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DYNAMIC BEHAVIOR OF ACOUSTIC ECHO CANCELLATION

submitted by

Haiyan Yuan, B. Sc (Eng)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master's of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering

Carleton University
Ottawa, Ontario, Canada K1S 5B6
January 4, 1994
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the Faculty of Graduate Studies and Research
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DYNAMIC BEHAVIOR OF
ACOUSTIC ECHO CANCELLATION

submitted by

Haiyan Yuan, B.Sc (Eng)

in partial fulfillment of the requirements
for the degree of Master's of Engineering

Thesis Supervisor

Chair, Department of Systems and Computer Engineering

Carleton University
January 4, 1994
ABSTRACT

This thesis studies the dynamic behavior of acoustic echo cancellation in hands-free communications. In hands-free desktop terminals, the loudspeaker and microphone are closely coupled in the same enclosure. In order to maintain stability, usually only half-duplex conversation is allowed. The technique of acoustic echo cancellation has been applied as a means of maintaining stability while allowing full-duplex conversation.

This work presents basic limitations on the acoustic echo cancellation system such as room noise, system nonlinearity, system under-modeling and room nonstationarities. This work investigates the tracking ability under various nonstationary conditions.

This thesis proposes ES-based (exponential step) algorithms to provide both good tracking during the nonstationary periods and a good level of cancellation during stationary periods. Computer simulations show this approach to be very promising. Another benefit of ES-based algorithms is their low computational complexity. Preliminary simulations of windowed ES-based algorithms were also conducted.

Adaptive step size algorithms based on ES-LMS, ES-NLMS, and NLMS algorithms were proposed also. So far, the simulations on these adaptive step size algorithms didn't show their performance to be as good as the regular ES-based algorithms.
To my husband Albert, and my father,
and for the memory of my mother
ACKNOWLEDGEMENTS

I would like to thank my supervisor, Dr. R. Goubran, whose guidance, insight and encouragement have been much appreciated over the course of my thesis. I would like to thank Neil Birkett for the many discussions about our work which benefited both of us greatly. I would also like to thank Rajeev Bector for the many hours he spent helping me with the experiments. Finally, I would like to thank Janet Redies for her proofreading this thesis.

I would also like to thank TRIO, Bell-Northern Research, NSERC, and Carleton University for their financial support for this research.
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<td>Acoustic Echo Canceller</td>
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<tr>
<td>AIR</td>
<td>Acoustic Impulse Response</td>
</tr>
<tr>
<td>ALE</td>
<td>Adaptive Line Enhancer</td>
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<td>AR</td>
<td>Auto Regressive</td>
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<td>BLMS</td>
<td>Block LMS algorithm</td>
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<td>CAPZ</td>
<td>Common-Acoustical-Pole and Zero</td>
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<td>DAT</td>
<td>Digital Audio Tape</td>
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<td>ERLE</td>
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<td>GAL</td>
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<td>GIVE</td>
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<tr>
<td>IIR</td>
<td>Infinite Impulse Response</td>
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<td>ILMSN</td>
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<td>LMS</td>
<td>Least Mean Squares algorithm</td>
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<tr>
<td>MA</td>
<td>Moving Average</td>
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<td>MLS</td>
<td>Maximum-LENGTH Sequences</td>
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<td>MMSE</td>
<td>Minimum Mean Squared Error</td>
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<td>ODE</td>
<td>Ordinary Differential Equation</td>
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<td>PIR</td>
<td>Periodic Impulse Response</td>
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<td>Room Transfer Function</td>
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<td>Sign Algorithm</td>
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<td>SFTF</td>
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</tr>
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<td>SRA</td>
<td>Signed Regressor Algorithm</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<td>TDS</td>
<td>Time Delay Spectrometry</td>
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CHAPTER 1

INTRODUCTION

1.1 Brief Description of Problem

Hands-free telephones are very useful in many applications such as speaker phones, conference phones, and cellular systems installed in cars.

The problem of providing the means for a hands-free telephone conversation has been investigated for many years [7]. But this problem remains unsolved. In the hands-free system, in order to maintain comfortable receive and transmit levels for the near end and far end users, high gains are required (see Figure 1). These high fixed gains and leaks in hybrid can cause electro-acoustic instability.

![Diagram of Hands-free System]

**FIGURE 1 Hands-free System.**

Even if stability is maintained, significant talker echo would be a problem. Talker echo is the return of a talker’s delayed speech to him or herself. Talker echo would become a problem for the far end user connected to the hands-free user across the network, as the
received signal at the hands-free telephone would be directly coupled back to the far end talker via the microphone, with little loss and possibly gain.

The traditional technique of voice controlled switching has been addressed in Hansler's paper [74] (for example, using the Motorola chip). Problems arise in cases of environmental noise that may erroneously be sensed as near end speed. A second major source of errors is poorly balanced hybrids causing a strong near end (electrical) echo of the signal transmitted [74]. Although the electrical echo could be removed by applying a hybrid echo canceller, this switching still leads to a half-duplex hands-free system (only one voice path available at a time). It is hoped that the restriction in half-duplex conversations would not be present in handset (full-duplex) conversations.

Other conventional methods applied to prevent feedback problems, such as echo suppression or gain control may cause degradation of speech quality, especially in double-talk situations [75].

1.2 Thesis Objective

To implement a "satisfactory" solution means to design a system that allows a hands-free telephone conversation without any loss in speech quality and without any restrictions to the talking parties. During their conversation, both speakers should be able to move freely in ordinary offices or living rooms using only conventional loudspeakers and microphones. A system offering this degree of convenience can be achieved by applying two echo cancellers (see Figure 2).

In this case, leaks in hybrid could be removed by the hybrid echo canceller (HEC); the part
of the microphone output signal which is caused by the loudspeaker would be cancelled by the output of the acoustic echo canceller (AEC), resulting in a perfect decoupling of the loudspeaker and microphone. It is hoped that introducing an adaptive filter would improve the performance considerably.

![Diagram of echo cancellation]

**FIGURE 2 Echo Cancellation in Hands-free Environment.**

The acoustic echo cancellation has turned out to be a more involved and complex problem than electrical hybrid echo cancellation for several reasons:

1. The acoustic echo path is affected by any movement within its acoustic surroundings.
2. The length of cancellation required is very long.
3. The presence of background acoustic noise in the room.
4. The acoustic echo path has at least one significant non-linear component (the loudspeaker) [86].

This thesis investigated several adaptive filtering algorithms suitable for acoustic echo cancellation in hands-free phones (exponential step LMS family algorithms, and variable
step size LMS family algorithms). The impact of filter length and nonstationarities was studied. A digital recording system was set up to obtain high-quality two-channel recordings through actual hands-free terminal enclosures and transducers, from which various algorithms could be applied to this data in non-real time.

1.3 Thesis Organization

This thesis is organized as follows. Chapter 2 presents the background in acoustic echo cancellation techniques. Chapter 3 describes the capture of the real telephony data (reference and primary signal) that were used in the simulations. Chapter 4 describes the various methods used to measure the impulse response.

Environmental conditions such as room noise, the acoustic impulse length, and system non-linearities will be shown to be basic limitations on canceller performance, and are discussed in Chapter 5.

Chapter 6 investigates tracking ability comparison between LMS and RLS family algorithms, some characteristics of ES-based (exponential step) LMS family algorithms, and the dynamic performance issues such as convergence speed and tracking ability for several different adaptive filtering algorithms. The performance of adaptive step gain of ES-based algorithms and NLMS algorithms is also investigated.

Finally, Chapter 7 presents the conclusions of this thesis and the suggested further work.

Additional simulations that were performed with ES_NLMS (compared with the regular NLMS) are shown in the Appendices.
CHAPTER 2

ACOUSTIC ECHO CANCELLATION TECHNIQUES

Adaptive filters can be classified into Finite Impulse Response (FIR) filters and Infinite Impulse Response (IIR) filters. This chapter examines both structures.

2.1 Adaptive FIR Filtering Algorithms

Adaptive FIR filters are discussed in detail by Widrow [2] and Haykin [4] and are shown in Figure 3. Part (a) shows the block diagram of an FIR adaptive filter; part (b) shows the tapped delay line model.

![Block diagram of finite impulse response adaptive filter](a)

![Tapped delay line model](b)

**FIGURE 3** (a) Block diagram of finite impulse response adaptive filter. (b) Tapped delay line model.
2.1.1 Least-Mean Squares (LMS)

Of the many algorithms for performing the filter adaptation, the most popular one is the Least Mean Squares (LMS) algorithm [4]. According to Figure 3, the algorithm is as follows:

\[ y(n) = \sum_{i=0}^{M-1} w[i] x(n-i) = W^T(n) X(n) \]  \hspace{1cm} (1)

\[ e(n) = d(n) - y(n) \]  \hspace{1cm} (2)

\[ W(n+1) = W(n) + \mu e(n) X(n) \]  \hspace{1cm} (3)

where

- \( x(n) \) is the current input signal (reference signal)
- \( d(n) \) is the desired signal
- \( y(n) \) is the filter output
- \( e(n) \) is the error signal
- \( M \) is the filter order
- \( w_i(n) \) is the filter coefficient, \( i = 0, 1, ..., M-1 \)
- \( \mu \) is the step size, which must be chosen so that:

\[ 0 < \mu < \frac{2}{M-1} = \frac{2}{M \sigma_x^2} \sum_{i=0}^{\lambda_i} \]  \hspace{1cm} (4)

where \( \lambda_i \) is the eigenvalue of the input correlation matrix, and \( \sigma_x^2 \) is the power of the input signal.
2.1.2 Normalized Least-Mean Squares (NLMS)

In applications where the exact input power is not known or is highly variable, a more appropriate choice is to calculate the exact value for \( \mu \) at each iteration. The following form for the variable step size calculation is suggested according to Equation 4:

\[
\mu = \frac{\alpha}{\varepsilon + \sum_{i=0}^{M-1} x^2(n-i)}
\]

where \( \alpha \) is a number between 0 and 2, \( \varepsilon \) is a small positive constant used to prevent the step size from becoming too large. This version of the LMS algorithm is known as the normalized LMS algorithm, or NLMS [2][3][4].

2.1.3 Recursive Least Squares (RLS)

The recursive least squares (RLS) is a more elaborate algorithm than LMS family algorithms. An important feature of the RLS algorithm is that it utilizes information contained in the input data, extending back to the instant when the algorithm is initiated. According to Figure 3, the algorithm is as follows [4]:

\[
k(n) = \frac{\lambda^{-1}P(n-1)X(n)}{1 + \lambda^{-1}X^T(n)P(n-1)X(n)}
\]

\[
\alpha(n) = d(n) - W^T(n-1)X(n)
\]

\[
W(n) = W(n-1) + k(n)\alpha(n)
\]

\[
P(n) = \lambda^{-1}P(n-1) - \lambda^{-1}k(n)X^T(n)P(n-1)
\]

*Initialization* \( P(0) = \delta^{-1}I \quad \delta = \text{small positive constant} \)

\( W(0) = 0_{\text{vector}} \)
where
\lambda is the forgetting factor (set between 0 and 1)
P(n) is the M by M inverse correlation matrix
k(n) is the M by 1 gain vector
\alpha(n) is the estimation error (a priori)

2.2 Adaptive IIR Filtering Algorithms

Adaptive filters could also be based on the use of Infinite Impulse Response (IIR) filters. Falconer [31] investigated the performance of a pole-zero adaptive filter in echo cancellation. The LMS type algorithm was used to update the coefficients of the pole and zero equations.

![Diagram of pole-zero adaptive echo canceller](image)

**FIGURE 4** Basic structure of pole-zero adaptive echo canceller.

The transfer function realized by the echo canceller is as follows (refer to Figure 4):

\[
\frac{A(z)}{1 - B(z)} = \frac{\sum_{t=0}^{n_d} a_t(n) z^{-t}}{1 - \sum_{j=1}^{n_b} b_j(n) z^{-j}}
\]  

(10)
where

\( n_a \) is the order of zeros

\( n_b \) is the order of poles, i.e. order of IIR filter

\( A(n) \) is the \((n_a+1)\) by 1 zero coefficient vector, i.e., \( A(n) = [a_0(n) \ a_1(n) \ldots \ a_{n_a}(n)]^T \)

\( B(n) \) is the \(n_b\) by 1 pole coefficient vector, i.e., \( B(n) = [b_1(n) \ b_2(n) \ldots \ b_{n_b}(n)]^T \)

Algorithm:

\[
e(n) = d(n) - \sum_{i=0}^{n_a} a_i(n)x(n-i) - \sum_{j=1}^{n_b} b_j(n)d(n-j) \tag{11}
\]

\[
A(n+1) = A(n) + \mu_a e(n)X(n) \tag{12}
\]

\[
B(n+1) = B(n) + \mu_b e(n)D(n) \tag{13}
\]

The current step sizes are normalized as:

\[
\mu_a(n) = \frac{\mu}{P_x(n)} \tag{14}
\]

\[
\mu_b(n) = \frac{\mu}{P_d(n)} \tag{15}
\]

where \( \mu \) is the fixed step size, and \( P_x(n) \) and \( P_d(n) \) are estimates of the current power levels of \( x(n) \) and \( d(n) \).

2.3 Variants of LMS Adaptation Algorithms

An analysis of the performance of LMS must consider the convergence and the misadjustment due to noise and to parameter variation (so called "lag misadjustment") which were discussed in Widrow [2][10]. Although the deficiency of the LMS algorithm in dealing with colored inputs was pointed out and several ways were suggested to combat it [1][3][41], the LMS algorithm has always been attractive to most researchers in the field of adaptive signal processing, such as echo cancellation [24][40][46][47][76], line equal-
ization [29], and adaptive noise cancellation [8][13][71]. At the same time, there has been some work done to study the statistical efficiency of LMS algorithm [19][54][97]. Davila presented simulation results to show that LMS type algorithms are more robust than the least squares to sudden variation of the environment parameters [54].

