the-wall. Clauser (1954) replotted Eqn. 2-6 using non-dimensional velocity \( \frac{u}{U} \) as an ordinate rather than \( \frac{u}{u_\tau} \). He found that families of curves could be plotted with \( C_f \) as a variable. These curves are called Clauser plots. A typical Clauser plot is seen in Figure 2.5. Experimental data could then be plotted on a Clauser plot, and from the location of the experimental data relative to the Clauser plot curves, skin friction coefficient can be determined.

Clauser also noted that the defect law for zero pressure gradient conditions was identical for both smooth and rough surfaces.

Using Eqn. 2-14, on the defect law parameter, \( \frac{u-U}{u_\tau} \) becomes:

\[
\frac{u-U}{u_\tau} = (\frac{u}{U} - 1) \sqrt{\frac{C_f}{2}} \tag{2-15}
\]

From Eqn. 2-4a, assuming zero pressure gradient, the defect law can then be used as a tool to determine the skin friction coefficient if one plots the measured non-dimensional velocity data, \( \frac{u}{U} \), versus non-dimensional height, \( \frac{\nu}{\delta} \), for a given boundary layer. The boundary layer height is generally defined as the height at which the local velocity becomes 99% or 99.5% of freestream. The height at which the local velocity was 99% of freestream was used in this research but negligible changes in results occur if the 99.5% of freestream definition is used. The defect-law method was used in the present work to experimentally
determine $C_f$ from the experimental data; an example is shown in Figure 2.6.

Another method that can be used to determine $C_f$ is the momentum integral method derived by Von Kármán (1921). The equation is:

$$\frac{d\theta}{dx} + (2+H)\frac{\theta}{U} \frac{dU}{dx} = \frac{C_f}{2}$$  \hspace{1cm} (2-16a)

where:

$$\theta = \text{momentum thickness} = \int_0^\infty \frac{u}{U} \left(1-\frac{u}{U}\right) dy$$

$$\delta^* = \text{displacement thickness} = \int_0^\infty \left(1-\frac{u}{U}\right) dy$$

$$H = \text{boundary layer shape factor} = \frac{\delta^*}{\theta}$$

By the Bernoulli equation for inviscid flow, one could replace the velocity gradient term in eqn 2-16a by an equivalent pressure gradient term given by:

$$\frac{dU}{dx} = -\frac{1}{\rho U} \frac{dP_e}{dx}$$  \hspace{1cm} (2-16b)

Eqn. 2-16a then becomes:

$$\frac{d\theta}{dx} - (2+H)\frac{\theta}{\rho U^2} \frac{dP_e}{dx} = \frac{C_f}{2}$$  \hspace{1cm} (2-16c)

Using the theory outlined in this Chapter, experimental skin friction coefficients will be found and compared.
3.0 DISCUSSION OF EXPERIMENT

This chapter presents an overview of the apparatus and procedures employed to obtain the experimental data. Calibration techniques will also be outlined. The basic procedure of this experimental work was to traverse vertically, at a given probing station, through a turbulent boundary layer in the flow over a flat rough plate. Velocity profile data was taken and then reduced to obtain the desired results, such as skin friction coefficient.

3.1 Experimental Apparatus

3.1.1 Wind Tunnel and Test Section

The experiments were performed in a low speed, closed circuit wind tunnel at Carleton University. The wind tunnel has a contraction ratio of 12:1 and a test section 20" x 30" x 72" long (0.51m x 0.76m x 1.83m). The tunnel is powered by a 45 kW induction motor and is capable of speeds up to approximately 75 m/s (170 mph). The fan speed and thus the wind speed are controlled by a magnetic clutch located between the motor and the wind tunnel fan. The testing was performed at a wind speed of approximately 45-50 m/s which translates to a Reynold's number of 2.8-3.0 x 10^6 based on a unit length of 1m. The dynamic pressure in the test section was determined by a calibration relation for this wind tunnel.
The relation is (Nituch, 1972):

\[
\frac{P_{c_1} - P_{c_2}}{\frac{1}{2} \rho V_{\infty}^2} = 0.91
\]  

(3-1)

where \( P_{c_1} \) is the static pressure recorded in the tunnel settling chamber and \( P_{c_2} \) is the static pressure measured just upstream of the test section.

The test section has a Plexiglas side wall for viewing of the experiment. Holes were drilled along the centerline of the test section floor at 5 different streamwise locations to enable a probe to traverse the boundary layer formed on the test section floor. These locations are measured from the test section leading edge and distances are as follows: 0.363 m, 0.713 m, 1.063 m, 1.413 m, 1.763 m. These probing stations will be designated stations 1 thru 5. At the first and third stations, 2 additional span-wise holes located at ± 0.1651 m from the centerline were added to enable checks of spanwise uniformity of the flow. At station 5, four spanwise holes were added at ± 0.1651 m and ± 0.254 m from the centerline. Figure 3.1 shows a schematic of the test section floor and the probing stations. Corresponding to each of the five centerline stations, 0.065" I.D. static pressure taps were installed in the test section ceiling.
3.1.2 Test Plates

Three experimental test plates were machined by Carleton University Science Technology Centre (STC). These plates are composed of 1/4" thick PVC plastic with dimensions 30" x 72". Each of the three test plates had probing station holes drilled at the locations corresponding to the test section probing stations. In addition, various countersunk holes were drilled along the periphery of the plates and internally to allow the plates to be fixed to the test section floor with No. 8 woodscrews. Once the plate was secured, these holes were filled with plasticine to eliminate any disturbances to the flow. Each plate also had its leading edge filed down to a bullnose rounding so that the flow would not encounter a square leading edge. Once the test section was suitably positioned in the wind tunnel circuit, smooth duct tape was applied at the joint between the test section and the contraction exit to ensure that no suction or blowing at the test section/contraction joint occurred. To create the depression roughness, circular cavities were machined into the test plates. 1/4" end mill cavities, 1/8" deep, were machined into two of the test plates. 1/4" ball mill cavities, also 1/8" deep, were machined into the third. Side profiles of these cavities or depressions are shown in Figures 3.2a and 3.2b. The end mill depressions were cylindrical pits while the ball mill created hemispherical pits. Adjacent rows were staggered, even though no conclusive effect of staggering had been previously found (Waigh, 1996). Each individual pit or depression will be considered to be a roughness element. A general roughness element and surrounding area are shown in Figure 3.3. The spacing between roughness elements was chosen to obtain
certain values of Simpson’s (1973) three dimensional spacing parameter, \( \lambda = \frac{A_S}{A_{XF}} \). However as depressions have no definable positive frontal area, a spacing parameter \( \frac{A_S}{A_{XS}} \) is used as the spacing variable. For this purpose, \( A_{XS} \) was taken to be the depression cross section area perpendicular to the flow, that is: \((1/4 \text{ in}) \times (1/8 \text{ in}) = (1/32 \text{ in}^2)\) for the cylindrical depressions and \((0.5\pi) \times (1/8 \text{ in})^2 = (\pi/128 \text{ in}^2)\) for the hemispherical depressions. One plate, having cylindrical depressions was given a spacing parameter of \( \frac{A_S}{A_{XS}} = 10 \). This value led to a distance between adjacent rows and columns of roughness elements of 0.559" for the cylindrical cavities. To minimize machine setup time, a distance of 0.559" was also used for the plate with the hemispherical depressions. This distance corresponds to a spacing parameter of \( \frac{A_S}{A_{XS}} = 12.7 \), based on the hemispherical cross-sectional area. The second plate containing cylindrical depressions was machined such that a spacing parameter of \( \frac{A_S}{A_{XS}} = 100 \) was obtained. This value corresponds to a distance of 1.768" between adjacent rows and columns of roughness elements. The spacing and staggering of depressions in the three plates are shown in Figures 3.4a and 3.4b.

In addition, two flat aluminum plates covered with two different types of vinyl flooring were studied. These plates also had dimensions 72" x 30" x 1/8". The vinyl floor coverings were cut to fit the base aluminum plates and were attached using standard vinyl flooring adhesive. The vinyl surfaces had small dimples throughout which occurred randomly over the surface. Figure 3.4c and 3.4d show digital photographs of the first and second vinyl plates respectively. These plates were tested to generate results for flow over plates with random
CHAPTER THREE: DISCUSSION OF EXPERIMENT

depression roughness rather than the regular roughness patterns on the three test plates mentioned earlier. To determine the mean depth of the cavities, a straight edge was laid on the vinyl surface and estimates were made of the depth. These estimates were then averaged. Also a roughness density was calculated by counting the number of dimples in a 6" x 6" planar area. The first vinyl plate had average depression depth of 0.005" with a roughness density of 20 depressions/in², while the second vinyl plate tested had an average depression depth of 0.004" with a roughness density of 6 depressions/in². The dimples measured for both plates had varying irregular shapes. An averaged estimated shallow rounded cross-section was measured for each plate. The two vinyl plates had average dimple rounded cross-sectional areas of 0.1" and 0.15" respectively. The flows over these two plates were probed at stations 1 thru 5 as mentioned above. The vinyl plates were mounted individually into the test section using woodscrews and the joint between the leading edge of the plates and the wind tunnel contraction were taped as for the machined PVC test plates.