Reduction of the complexity of LMS algorithm has received attention in the area of adaptive filtering. This reduction is usually done by clipping either the estimation error or the input data. The algorithm based on clipping the estimation error is known as the sign algorithm (SA). The coefficients are adapted as follows (refer to Equation 1 and 2):

\[ W(n+1) = W(n) + \mu X(n) \text{ sgn}(e(n)) \]  \hspace{1cm} (16)

Eweda [38] studied the convergence of a SA with decreasing gain when governing the weights of an adaptive filter in the noiseless case. A rigorous proof of almost sure convergence to the optimal filter is attained under a weak ergodicity assumption that includes the case of correlated observations. Furthermore, Eweda [55] gave the analysis of the SA in the case of nonstationary and correlated data. The optimum step size \( \mu_{\text{opt}} \) is derived under the assumption that increments of optimal filter weights are white.

The algorithm based on clipping the input data is known as the signed regressor algorithm (SRA) [54], and is given by (refer to Equation 1 and 2):

\[ W(n+1) = W(n) + \mu e(n) \text{ sgn}(X(n)) \]  \hspace{1cm} (17)

\[ \text{sgn}(X(n)) = [\text{sgn}(x(n)), \text{sgn}(x(n-1)), \ldots, \text{sgn}(x(n-M+1))]^T \]  \hspace{1cm} (18)

One of Davila's main contributions in the paper [54] is concerned with the nonstationary case. It is shown that the tracking ability of the SRA increases as the tolerable mean square
excess estimation error is increased, and decreases as the plant noise power and the filter length are increased. In the case of slow variations, the minimum mean square excess estimation error the SRA is \( \sqrt{\pi/2} \) times greater than that of the LMS algorithm. But the simulation was done only with artificial data and time varying plant with tap number \( M = 5 \).

In the acoustic echo cancellation, the echo path should be modelled at least several hundred taps. Based on the analysis, it is noticed that as the filter length is increased, the tracking ability decreases.

Conventional gradient-based adaptive filters, such as LMS, use an instantaneous gradient estimate to update the filter coefficients. It leaves the algorithms vulnerable to impulsive interference. Haweel and Clarkson [65] demonstrated the principle of Order Statistic Least Mean Square (OSLMS) filters. A general class of OSLMS algorithms has been shown to provide greatly superior performance to LMS and other linear update algorithms in impulsive and other non-Gaussian environments, and slightly inferior performance for Gaussian inputs. In our study of the dynamic behavior of acoustic echo cancellation in a teleconference room, there is little chance for impulsive disturbance to appear.

Boray and Srinath [66] proposed a conjugate gradients algorithm. In this algorithm, there is an adaptive forgetting factor \( \theta(n) \) to adjust the direction vector. Instability occurred whenever \( \theta(n) \) exceeded unity. In nonstationary cases, the error will jump due to nonstationarities and \( \theta(n) \) will exceed unity. Unless some modifications are offered, this algorithm is not appropriate for nonstationary cases.

Leon, Kerchief and Kitzen [79] investigated the problem of tracking an acoustic, time-varying impulse with an adaptive filter. The approach of using mirror sources and mirror
microphones to account for reflections against the walls of a room, and using the theory of scattering to account for the influence of an object moving through the room, leads to a simple expression from which the benefit of applying an adaptive filter can be judged. It was suggested that the basic LMS adaptive filter is not able to track the nonstationarities due to a person walking through a normal living room if the length of the filter is so large that the error due to the nonstationarities is greater than the error due to the finite length of the adaptive filter. In other words, the length of the adaptive filter is limited in order to achieve a good tracking performance.

Cioffi and Ho [78] give a finite precision analysis of the block-gradient adaptive data-driven echo canceller. The general configuration is shown as follows (refer to Equation 1 and 2):

$$W(n + L) = W(n) + \mu \sum_{k = n}^{n + L - 1} e(k) X(k)$$

(19)

where L is the block length of the BLMS$^1$ (L = 1 for LMS).

The BLMS requires significantly less precision than the standard LMS algorithm, and requires almost exactly the same amount of computation in the data echo canceller. Cioffi and Ho's analysis shows:

- There exists an optimum trade-off between reducing finite-precision degradation (by increasing the step size $\mu$), and averaging the noise due to the gradient estimate (which traditionally dictates that the step size $\mu$ should be as small as possible).

---

1. The block LMS algorithm will be referred to as the BLMS; likewise, the LMS algorithm will be referred to as the LMS.
- As L increases, the degradation due to finite precision error decreases.

- For a given fixed µ and steady-state performance level, we can reduce the required precision by increasing L.

Ciofí et al suggested that the BLMS could be used effectively to reduce the storage requirements of the echo canceller, by reducing the coefficient wordlength 1 bit per factor of 4 reduction in tracking rate. But unfortunately, this implies that the BLMS algorithm is not a suitable candidate in cases where the tracking ability is the greatest concern (also refer to [96]).

2.3.1 Algorithms with Dynamic Adaptation Step-size

The LMS algorithm [2] has a trade-off in the choice of the adaptation step size. A large step size gives faster convergence whereas a smaller one is necessary for a smaller final misadjustment from the optimum Wiener solution.

Harris proposed a Variable Step (VS) algorithm to overcome this problem [23]. This algorithm controls the step size by an estimate of the distance to the mean-squared error minimum. Sugiyama pointed out that this algorithm has the disadvantage of step size instability [44]. It uses the sign of the gradient component of the squared error. Ideally, this component should contain only the squared error. However, in some applications such as echo cancellation, this component is “contaminated” by the signal to be received. As a result, an incorrect sign is often detected, causing undesirable change of the step size. More error in sign detection will occur when the signal has more peaks and dips in its waveform. To make the effect of these wrong sign detections smaller, the limited range of
parameters in turn degrades the superiority of the VS algorithm. Also, due to the step-size instability, the residual noise level of the VS algorithm is high.

Kwong and Johnston [80] proposed a Variable Step Size (VSS) LMS algorithm, where the step size increases or decreases as the mean-squared error increases or decreases, allowing the adaptive filter to track changes in the system as well as producing a small steady-state error. The choice of the step size reflects a trade-off between misadjustment and the speed of adaptation. When the prediction error is large, increasing the step size provides faster tracking; when the prediction error is small, decreasing step size yields smaller misadjustment.

Algorithm:

\[ W(n+1) = W(n) + \mu(n) X(n) e(n) \]  \hspace{1cm} (20)

\[ \mu'(n+1) = \alpha \mu(n) + \gamma e^2(n) \quad 0 < \alpha < 1, \gamma > 0 \] \hspace{1cm} (21)

\[ \mu(n+1) = \mu_{\text{max}} \quad \text{if } (\mu'(n+1) > \mu_{\text{max}}) \]
\[ \mu_{\text{min}} \quad \text{if } (\mu'(n+1) < \mu_{\text{min}}) \]
\[ \mu'(n+1) \quad \text{otherwise} \] \hspace{1cm} (22)

Parameters are suggested to be \( \mu_{\text{max}} = 0.1 \), \( \mu_{\text{min}} = 10^{-5} \), \( \alpha = 0.97 \), \( \gamma = 4.8 \times 10^{-4} \). Simulation results in the paper indicate that it tends to be less sensitive to the eigenvalue spread of the input data, and it has nonstationary tracking potential. This is achieved with the addition of an extra tap update, but basically the VSS LMS algorithm has the same computational complexity as the conventional LMS algorithm. This idea could also be extended to the NLMS and later discussed exponential step LMS family algorithms.
Chen and Vandewalle [33] proposed a variant of the NLMS, in which the traditionally scalar step-size constant \( \mu \) is replaced by a time varying \( \mu \)-vector, in which more recent taps are given larger values of \( \mu \).

Makino and Kaneda [51] have taken the basic idea of a \( \mu \)-vector NLMS algorithm and adapted it for use in an acoustic echo cancellation application. A variant of the conventional NLMS, termed the \( ES \) (Exponential Step) algorithm was proposed. In Makino and Kaneda's study, the room impulse response was measured repeatedly, and the impulse response variation was studied to determine the statistics of variation in the room impulse response. They noted that in typical teleconferencing situations, the impulse response coupled between the loudspeaker and microphone attenuates exponentially, and that the variation in this impulse response attenuates by the same exponential ratio. For this reason, they initialized their algorithm to have an exponentially decaying \( \alpha \)-vector, i.e.:

\[
\alpha(n) = \begin{bmatrix} \alpha_0(n) & \alpha_1(n) & \cdots & \alpha_{M-1}(n) \end{bmatrix}
\]

(23)

\[
\alpha_i(n) = \alpha_0(Base)^{-\text{(DecayFactor)}}i, \quad i = 0, 1, \ldots, M - 1
\]

(24)

The \( \alpha \)-vector should be updated with respect to the mean-squared error of the corresponding coefficient. For practical reasons, however, simulations were performed with a time-invariant exponentially decaying \( \alpha \)-vector. Based on the results, the newly proposed ES algorithm showed a tripling of convergence speed for white noise inputs and a doubling convergence speed for speech signals, but the same computational load as the conventional NLMS algorithm. This principle could be applied to the LMS algorithm, in which case the regular step size \( \mu \) is replaced by an exponentially decaying \( \mu \)-vector.
2.4 Variants of RLS Adaptation Algorithms

There are two much-studied methods of solving the minimum-mean-square problem, which is to minimize the variance of the error signal. One solution is a stochastic Gauss-Newton algorithm, which leads to the least mean squares (LMS) problem described above; its solution can be computed recursively in time, leading to the recursive least squares (RLS) algorithms derived from least squares cost functions. In high or median SNR environments, the problem to be solved is essentially quadratic in nature. Since the RLS algorithm solves quadratic problems exactly, it can find the optimal solution quickly [58]. Alternatively, it can be shown that the convergence behavior of the RLS algorithm is independent of the spectral characteristics of the input signal.

However, in practice, the use of LMS algorithms is widespread mainly due to their computational simplicity [4]. More recently, fast RLS algorithms have been introduced that circumvent the computational burden of the Riccati equation in the conventional RLS algorithm. There are two families of such fast algorithms, corresponding to two possible filter structures: the fast lattice (FLA) and the fast transversal filter (FTF) algorithms. Both algorithms, like the LMS algorithm, lead to a computational complexity of $O(M)$, but FTF has a significantly lower computation than FLA. A comparison of some adaptive algorithms is summarized in [58] (see Table 1).

**TABLE 1 Comparison of Some Adaptive Algorithms**

<table>
<thead>
<tr>
<th></th>
<th>Problem Solved</th>
<th>Tracking Speed</th>
<th>Computational Complexity ($\propto$)</th>
<th>Numerical Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS</td>
<td>LMS</td>
<td>slow</td>
<td>$2M$</td>
<td>exponential</td>
</tr>
<tr>
<td>NLMS</td>
<td>LMS</td>
<td>slow</td>
<td>$(2M+1)\propto + M(\propto) \approx (10M+1)\propto$</td>
<td>exponential</td>
</tr>
<tr>
<td>RLS</td>
<td>RLS</td>
<td>fast</td>
<td>$2M^2 + 6M$</td>
<td>exponential</td>
</tr>
<tr>
<td>FLA</td>
<td>RLS</td>
<td>fast</td>
<td>$14M(\propto) + 2M(\propto) \approx 30M(\propto)$</td>
<td>exponential</td>
</tr>
</tbody>
</table>
TABLE 1 Comparison of Some Adaptive Algorithms

<table>
<thead>
<tr>
<th></th>
<th>Problem Solved</th>
<th>Tracking Speed</th>
<th>Computational Complexity $(\times)$</th>
<th>Numerical Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTF</td>
<td>RLS</td>
<td>fast</td>
<td>$7M$</td>
<td>unstable</td>
</tr>
<tr>
<td>SFTF</td>
<td>RLS</td>
<td>fast</td>
<td>$8M$</td>
<td>exponential</td>
</tr>
</tbody>
</table>

Stock and Kailath [58] indicated that although the FTF algorithm is a very attractive adaptive algorithm, numerical considerations disturb this picture. In their paper, the error feedback mechanism was introduced and the influence of the different feedback parameters was summarized. The analysis of the error propagation was described in several steps. Furthermore, it was stated that no numerically stable FTF algorithm could exist with computational complexity less than $8M$ ($6M$ for the prediction problem). The complete range of exponential weighting factor $\lambda$ was given, $\lambda \in [1-1/2M, 1)$, where $M$ is the order of the adaptive filter. If one would consider optimizing the numerical behavior with respect to $\lambda$, then $\lambda_{opt} = 1 - 0.4/M$ is about the optimal choice. The initial transient could be decreased by increasing the initialization constant $\mu$ (note that this $\mu$ is not the step size parameter as we see previously in the LMS type algorithm). It was found that after an initial transient, the error level for the stabilized FTF algorithm coincides with that of the conventional RLS algorithm, and both do slightly better than the fast lattice algorithm. It was finally noted that in the SFTF, it was possible to deviate slightly from the rigorous FTF equations, for example, by varying the weighting factor $\lambda$ very slowly in time. Indeed, any deviation from the strict algorithm structure acts like the insertion of possibly large numerical errors. However, with a stable error propagation system, the effects of these errors will decay exponentially fast. A Fortran 77 program listing is provided in the appendix for the 8N stable FTF algorithm.
Although researchers have generally made an effort to present algorithms in the general multi-channel context, it appears that single-channel algorithms have been substantially more popular with practitioners [72]. This discrepancy between theory and practice is at least partly due to the fact that multi-channel algorithms (in their straightforward extension of the single-channel forms) require matrix computations. Also, potential numerical difficulties are inherently associated with these matrix operations. The difficulties in the implementation of multi-channel algorithms spurred a growing interest in scalar implementations of multi-channel recursions, namely, implementations that require no matrix processing. Moreover, the increasing interest in dedicated VLSI hardware implementation favors algorithms that can be implemented in modular architectures with a regular and highly parallel structure.

In Slock's paper [72], a general principle for modular decomposition of multi-channel recursions was outlined and then applied to multi-channel lattice algorithms; modular multi-channel lattice algorithms were also independently derived. Slock applied a modular decomposition technique to two types of block processing that can occur in FTF algorithms: multi-channel and multi-experiment filtering [72]. By processing the different channels and/or experiments sequentially (i.e. one at a time) the multi-channel and/or multi-experiment algorithm are/is decomposed into a set of interwined single-channel single-experiment algorithms. The principle of the modular decomposition technique has surfaced (explicitly) in several least squares problems [21][22][26][30]. However, it further appears to be the (implicit) basis for all systolic arrays for matrix computations, such as matrix multiplication, triangular factorization, and QR decomposition.
Alexander and Ghirnikar [88] proposed a new computationally efficient algorithm for recursive least squares filtering, which is based upon an inverse QR decomposition. The method solves directly for the time-recursive least squares filter vector, while avoiding the highly serial backsubstitution step required in previously derived direct QR approaches. Furthermore, it employed orthogonal rotation operations to recursively update the filter, and thus preserved the inherent stability properties of QR approaches to recursive least squares filtering. It is worth noting here the important benefits achieved by the QR-based approaches: first, the time-recursive filtering problem exhibits more stable properties when implemented in the QR decomposition form [4] [50] [42], and second, the rotation computations are easily mapped onto systolic array structures for a parallel implementation [4] [15].

While the simulation results did not rigorously prove the stability of the inverse QR algorithm, the authors did suggest that the algorithm has outstanding numerical properties and thus is an excellent candidate for applications requiring stable performance. Simulations strongly suggested the method preserved the rapid initial convergence of RLS, while maintaining the long-term stability properties of orthogonal rotation methods.

Notice that the new algorithm (inverse QR) proposed in this paper is only a computationally efficient stable algorithm for the RLS algorithm, and the same ERLE performance (defined in Section 4.5 Equation 28) is achieved. RLS, inverse QR and SFTF[58] algorithms were simulated on a PC486 over the course of this thesis (see the following table).

<table>
<thead>
<tr>
<th>Adaptation period [8000] for 70 taps</th>
<th>RLS</th>
<th>Inv-QR</th>
<th>SFTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-real time processing time [second]</td>
<td>740</td>
<td>561</td>
<td>22</td>
</tr>
</tbody>
</table>
Finite memory RLS algorithms have also been presented to facilitate adaptive filtering in nonstationary environments [11]. The introduced short-term sequential regression (STSER) algorithm updates the filter weights using information related to “local” statistics of the input data. The corresponding correlation matrix at any instant depends only on a specified number of previous input samples. According to experimental results, the authors suggested that the STSER algorithm is a promising candidate for adaptive filtering in a nonstationary environment. But aside from their excessive computational requirements, these algorithms also suffer from the same problem as exponential weighting algorithms (refer to Section 2.6).

RLS lattice filters exhibit excellent numerical behavior and are capable of tracking signals with time-varying characteristics. But various phenomena may disturb the normal operation of an adaptive lattice filter and cause a degradation in its performance [64]. In Lev-Ari’s paper [64], sufficient conditions for the stability of several adaptive lattice filter configurations are specified. The feasible domains of two particular adaptive filters are established: the normalized RLS lattice and the unnormalized RLS lattice. Similar results may hold for other adaptive RLS filters such as the QRD lattice of the fast transversal filter (FTF). The primary objective is to underscore the importance of introducing problem-specific constraints in the context of performance analysis for all adaptive filters.

Fan and Liu [89] developed a global convergence proof for both the gradient adaptive lattice (GAL) and the least squares lattice (LSL) using the well-known ordinary differential equation (ODE) approach. The same technique is also used to study the convergence speed of each algorithm. In contrast to the general belief [4] [16], it is concluded, based on
both analysis and simulation that neither algorithm is completely immune to the input data condition or the eigenvalue spread of the input correlation matrix. The reason is imperfect estimation of prediction error covariances before convergence. Previously, Honig and Messerschmitt also argued against this belief [14]. Admittedly though, such dependence on the data condition is far less than the LMS algorithm so that for not too ill-conditioned data, such dependence is simply not observed [4] [16].

2.5 Variants of IIR Adaptation Algorithms

Traditionally, adaptive filters are implemented using finite impulse response (FIR) structures. This is largely due to the simplicity of the adaptation of FIR systems, guaranteed stability, and unimodal mean square error surface [4]. During the adaptation of IIR systems, it is difficult to ensure stability. Moreover, gradient descent adaptive procedures are not guaranteed to find global optima in the nonconvex error surfaces of IIR systems [42]. Indeed, the analysis of IIR algorithms is much more complex than that of FIR algorithms because of the nonlinear property of the output error [20].

However, the acoustic echo path is characterized by a pole-zero system. As a result of modeling this system with an all-zero structure, the number of weights in the cancellers must be large in order to achieve satisfactory performance. The FIR structure results in high cost, high computational complexity and slow convergence. IIR filters are more efficient than FIR filters, and they offer potential performance improvements over their FIR counterparts due to the superior system modeling ability afforded by the poles of an IIR transfer function. This motivates the investigation of adaptive infinite impulse response (IIR) filters for the acoustic echo cancellation, with the hope of a significant reduction of
complexity.

A first attempt consisted of comparing the performances of FIR and IIR adaptive filters to identify an actual echo path. Fan and Jenkins [39] suggested that it might not be advantageous to use an IIR canceler if: 1) the order of the echo path transfer function is not known exactly; 2) there is measurement noise; 3) fast convergence is required. For the acoustic echo cancellation, all three points are true which likely makes the use of this particular adaptive IIR algorithm unsuitable. Other researchers [57] [61] [63] [90] [36] [69] [85] [68] still believe IIR adaptive filters are good candidates.

Extensive simulations show that the IIR canceler performs better than the FIR canceler after convergence [39]. In fact, the slow convergence rate becomes quite troublesome when the order of the IIR canceler is more than two. Mboup and Bonnet [63] investigated the fundamental question of whether the poor performance stems from a sub-optimality of the updating procedures or from an irrelevancy of the pole-zero structure itself. The encouraging results suggested that IIR adaptive filtering must be considered as a possible alternative to FIR adaptive filtering for the acoustic echo cancellation problem.

In addition, IIR systems have an important advantage over FIR systems [90]. For a $K^{th}$ order FIR system, both the region of support of the impulse response and the number of adaptive parameters equal $K$. For an IIR system, the length of the impulse response is uncoupled from the order (or number of parameters) of the system. Since the length of the impulse response of a filter is closely related to the depth of system memory, IIR systems are preferred over FIR systems for modeling systems and signals characterized by deep memory and a small number of free parameters. Principle, Vries and Oliveira [90] intro-
duced the generalized feedforward filter, an IIR filter with restricted feedback architecture (Figure 5).

![Diagram of generalized feedforward filter]

**FIGURE 5** The generalized feedforward filter.

$G(z)$ is the generalized delay operator. The gamma filter is a particular instance of the generalized feedforward filter. In this case, $G(z)$ is the gamma delay operator:

$$G(z) = \frac{\mu}{z - (1 - \mu)}$$

(25)

where $\mu$ is an adaptive memory parameter ($0 < \mu < 2$ for stability). When $\mu = 1$, the gamma filter reduces to Widrow's adaptive transversal filter. The choice of $\mu = 1$ represents a memory structure with maximum resolution and minimum depth. For $\mu \neq 1$, the gamma filter transfer function is of the IIR type.

From the Table 2 [90], it is clear that the gamma filter shares the trivial stability, easy adaptation with FIR filters, and shares the uncoupling of the region of support of the impulse response and the filter order with IIR filters.

<table>
<thead>
<tr>
<th>$K^{th}$ Order Filter</th>
<th>FIR</th>
<th>Gamma</th>
<th>IIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability</td>
<td>Always stable</td>
<td>Trivial stability $0 &lt; \mu &lt; 2$</td>
<td>Nontrivial stability</td>
</tr>
</tbody>
</table>
TABLE 2 Comparison of FIR, IIR, and Gamma filter properties for Kth order filter.

<table>
<thead>
<tr>
<th>Kth Order Filter</th>
<th>FIR</th>
<th>Gamma</th>
<th>IIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complexity of adaptation</td>
<td>(O(K^2))</td>
<td>(O(K))</td>
<td>(O(K))</td>
</tr>
<tr>
<td>Memory depth &amp; order</td>
<td>Coupled K</td>
<td>Uncoupled (K/\mu)</td>
<td>Free</td>
</tr>
</tbody>
</table>

It is suggested that the gamma filter is preferable if the processing problem involves signals with energy concentrated at low frequencies and relatively few degrees of freedom [90].

Chao, Kawabe and Tsujii [85] presented a novel IIR adaptive Gradient Instrumental Variable Echo Canceller (GIVE) using the equation-error method, which is known for its guaranteed convergence and good stability. Two echo cancellation structures were investigated: parallel GIVE structure and series-parallel GIVE structure. Computer simulations were presented under assumption that the echo path is a rational function of 10 over 10 degrees with artificial data.

As analyzed in the beginning of this section, some practical obstacles to the use of adaptive IIR filters include the possibility of local minima in the performance surface, and the potential for unstable behavior during the adaptation. The problem of local minima can be averted by using adaptive algorithms distinct from the output error method, such as the Steiglitz-McBride method [5] or the model reference hyperstable approach [9][12]. The problem of stability, on the other hand, depends strongly on the filter structure used for the IIR system. The stability and numerical advantages of the lattice filter over the direct form have been recognized for some time. Previous attempts at applying lattice structures to adaptive IIR filtering have met with gradient computations of \(O(M^2)\) complexity. To overcome this computational burden, Regalia [68] proposed two new lattice-based algorithms for adaptive IIR filtering and system identification, with both algorithms of \(O(M)\) com-
plexity. The first algorithm is a reinterpretation of the Steiglitz-McBride method, while the second is a variation of the output error method.

2.6 Comparisons Between Various Adaptation Algorithms

LMS and RLS are the two major families of adaptive filtering algorithms. Haykin [4] and some other researchers [27][45][53] suggested that the LMS algorithm has a superior tracking performance compared with the RLS algorithm. Macchi [59] showed, both analytically and by simulation, that the RLS algorithm has a worse steady-state tracking performance than the LMS algorithm for practical situations.

The RLS algorithm has very useful convergence properties after its initial start-up at $t = 0$. However, as the number of iterations grows larger, the RLS algorithm's adaptation capacity decreases substantially. The most widely used solution is called “exponential weighting”. This method uses a scalar constant to attenuate the effects of data from the more distant past. One drawback of exponential weighting is that it may cause the deterministic correlation matrix to become near-singular. As a result, finite precision effects can adversely affect the accuracy of the RLS filter coefficient vector [54].

Echo cancellation in teleconferencing systems or loudspeaking telephones requires the modelling of the acoustic impulse responses (AIR) of rooms. It is hoped that for reverberant situations a pole-zero representation would be much more economical than an all-zero one. Gudvangen and Flockton [87] showed that this is not the case. Contrary to expectations, it is impossible to obtain the same echo return loss with significantly less parameters by using IIR rather than FIR filters to model a reverberant acoustic path.
2.7 Literature Review Summary

This review of previous work on the acoustic echo cancellation shows that many issues remain to be solved. The issues of improving the steady-state performance and improving tracking capabilities during periods of movement in the acoustical environment are two significant problems that must be addressed. Taking this into consideration, in addition to the simplicity of implementation, this thesis concentrates on the FIR adaptive filtering with LMS type algorithms. This thesis explores the dynamic behavior of an acoustic echo canceller under various physical conditions.
CHAPTER 3

EXPERIMENTAL SETUP AND ROOM IMPULSE RESPONSES

3.1 Basic Recording Setup

The measurement setup is shown in Figure 6. The noise source was injected into the microphone end of Handset No.1. This noise was sent to the loudspeaker of Handset No.2 through the telephone switching network. One Digital Audio Tape (DAT) recorder was employed to record simultaneously the primary signal on one channel and the reference signal on the other channel. After recording, the data was downloaded from DAT to PC files.
FIGURE 6 Experimental setup.

Two Hands-free Telephone Sets, a low cost and a medium cost handset as listed in Table 3, were modified to allow proper injection and measurement of signals.

TABLE 3 Two modified hands-free telephone sets.

<table>
<thead>
<tr>
<th>Handset Cost</th>
<th>Handset Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$120</td>
<td>Panasonic KX-T2315 with power adapter unit KX-A09C</td>
</tr>
<tr>
<td>$40</td>
<td>Telemax CP268A</td>
</tr>
</tbody>
</table>

Handset modifications were introduced to provide access to the handset microphone
(transmit) so that external signals/noise could be injected for testing, to provide power to the microphone externally to eliminate 60 Hz and harmonics from being amplified, and to provide an external connection to the speaker terminals for the measurement of speaker driving voltage. In order to allow volume changes without perturbing the measurement setup, the speakerphone volume control and mode switch were modified to locate externally. This is particularly necessary for measurements done inside a noise shielded box or the anechoic chamber.