A sixth, aerodynamically smooth, plate was also tested for validating the measurement and data reduction techniques. The smooth plate was made of aluminum and its dimensions were 72" x 30" x 1/8". This plate was fixed to the test section by No. 8 wood screws which were countersunk into the plate. These countersunk depressions were covered with plasticine when the plate was suitably positioned. The joint at the leading edge of the plate and the wind tunnel contraction was covered with duct tape for the reasons outlined above. The smooth plate was tested at three probing stations, numbers 1, 3 and 5.
3.1.3 Traverse Gear and Pitot Probe Assembly

A stepper motor driven traverse gear/probe assembly was used to traverse a Pitot tube vertically through the boundary layer on the test plate. The assembly was fastened to the underside of the test section so that the traverses would be made from the test plate surface to free stream conditions. As mentioned earlier, the probe stem passed through the test section floor and test plate via the holes on the the test section centerline and spanwise probing stations.

The Pitot mouth extension was made using stainless steel hypodermic tubing, 0.04" O.D. x 0.02" I.D.. As it is imperative to achieve measurements as close to the wall as possible, the Pitot probe mouth was flattened, sanded and polished making a flat ovoid shape. Figure 3.5 shows the probe dimensions after flattening. The Pitot mouth height measured an outside height of 0.020", an inside height of 0.011" and width of 0.05" after flattening. A digital photograph showing the Pitot probe is shown is Figure 3.6.

To ensure consistency of results, the Pitot probe mouth was placed in the same position relative to the 4 surrounding depressions at each station for each plate. It was positioned so that the probe mouth would touch in the centre of the planar diamond shape created by the 4 surrounding roughness depressions. This positioning is shown in Figure 3.7. To allow for this placement, a mounting block was constructed for the stepper motor unit
such that when loosely fixed to the underside of the test section, the probe mouth position could be adjusted to within 1/16" of the position shown in Figure 3.7 for all stations. Figure 3.8 shows this mounting block. Two rectangular strips of aluminum acted as washers to secure the motor assembly to the test section underside. As seen in Figure 3.8, this block allows for a travel of ± 0.625" in any given direction and the entire assembly was tightened down to the test section when the suitable placement was achieved.

The stepper motor consists of an eight-wire Eastern Air Devices motor rated at 5.1V, 1.0A. Attached to the stepper motor was a lead screw which allowed a ratio of 8000 steps/inch. Calibration on the traversing distance was done using a height gauge. Position changes were accurate to within 0.02mm. A schematic of the transverse gear assembly is shown in Figure 3.9. The traverse gear was computer controlled by a controller/interpreter unit (see section 3.1.4)
3.1.4 Data Acquisition System and Interpretation

3.1.4.1 Real Time Flow Analysis (RTFA)

The experimental data was obtained at five different streamwise measuring stations for each plate. This data was captured using a computer program called Real Time Flow Analysis (RTFA). The code was written and modified in Turbopascal format by various authors, the last being in 1997 by Pajayakrit (1997). At the time of the present tests, simple modifications were made to allow for an increased number of vertical steps. This program was fully automatic; it controlled probe location and recorded all data. It also allowed for variations in sampling time to be made, to be discussed in section (3.2.2). The code simultaneously recorded, at each step, dynamic pressure at the probe position along with a corresponding free stream reference dynamic pressure. The free stream reference dynamic pressure was determined from the contraction pressure difference $P_{c1} - P_{c2}$, as mentioned earlier. The code was programmed such that the data acquisition system's analog differential voltage setting was used. Zero offsets were measured before and after every run at zero wind velocity and were used in the data reduction phase (see section 3.4). A sample output of the RTFA software is shown in Appendix A.
3.1.4.2 Pressure Transducers

At each step, during each test run, pressure transducers measured the Pitot pressures within and outside of the boundary layer. Two Setra pressure transducers (Model 239) were used. One was used for boundary layer measurements, which measured the differential pressure of the Pitot probe and static pressure corresponding to the probing station. The other measured the reference pressure difference, \((P_{c1} - P_{c2})\). Both transducers have a nominal output of \(\pm 2.5\) VDC for an applied pressure of \(\pm 0.5\) psid. The zero pressure and full scale outputs are \(0.0\) VDC \(\pm 0.4\%\) FS and \(5.0\) VDC \(\pm 0.4\%\) FS, respectively, with a quoted non-linearity of \(\pm 0.1\%\) FS (using the Best Straight Line method). These transducers were attached to the Pitot probe assembly and static taps using flexible Tygon tubing of various size. They were re-calibrated prior to every day’s set of test runs. The calibration techniques are discussed in section (3.2.1).

3.1.4.3 Data Acquisition System

The data acquisition system used was the Analog/Digital converter, Scientific Instruments Model 81 Electronic Measurement system. This system has the capability of eight analog, 4 digital and 1 counter channels. Of the eight analog channels three were used. Two for the differential voltage measurements of the transducers while a third was used for auto-zeroing. The system required 12 VDC power input and was housed in a separate
cabinet. It was connected to the computer by an RS232C serial interface. This interface option is accessed through a standard 25-pin D-style connector. The system has 4 VDC input range capabilities. The experimental data for the boundary layer transducer ranged from 0.3 - 0.9V while the reference dynamic pressure transducer voltage remained at approximately 0.9V for all trials. This input range dictated the use of the data acquisition systems Range 0, which is ± 5.000V. At this setting, the weight of the least-significant A/D bit is 1.22 mV.

The Analog/Digital converter uses a 12 bit-plus-sign dual slope integrator to perform all analog measurements. The 12 bit convertor leads to a digital answer between 0 and 4095. As the convertor is of the integrating type, it integrates a measured voltage value over a period of time. The standard factory condition uses a 3.5795 MHz crystal to clock the A/D, and results in an integration period of 1/30th of a second. This period permits a conversion speed of approximately 10 conversions/second (7.5 conversions/second minimum). The sampling rate of the system was limited by the controlling computer to approximately 5-6 samples/second.

A schematic diagram of the experimental setup is shown in Figure 3.10.
3.1.5 Miscellaneous Apparatus

For data reduction purposes, a barometer was used to measure the ambient atmospheric pressure prior to each run. Also, a dial thermometer embedded in the wind tunnel was used to record the initial and final air temperatures for each run to account for air density variations.

3.2 Calibration Techniques

3.2.1 Pressure Transducers

The two Setra pressure transducers described in section 3.1.3.1 were calibrated regularly, and in a wide variety of ambient conditions to ensure accurate results. The transducers were connected in parallel, by Tygon tubing, with a U-tube manometer and an air squeeze bottle. For calibration, transducer output voltage readings were taken using a Digital Multimeter. The air squeeze bottle was used to produce pressures for calibration purposes. Use of human breath for this purpose is to be avoided as the transducers are sensitive to humid air. The voltage readings and fluid level of the manometer were recorded simultaneously. A computed linear trend curve was fit through the calibration data points. Typical results for both transducers are shown graphically in Figures 3.11a and 3.11b. It
should be mentioned that these results are in accordance with previous work (Chinoy 1998, Lawrysyn 1989) using the same transducers. Due to the crude scaling on the U-tube manometer, the accuracy of this technique is limited to ± 0.25mm H₂O (3.48 x 10⁻⁴ psi). As mentioned previously, tests were done with varying ambient conditions. Current transducer calibration data were always used in the reduction of experimental data (see section 3.4).