Interface circuits were built up. Noise/Signal Injector circuits were used to inject noise/voice into the microphone end of Handset No.1 and provide a DC block. National TP3040 Monolithic fifth order elliptic lowpass/fourth order Chebyshev high pass filters provide flat response in telephony passband 200 Hz ~ 3.4 kHz. Switched capacitor filters allow a reduced sampling rate, 8 kHz, without aliasing and help to reduce noise contribution. Adjustable gain and phase inversion amplifiers are necessary for miscellaneous amplification of signals. TL074 quadratic low-noise operational amplifiers are used. Analog devices AD524 Instrumentation amplifier is used to interface the telephone set to a DAT and the DSP board (Ariel DSP96002).

The noise shielded box consists of two boxes, one inside the other to provide noise immunity from the external environment. The box was constructed with 0.25" thick cardboard with corrugated bedding foam glued to both the inside and outside surfaces. The box provides a 21.9 dB attenuation, however, some reflections were noticed.

To minimize the effect of any other measurement errors, a portable recording system was set up using a DAT recorder. Low noise balanced (XLR) cables were used to connect the
loudspeaker and microphone to the DAT. The DAT recorder samples data at a 48 kHz rate (20 kHz bandwidth) with an approximate SNR of 90 dB.

3.2 DAT to PC Transfer

As the DAT recorder allowed access to the recorded data directly in digital form, the data ideally could have been transferred digitally to a PC. However, this requires a specialized DAT interface board for the PC.

It is also possible to playback the data in analog form, re-digitizing this signal using the analog to digital converters (ADC) of a PC-based DSP board. With use of sufficient accuracy (16-bit precision), this re-sampling has a negligible effect on algorithm performance. The re-sampling of the data also allows a simple method of reducing the full bandwidth of the original signal (20 kHz) to a more appropriate 4 kHz telephony bandwidth.

For this thesis, an Ariel DSP-96 System Board was used to re-sample the data. The large memory capacity of the board (256 kwords) allowed up to 15 seconds of data at a sampling rate of 16 kHz to be recorded. Once recorded, the data was downloaded across the DSP host port to a left and right channel PC file (refer to [86] Appendix C about the file format). The required data was later processed on the PC in floating point format.

3.3 Some Techniques of Measuring The Impulse Response

3.3.1 Impulsive Excitation

In measuring the impulse response of a linear system, the most direct approach is to apply an impulse excitation to the system and observe the response. There are two basic difficulties with this approach [17]. The first is generating the impulsive excitation, and the sec-
ond is obtaining adequate dynamic range. This method lacks reproducibility and the noise spectrum is limited. Although techniques exist for producing an impulsive acoustical excitation, such as electronic spark gaps, pistol shots, or exploding balloons, it is difficult to ensure that the energy is equally distributed over all frequencies of interest. Because the duration of the impulse, by definition, is very short, it is difficult to deliver enough energy to the system to overcome the noise that is present. The amplitude of the impulse is limited by the range of linearity of the system and its duration is limited by the range of frequencies of interest.

3.3.2 Maximum-Length Sequences (MLS)

Measuring the impulse response using a noise-like excitation is capable of providing much greater dynamic range than can be obtained using an impulsive excitation. When the noise-like excitation is chosen to be a binary maximal-length shift register sequence\(^1\), several other advantages accrue. Foremost among these is that the cross correlation can be performed very efficiently.

---

\(^1\) A binary sequence whose length is \(2^m - 1\) is called a maximal-length sequence, where \(m\) is any integer.
The measurement setup is shown in Figure 7.

![Diagram of measurement setup](image)

**FIGURE 7 Impulse response measurement with MLS.**

An MLS signal, \( n(k) \), is fed to a loudspeaker. The observed signal at the microphone, \( y(k) \), can be written as

\[
y(k) = n(k) \otimes h(k)
\]  
(26)

where \( h(k) \) is the impulse response between the loudspeaker and microphone.

The cross correlation of the input and the output is related to the autocorrelation of the input by a convolution with the impulse response:

\[
\Phi_{ny}(k) = \Phi_{nn}(k) \otimes h(k)
\]  
(27)

where

\( \Phi_{nn}(k) \) - autocorrelation of input signal

\( \Phi_{ny}(k) \) - cross correlation of the input and the output signal

When the input autocorrelation \( (\Phi_{nn}(k)) \) is equal to the Dirac delta function \( (\delta(k)) \), the
result of convolving autocorrelation ($\Phi_{nn}(k)$) with any function is the function itself, in this case, the impulse response. Thus the impulse response can be recovered by cross-correlating the noise input $n(k)$ with the output $y(k)$.

MLS [17] [48] measures the periodic impulse response (PIR). The PIR can be considered identical to the true impulse response provided that the MLS period equals or exceeds the duration of the system impulse response. In the face of external noise and nonlinearities, the MLS approach [48] is shown to be as robust as time-delay spectrometry\(^1\) (TDS) [6].

### 3.3.3 Adaptive Filtering

One important basic class of adaptive filtering applications is system identification. An adaptive filter is used to provide a linear model that represents the best fit to an unknown system [4] (Figure 8)\(^2\).

The system and the adaptive filter are driven by the same input (the reference signal, $u(n)$). The system output (the primary signal, $d(n)$) is formed by the echo signal and the local speech signal. The adaptive echo canceller synthesizes in real time a replica of the acoustic echo path, $y(n)$, and subtracts it from $d(n)$ to isolate the local speech. The model of the acoustic echo path usually takes the form of a very large transversal filter, which simulates the system impulse response coupled between the loudspeaker and the microphone. In this thesis, adaptive filtering is applied to measure the impulse response of the room.

---

1. Drawback with TDS: long processing time, plus hardware and software complexity [6].
2. Referring to Figure 6, it needs to be clarified that the noise shielded box is not used in this set of experiments.
FIGURE 8 (a) Impulse response measurement with adaptive filtering. (b) Echo canceller modelling of non-linear response.
3.4 Confirmation Experiments

The Echo Return Loss Enhancement (ERLE), a measure of power reduction between the primary signal $d(n)$ and the error signal $e(n)$, is formally defined in Equation 28.

$$ERLE \ (dB) = 10 \log \frac{E[d^2(\text{)}(n)]}{E[e^2(\text{)}(n)]}$$  \ (28)

$E[\text{] is the expectation operator. For each iteration, the primary file and the generated error file were run through an 'ERLE filter', in which this usual long-term average definition of ERLE was substituted with the following moving average definition to obtain a smoothed measure of local ERLE performance:

$$ERLE \ (dB) = 10 \log \frac{\sum_{i=0}^{255} d^2(n-i)}{\sum_{i=0}^{255} e^2(n-i)}$$  \ (29)

The basic structure of the acoustic echo canceller is illustrated in Figure 2. This is a classical system identification problem where the adaptive filter adjusts its coefficients to model the echo path between the loudspeaker and microphone. In this section, experiments were conducted to verify the modelling ability of adaptive filtering under stationary conditions.

3.4.1 Impulse Response Measurement

Using the configuration of Figure 8(a), the impulse response of a conference room ME3033 was measured. A 1500_tap_FIR model with a 16 kHz sampling frequency was used. Adaptation was allowed to continue for 2 seconds (32,000 samples). Tap weights were averaged over the last 8,000 samples, where the convergence was guaranteed.
FIGURE 9 Empty room's configuration.

FIGURE 10 ERLE performance for empty room.

FIGURE 11 Impulse response of empty room.
FIGURE 12 Room’s configuration.

FIGURE 13 Impulse response with table at position A.

FIGURE 14 Impulse response with table at position B.
It is well-known that all the taps of the filter are interpreted as static peaks. Define the corresponding parameters as follows:

\[ D = \text{Distance from obstacle to speakerphone [m]} \]

\[ \Delta L = \text{Path length of direct wave} = 2D [\text{m}] \]

\[ \tau = \text{Delay} = \Delta L/340 [\text{sec}] \]

\[ \text{Tap # delayed} = \tau \times \text{Sampling Frequency} = \tau \times 16 \text{ kHz} \]

Then, Table 4 is derived as:

**TABLE 4 Spatial position of obstacle corresponding to the location of a peak in the impulse response.**

<table>
<thead>
<tr>
<th>Spatial position [m]</th>
<th>D [m]</th>
<th>( \tau ) [m]</th>
<th>Tap # delayed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.59</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>1.2</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>1.8</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>2.4</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>188</td>
<td></td>
</tr>
</tbody>
</table>

Note: size of the table face is 123L×61W cm\(^2\).

The experiments in Figure 12 ~ 14 show that the reflections of the direct wave on the table appear exactly at the anticipated locations in the impulse responses.

The above confirmation experiments are encouraging for use of the adaptive filter technique to model the impulse response.
3.4.2 Measuring the Delay Time from the Loudspeaker to the Microphone

This experiment was done for another project to measure the impulse response of a tube using the MLS method. To verify the identification ability of adaptive filtering, the NLMS algorithm was applied to this set of data\(^1\) to get the impulse response.

The configuration is shown in Figure 15. The white noise was fed to the loudspeaker. The observed signal at the microphone was the primary signal.

![Diagram of the measurement setup](image)

FIGURE 15 Configuration for measuring the impulse response of a tube.

The filter was allowed to adapt for 44,000 samples (1 second). The impulse response was averaged over the last 8,000 samples, where it is guaranteed to achieve a stable ERLE.

As shown in Figure 16, the first and second peaks appear at the 65\(^{th}\) and 140\(^{th}\) taps. Similarly to Table 4,

\[
\Delta L = \text{Path length (of the direct wave/first reflection)} \ [\text{m}]
\]

---

1. This set of data was recorded by Bruno Korst-Fagundes, a M.Eng. student in Department of Electronics, Carleton University.
\[ \tau = \text{Delay} = \frac{\Delta L}{340} \text{ [sec]} \]

Tap \#_{\text{delayed}} = \tau \times \text{SamplingFrequency} = \tau \times 44 \text{ kHz}.

The 65th and 140th taps correspond to the distances of 50 cm and 108 cm, respectively.

By measuring the actual distance from the source to the microphone for the direct wave and the first reflection, it has turned out to be perfectly matched, which gave us much confidence on the modelling ability of adaptive filtering technique.

**FIGURE 16** Impulse response for a tube with sampling frequency 44 kHz.
3.5 Speaker-Microphone Impulse Response Profile Variations Corresponding to Different Configurations

In order to examine and understand the performance of an acoustical echo canceller, the impulse response coupled between the loudspeaker and microphone needs to be studied. For a better view, the first 100 taps of the impulse response in a furnished conference room are plotted in Figure 17. The first 100 taps of the impulse response in an anechoic chamber are plotted in Figure 18. The first 100 taps of the impulse response in an anechoic chamber using separated microphone is plotted in Figure 19.

Define three types of waves transmitted from the loudspeaker to the microphone (see Section 3.3.3 Figure 8(b)):

1. the direct wave and reverberation between the loudspeaker and the microphone through the air;

2. the conducted wave, which is coupled between the loudspeaker and the microphone within the terminal enclosure; and,

3. the reflections outside the terminal enclosure, which are transmitted from the loudspeaker, reflected by the table, walls, furniture etc., and picked up by the microphone.

When the impulse response of a full conference room is measured, all of the above three components are included, as shown in Figure 17. While measuring in an anechoic chamber using the original phoneset, no reflection contributes to the impulse response, as shown in Figure 18. And when the measurement is done in the anechoic chamber using another separated microphone, the impulse response is caused only by the direct wave and
air reverberation, as shown in Figure 19. It is noticed that the contribution of the direct wave and air reverberation is less than that of the conducted wave, and the reflections from the furniture and walls are the strongest.

FIGURE 17 The first 100 taps of the impulse response in a furnished conference room, phoneset on the table.

FIGURE 18 The first 100 taps of the tap impulse response in an anechoic chamber, phoneset on thick foam.
In the following plots, i.e. Figure 21 and 22, we will get the impulse response changing with the object moving above the phoneset. Since the plots are done by the first 50 taps with only six positions for a clear view, the plots look somewhat discontinuous plotted in these coarse locations. Notice that the static peaks appear at the corresponding positions.

FIGURE 20 Configuration for experimental setup, obstacle with good reflection.
FIGURE 21 Set of impulse responses for an object moving 24 - 30 cm above the phone in a conference room.
FIGURE 22 A set of impulse responses for object moving 40 – 45 cm above the phoneset in the anechoic chamber.
CHAPTER 4
LIMITATIONS OF ACOUSTIC ECHO CANCELLATION

4.1 Adaptive Echo Canceller Structure.

Knappe's thesis [86] proposed that the obtainable ERLE (definition in Section 4.5, Equation 28) for a given filter length is limited by the following four factors:
1. Noise: caused by acoustic noise such as fans, A/C and vibrations; thermal and impulsive circuit noise; and DSP-related noise such as truncation, finite word length, and algorithms.

2. Undermodelling of room impulse response: this becomes the limiting factor of the achievable ERLE for adaptive filter length significantly less than the terminal/room impulse response. An increased number of taps results in complexity, noise and slower convergence. Subband processing (filter banks or wavelets) could be used to improve performance.

3. System non-linearities: the non-linearities in the transfer function is mainly due to the loudspeaker. This factor becomes the asymptotic limit for the achievable ERLE if it is significantly greater than the effects of room/measurement noise.

4. Dynamic performance (nonstationarity): the initial convergence and tracking are compromised by people moving in the room or objects moving above the phoneset.

Knappe [86] analyzed the limitations on the obtainable ERLE due to room noise and adaptive filter lengths, and provided derivations for the obtainable ERLE performance of a system. For a linear time invariant physical system, a converged adaptive FIR with M filter taps would reach the asymptote [86] (refer to definition in Equation 28):

\[
\lim_{M \to \infty} (ERLE (dB)) = \lim_{M \to \infty} \left[ 10 \log \frac{E[d^2(i, \cdot)]}{E[e^2(n)]} \right] \\
= \lim_{M \to \infty} \left[ 10 \log \frac{\sigma_d^2}{\sigma_e^2} \right] \\
\equiv 10 \log \frac{\sigma_d^2}{\sigma_e^2} \\
\equiv \frac{(Signal + Noise)}{Noise} \quad \text{Ratio (dB) of Primary (d(n))}
\] (30)
where

d(n) is the primary signal,
e(n) is the estimate error,
v(n) is the noise.

4.2 Noise Sources in Measurement Setup

![Diagram of noise sources]

**FIGURE 24 Residual Noise.**

The various residual noise sources are as following:
$N_1'$  circuit noise from the stereo (i.e. input signal is voice amplified by stereo) or noise source (i.e. input signal is white noise). The spectral content of this noise is essentially a 60 Hz hum and its harmonics which are superimposed on broadband white noise.

$N_2'$  environmental noise picked up by the microphone circuit on the transmit end.

$N_3'$  phone line noise introduced by the telephone circuitry during a call.

$N_4'$  internal circuit noise of the hands-free telephone.

$N_5'$  noise introduced by the differential amplifier.

$N_6'$  noise introduced by the Ariel DSP board during A/D conversion.

Experiments showed that the phone line noise and internal circuit noise of the hands-free phoneset are very low. A 1 the noise caused by the amplifier is even less, which means that the designed interface circuit is working quite well. The sampling frequency is chosen to be 16 kHz with the purpose of decreasing the noise contributed by A/D conversion.

### 4.3 Effects of Filter Length (LMS as an Example)

Simulations of the LMS algorithm of varying length, 25 to 1500 tap, with an adaptation stepsize, $\mu = 0.001$, were conducted using the same experimental data (files AP403.TIM and AR403.TIM\(^1\), stationary condition). In these simulations, the convergence speed was not important as long as the filter converged within the given data window. Adaptation was allowed to continue for 2 seconds (32,000 iterations). The final converged ERLE for each simulation was averaged over the last 8000 ERLE samples, where convergence was guaranteed. This converged ERLE data is summarized below in Table 5 and plotted in

---

\(^1\) Recorded in the anechoic chamber.
Figure 25. It shows the achievable ERLE versus the number of taps for the real data. At the beginning, the system was under-modelled. Starting from the smallest number of taps (i.e. 25), the achievable ERLE increased as more filter taps were added. At a certain point, however, the achievable ERLE curve approached an asymptote, after which adding additional filter taps gave no improvement in the steady-state performance.

![Achievable ERLE vs. number of taps for LMS μ=0.001, Real Data.](image)

**TABLE 5** Converged ERLE vs. Number of taps for LMS $\mu=0.001$, Real Data (signal+noise) to noise ratio is 27.2 dB.

<table>
<thead>
<tr>
<th># of Taps</th>
<th>ERLE (dB) from AP403 TIM and AR403 TIM $\mu=0.001$</th>
<th># of Taps</th>
<th>ERLE (dB) from AP403 TIM and AR403 TIM $\mu=0.001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>6.19</td>
<td>550</td>
<td>27.62</td>
</tr>
<tr>
<td>50</td>
<td>14.02</td>
<td>600</td>
<td>27.03</td>
</tr>
<tr>
<td>75</td>
<td>18.63</td>
<td>650</td>
<td>27.06</td>
</tr>
<tr>
<td>100</td>
<td>20.54</td>
<td>700</td>
<td>27.08</td>
</tr>
<tr>
<td>125</td>
<td>22.76</td>
<td>750</td>
<td>27.11</td>
</tr>
<tr>
<td>150</td>
<td>24.63</td>
<td>800</td>
<td>27.12</td>
</tr>
</tbody>
</table>
TABLE 5  Converged ERLE vs. Number of taps for LMS $\mu=0.001$, Real Data (signal+noise) to noise ratio is 27.2 dB.

<table>
<thead>
<tr>
<th># of Taps</th>
<th>ERLE (dB) from AP403.TIM and AR403.TIM $\mu=0.001$</th>
<th># of Taps</th>
<th>ERLE (dB) from AP403.TIM and AR403.TIM $\mu=0.001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>175</td>
<td>25.32</td>
<td>850</td>
<td>27.14</td>
</tr>
<tr>
<td>200</td>
<td>25.59</td>
<td>900</td>
<td>27.15</td>
</tr>
<tr>
<td>250</td>
<td>26.20</td>
<td>1000</td>
<td>27.16</td>
</tr>
<tr>
<td>300</td>
<td>26.46</td>
<td>1100</td>
<td>27.16</td>
</tr>
<tr>
<td>350</td>
<td>26.64</td>
<td>1200</td>
<td>27.16</td>
</tr>
<tr>
<td>400</td>
<td>26.79</td>
<td>1300</td>
<td>27.17</td>
</tr>
<tr>
<td>450</td>
<td>26.90</td>
<td>1400</td>
<td>27.18</td>
</tr>
<tr>
<td>500</td>
<td>26.99</td>
<td>1500</td>
<td>27.18</td>
</tr>
</tbody>
</table>

Notice that in Figure 25, the (signal+noise) to noise ratio of the primary signal, 27.2 dB, acted as the ERLE asymptote. Since the volume was very low in the experiment (for AP403.TIM and AR403.TIM), there is no system non-linearity involved. As the length of the filter increases, the uncancelled tail takes on less significance as an asymptote is reached, beyond which point the ERLE does not improve. The maximum achievable ERLE will be equivalent to the (signal + noise) to noise ratio of the returning echoed signal (primary) (see Equation 30).

4.4 Effect of Non-Linearity on ERLE

This section examines the effect of the non-linearity of the loudspeaker on the ERLE performance\(^1\). With the level of background noise fixed, increasing the volume of the signal means that the (signal+noise) to noise ratio is increased. Intuitively speaking, the achievable ERLE should increase also (refer to Equation 30).

Figure 26 shows the achievable ERLE while increasing the (signal+noise) to noise ratio

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\(^1\) This set of experimental data was collected in the anechoic chamber.
(refer to Table 6). At the beginning, when the volume is increased, the ERLE also increases. However, when the volume exceeds the linear range of the loudspeaker, it is clear that the achievable ERLE decreases.

**Figure 26** Achievable ERLE vs. voltage before the loudspeaker for 750 tap NLMS, $\alpha = 1.0$, white noise as input signal.
TABLE 6 Effect of non-linearity of loudspeaker on ERLE (dB).

<table>
<thead>
<tr>
<th>DAT REC LEVEL</th>
<th>Volume</th>
<th>RMS voltmeter (mV) before loudspeaker</th>
<th>Converged ERLE (dB) (last 8,000 out of 32,000 iteration, 16 KHz) 750 tap NLMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>0.05</td>
<td>26.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2</td>
<td>26.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.75</td>
<td>26.90</td>
</tr>
<tr>
<td>White noise</td>
<td>2</td>
<td>2</td>
<td>27.35</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>27.60</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>2</td>
<td>26.32</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>2</td>
<td>24.63</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>2</td>
<td>22.58</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>2</td>
<td>21.35</td>
</tr>
<tr>
<td>Medium</td>
<td>40</td>
<td>2</td>
<td>21.19</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>75</td>
<td>20.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>20.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>130</td>
<td>19.83</td>
</tr>
</tbody>
</table>

This problem is mainly due to the non-linearity of the loudspeaker. Detailed discussion is beyond the scope of this thesis. The basic idea to solve this problem is training a nonlinear system such as neural network, Volterra filter etc., to model the loudspeaker, and applying this nonlinear system before the regular linear adaptive filter or adding the inverse nonlinear system before the loudspeaker. Figure 27 shows different possible configurations to solve the non-linearity problem.
FIGURE 27 Modelling speaker with a nonlinear system to improve the performance of echo cancellation.
CHAPTER 5

DYNAMIC PERFORMANCE OF ACOUSTIC ECHO CANCELLATION

This chapter examines the initial convergence speed, the steady-state performance (after convergence of the adaptive filter, i.e., converged ERLE), and the tracking ability of acoustic echo cancellation. The initial convergence speed is the time required to reach a steady-state mean-squared error variance from the algorithm initialization. The tracking ability is the ability to maintain a constant mean-squared error variance while changes are occurring in the impulse response to be modelled. The tracking ability appears to be similar to the initial convergence speed. But it is not the case that good performance for one implies good performance for the other [4]. In the area of acoustic echo cancellation, the tracking ability turns out to be a more important consideration than the initial convergence speed.

The experiment was made by applying a reference signal to the loudspeaker and recording the primary signal coupled to the microphone (see Section 3.1, Figure 6). This was performed in two different room environments: one is a regular conference room\(^1\), and the other an anechoic chamber\(^2\). In order to study the tracking performance of algorithms, nonstationarities were imitated by passing objects one foot above the phoneset and by people moving one foot away from the phoneset.

---

1 ME3033, in Minto CASE, Department of Systems and Computer Engineering, Carleton University.
2 In Department of Psychology, Carleton University.
During the course of this thesis, simulations of several algorithms (LMS, NLMS, VSS LMS, ES_LMS, ES_NLMS, RLS, SUTF) were conducted. The LMS algorithm was described in Section 2.1.1. The NLMS algorithm was described in Section 2.1.2. The RLS algorithm was described in Section 2.1.3. The VSS_LMS algorithm was described in Section 2.3.1. The SUTF[58] and ES-based algorithms [51] were also studied. The adaptations of ES_LMS and ES_NLMS algorithm were listed as the following (refer to Section 2.1.1, Equation 1 and Equation 2).

**ES_LMS algorithm:**

\[ W(n+1) = W(n) + \mu e(n) X(n) \]  
\[ (31) \]

where

\( \mu \) is the diagonal step size matrix, \( \mu = \text{diag}(\mu_0, \mu_1, ..., \mu_{M-1}) \), each step size \( \mu_i = \mu_0 \cdot (\text{decay\_factor})^i, i = 0, 1, ..., M-1. \)

**ES_NLMS algorithm:**

\[ W(n+1) = W(n) + \alpha e(n) X(n) \]  
\[ (32) \]

where

\( \alpha \) is the diagonal step gain matrix, \( \alpha = \text{diag}(\alpha_0, \alpha_1, ..., \alpha_{M-1}) \), each gain \( \alpha_i = \alpha_0 \cdot (\text{decay\_factor})^i, i = 0, 1, ..., M-1. \)
5.1 Steady-State Performance of LMS-Based Algorithms

Simulations of several LMS-based algorithms were conducted using experimental data (files NP1001.TIM and NR1001.TIM, stationary conditions): 750 tap LMS algorithm with \( \mu = 0.0015 \) (Figure 28), 750 tap ES_LMS algorithm with \( \mu_n = 0.003E-0.005n \) (Figure 29), 750 tap NLMS algorithm with \( \alpha = 1.0 \) (Figure 30), 750 tap ES_NLMS algorithm with \( \alpha_n = 1.2E-0.005n \) (Figure 31).

Adaptation was allowed to continue for 0.5 second (8,000 samples). Corresponding parameters were chosen to gain the same level ERLE performance. The final converged ERLE was averaged over the last 2,000 ERLE samples, where the convergence was guaranteed (Table 7).

<table>
<thead>
<tr>
<th># of Taps</th>
<th>Converged ERLE (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NLMS</td>
</tr>
</tbody>
</table>

The comparison between Figure 28 and 29/Figure 30 and 31 demonstrates that the ES_LMS/ES_NLMS algorithm provides a faster initial convergence speed for the same level of misadjustment than the LMS/NLMS algorithm. The comparison between Figure 28 and 30/Figure 29 and 31 demonstrates that the NLMS/ES_NLMS algorithm has a faster initial convergence speed for the same level of misadjustment than the LMS/ES_LMS algorithm.
FIGURE 28 ERLE for 750 tap LMS with $\mu = 0.001$, for NP1001.TIM and NR1001.TIM stationary conditions.

FIGURE 29 ERLE for 750 tap ES_LMS with $\mu_n = 0.002E-0.005n$, for NP1001.TIM and NR1001.TIM stationary conditions.
FIGURE 30 ERLE for 750 tap NLMS with $\alpha=0.1$, for NP1001.TIM and NR1001.TIM stationary conditions.

FIGURE 31 ERLE for 750 tap ES_NLMS, $\alpha(n)=1.2E-0.005n$, for NP1001.TIM and NR1001.TIM stationary conditions.
5.1.1 Characteristics of ES_LMS

5.1.1.1 Effects of Filter Length

Simulations of the ES_LMS algorithm of varying lengths, 25 to 1500 taps, with an adaptation stepsize, $\mu_n = 0.0017E-0.005n$, were conducted using experimental data (files NP1001.TIM and NR1001.TIM\(^1\), stationary condition). Adaptation was allowed to continue for 2 seconds (32,000 iterations). In these simulations, the convergence speed was not important as long as the filter converged within the given data window. The final converged ERLE for each simulation was averaged over the last 8000 ERLE samples, where convergence was guaranteed. This converged ERLE data is summarized below in Table 8 and plotted in Figure 32. It shows the achievable ERLE versus the number of taps for the real data. It shows the same tendency as did the LMS filtering (see Figure 25 in Section 4.3).