3.2.2 Determination of RTFA Sampling Time

As mentioned, the sampling rate of the data acquisition system, limited by the computer’s performance, was approximately 5-6 samples/second. The RTFA software allowed for varying the sampling time. An optimum sampling time was chosen based on trials conducted in three preliminary tests. Samples were taken for 20, 25 and 100 seconds and running averages were plotted in each case to observe the sampling trends. These plots are shown in Figures 3.12a, 3.12b and 3.12c, respectively. From these plots a sampling time of 30 seconds was chosen. A feature of the data acquisition code RTFA is that it provides the user with real time access to samples taken. Over a given sampling time, all the samples taken in that time can be viewed and accepted or rejected based on the conformity of the average of values at a given time with the cumulative average. At each traversing step, an arbitrary waiting time of 10 seconds was used to allow the system to reach steady state before initiating the sampling process.
3.2.3 Ceiling Static Pressure Distribution

Five evenly spaced static pressure taps, along the centerline of the test section ceiling at streamwise positions corresponding to each probing station, were used to determine static pressure distribution along the test section. The static pressures at each tap and simultaneous readings of the contraction pressures, $P_{c1}$ and $P_{c2}$ were taken using a manometer board which was referenced to the atmosphere. The recorded static pressures were then nondimensionlized with respect the dynamic tunnel pressure as per eqn. 3-1. The normalizing equation is shown below:

$$C_p = \frac{P_{\text{static}} - P_{c2}}{(P_{c1} - P_{c2})(\frac{1}{0.91})}$$  \hspace{1cm} (3-2)

The results of the static pressure distribution measurements are shown in Figure 3.13. A simple second order curve fit was generated to produce an expression for the streamwise pressure distribution. The expression matches the acquired data very well. As will be seen later, the very mild pressure gradient shown has minimal effects on the results obtained for $C_p$. It should be noted that this procedure was only used for obtaining the static pressure gradient along the test section flow. For boundary layer traverses, the ceiling static tap at the streamwise station of the traverse was connected directly to one side of the pressure transducer used for the Pitot measurements, as mentioned earlier.
3.2.4 Spanwise Uniformity of the Flow

As the experiments conducted were over flat plates with a uniform roughness pattern, and the data reduction was based on the assumption that the flow is two dimensional, it was necessary to confirm that the flow was indeed two dimensional. As discussed earlier, all plates were checked for spanwise uniformity at the first, third and fifth measuring stations. Figures 3.14a, 3.14b and 3.14c indicate the spanwise uniformity for the three machined plates at the third measuring station. The results show adequate spanwise uniformity and fully supports the assumption of two dimensionality of the flow. The other spanwise probing stations of the machined plates indicate a similar level of two dimensionality. Spanwise uniformity checks were not done on the vinyl flooring test plates as the roughness patterns on these plates was random and not very pronounced.

3.3 Boundary Layer Measurements

The boundary layer measurements were performed at five equal streamwise distance intervals along the plate. During any given trial the other probing station holes were covered using standard thin, transparent tape to ensure that no suction or blowing occurred at these sites. The data acquisition software allows for equidistant vertical steps through the boundary layer to be taken. Typically measurements were made at 70 - 90 points. The final 10 - 15 measurements were normally taken outside the boundary layer and those results were used
to determine freestream conditions. The use of these values will be discussed later.

As it is impractical to manually place the Pitot probe mouth as close to the wall as
visually possible and subsequently retrieve data from this point, it was proposed that the probe
be started with its mouth touching the plate. The probe was lowered until its mouth was
touching the surface and a flexure of its body noticed. This procedure allows the user to see
immediately when the mouth has lifted off the surface as this will correspond with a noticeable
pressure rise; the step at which the probe lifted off the surface was taken as the first step in
the data reduction phase.

3.4 Data Reduction

The data processing software used to reduce the experimental data was entitled
DATARED.FOR. This is a FORTRAN based program written by Lawrysyn (1989). It was
slightly modified to output required data and to take the first step as the step at which the
probe sensed the initial pressure rise, as mentioned earlier. The code was verified using a
1/7th power law velocity profile. The full code for DATARED.FOR is shown in APPENDIX
A. This software determined the boundary layer velocity, $u$, and freestream velocity, $U_\infty$, at
each step. Also, averaged dimensionless freestream velocity, $\frac{U}{U_\infty}$, was found at each step
using the aforementioned 10 - 20 measurements taken outside the boundary layer. The
Reynolds number, \( \frac{\nu u}{v} \), based on vertical distance from the wall, \( y \), and boundary layer velocity, \( u \), was also calculated. The momentum and displacement thicknesses were also found using the trapezoidal rule integration technique. A simple average of initial and final wind tunnel temperatures was used to determine average air density during a test run and initial and final transducer zero voltages recorded accounted for any transducer shift. A second order polynomial was fit to the momentum thickness distribution data (\( \Theta \) versus \( x \)) at the five measuring stations. This equation was differentiated and using the polynomial fit of the static pressure distribution discussed in section 3.2.3, the skin friction coefficients at each probing station were determined using the momentum integral equation, eqn. 2-16c. In addition, the DATARED.FOR output was imported into MS Excel and the skin friction coefficient was also determined graphically by the defect-law procedure outlined in section 2.4.

As the data reduction software required a set input format and the RTFA output data did not conform to this, a simple conversion algorithm was written to change the RTFA output into a DATARED.FOR input. This algorithm, CONVERT.FOR, is shown in APPENDIX A, as well as the converted results.
4.0 EXPERIMENTAL RESULTS AND DISCUSSION

This chapter presents the experimental results of the wind tunnel testing performed. Results for six plates will be shown, three machined plates which have patterned depression roughness, two plates covered with vinyl flooring, simulating random depression roughness, and an aerodynamically smooth plate, as discussed in section 3.1.2. In presenting each plate, experimental values for skin friction coefficient, momentum thickness, displacement thickness and shape factor will be shown for each probing station. Defect law plots which yielded the experimental skin friction coefficients will be shown. Also, skin friction coefficients obtained using Von Kármán's momentum integral equation will be presented for an assumed zero pressure gradient and with the experimental mild pressure gradient term added. For the latter approach, plots of streamwise variation of momentum thicknesses will be presented. This will be followed by a display of log law-of-the-wall intercept shifts, $\frac{\Delta u}{u_z}$, at each probing station, based on the defect-law skin friction coefficient. All results are tabulated. Finally, the chapter concludes with a discussion regarding all observed experimental results.
4.1 Results for Test Plates

4.1.1 Smooth Plate

An aerodynamically smooth plate was tested to verify that all aspects of the data retrieval and data reduction were valid. Only three probing stations were used as it was argued that three stations would produce adequate data to monitor important trends. Figures 4.1.1a thru 4.1.1c show the defect law plots for each of the three probing stations used for the smooth plate. Figure 4.1.1d represents the smooth wall Clauser plot at probing station 3. Table 4.1 shows the skin friction coefficients obtained using the defect law approach, the Clauser plot approach as well as displacement thicknesses, momentum thicknesses and shape factors obtained at each station.

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>$\delta^*$ (mm)</th>
<th>$\theta$ (mm)</th>
<th>$H$</th>
<th>$C_r$ (defect-law)</th>
<th>$C_r$ (Clauser Plot)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>3.287</td>
<td>2.418</td>
<td>1.359</td>
<td>0.00280</td>
<td>0.00280</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>3.552</td>
<td>2.558</td>
<td>1.389</td>
<td>0.00275</td>
<td>0.00275</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>3.87</td>
<td>2.907</td>
<td>1.331</td>
<td>0.00270</td>
<td>0.00265</td>
</tr>
</tbody>
</table>

The average shape factor of 1.36 is consistent with accepted turbulent boundary layer behaviour with zero pressure gradient.
Figure 4.1.1e shows the streamwise variation of momentum thickness. The second order curve fit is also shown. The results for skin friction coefficient deduced using the momentum integral equation are shown in Table 4.2. Note that these are excellent agreement with the $C_r$ values obtained using the defect-law method, Table 4.1. This result is typical of all plates tested.

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>$C_r$ (press. gradient neglected)</th>
<th>$C_r$ (with press. gradient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.313</td>
<td>0.00282</td>
<td>0.00281</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>0.00279</td>
<td>0.00275</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>0.00266</td>
<td>0.00269</td>
</tr>
</tbody>
</table>

As seen from Table 4.2, the test section pressure gradient had less than a 1.5% effect on the results, thereby justifying the use of the zero pressure gradient form of the velocity defect law for determining $C_r$. This result is also consistent for all plates tested, and will be discussed in greater detail later.

Figures 4.1.1f - 4.1.1h show the log law-of-the-wall plot for this smooth plate. As expected, no noticeable shift can be distinguished from the smooth wall log law line. In all the experimental cases, it is judged that the skin friction coefficients determined from the
CHAPTER FOUR: EXPERIMENTAL RESULTS AND DISCUSSION

defect law could not be resolved to an accuracy any better than ± 0.00005. This uncertainty results in a log law intercept shift of approximately ± 0.1.

4.1.2 Machined Plate 1: \( \frac{A_s}{A_{xc}} = 10 \), Cylindrical Depressions

The first test plate analysed was the plate which featured the cylindrical depressions, spaced such that \( \frac{A_s}{A_{xc}} = 10 \). From section 3.1.2, \( A_s \) is defined as the planar surface the roughness element occupies and \( A_{xc} \) is the depression cross-section perpendicular to the flow. Tests were performed at the five probing stations and the resulting defect law plots are shown in Figure 4.1.2a thru 4.1.2e. The reduced results for machined plate 1 are shown below in Table 4.3

<table>
<thead>
<tr>
<th>Station</th>
<th>( x ) (m)</th>
<th>( \delta^* ) (mm)</th>
<th>( \Theta ) (mm)</th>
<th>( H )</th>
<th>( C_r ) (defect-law)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>2.003</td>
<td>1.393</td>
<td>1.438</td>
<td>0.00450</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>3.108</td>
<td>2.188</td>
<td>1.420</td>
<td>0.00425</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>3.703</td>
<td>2.646</td>
<td>1.399</td>
<td>0.00415</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>4.811</td>
<td>3.456</td>
<td>1.392</td>
<td>0.00400</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>5.545</td>
<td>3.983</td>
<td>1.392</td>
<td>0.00390</td>
</tr>
</tbody>
</table>

The experimental momentum thickness curve fit for this plate is shown in Figure 4.1.2f. Using this result with the momentum integral equation yields the results in Table 4.4
Table 4.4: Skin Friction Coefficients Deduced using Momentum Integral Method  
(Machined Plate 1)

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>$C_r$ (press. gradient neglected)</th>
<th>$C_r$ (with press. gradient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>0.00445</td>
<td>0.00445</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>0.00431</td>
<td>0.00430</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>0.00416</td>
<td>0.00416</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>0.00402</td>
<td>0.00401</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>0.00387</td>
<td>0.00386</td>
</tr>
</tbody>
</table>

From these results, it is observed that the momentum integral equation yields results virtually identical to those of the defect-law approach. It is noted however that the variations in the values at stations 1 and 2 may be a result of the boundary layer having not yet settled from the entry into the test section. At stations 3 thru 5, the results are somewhat more consistent, within the defect-law $C_r$ resolution range of $\pm 0.00005$.

Table 4.5 shows the values of the log law intercept shift for this plate. These values are taken from Figures 4.1.2g thru 4.1.2k.
Table 4.5: Log Law Intercept Shifts for Machined Plate 1

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>Log Law Intercept shift, $\frac{\Delta u}{u_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.313</td>
<td>-5.3</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>-5.4</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>-5.5</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>-5.6</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>-5.8</td>
</tr>
</tbody>
</table>

4.1.3 Machined Plate 2: $\frac{A_s}{A_{xc}} = 12.7$, Hemispherical Depressions

Figure 4.1.3a thru 4.1.3e represent the defect-law plot for the five probing stations of this test plate. A summary of the reduced experimental data is listed in Table 4.6.

Table 4.6: Experimental Data for Machined Plate 2

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>$\delta^*$ (mm)</th>
<th>$\theta$ (mm)</th>
<th>H</th>
<th>$C_r$ (defect-law)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>2.182</td>
<td>1.488</td>
<td>1.467</td>
<td>0.00505</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>3.543</td>
<td>2.419</td>
<td>1.466</td>
<td>0.00480</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>4.511</td>
<td>3.111</td>
<td>1.450</td>
<td>0.00460</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>5.749</td>
<td>4.010</td>
<td>1.434</td>
<td>0.00430</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>6.858</td>
<td>4.703</td>
<td>1.458</td>
<td>0.00415</td>
</tr>
</tbody>
</table>
The calculated momentum thickness distribution plot for machined plate 2 is shown in Figure 4.1.3f, and the momentum integral equation skin friction coefficients are seen in Table 4.7.

Table 4.7: Skin Friction Coefficients Deduced using Momentum Integral Method (Machined Plate 2)

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>$C_f$ (press. gradient neglected)</th>
<th>$C_f$ (with press. gradient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>0.00501</td>
<td>0.00501</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>0.00480</td>
<td>0.00479</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>0.00459</td>
<td>0.00458</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>0.00438</td>
<td>0.00437</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>0.00417</td>
<td>0.00416</td>
</tr>
</tbody>
</table>

The log law intercept shifts for machined plate 2 can be seen in Table 4.8. The corresponding log law-of-the-wall plots are shown in Figures 4.1.3g - 4.1.3k.
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Table 4.8: Log Law Intercept Shifts for Machined Plate 2

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>Log Law Intercept shift, $\frac{\Delta u}{u_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.313</td>
<td>-6.5</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>-6.9</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>-7.0</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>-6.9</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>-6.9</td>
</tr>
</tbody>
</table>

4.1.4 Machined Plate 3: $\frac{A_s}{A_{xc}} = 100$, Cylindrical Depressions

The third plate tested consisted of cylindrical depressions with spacing between elements such that the Simpsons spacing parameter, $\frac{A_s}{A_{xc}} = 100$. The experimental results obtained are shown in Table 4.9. The defect-law plots corresponding to these values are seen in Figures 4.1.4a - 4.1.4e.
Table 4.9: Experimental Data for Machined Plate 3

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>δ' (mm)</th>
<th>θ (mm)</th>
<th>H</th>
<th>C_r (defect-law)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>1.845</td>
<td>1.417</td>
<td>1.335</td>
<td>0.00315</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>2.613</td>
<td>1.940</td>
<td>1.346</td>
<td>0.00300</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>3.493</td>
<td>2.477</td>
<td>1.405</td>
<td>0.00290</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>4.082</td>
<td>2.988</td>
<td>1.358</td>
<td>0.00280</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>4.799</td>
<td>3.475</td>
<td>1.376</td>
<td>0.00270</td>
</tr>
</tbody>
</table>

The streamwise momentum plot for machined plate 3 is shown in Figure 4.1.4f and the results obtained from the momentum integral equation are found in Table 4.10.

Table 4.10: Skin Friction Coefficients Deduced using Momentum Integral Method (Machined Plate 3)

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>C_r (press. gradient neglected)</th>
<th>C_r (with press. gradient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>0.00311</td>
<td>0.00311</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>0.00301</td>
<td>0.00301</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>0.00292</td>
<td>0.00291</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>0.00282</td>
<td>0.00281</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>0.00272</td>
<td>0.00271</td>
</tr>
</tbody>
</table>

The log law-of-the-wall intercept shifts corresponding to machined plate 3 are shown in Table 4.11. The plots showing the intercepts for this plate are given in Figures 4.1.4g - 4.1.4k.
Table 4.11: Log Law Intercept Shifts for Machined Plate 3

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>$\frac{\Delta u}{u_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.313</td>
<td>-0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>-0.4</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>-0.3</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>-0.4</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

The seen log law shifts are slight which implies that it is perhaps inappropriate to assume the plate to be fully rough.

4.2 Results for Vinyl Covered Test Plates

As mentioned, two test plates covered with vinyl floor covering were experimentally tested to obtain data for flow over flat plates with random depression roughness. Values of mean depression depth and depression density were estimated for each plate (see section 3.1.2). The first plate tested, denoted as vinyl plate 1, had a mean depression depth of 0.005" and a roughness density of 20 depressions/in², while the second plate tested, vinyl plate 2, had a mean depression depth of 0.004" and a roughness density of 6 depressions/in². The two plates had average rounded cross-sections of 0.1" and 0.15" respectively.
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The results for both vinyl plates tested are very near the results obtained from the smooth plate tested (see section 4.1.1). As a result defect-law plots and log-law intercept shift plots at stations 1, 3 and 5 only will be shown for each of the two vinyl plates. Defect-law plots and log-law intercept shift plots at other stations for these two plates show the same behaviour. Experimental results at all probing stations will be given.

4.2.1 Vinyl Plate 1: Vinyl Test Plate, Average Depression Height = 0.005", Estimated Roughness Density = 20 depressions/inch²

Experimental results for vinyl plate 1 are shown in Table 4.12. The defect-law plot at probing stations 1, 3 and 5 are shown in Figures 4.2.1a thru 4.2.1c.

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>δ* (mm)</th>
<th>θ (mm)</th>
<th>H</th>
<th>C_r (defect-law)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>1.786</td>
<td>1.313</td>
<td>1.361</td>
<td>0.00325</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>2.649</td>
<td>1.947</td>
<td>1.361</td>
<td>0.00305</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>3.083</td>
<td>2.280</td>
<td>1.352</td>
<td>0.00300</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>3.931</td>
<td>2.964</td>
<td>1.330</td>
<td>0.00290</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>4.543</td>
<td>3.331</td>
<td>1.364</td>
<td>0.00280</td>
</tr>
</tbody>
</table>

The streamwise momentum thickness distribution is shown in Figure 4.2.1d. Skin friction coefficients calculated using the momentum integral method are seen in Table 4.13.
Table 4.13: Skin Friction Coefficients Deduced using Momentum Integral Method
(Vinyl Plate 1)

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>( C_r ) (press. gradient neglected)</th>
<th>( C_r ) (with press. gradient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>0.00329</td>
<td>0.00328</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>0.00316</td>
<td>0.00316</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>0.00303</td>
<td>0.00303</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>0.00291</td>
<td>0.00290</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>0.00278</td>
<td>0.00278</td>
</tr>
</tbody>
</table>

Table 4.14 shows the log-law intercept shift for vinyl plate 1. The shifts seen are slight, comparable to those obtained for machined plate 3 (see section 4.1.4). A plot representing the log-law intercept shift at probing stations 1, 3 and 5 can be found in Figures 4.2.1e - 4.2.1g.