It is interesting to note the differences between Figures 25 and 32. In Figure 25, the converged ERLE curve approaches the asymptote at around the number of taps equal to 400, and in Figure 32 the corresponding number of taps is around 1000. This is because the experimental data for Figures 25 and 32 were collected in the anechoic chamber and the conference room, respectively. The impulse response coupled between the loudspeaker and the microphone is much shorter in the anechoic chamber than in the conference room.

\(^1\) Recorded in the conference room. ME3033
TABLE 8 Converged ERLE vs. Number of taps for ES_LMS, Real Data.

<table>
<thead>
<tr>
<th># of Taps</th>
<th>ERLE (dB) from NP1001 TIM and NR1001 TIM, $\mu_n = 0.0017E-0.005n$</th>
<th># of Taps</th>
<th>ERLE (dB) from NP1001 TIM and NR1001 TIM, $\mu_n = 0.0017E-0.005n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>10.94</td>
<td>600</td>
<td>21.62</td>
</tr>
<tr>
<td>50</td>
<td>14.50</td>
<td>650</td>
<td>21.76</td>
</tr>
<tr>
<td>75</td>
<td>16.18</td>
<td>700</td>
<td>22.12</td>
</tr>
<tr>
<td>100</td>
<td>16.71</td>
<td>750</td>
<td>22.39</td>
</tr>
<tr>
<td>150</td>
<td>17.48</td>
<td>800</td>
<td>22.50</td>
</tr>
<tr>
<td>200</td>
<td>18.36</td>
<td>900</td>
<td>22.80</td>
</tr>
<tr>
<td>250</td>
<td>19.25</td>
<td>1000</td>
<td>22.94</td>
</tr>
<tr>
<td>300</td>
<td>19.75</td>
<td>1100</td>
<td>23.03</td>
</tr>
<tr>
<td>350</td>
<td>20.08</td>
<td>1200</td>
<td>23.08</td>
</tr>
<tr>
<td>400</td>
<td>20.24</td>
<td>1300</td>
<td>23.12</td>
</tr>
<tr>
<td>450</td>
<td>20.62</td>
<td>1400</td>
<td>23.13</td>
</tr>
<tr>
<td>500</td>
<td>21.17</td>
<td>1500</td>
<td>23.14</td>
</tr>
<tr>
<td>550</td>
<td>21.34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 32 ERLE vs. Number of Taps for ES_LMS $\mu_n = 0.0017E-0.005n$, Real Data.
5.1.2.2 Effects of Decay Factor

The simulation data is summarized in Table 13. The corresponding curve in Figure 37 (the first ten data points are plotted in Figure 37(a)) shows the same features as that with ES_LMS plotted in Figure 33.

**TABLE 13 Converged ERLE vs. Decay Factor for 800 tap ES_NLMS, Real Data.**

<table>
<thead>
<tr>
<th>Decay Factor</th>
<th>$\xi_{\text{RLE}}$ (dB) from NP1001.TIM and NR1001.TIM $\alpha_n = 0.2E-(\text{factor})^n$</th>
<th>Decay Factor</th>
<th>$\xi_{\text{RLE}}$ (dB) from NP1001.TIM and NR1001.TIM $\alpha_n = 0.2E-(\text{factor})^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>22.40</td>
<td>0.005</td>
<td>22.79</td>
</tr>
<tr>
<td>1.0e-16</td>
<td>22.41</td>
<td>0.006</td>
<td>22.68</td>
</tr>
<tr>
<td>1.0e-8</td>
<td>22.44</td>
<td>0.007</td>
<td>22.44</td>
</tr>
<tr>
<td>1.0e-7</td>
<td>22.45</td>
<td>0.008</td>
<td>21.79</td>
</tr>
<tr>
<td>1.0e-6</td>
<td>22.46</td>
<td>0.009</td>
<td>21.38</td>
</tr>
<tr>
<td>1.0e-5</td>
<td>22.47</td>
<td>0.01</td>
<td>20.32</td>
</tr>
<tr>
<td>5.0e-5</td>
<td>22.50</td>
<td>0.016</td>
<td>19.21</td>
</tr>
</tbody>
</table>
Simulation results are listed in Table 10. Notice in the first case, when the decay factor is 0, the ES_LMS algorithm becomes the regular LMS algorithm, and the steady-state ERLE is not as good as the ES_LMS algorithm with a decay factor in the range of 0.00005 – 0.006.

In order to get a clear plotting, the first four data points are plotted in Figure 33(a). It is shown in Figure 33 that the performance of the ES_LMS algorithm is relatively stable for a wide range of decay factors (0.00005 ~ 0.006).

<table>
<thead>
<tr>
<th>Decay Factor</th>
<th>ERLE (dB) from NPI0101 TIM and NR1001 TIM</th>
<th>Decay Factor</th>
<th>ERLE (dB) from NPI0101 TIM and NR1001 TIM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \mu_n = 0.0017 \text{E-(factor)} \times n )</td>
<td></td>
<td>( \mu_n = 0.0017 \text{E-(factor)} \times n )</td>
</tr>
<tr>
<td>0</td>
<td>21.80</td>
<td>0.016</td>
<td>19.35</td>
</tr>
<tr>
<td>1.0E-8</td>
<td>21.82</td>
<td>0.020</td>
<td>18.89</td>
</tr>
<tr>
<td>0.00001</td>
<td>21.83</td>
<td>0.024</td>
<td>18.32</td>
</tr>
<tr>
<td>0.00003</td>
<td>21.84</td>
<td>0.027</td>
<td>17.96</td>
</tr>
<tr>
<td>0.00005</td>
<td>21.85</td>
<td>0.03</td>
<td>17.64</td>
</tr>
<tr>
<td>0.0001</td>
<td>21.86</td>
<td>0.04</td>
<td>16.79</td>
</tr>
<tr>
<td>0.0003</td>
<td>21.87</td>
<td>0.05</td>
<td>16.00</td>
</tr>
<tr>
<td>0.0005</td>
<td>21.97</td>
<td>0.07</td>
<td>14.92</td>
</tr>
<tr>
<td>0.001</td>
<td>22.04</td>
<td>0.1</td>
<td>13.29</td>
</tr>
<tr>
<td>0.002</td>
<td>22.16</td>
<td>0.12</td>
<td>12.20</td>
</tr>
<tr>
<td>0.003</td>
<td>22.23</td>
<td>0.14</td>
<td>11.20</td>
</tr>
<tr>
<td>0.004</td>
<td>22.34</td>
<td>0.16</td>
<td>10.33</td>
</tr>
<tr>
<td>0.005</td>
<td>22.50</td>
<td>0.20</td>
<td>8.84</td>
</tr>
<tr>
<td>0.006</td>
<td>21.89</td>
<td>0.25</td>
<td>7.38</td>
</tr>
<tr>
<td>0.007</td>
<td>21.58</td>
<td>0.3</td>
<td>6.26</td>
</tr>
<tr>
<td>0.008</td>
<td>21.21</td>
<td>0.5</td>
<td>2.38</td>
</tr>
<tr>
<td>0.009</td>
<td>20.85</td>
<td>1.0</td>
<td>0.156</td>
</tr>
<tr>
<td>0.01</td>
<td>20.54</td>
<td>1.5</td>
<td>0.021</td>
</tr>
<tr>
<td>0.013</td>
<td>19.87</td>
<td>2.0</td>
<td>0.008</td>
</tr>
</tbody>
</table>
FIGURE 33 Converged ERLE vs. Step Size Decay Factor for 800 tap ES_LMS,
$\mu_n = 0.0017E^-(\text{factor})^n$, Real Data.
The exponential decay curve for the optimum decay factor for the tested conference room is shown in Figure 34.

![Exp. decay curve](image)

**FIGURE 34** Exponential decay curve of step gain (stepsize) element $\mu_n$ with initial $\mu_0 = 0.0017$, and decay factor $0.005$, i.e. $\mu_n = 0.0017E-0.005n$.

### 5.1.1.3 Effects of Initial Step Size

For the regular LMS, it is suggested that the conservative choice of a value for the stepsize parameter $\mu$ less than twice the reciprocal of the total input power offers a practical method for ensuring the convergence of the mean-squared error [4]:

$$0 < \mu < \frac{2}{M - 1} = \frac{2}{\sum_{i=0}^{\text{total} - \text{input} - \text{power}}}$$  \(33\)

where $\lambda_i$ is the eigenvalue of the input correlation matrix, the total input power is equivalent to $\text{Mr}(0)$, and $\text{r}(0)$ is the variance of the input signal, and $M$ is the filter order. In this
simulation the input signal is white noise with zero mean and unity variance. This implies that \( \mu \) must be chosen to satisfy the criterion \( 0 < \mu < 2/M \). For a 800 tap LMS, the theoretical maximum \( \mu \) is 0.0025.

According to the characteristics of the ES_LMS algorithm, the convergence condition is derived to be (Appendix A)

\[
0 < \mu_0 < \frac{2}{\sum_{i=0}^{M-1} \lambda_i E - Di}
\]

(34)

where \( \lambda_0 > \lambda_1 > ... > \lambda_{M-1} \), \( D \) is the decay factor for the step size, \( \mu_i = \mu_0 E - Di \). The flexible range for the initial step size \( \mu_0 \) has been largely increased, which is valuable in practice. This analysis correlates well with the simulation results (Table 11, Figure 35).
TABLE 11 Converged ERLE vs. Initial Step Size for 800 tap ES_LMS with decay factor 0.005. Real Data.

<table>
<thead>
<tr>
<th>Initial step size $\mu_0$</th>
<th>ERLE (dB) from NP1001.TIM and NR1001.TIM $\mu_n = \mu_0 e^{-0.005n}$</th>
<th>Initial step size $\mu_0$</th>
<th>ERLE (dB) from NP1001.TIM and NR1001.TIM $\mu_n = \mu_0 e^{-0.005n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.05E-4</td>
<td>22.55</td>
<td>0.0066</td>
<td>22.21</td>
</tr>
<tr>
<td>0.0012</td>
<td>22.54</td>
<td>0.0073</td>
<td>22.09</td>
</tr>
<tr>
<td>0.0016</td>
<td>22.52</td>
<td>0.008</td>
<td>21.93</td>
</tr>
<tr>
<td>0.002</td>
<td>22.48</td>
<td>0.0089</td>
<td>21.67</td>
</tr>
<tr>
<td>0.0022</td>
<td>22.47</td>
<td>0.0098</td>
<td>21.33</td>
</tr>
<tr>
<td>0.0027</td>
<td>22.46</td>
<td>0.0103</td>
<td>21.12</td>
</tr>
<tr>
<td>0.0031</td>
<td>22.46</td>
<td>0.0107</td>
<td>20.88</td>
</tr>
<tr>
<td>0.0036</td>
<td>22.45</td>
<td>0.0112</td>
<td>20.60</td>
</tr>
<tr>
<td>0.0041</td>
<td>22.43</td>
<td>0.0116</td>
<td>18.29</td>
</tr>
<tr>
<td>0.0047</td>
<td>22.41</td>
<td>0.0121</td>
<td>11.89</td>
</tr>
<tr>
<td>0.0052</td>
<td>22.37</td>
<td>0.0125</td>
<td>6.07</td>
</tr>
<tr>
<td>0.0059</td>
<td>22.30</td>
<td>0.0130</td>
<td>-6.99</td>
</tr>
</tbody>
</table>

FIGURE 35 Converged ERLF vs. Initial Step size $\mu_0$ for 800 tap ES_LMS with decay factor 0.005. Real Data, SNR = 33.1 dB.
5.1.2 Characteristics of ES_NLMS

5.1.2.1 Effects of Filter Length

Simulations of the ES_NLMS algorithm of varying length, 25 to 1500 tap, with an adaptation step gain, \( \alpha_n = 0.2E-0.005n \), were conducted using experimental data (files NP1001.TIM and NR1001.TIM, stationary condition). This converged ERLE data is summarized below in Table 8 and plotted in Figure 36. The data shows the achievable ERLE versus the number of taps for the real data. It shows the same tendency as in the ES_LMS filtering (see Section 5.1.1.1, Figure 32).

<table>
<thead>
<tr>
<th># of Taps</th>
<th>ERLE (dB) from NP1001.TIM and NR1001.TIM ( \alpha_n = 0.2E-0.005n )</th>
<th># of Taps</th>
<th>ERLE (dB) from NP1001.TIM and NR1001.TIM ( \alpha_n = 0.2E-0.005n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>11.70</td>
<td>650</td>
<td>22.35</td>
</tr>
<tr>
<td>50</td>
<td>15.04</td>
<td>700</td>
<td>22.52</td>
</tr>
<tr>
<td>75</td>
<td>16.57</td>
<td>750</td>
<td>22.63</td>
</tr>
<tr>
<td>100</td>
<td>16.99</td>
<td>800</td>
<td>22.79</td>
</tr>
<tr>
<td>150</td>
<td>17.92</td>
<td>900</td>
<td>22.95</td>
</tr>
<tr>
<td>200</td>
<td>18.87</td>
<td>1000</td>
<td>22.99</td>
</tr>
<tr>
<td>250</td>
<td>19.75</td>
<td>1100</td>
<td>22.96</td>
</tr>
<tr>
<td>300</td>
<td>20.32</td>
<td>1200</td>
<td>22.92</td>
</tr>
<tr>
<td>350</td>
<td>20.58</td>
<td>1300</td>
<td>22.87</td>
</tr>
<tr>
<td>400</td>
<td>20.77</td>
<td>1400</td>
<td>22.82</td>
</tr>
<tr>
<td>450</td>
<td>21.18</td>
<td>1500</td>
<td>22.66</td>
</tr>
<tr>
<td>500</td>
<td>21.72</td>
<td>1600</td>
<td>22.59</td>
</tr>
<tr>
<td>550</td>
<td>22.01</td>
<td>1700</td>
<td>22.45</td>
</tr>
<tr>
<td>600</td>
<td>22.29</td>
<td>1800</td>
<td>22.36</td>
</tr>
</tbody>
</table>
FIGURE 36 ERLE vs. Number of taps for ES_NLMS $\alpha_n=0.2E-0.005n$, Real Data.