Table 4.14: Log Law Intercept Shifts for Vinyl Plate 1

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>Log Law Intercept shift, ( \frac{\Delta u}{u_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.313</td>
<td>~ 0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>~ 0.4</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>~ 0.6</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>~ 0.5</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>~ 0.3</td>
</tr>
</tbody>
</table>
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Based on the shifts seen in Table 4.13, it can be argued that perhaps the flow over vinyl plate 1 is not fully rough.

4.2.2 Vinyl Plate 2: Vinyl Test Plate, Average Depression Height = 0.004", Estimated Roughness Density = 6 depressions/inch²

Summarized experimentally determined results for vinyl plate 2 are seen in Table 4.15. A defect-law plot corresponding to the data obtained at probing stations 1, 3 and 5 are shown in Figures 4.2.2a - 4.2.2c.

Table 4.15: Experimental Data for Vinyl Plate 2

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>δ* (mm)</th>
<th>θ (mm)</th>
<th>H</th>
<th>(C_r) (defect-law)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>2.103</td>
<td>1.532</td>
<td>1.370</td>
<td>0.00305</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>2.826</td>
<td>2.069</td>
<td>1.366</td>
<td>0.00295</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>3.586</td>
<td>2.676</td>
<td>1.340</td>
<td>0.00280</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>4.156</td>
<td>3.122</td>
<td>1.331</td>
<td>0.00270</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>4.994</td>
<td>3.802</td>
<td>1.313</td>
<td>0.00265</td>
</tr>
</tbody>
</table>

The skin friction coefficient results calculated using the momentum integral method for vinyl plate 2 are seen in Table 4.16. The streamwise momentum thickness variation is seen in Figure 4.2.2d.
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Table 4.16: Skin Friction Coefficients Deduced using Momentum Integral Method
(Vinyl Plate 2)

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>$C_f$ (press. gradient neglected)</th>
<th>$C_f$ (with press. gradient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.318</td>
<td>0.00301</td>
<td>0.00301</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>0.00291</td>
<td>0.00291</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>0.00282</td>
<td>0.00281</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>0.00272</td>
<td>0.00271</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>0.00262</td>
<td>0.00261</td>
</tr>
</tbody>
</table>

Similar to plates 3 and 4, the log-law intercept shifts for vinyl plate 2 are small, and comparable to the smooth plate tested (see section 4.1.1). The log-law intercept shifts at the five probing stations of vinyl plate 2 are shown in Table 4.17. The log-law intercept plot at stations 1, 3 and 5 are seen in Figure 4.2.2e thru Figure 4.2.2g.

Table 4.17: Log Law Intercept Shifts for Vinyl Plate 2

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>Log Law Intercept shift, $\frac{\Delta u}{u_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.313</td>
<td>-0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.668</td>
<td>-0.6</td>
</tr>
<tr>
<td>3</td>
<td>1.018</td>
<td>-0.3</td>
</tr>
<tr>
<td>4</td>
<td>1.368</td>
<td>-0.1</td>
</tr>
<tr>
<td>5</td>
<td>1.718</td>
<td>-0.2</td>
</tr>
</tbody>
</table>
4.3 Discussion of Results

The results seen for all plates are consistent with known boundary layer behaviour. Seen in Table 4.1 for the smooth plate and Tables 4.3, 4.6, 4.9 for the machined plates, and Tables 4.12 and 4.15 for the vinyl covered plates, the experimental shape factors, $H$, are in the range of 1.3 - 1.5. This is consistent with turbulent boundary layer shape factors in zero or mild pressure gradients. Also seen in Tables 4.3, 4.6, 4.9, 4.12, and 4.15 is the trend of decreasing skin friction coefficient with streamwise distance from the leading edge. This is also as expected, since skin friction coefficient typically decreases with streamwise distance, due to increasing thickness of the boundary layer.

The skin friction coefficients of the three machined plates, machined plates 1, 2 and 3, are of the same order of magnitude with those for flat plates with similar depression type roughness (Kithcart, 1995) and plates with very small protrusion roughness heights similar to that of frost on a flat plate (Lawrysyn, 1989). The skin friction coefficients for machined plate 2, with hemispherical depressions, are on average, half of the values seen for protrusion geometry of similar cross-sectional dimensions under similar flow conditions (Scaggs et al., 1988) The same trend of smaller skin friction coefficient for depression compared to protrusion roughness, is seen for machined plate 1, densely packed cylindrical depressions. Similar behaviour has been observed previously for depression type roughness (Kithcart and Klett, 1995; Choi and Fujisawa, 1993). This result suggests that perhaps protrusion roughness
calculation techniques can also be employed for depression-type roughness if an ‘equivalent’ protrusion height can be found for the depressions. This notion will be discussed in greater detail in Chapter 5.

It is interesting to notice that the skin friction coefficient with the hemispherical depressions is somewhat larger in magnitude than that with cylindrical depressions when equivalent spacing is used. This is contrary to the case for protrusion roughness, where it is well known that the drag on a cylinder is considerably higher than that on a hemisphere. As the physics of flows over and in a cavity are very complex and not well understood, only tentative hypotheses can be made regarding this skin friction result. Experimental results found by Kithcart and Klett (1995) are interesting in this respect. They relate the experimental error obtained for flows over cavities to the physical nature of the plates themselves. He found that the machining of the cavities is very important to experimental results and slight imperfections in the machining process can lead to erroneous results. In their list of slight imperfections he included variations in hole diameter due to skewed machining of the entire hole. Based on the uniformity of the present data at each probing station with the defect-law plots, Figures 4.1.2a - 4.1.2e, Figures 4.1.3a - 4.1.3e, and Figures 4.1.4a - 4.1.4e, and the smoothly decreasing trend of skin friction coefficients with streamwise distance, it is assumed that such physical imperfections are insignificant for the present plates, as there are no inconsistent data points along any of the plates or at any given probing station. One possible explanation of the higher skin friction coefficient for hemispherical depressions compared to
cylindrical depressions is the effect of vortices in the cavities on the flow. For flow over both types of depression, the streamlines very near the wall will deflect into the cavity. However, due to the shape of the hemispherical cavity the flow may follow the contour of the depression and re-emerge acting against the flow very near the wall. This ‘reverse’ flow would act as a retarding force which might act as an addition to the effective wall friction that the flow experiences and that the Pitot probe ultimately detects. Such a reverse flow would not be as pronounced for a cylindrical depression as the vortices created would probably be trapped within the cavity itself, with the flow simply turning on itself within the depression. A diagram presenting this hypothesis is seen in Figure 4.3. As mentioned, the physics of flow in cavities is not well understood. Flow and vortices within a cavity could be an unsteady system with vortices from the cavity ejecting into the flow, at irregular intervals, when enough energy has been stored.

The skin friction coefficients seen in Table 4.9, for machined plate 3 (sparsely distributed cylindrical depressions) are very nearly the same as those for the smooth flat plate (Figures 4.1.1a - 4.1.1c and Figures 4.1.3a - 4.1.3f). This implies that sparsely distributed depressions of this type have little effect on the flow. This has ramifications in engineering design practice as spacing patterns similar to this could be used to attach skins subjected to free stream flow with minimal skin friction increase. Drag reduction and heat transfer increases have even been discussed as potential gains from using depressions (Choi and Fujisawa, 1993; Kithcart and Klett, 1995). The results would also seem to suggest that the
skin friction for machined plate 3 is generated mainly by the flat plate flow surrounding the depressions with little effect from the cavities.

Results similar to those for machined plate 3 were found for the two vinyl plates. The skin friction coefficients obtained, as seen in Tables 4.12 and 4.15, are very close to those obtained for the smooth plate tested. This shows that random patterns of depression roughness with small depths have little effect on the flow. This conclusion can also be drawn by noticing that the log-law intercept shifts for both vinyl plates are small, which implies the flow over the plates may not be fully rough.

From Tables 4.2, 4.4, 4.7, 4.10, 4.13 and 4.16, it is seen that the skin friction coefficients obtained by using the momentum integral approach are closely similar to those obtained using the defect-law. Use of the momentum integral approach to determine skin friction does not suffer from some of the ambiguity linked to the defect-law method as the momentum integral technique does not rely on determining the boundary layer edge position. The definition of boundary layer edge height varies from author to author and fluctuations in freestream conditions may lead to inaccurate values of boundary layer thickness, which then would lead to incorrect defect-law $C_f$ values. The momentum integral method uses momentum and displacement thicknesses as key inputs; their values do not change significantly with the value of boundary layer thickness chosen. The momentum integral method is, on the other hand, sensitive to departure from two dimensionality. As the
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momentum integral method yields comparable results to the defect-law approach, one concludes that the defect-law approach in determining skin friction as well as the experimental techniques used, are valid in this case. The small differences in $C_r$ values are mainly caused by the lack of resolution in the defect-law plots. The values obtained from the momentum integral technique certainly fall within the range of defect-law skin friction coefficients when the previously discussed uncertainty of ± 0.00005 is considered. Also seen are the comparisons between skin friction coefficients calculated with and without pressure gradient effects. The results agree very well which implies that the experimental pressure gradient effects are small and validates the use of zero pressure gradient log law and defect-law techniques.

The summarized results of the log law intercept shifts seen in Tables 4.5, 4.8 and 4.11 show results that are consistent with flow over protrusion-roughened plates. The log law shifts for machined plates 1 and 2 are far more pronounced than for machined plate 3 which is expected as the holes are more densely packed. The log law shifts for machined plate 3, seen in Table 4.11, are small in comparison which implies that the sparse roughness array has little effect on the flow. It also suggests that fully rough calculation techniques may be inappropriate as the flow is mostly undisturbed by the presence of the depressions.

Protrusion roughness has a well defined positive height. Knowing the roughness height and the log law shift, eqn. 2-8 can be used to determine a correlation between $C(\lambda)$ and
spacing parameter, \( \lambda \). Depression roughness has no well-defined positive roughness height so correlations for \( C(\lambda) \), using an arbitrary height, would be meaningless. This again suggests the concept of using an equivalent positive roughness height for depressions such that the results for depression roughness can be correlated with those for protrusion roughness types. As mentioned, this concept will be considered in greater detail in Chapter 5.
5.0 COMPUTATIONAL WORK

5.1 Background and Objectives

As mentioned previously, it is argued that there may exist an equivalent protrusion or positive roughness height for depression or negative type roughness. It is also argued that when the equivalent roughness height is found, depression roughness data may be compared to protrusion roughness data. For the purpose of finding such equivalent heights, computations of rough-walled boundary layer development were employed. It must be mentioned however that any conclusions based on the calculated equivalent heights are speculative in nature, as, to the author’s knowledge, no literature exists to validate this technique.

A discrete element method was used for the aforementioned computations. The concept of the discrete element method dates back to Schlichting (1936) in which he argued that drag incurred by flow over a roughened surface can be divided into two categories, form drag on the roughness elements and viscous shear on the surface between roughness elements. From this foundation, many authors have proposed ways to implement the roughness element effects in the governing boundary layer equations. Taylor et al. (1985) used work by Finson and Clarke (1980) and added terms to the governing equations which account for the blockage effects of the roughness elements. As it was not the intent of the current research
to modify the discrete element approach proposed by Taylor et al., a thorough discussion of the blockage factor definitions and implementation into the governing flow equations is not provided as it is beyond the scope of this thesis. For a more detailed definition of these factors and their implementation in the computer code used for the present work, see Gagné (1998).

The code used for this experiment was TSL.FOR, a FORTRAN code that solved the parabolized thin-layer Navier-Stokes equations using a streamwise marching algorithm. This code that was initially developed for smooth walls by Pajayakrit (1997). The discrete element roughness model was implemented in the TSL code by Gagné (1998). The code allows for various geometries of protrusion roughness elements to be input as well as flow conditions. Three turbulence models are also available in the code, the Baldwin Lomax method (1978), the k - $\epsilon$ method and the k - $\omega$ method. Of the three, the Baldwin Lomax (1978) option was chosen for the present work as it is a fast, reliable turbulence model for flow over flat plates.

As mentioned, it was the intent of this exercise to determine, if possible, the existence of an equivalent protrusion roughness height for depression type roughness. If this equivalent height did indeed exist, then correlations for other log-law parameters, such as the variation of the log-law roughness parameter, $C(\lambda)$, could be expected in terms of it. The correlations would be expected to be the same as those for actual protrusion roughness.
5.2 Results and Discussion

The method used for determining the equivalent roughness height was to use the TSL code supplied by Gagné (1998) to compute the flow for each experimental roughness plate. The code was verified by conducting benchmark cases and comparing the benchmark results to those obtained by Gagné (1998). The goal of this procedure was to find, by trial and error, an equivalent roughness height for the depression roughnesses such that the computational skin friction coefficients found using the equivalent height were comparable to the experimentally determined defect law skin friction coefficients. The code requires an initial or starting profile of variation of dimensionless boundary layer velocity, $\frac{u}{U}$, with dimensionless height, $\frac{Y}{\delta}$. For each trial computation conducted for each plate, the velocity profile measured at the first probing station was used as the starting profile. Also, inputs of roughness geometry are needed. For the two plates with cylindrical depressions, it would seem reasonable that the equivalent protrusion geometry would consist of a positive protrusion cylinder. A protrusion cylinder was also chosen as the equivalent protrusion geometry for the hemispherical depressions in order to keep comparisons between plates simple. Computational runs were performed by trial and error on each plate using the equivalent roughness height as a variable input parameter. Once the predicted skin friction coefficients at each probing station were deemed adequately close to the experimental skin friction coefficients obtained via the defect law, the corresponding trial value of the equivalent protrusion height, $k_{eq}$, was identified as the correct one. Corresponding log-law intercept shifts, $\frac{\Delta u}{u_*}$, (see Chapter four)
were the experimentally determined ones. Using the $k_{eq}$ and $\frac{\Delta u}{u_c}$ values, a corresponding value of the roughness constant, $C(\lambda_{eq})$, could be calculated by use of Eqn. 2-8. The equivalent heights determined for each plate were also used in Simpson's' (1973) definition of roughness spacing parameter, Eqn. 2-12, to determine an equivalent spacing parameter $\lambda_{eq}$. As mentioned, both the cylindrical and the hemispherical depressions were taken to have the equivalent roughness geometry of a protruded cylinder, hence the frontal area, $A_F$, of the equivalent element is defined as, $(k_{eq})(1/4'')$, where (1/4'') is the nominal diameter of the depression and $k_{eq}$ is the equivalent height. The planar surface area which the element occupies, $A_S$, is the fixed value dependent on the spacing between elements. From section 3.1.2, the spacing between roughness elements is 0.559" for the plates 1 and 2 and the spacing between roughness elements for plate 3 is 1.768"; Planar surface area, $A_S$, values are thus 0.312 sq. inches and 3.123 sq. inches, respectively.

5.2.1 Equivalent Height Results based on Skin Friction Equivalence

Tables 5.1, 5.2 and 5.3 show a comparison of computed $C_f$ values for machined plates 1, 2 and 3 respectively, obtained from the TSL.FOR code with the identified equivalent protrusion height, and experimental $C_f$ values obtained by the defect-law technique. The $k_{eq}$ and $d_{act}$ values also shown in the Tables denote the equivalent height and the actual depression depth respectively. To indicate sensitivity to the chosen $k_{eq}$ value, Figure 5.1 shows a comparison between computed and experimental $C_f$ values for the identified optimum $k_{eq}$ and
for $k_{eq}$ values 10% larger and 10% smaller than this.