5.1.2.2 Effects of Decay Factor

The simulation data is summarized in Table 13. The corresponding curve in Figure 37 (the first ten data points are plotted in Figure 37(a)) shows the same features as that with ES_LMS plotted in Figure 33.

<table>
<thead>
<tr>
<th>Decay Factor</th>
<th>$\Delta$ERLE (dB) from NP1001.TIM and NR1001.TIM $\alpha_n = 0.2E-(factor)\times n$</th>
<th>Decay Factor</th>
<th>$\Delta$ERLE (dB) from NP1001.TIM and NR1001.TIM $\alpha_n = 0.2E-(factor)\times n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>22.40</td>
<td>0.005</td>
<td>22.79</td>
</tr>
<tr>
<td>1.0e-16</td>
<td>22.41</td>
<td>0.006</td>
<td>22.68</td>
</tr>
<tr>
<td>1.0e-8</td>
<td>22.44</td>
<td>0.007</td>
<td>22.44</td>
</tr>
<tr>
<td>1.0e-7</td>
<td>22.45</td>
<td>0.008</td>
<td>21.79</td>
</tr>
<tr>
<td>1.0e-6</td>
<td>22.46</td>
<td>0.009</td>
<td>21.38</td>
</tr>
<tr>
<td>1.0e-5</td>
<td>22.47</td>
<td>0.01</td>
<td>20.32</td>
</tr>
<tr>
<td>5.0e-5</td>
<td>22.50</td>
<td>0.016</td>
<td>19.21</td>
</tr>
</tbody>
</table>

TABLE 13 Converged ERLE vs. Decay Factor for 800 tap ES_NLMS, Real Data.
### TABLE 13 Converged ERLE vs. Decay Factor for 800 tap ES_NLMS, Real Data.

<table>
<thead>
<tr>
<th>Decay Factor</th>
<th>ERLE (dB) from NP1001 TIM and NR1001 TIM $\alpha_n = 0.2E-(factor)*n$</th>
<th>Decay Factor</th>
<th>ERLE (dB) from NP1001 TIM and NR1001 TIM $\alpha_n = 0.2E-(factor)*n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>22.53</td>
<td>0.027</td>
<td>17.54</td>
</tr>
<tr>
<td>0.0003</td>
<td>22.54</td>
<td>0.04</td>
<td>16.41</td>
</tr>
<tr>
<td>0.0005</td>
<td>22.55</td>
<td>0.07</td>
<td>14.84</td>
</tr>
<tr>
<td>0.001</td>
<td>22.62</td>
<td>0.1</td>
<td>13.14</td>
</tr>
<tr>
<td>0.002</td>
<td>22.68</td>
<td>0.15</td>
<td>10.63</td>
</tr>
<tr>
<td>0.003</td>
<td>22.77</td>
<td>0.25</td>
<td>7.32</td>
</tr>
<tr>
<td>0.004</td>
<td>22.78</td>
<td>0.5</td>
<td>2.32</td>
</tr>
</tbody>
</table>

### FIGURE 37 Converged ERLE vs. Step Gain Decay Factor for 800 tap ES_NLMS, $\alpha_n = 0.2E-(factor)*n$, Real Data.
The exponential decay curve for the optimum decay factor for the tested conference room is plotted in Figure 38.

![Exponential decay curve of step gain element $\alpha$, with initial $\alpha_0 = 0.2$, and decay factor 0.005, i.e. $\alpha_n = 0.2E^{-0.005n}$.](image)

**FIGURE 38 Exponential decay curve of step gain element $\alpha$, with initial $\alpha_0 = 0.2$, and decay factor 0.005, i.e. $\alpha_n = 0.2E^{-0.005n}$.**

5.1.2.3 **Effects of Initial Step Gain**

The convergence condition for ES_NLMS is derived to be $0 < \alpha_0 < 2$ (Appendix B), which is the same as the conventional NLMS and correlates well with the simulation results (Table 14 and Figure 39).

**TABLE 14 Converged ERLE vs. Initial Step gain for 800 tap ES_NLMS with decay factor 0.005, Real Data.**

<table>
<thead>
<tr>
<th>Initial step gain $\alpha_0$</th>
<th>ERLE (dB) from NP1001.TIM and NR1001.TIM $\alpha_n = \alpha_0 E^{-0.005n}$</th>
<th>Initial step gain $\alpha_0$</th>
<th>ERLE (dB) from NP1001.TIM and NR1001.TIM $\alpha_n = \alpha_0 E^{-0.005n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>22.80</td>
<td>1.4</td>
<td>22.45</td>
</tr>
<tr>
<td>0.1</td>
<td>22.79</td>
<td>1.6</td>
<td>22.33</td>
</tr>
<tr>
<td>0.3</td>
<td>22.78</td>
<td>1.8</td>
<td>22.03</td>
</tr>
</tbody>
</table>
TABLE 14 Converged ERLE vs. Initial Step gain for 800 tap ES_NLMS with decay factor 0.005, Real Data.

<table>
<thead>
<tr>
<th>Initial step gain $\alpha_0$</th>
<th>$\text{ERLE (dB)}$ from NP1001.TIM and NR1001.TIM $\alpha_n = \alpha_0 e^{-0.005n}$</th>
<th>Initial step gain $\alpha_0$</th>
<th>$\text{ERLE (dB)}$ from NP1001.TIM and NR1001.TIM $\alpha_n = \alpha_0 e^{-0.005n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>22.78</td>
<td>2.0</td>
<td>20.83</td>
</tr>
<tr>
<td>0.7</td>
<td>22.76</td>
<td>2.2</td>
<td>19.99</td>
</tr>
<tr>
<td>0.9</td>
<td>22.63</td>
<td>2.4</td>
<td>16.06</td>
</tr>
<tr>
<td>1.0</td>
<td>22.56</td>
<td>2.8</td>
<td>-4.36</td>
</tr>
<tr>
<td>1.2</td>
<td>22.48</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 39 Converged ERLE vs. Initial Step gain $\alpha_0$ for 800 tap ES_NLMS with decay factor 0.005, i.e. $\alpha_n = \alpha_0 e^{-0.005n}$, Real Data.
5.2 Tracking Performance of LMS-Based Algorithms

5.2.1 Performance of Exponential Step Algorithms

This set of simulations investigated the tracking ability of several LMS-based algorithms (LMS, ES_LMS, NLMS, ES_NLMS) using the same real data (files NP1003.TIM and NR1003.TIM\(^1\), nonstationary conditions).

The performance comparison between the NLMS and the ES_NLMS algorithms for a 1500 tap FIR adaptive filter is shown in Figure 40. The solid line is for the NLMS algorithm, and the dashed line for the ES_NLMS algorithm. Corresponding parameters are chosen to gain the same level of ERLE performance. The final converged ERLE was averaged over the last 16,000 ERLE samples, i.e. 1 second, where the convergence is guaranteed (Table 15). In Figure 40, the final ERLEs for the NLMS and the ES_NLMS algorithms are 23.35 dB and 23.31 dB, respectively. The comparison demonstrated that ES_NLMS algorithm not only provides faster initial convergence speed than the conventional NLMS algorithm for the same level of misadjustment, but also offers its impressive tracking ability when nonstationarity occurs.

<table>
<thead>
<tr>
<th># of Taps</th>
<th>NLMS</th>
<th>ES_NLMS</th>
<th>LMS</th>
<th>ES_LMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>23.35</td>
<td>23.31</td>
<td>23.29</td>
<td>23.92</td>
</tr>
<tr>
<td>750</td>
<td>21.90</td>
<td>21.82</td>
<td>22.25</td>
<td>22.44</td>
</tr>
<tr>
<td>375</td>
<td>19.94</td>
<td>19.98</td>
<td>20.15</td>
<td>20.26</td>
</tr>
</tbody>
</table>

Similar performance comparisons are shown in Figure 41 for a 750 tap NLMS algorithm

1. Recorded in the conference room, ME3033.
and an ES_NLMS algorithm, Figure 42 for a 375 tap NLMS algorithm and an ES_NLMS algorithm, Figure 43 for a 1500 tap LMS algorithm and an ES_LMS algorithm, Figure 44 for a 750 tap LMS algorithm and an ES_LMS algorithm, and Figure 45 for a 375 tap LMS algorithm and an ES_LMS algorithm.

The comparisons lead to a conclusion that the ES type algorithms outperform their regular counterparts in the initial convergence speed and tracking ability.

The principle of ES_based algorithm is to set the factor for adaptation speed (step size for ES_LMS, or step gain for ES_NLMS) as an exponentially decaying vector according to the characteristics of the room impulse response and the variations in this room impulse response.

There is another advantage in the ES_based algorithms. Figures 34 and 38 imply that the ES_based algorithms work as a combination of short fast adapted filtering and long slow adapted filtering. The shorter part is adapted with fairly large step gains, and provides fast tracking ability, while the longer part is adapted with small step gains, so the residual error is very low and the steady-state ERLE performance is high. This analysis has been verified by the computer simulations (see Figures 40 to 45).
NLMS Family Algorithms

I. 1500 tap ES_NLMS & NLMS

FIGURE 40 ERLE performance comparison between 1500 tap NLMS (solid line) with $\alpha=0.3$, and 1500 tap ES_NLMS (dashed line) with $\alpha_n=1.2E^{-0.005n}$ for NP1003 TIM and NR1003.TIM nonstationary conditions.

II. 750 tap ES_NLMS & NLMS

FIGURE 41 ERLE performance comparison between 750 tap NLMS (solid line) with $\alpha=0.3$, and 750 tap ES_NLMS (dashed line) with $\alpha_n=1.2E^{-0.005n}$ for NP1003.TIM and NR1003.TIM nonstationary conditions.
III. 375 tap ES_NLMS & NLMS

FIGURE 42 ERLE performance comparison between 375 tap NLMS (solid line) with $\alpha=0.3$, and 375 tap ES_NLMS (dashed line) with $\alpha_n=1.3E-0.005n$ for NP1003.TIM and NR1003.TIM nonstationary conditions.

LMS Family Algorithms

IV. 1500 tap ES_LMS & LMS

FIGURE 43 ERLE performance comparison between 1500 tap LMS (solid line) with $\mu=8.3E-4$ and 1500 tap ES_LMS (dashed line) with $\mu_n=0.0013E-0.005n$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
V. 750 tap ES_LMS & LMS

FIGURE 44 ERLE performance comparison between 750 tap LMS (solid line) with $\mu=0.0015$ and 750 tap ES_LMS (dashed line) with $\mu_a=0.002E-0.005n$, for NP1003.TIM and NR1003.TIM nonstationary conditions.

VI. 375 tap ES_LMS & LMS

FIGURE 45 ERLE performance comparison between 375 tap LMS (solid line) with $\mu=0.003$ and 375 tap ES_LMS (dashed line) with $\mu_a=0.005E-0.005n$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
5.2.2 Performance of Windowed Step Gain/Size Algorithms

The ES-based algorithm (ES_NLMS/ES_LMS) has been shown to converge faster than the conventional NLMS/LMS algorithm. In the ES-based algorithm, the step gain \( \alpha_i \) and step size \( \mu_i \) element is decreased exponentially. In a practical system constructed with multiple Digital Signal Processor (DSP) chips, \( \alpha_i/\mu_i \) is set in discrete steps with one constant value per DSP chip as shown in Figure 46. This practical modification prompts the windowed step gain/step size algorithm.

In this section, simulations of the windowed ES_NLMS algorithm were conducted using the real data (files NP1003.TIM and NR1003.TIM, nonstationary conditions). Setting the \( \alpha \) vector to be \( \alpha_n=1.4E^{-0.005n} \) (0\( \leq n<128 \)), \( \alpha_n=0.8E^{-0.005n} \) (128\( \leq n<256 \)), \( \alpha_n=0.2E^{-0.005n} \) (257\( \leq n<750 \)) (Figure 47(b)), the ERLE curve was plotted in Figure 47(a). Compared with 750 tap NLMS and ES_NLMS (Figure 41), the 750 tap windowed ES NLMS algorithm has a faster initial convergence speed and better tracking ability than the NLMS algorithm, and performs similarly to the ES_NLMS algorithm.

FIGURE 46 Step gain element \( \alpha_i/\mu_i \) when \( \alpha_i/\mu_i \) is set in discrete steps with one constant value per DSP chip.
FIGURE 47 (a) ERLE for 750 tap windowed ES_NLMS algorithm. (b) Configuration of \( \alpha \).

Preliminary simulations of the windowed ES_LMS, LMS and NLMS algorithms were conducted. But the windowed algorithm does not outperform the ES_NLMS or ES_LMS algorithms. For the brevity of this thesis, these results were not plotted. Detailed study is beyond the scope of this thesis.
5.2.3 Performance of Time-Varying Step Gain Algorithms

The ES_NLMS algorithm has been shown to provide better tracking ability and convergence speed than the NLMS algorithm. The ES_LMS algorithm could be set to have equivalent or superior tracking performance to an ES_NLMS algorithm. It may be possible to incorporate an algorithm similar to the variable stepsize LMS [80] (time-varying) in the context of the NLMS, ES_LMS and ES_NLMS algorithms, where under conditions of low mean-squared error output, the magnitude of change in the already variable stepsizes of the NLMS, ES_LMS and ES_NLMS algorithms should be attenuated by an adaptive gain factor. Similarly, under high mean-squared error conditions (such as during a room nonstationarity), this gain could be adaptively increased. Let’s call them VS_NLMS, VES_LMS, and VES_NLMS algorithms respectively.