**Table 5.1 Skin Friction Comparison; $\frac{A_s}{A_{xs}} = 10$, Cylindrical Depression Plate - (Plate 1)**

<table>
<thead>
<tr>
<th>Station</th>
<th>x (m)</th>
<th>Computed $C_f$ per TSL</th>
<th>Experimental $C_f$ per Defect-Law</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>0.00444</td>
<td>0.0045</td>
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<tr>
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<td>1.018</td>
<td>0.00407</td>
<td>0.0041</td>
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<tr>
<td>4</td>
<td>1.368</td>
<td>0.00401</td>
<td>0.0040</td>
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<tr>
<td>5</td>
<td>1.718</td>
<td>0.00394</td>
<td>0.0039</td>
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</table>

$k_{eq} = 0.0161''$, \quad k^* = \frac{k_{eq} \mu \tau}{v} = 54.2$, \quad d_{act} = 0.125''

**Table 5.2 Skin Friction Comparison; $\frac{A_s}{A_{xs}} = 10$, Hemispherical Depression Plate - (Plate 2)**

<table>
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$k_{eq} = 0.021''$, \quad k^* = \frac{k_{eq} \mu \tau}{v} = 74.6$, \quad d_{act} = 0.125''
Table 5.3 Skin Friction Comparison; $\frac{A_s}{A_{xs}} = 100$, Cylindrical Depression Plate - (Plate 3)

<table>
<thead>
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<th>Station</th>
<th>x (m)</th>
<th>Computed $C_r$ per TSL</th>
<th>Experimental $C_r$ per Defect-Law</th>
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$k_{eq} = 0.0118''$  \hspace{1cm} $k^- = \frac{k_{eq} v_c}{v} = 33.2$  \hspace{1cm} $d_{act} = 0.125''$

From Tables 5.1, 5.2 and 5.3, it is seen that the skin friction coefficients obtained from the discrete element computations, with the equivalent protrusion roughness heights shown, agree closely with the experimental skin friction values obtained using the defect-law. Moreover the agreement is excellent at all 5 stations along the plates. It is interesting to notice that for both cylindrical-roughness plates, the values of equivalent protrusion roughness heights obtained are both on the order of one tenth the actual depression depth. This leads one to believe that for cylindrical cavities, there may a value of equivalent protrusion height that applies regardless of spacing between roughness elements, as in the case here. This should be the case if the flow in and near an individual cavity is unaffected by the presence of adjacent cavities. It is also seen that the equivalent protrusion height for the hemispherical cavities is larger in magnitude than that for the cylindrical cavities. This is expected as the skin friction
coefficients obtained via the defect-law for the hemispherical depression plate were larger than those of the cylindrical depression plate for the same roughness element spacing.

5.2.2 Simpsons Spacing Parameter, $\lambda_{eq}$ and Log-law Intercept Shift based on Equivalent Height

As mentioned, having determined an equivalent roughness height, $k_{eq}$, it is possible to calculate the log-law roughness parameter $C(\lambda_{eq})$ based on the equivalent height, via Eqn. 2-8, using the log-law intercept shifts, $\frac{\Delta u}{u_e}$, found in Chapter four. Also, as outlined above, it is possible to calculate a Simpsons (1973) spacing parameter (Eqn. 2-10) based on the equivalent height. That is $\lambda_{eq} = \frac{A_5}{k_{eq}(0.25\text{"})}$ The average values of $C(\lambda_{eq})$ and the values of the equivalent Simpsons (1973) spacing parameter $\lambda_{eq}$ for each plate are shown in Table 5.4.

<table>
<thead>
<tr>
<th>Machined Plate</th>
<th>Equivalent Simpsons Spacing Parameter, $\lambda_{eq} = \frac{A_5}{k_{eq}(0.25\text{&quot;})}$</th>
<th>Average Log-law Roughness Parameter, $C(\lambda_{eq})$</th>
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<td>1</td>
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<td>3</td>
<td>1060</td>
<td>-8.12</td>
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</table>
CHAPTER FIVE: COMPUTATIONAL WORK

The values seen in Table 5.4 for plates 1 and 2 fall on the correlation curve generated by Simpson (1973); this is seen in Figure 5.2. Using the concept of equivalent height, machined plates 1 and 2 move from the section of Simpson’s (1973) correlation curve that collapses densely spaced roughnesses to the section of the correlation for sparsely distributed roughness elements of the same geometry. It is noted however (see Table 5.1), that the value of the roughness Reynolds number \( \frac{k_{eq} u_r}{\nu} \) for machined plate 1 falls below the value of 60 expected for fully rough flow. However, the definition of fully rough flows seen in Chapter 3 was developed using standard sand roughness which does not necessarily apply here. Results for machined plate 3 also correlates with Simpson’s (1973) curve. The experimental data for machined plate 3 moves further into the sparse region of Simpson’s (1973) correlation curve. The experimental data for machined plate 3 is also shown in Figure 5.2. As with machined plate 1, the value of the equivalent roughness Reynolds number is below that required for fully rough flow. The same argument made for machined plate 1 applies here as well. Simpson’s (1973) correlation only applies to fully rough flow.

Research done by Waigh (1996) rework the correlation between the roughness parameter \( C(\lambda) \) and the spacing parameter \( \lambda \) to better collapse data from previous authors. Waigh (1996) separately the correlation between \( C(\lambda) \) and \( \lambda \) into densely spacing and a sparse spacing regime. As the experimentally determined values of \( \lambda_{eq} \) and \( C(\lambda_{eq}) \) correlate better with the sparse-regime portion of Simpson’s (1973) correlation, the correlation devised by Waigh (1996) for sparse spacing, \( \lambda_{eff} = (\lambda_{eq} Z_b^{0.55} Z_r^{1.38}) \), where \( Z_b = \frac{k_{eq}}{b} \) and \( Z_r = \frac{A_w}{A_F} \), will be
used. The values of \( k_e, b, \) and \( A_r \) are defined in Figure 2.4a. \( Z_r \) is called the roughness element bluntness parameter where \( A_w \) is defined as the upstream-facing wetted area of the roughness element. For cylindrical elements \( A_w = \frac{1}{2} A_F. \) The values of \( \lambda_{\text{eff}} \) for Waigh's (1996) correlation are seen in Table 5.5 for the three plates.

Table 5.5: Equivalent Waigh Spacing Parameter and Log Law Roughness Parameters of the three Machined test plates

<table>
<thead>
<tr>
<th>Machined Plate</th>
<th>Equivalent Waigh's Spacing Parameter, ( \lambda_{\text{eff}} = (\lambda_{eq} Z_b^{0.55} Z_r^{1.38}) )</th>
<th>Average Experimental Log-law Roughness Parameter, ( C(\lambda_{eq}) )</th>
</tr>
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<tr>
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<td>2</td>
<td>28.42</td>
<td>-3.62</td>
</tr>
<tr>
<td>3</td>
<td>368.65</td>
<td>-8.20</td>
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</table>

The current experimental data is compared with Waigh's (1996) correlation in Figure 5.3. All three plates correlate well with the revised Waigh (1996) correlation which further supports the idea of using equivalent heights for depression-type roughness analysis.
6.0 CONCLUSIONS

The conclusions that can be drawn from the present experimental work include:

1. **Experimental data reduction techniques for protrusion roughness can also be used for experiments on depression roughness.**

2. The defect law and the momentum integral techniques provide accurate means for determining the skin friction coefficient, within a resolution range of $\pm 0.00005$, for flow over flat plates containing arrays of cavities.

3. Cavity or depression roughness produces substantially lower skin friction coefficients than flow over protrusion roughness of similar element geometry and spacing.

4. Skin friction coefficients for flow over arrays of hemispherical depressions are higher than for flow over cylindrical depressions of similar depth and spacing.

5. For the experimental plates tested, equivalent protrusion roughness heights exist such that the skin friction coefficients computed using the equivalent height agree well with the experimental skin friction coefficients.
7.0 RECOMMENDED FUTURE WORK

Based on the experimental findings outlined within this thesis, recommendations for further study include:

1. Conduct similar experiments using plates with varying depression roughness geometry and spacing in order to better understand and possibly correlate results.

2. Perform more testing on the notion of equivalent protrusion height so that more conclusive remarks can be made as to the applicability of the technique.

3. Execute studies using depression roughness on engineering surfaces such as aerofoils and turbine blades

4. Conduct studies using plates with random roughness and attempt to determine, if possible, the skin friction coefficients using the experimental techniques discussed here.

5. Investigate the physics of flow near and within a hole/depression to better understand effects of cavity roughness on boundary layer development.
REFERENCES


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\[ U_e = 47 \text{m/s and } \delta = 32 \text{mm} \]
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\[ y = -0.0001x^2 + 0.0023x + 0.0008 \]
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\[ y = -0.00015x^2 + 0.0026x + 0.0007 \]
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\[ y = -7 \times 10^{-5}x^2 + 0.0016x + 0.0009 \]
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The graph shows a quadratic relationship between the Thera (m) and Streamwise Distance (m), with the equation:

\[ y = -0.0001x^2 + 0.0017x + 0.0008 \]
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\[ \Delta \]
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\[ y = -7E-05 x^2 + 0.0015 x + 0.0011 \]
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### A1: Sample Output of RTFA data acquisition software

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2-9-1998, 9:18

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A2: Source Code for DATARED.FOR

c c
this program accepts the output from the data conversion
program convert.for and determines the boundary layer velocity
profile values, reynolds number, and momentum and displacement
thickness at the given station.
c
program datared
dimension step(200),dist(200),v(200),vref(200),vv(200)
dimension rey(200),reylg(200),distln(200)
real ml,m2,o
integer jj,ii,kk
pitoth=0.508

height of flat pitot tube =pitoth in mm
open(unit=1,file='',status='old')
open(unit=3,file='',status='unknown').
c
reads in the raw data from 'convert.for'

read(1,*) date
read(1,*) bp
read(1,*) t1
read(1,*) t2
read(1,*) bkk
read(1,*) volt2
read(1,*) volt1
read(1,*) volt2f
read(1,*) volt1f
read(1,*)
read(1,*)
read(1,*)
vavg=0.0
n=int(bkk)

write(*,*) ' Enter the step where tube leaves plate surface'
read(*,*) nb
begin
write(*,*) ' enter the distance correction factor '
c
write(*,*) distcor
n4=n-nbegin

write(3,901)
write(3,*), n4

cc Reading in the Data from the input file
cc
do i=1,n
    read(1,*), step(i),dist(i),v(i),vref(i)
endo
do 200, i=1,n

cc Average temperature for data point

t1c=(t1-32)/1.8

t2c=(t2-32)/1.8

tc=t1c + (t2c-t1c)*i/n

t=t1+(t2-t1)*i/n

go=bp/(0.287*(273+(t-32)/1.8))

cc Correction due to initial and final Transducer voltages

cc Note: 'Values for when Tunnel is off'

v1=volt1+((volt1f-volt1)*i/n)

v2=volt2+((volt2f-volt2)*i/n)

cc Pressure Conversion from Volts to Pascals using Calibration Data

Transducer Serial No: Factor=1.371 Pa = 1 mV

Transducer Serial No: Factor=1.375 Pa = 1 mV

cc where: p = Boundary Layer Dynamic pressure

pref = PC1 - PC2

p=1.3626*(v(i) - v1)*1000

p=abs(p)

v(i)=abs(2/go*p)**0.5

pref=(1.3369*(vref(i)-v2)*1000)/0.91

vref(i)=abs(2/go*pref)**0.5

v(i)=v(i)/vref(i)

dist(i)=dist(i)*25.4

200 continue

c

continue

distx=dist(nbend)

do 230 i=1,n4

    dist(i)=dist(i+nbend-1) - distx + (pitoth/2)

    if ((dist(i)/pitoth).ge.1) then
        goto 205
    end if

230 continue

n=n4
c
The last 10 - 20 freestream velocities are averaged

```
nn=n-10
vvs=0

300 do i=nn , n
    vvs=vvs+vv(nn)
continue

vvavg=vvs/11
vavg=0.
theta=0.
deltas=0.

400 do i=1,n
    vv(i)=vv(i)/vvavg
    vref(i)=vv(i)*v(i)
    visc=((0.009157*tc)+1.319)/100000
    vavg=vavg+vref(i)
    rey(i)=(dist(i)/1000)*vref(i)/visc
    reylg(i)=log10(abs(rey(i)))
    distln(i)=log(abs(dist(i)))
write(3,900) i,dist(i),v(i),vref(i),vv(i),reylg(i),distln(i)
```

Calculates freestream vel and log of rey no.

```
Reynolds Number Based on Outer Velocity
rey(i)=(dist(i)/1000)*v(i)/visc

Reynolds Number Based on Local Velocity
rey(i)=(dist(i)/1000)*v(i)/visc
```

Determining theta,deltastar and h from expt data

```
vv(0)=0
theta=0
deltas=0
dist(0)=0

500 do i=1,n
    v4=((vv(i)+vv(i-1))/2)
    theta=theta+((v4*(1-v4))*(dist(i)-dist(i-1)))
    deltas=deltas+ ((1-v4)*(dist(i)-dist(i-1)))
continue
```

```
h1=deltas/theta
vavg=vavg/(n-1)
write(3,* ) vavg, ' m/s average velocity'
```
write(3,*) theta, ' mm theta'
write(3,*) deltas, ' mm delta star'
write(3,*) hl  , ' shape factor h'
do 600 i=1,n
    write(3,900) i,dist(i),v(i),vref(i),vv(i),reylg(i)
600 continue
899 format(g12.6)
900 format(3x,i3,3x,g12.6,3x,g12.6,3x,g12.6,3x,g12.6,3x,g12.6)
901 format('1',2x,'step',5x,'dist',11x,'v ',13x,'vref',10x,'v/vref',
* 9x,'log rey')

stop
end
A3: Source Code for CONVERT.FOR

program convert
C This program converts output file from RTFA.BAS to a usable
C input file for DATARED.FOR
integer step(200), span(200), rad(200), conv
real dist(200), v(200), vref(200), vv(200)
real p0(200), p1(200), p2(200), q0(200), q1(200), q2(200), pq(200)
real pq2(200), spanr(200), radr(200)
real bp, tl, t2, volt1, volt2, volt1f, volt2f
real calib1, calib2, slope, delx, title
C character *80 inname, oname, title

C write ('**') 'Enter *.raw filename you wish to convert'
C read ('**') inname
C write ('**') 'Enter the output filename'
C read ('**') oname

open (unit=1, file=' ', status='old')
open (unit=2, file=' ', status='unknown')

C write ('**') 'Enter Experiment Title'
C read ('**') title
write ('**') 'Enter Atm. Barometric Pressure (in kPa)'
read ('**') bp
write ('**') 'Enter Initial and Final Wind Tunnel Temps.(in degF)'
read ('**') tl, t2
write ('**') 'Enter total number of steps'
read ('**') bkk
write ('**') 'Enter value of Transverse Gear "Steps/Inch",'
C & (Note: Usually 8000 steps/inch)'
C read ('**') conv
cc
cc Negative 8000 since Gear is transversing from test-section floor to
BL edge
conv=-1*8000

write ('**') 'Enter the final voltage values of the transducers,
C & (Starting with BL value, in (mV)'
read ('**') volt1f, volt2f
write ('**') 'Enter Intitial Transducer Voltages in mV'
read ('**') volt1, volt2
cc
cc Information for Static tap offset value

c write ('**') 'Enter Calibration factor for Transd.BL (in Pa/mV)'
C read ('**') calib1
write(*,*) 'Enter Calibration factor for Transd. REF (in Pa/mV)'
read (*,*) calib2

write(*,*) 'Enter slope of Non-Dim. Pressure vs. Static tap & (in 1/m)'
read (*,*) slope

write(*,*) 'Enter streamwise dist. from tap position to pitot & mouth placement (in cm)'
read (*,*) delx

'slope' is the first derivative equation of the Non-Dim. Static pressure curve 'delx' is the streamwise position of the pitot mouth measured from the front of the test section

slope=((4e-6)*delx) - 0.0006

Changing Final Voltage values from mV into V
volt1f=volt1f/1000
volt2f=volt2f/1000

Beginning to read in values from *.RAW file
read(1,*)
read(1,*)
read(1,*)
read(1,*)

10 format(i3,x,i6,x,i6,x,f5.1,f6.1,x,f5.1,f6.1,x,f7.4,f7.4)
do i=1,bkk
  read(1,10) step(i),span(i),rad(i),p0(i),p1(i),q0(i),q1(i),
            & pql(i)
endo

Setting Initial Transducer Voltages
volt2=q0(1)/1000
volt1=p0(1)/1000

Writing Initial Boundary Conditions to Output file
write (2,*) title
write (2,*) bp  
write (2,*) t1  
write (2,*) t2  
write (2,*) bkk  
write (2,*) volt2  
write (2,*) volt1  
write (2,*) volt2f  
write (2,*) volt1f  
write (2,*)
write(2,*), 'Step    Dist (inches)    Q (volts)    Qref (volts)''

CC Converting Integer Steps into Real Steps for Easier Manipulation

DO k=1,bkk
   spanr(k)=span(k)/1.
   radr(k)=rad(k)/1.
ENDDO

C 20 FORMAT(i3,x,f9.4,x,f8.4,x,f5.4,x,f7.4,x,f7.4,x,f7.6,x,
         & f7.4,x,f7.4,x,f7.4,x,f7.4)
20 FORMAT(i3,4x,f9.4,8x,f9.4,5x,f9.4)

C  
C
C
C
CC Converting number of steps to inches and Millivolts to volts 
CC Using Conversion of -8000 steps/inch for distance

DO j=1,bkk
   spanr(j)=spanr(j)/conv
   radr(j)=radr(j)/conv
   p0(j)=p0(j)/1000

CC Implementing static tap offset by adding slope of Non Dim. 
CC static pressure vs. Static tap position and 
CC redimensionalizing into volts by multiplying q1(j)
CC 
CC where q1 = PC1 - PC2

   p1(j)=(p1(j)/1000)+((slope*q1(j)/(0.91*1000)))
   p2(j)=p2(j)/1000
   q0(j)=q0(j)/1000
   q1(j)=q1(j)/1000
   q2(j)=q2(j)/1000

CC DATARED.FOR requires these as inputs
A3: SOURCE CODE FOR CONVERT.FOR

write(2,20) step(j), radr(j), pl(j), ql(j)
enddo
stop
end
### A4: Sample Output of CONVERT.FOR

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