\textit{VS_NLMS algorithm:}

\[
\ddot{\alpha}(n+1) = 0.97\alpha(n) + \gamma e^2(n) \tag{35}
\]

\[
\alpha(n+1) = \begin{cases} 
\alpha_{\text{max}} & \ddot{\alpha}(n+1) \geq \alpha_{\text{max}} \\
\alpha_{\text{min}} & \ddot{\alpha}(n+1) \leq \alpha_{\text{min}} \\
\ddot{\alpha}(n+1) & \text{otherwise}
\end{cases} \tag{36}
\]

where

0.97 is the forgetting factor

e(n) is the error signal (refer to Section 2.1.1, Equation 1 and 2)

\(\gamma\) is a constant factor

\(\alpha_{\text{max}}\) is the maximum step gain

\(\alpha_{\text{min}}\) is the minimum step gain.
Unscaled VES_NLMS algorithm (refer to Equation 32):

\[ \tilde{\alpha}_i(n+1) = 0.97\alpha_i(n) + \gamma e^2(n) \quad i = 0, 1, \ldots, M - 1 \]  

\[ \alpha_i(n+1) = \begin{cases} \alpha_{\text{max}} & \tilde{\alpha}_i(n+1) \geq \alpha_{\text{max}} \\ \alpha_{\text{min}} & \tilde{\alpha}_i(n+1) \leq \alpha_{\text{min}} \\ \tilde{\alpha}_i(n+1) & \text{otherwise} \end{cases} \]

where

\( \alpha(n) \) is the diagonal step gain matrix, \( \alpha(n) = \text{diag}[\alpha_0(n) \alpha_1(n) \ldots \alpha_{M-1}(n)] \).

Notice that in the unscaled VES_NLMS algorithm, the step gain curve loses its exponential decay characteristic at the first hundred taps (shown in Figure 55). In order to sustain the exponential decay shape, a scaled factor could be used to apply to all the step gains \( \alpha_i(n), i=0, 1, \ldots, M-1 \). This scheme is named the scaled VES_NLMS algorithm.

Scaled VES_NLMS algorithm:

\[ \tilde{\alpha}_0(n+1) = 0.97\alpha_0(n) + \gamma e^2(n) \]  

\[ S_0(n+1) = \frac{\alpha_{0,\text{max}}}{\alpha_0(n)} \tilde{\alpha}_0(n+1) \geq \alpha_{0,\text{max}} \]

\[ \frac{\alpha_{0,\text{min}}}{\alpha_0(n)} \tilde{\alpha}_0(n+1) \leq \alpha_{0,\text{min}} \]

\[ \frac{\tilde{\alpha}_0(n+1)}{\alpha_0(n)} \text{otherwise} \]

\[ \alpha_i(n+1) = S_0(n+1) \times \alpha_i(n) \quad i = 0, 1, \ldots, M - 1 \]
where

$\alpha_{0,\text{max}}$ is the maximum value for the first step gain $\alpha_0$ (Figure 48)

$\alpha_{0,\text{min}}$ is the minimum value for the first step gain $\alpha_0$ (Figure 48)

$M$ is the filter order

$S_0(n)$ is the intermediate scaling factor.

![Graph showing $\alpha_0$ as a function of tap number, $n$.](image)

**FIGURE 48** Configuration of $\alpha$ adapted range, $\alpha_0$ is the initial step gain vector.

Preliminary simulations showed that it was not possible to give the VES_NLMS a sufficient range of $\alpha$ values to match the convergence speed and tracking ability of ES_NLMS without causing instability. In order to offer a wide range of step gain for the nonstationary environment, we could clip the step gain for the first one second, after which the algorithm has converged. Then the adaptation is switched to variable step gain mode. This is quite reasonable in practice. Since compared with the tracking ability, the initial convergence speed is not so important as long as the algorithm converges to some required ERLE early enough in the conversation or during some pre-conversation training period. We call it
ESVES\_NLMS here since the normal ES\_NLMS is used during the first one second period.

Given $\gamma = 2.8 \times 10^{-5}$, $\alpha_{\text{max}} = 1.8$, $\alpha_{\text{min}} = 0.2$, the performance of the VS\_NLMS (variable step gain NLMS algorithm) is shown in Figures 49 and 50. Given $\gamma = 7.1 \times 10^{-5}$, $\alpha_{\text{max}} = 1.8$, $\alpha_{\text{min}} = 0.4$, the performance of the VS\_NLMS is shown in Figures 51 and 52. Notice it doesn’t outperform the conventional NLMS.

Given $\gamma = 1.41 \times 10^{-4}$, $\alpha_{\text{max}} = 1.8$, $\alpha_{\text{min}} = 0.2$, the performance of the unscaled VES\_NLMS is plotted in Figures 53 to 56. Its converged mean-squared error performance for stationary environments is not as good as the conventional NLMS algorithm, even if it does outperform the NLMS algorithm during nonstationary conditions.

Given $\gamma = 1.41 \times 10^{-4}$, $\alpha_{0, \text{max}} = 2.0$, $\alpha_{0, \text{min}} = 0.4$, the performance of the scaled VES\_NLMS is plotted in Figures 57 to 60. Given $\gamma = 1.41 \times 10^{-4}$, $\alpha_{0, \text{max}} = 4.0$, $\alpha_{0, \text{min}} = 0.2$, the performance of the ESVES\_NLMS is plotted in Figures 61 to 64.

Contrary to the expected, simulation data with variable step gain (time-varying) NLMS and ES\_NLMS are not an improvement on the regular ES\_NLMS algorithm (see Figure 41). Considering the extra calculations and comparisons existing with the variable step gain (time-varying), the achievable performance is not worth it.

The principle of unscaled/scaled VES\_NLMS algorithms could be extended to the ES\_LMS algorithm and to the unscaled/scaled VES\_LMS algorithms. However, for simulations performed with the counterparts of LMS type algorithms, none of these algorithms appears to give any convergence and tracking improvement. For the brevity of this thesis, these simulation results have been omitted.
FIGURE 49  ERLE performance for 750 tap variable step gain NLMS, \( a=0.97, \gamma=2.8e-5, \alpha_{\text{max}}=1.8, \alpha_{\text{min}}=0.2 \) for NP1003.TIM and NR1003.TIM Nonstationary condition.

FIGURE 50  Step gain \( \alpha \) vs. time for 750 tap variable step gain NLMS (VS_NLMS) shown in Figure 49.
FIGURE 51  ERLE performance for 750 tap variable step gain NLMS, $a=0.97$, $\gamma=7.1e-5$, $\alpha_{\text{max}}=1.8$, $\alpha_{\text{min}}=0.4$ for NP1003.TIM and NR1003.TIM Nonstationary condition.

FIGURE 52  Step gain $\alpha$ vs. time for 750 tap variable step gain NLMS (VS.NLMS) shown in Figure 51.
FIGURE 53 ERLE performance for 750 tap unscaled VES_NLMS, \( \alpha=0.97, \gamma=1.41e^{-4}, \alpha_{\text{max}}=1.8, \alpha_{\text{min}}=0.2 \) for NP1003.TIM and NR1003.TIM Nonstationary condition.

FIGURE 54 The first step gain \( \alpha_0 \) vs. time for 750 tap unscaled VES_NLMS shown in Figure 53.
FIGURE 55 Steady-state step gain α exponential decay curve for 750 tap unscaled VES_NLMS shown in Figure 53.

FIGURE 56 ERLE performance comparison between 750 tap NLMS (solid line) with $\alpha=0.3$, and 750 tap unscaled VES_NLMS (dashed line) with $a=0.97, \gamma=1.41e-4, \alpha_{\text{max}}=1.8, \alpha_{\text{min}}=0.2$ shown in Figure 53.
FIGURE 57 ERLE performance for 750 tap scaled VES_NLMS, $a=0.97, \gamma=1.41\times10^{-4}$,
$\alpha_{\text{max}}=2.0, \alpha_{\text{min}}=0.4$ for NP1003.TIM and NR1003.TIM Nonstationary condition.

FIGURE 58 The first step gain $\alpha_0$ vs. time for 750 scaled tap VES_NLMS shown in Figure 57.
FIGURE 59 Steady-state step gain $\alpha$ exponential decay curve for 750 tap scaled VES NLMS shown in Figure 57.

FIGURE 60 ERLE performance comparison between 750 tap NLMS (solid line) with $\alpha=0.3$, and 750 tap scaled VES NLMS (dashed line) with $s=0.97$, $\gamma=1.41\times10^{-4}$, $\alpha_{\max}=2.0$, $\alpha_{\min}=0.4$ shown in Figure 57 for NP1003.TIM and NR1003.TIM nonstationary conditions.
FIGURE 61 ERLE performance for 750 tap ESVES_NLMS, $\alpha=0.97, \gamma=1.41\times10^{-4}, \alpha_{\text{max}}=4.0, \alpha_{\text{min}}=0.2$ for NP1003.TIM and NR1003.TIM Nonstationary condition.

FIGURE 62 The first step gain $\alpha_0$ vs. time for 750 tap ESVES_NLMS shown in Figure 61.
FIGURE 63  Steady-state step gain vs exponential decay curve for 750 tap ESVES NLMS shown in Figure 61.

FIGURE 64  ERLE performance comparison between 750 tap NLMS (solid line) with $\alpha=0.3$, and 750 tap VES NLMS (dashed line) with $\alpha=0.97, \gamma=1.41 \times 10^{-4}, \alpha_{\text{max}}=4.0, \alpha_{\text{min}}=0.2$ shown in Figure 61 for NP1003.TIM and NR1003.TIM nonstationary conditions.
5.3 Performance of RLS-Based Algorithms

5.3.1 Steady-State Performance

This section examines the steady state performance of RLS-based algorithms. During the simulations with the SFTF algorithm, when the chosen tap number was larger than 100, the algorithm diverged (the maximum tap number was suggested to be 100 [58]). The computational storage requirements restricted the RLS algorithm to 230 taps. For real data, this prompted the undermodelling (see Figure 12 in Chapter 3). In this section, the tap number was chosen to be 70, which is still acceptable for the purposes of this section.

Simulations of two RLS-based algorithms were conducted using experimental data (files NP1001.TIM and NR1001.TIM, stationary condition): a 70 tap RLS algorithm with forgetting factor 0.9995 and initialization constant $\mu=10$ (Figure 66), and a 70 tap SFTF algorithm with forgetting factor 0.9995 and initialization constant $\mu=10$ (Figure 67).

In order to compare the performance of RLS-based algorithms with that of LMS-based algorithms, simulations of several LMS-based algorithms were conducted using the same experimental data (files NP1001.TIM and NR1001.TIM, stationary conditions): 70 tap LMS algorithm with $\mu = 0.001$ (Figure 68), 70 tap ES_LMS algorithm with $\mu_n = 0.002E-0.005n$ (Figure 69), 70 tap NLMS algorithm with $\alpha = 0.1$ (Figure 70), 70 tap ES_NLMS algorithm with $\alpha_n = 0.2E-0.005n$ (Figure 71).

Adaptation was allowed to continue for 0.5 second (8,000 iterations). The final converged ERLE was averaged over the last 2,000 ERLE samples, where convergence was guaranteed (Table 16).
TABLE 16 Converged ERLE of different algorithms for NP1001.TIM and NR1001.TIM.

<table>
<thead>
<tr>
<th># of Taps</th>
<th>RLS</th>
<th>SFTF</th>
<th>NLMS</th>
<th>ES_NLMS</th>
<th>LMS</th>
<th>ES_LMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>15.52</td>
<td>15.77</td>
<td>15.32</td>
<td>15.53</td>
<td>15.06</td>
<td>15.19</td>
</tr>
</tbody>
</table>

Comparisons of the ERLE curves in Figures 66 and 67 with those in Figures 68 to 71 show that RLS-based algorithms have a faster rate of initial convergence than LMS-based algorithms in the case of a stationary environment.

FIGURE 66 ERLE performance for 70 tap RLS, $\lambda=0.9995$, for NP1001.TIM and NR1001.TIM stationary conditions.

FIGURE 67 ERLE performance for 70 tap SFTF, $\lambda=0.9995$, for NP1001.TIM and NR1001.TIM stationary conditions.
FIGURE 68 ERLE for 70 tap LMS with $\mu=0.001$, for NP1001.TIM and NR1001.TIM stationary conditions.

FIGURE 69 ERLE for 70 tap ES_LMS with $\mu_n=0.002E-0.005n$, for NP1001.TIM and NR1001.TIM stationary conditions.
5.3.2 Tracking Performance

This section examines the tracking ability of RLS-based algorithms compared with LMS-based algorithms. Simulations of LMS family algorithms (LMS, ES_LMS, NLMS, ES_NLMS) and RLS family algorithms (RLS and SFTF) were conducted using the same real data (files NP1003.TIM and NR1003.TIM, nonstationary conditions). Adaptation was allowed to continue for the whole duration of the input files (10 seconds, i.e., 160,000 samples). The nonstationarity was caused by objects moving above the phoneset.
At first, simulations were run for 70 tap NLMS (with two different step gains 0.1 and 1.0), ES_NLMS ($\alpha_n=1.2E-0.005n$), LMS (with two different step sizes 0.001 and 0.014), ES_LMS ($\mu_n=0.03E-0.005n$), RLS (with four different forgetting factors 1.0, 0.9995, 0.9970, 0.9943), and SFTF (with four different forgetting factors 1.0, 0.9995, 0.9970, 0.9943, the same initialization constant $\mu=10$) (The optimum forgetting factor for 70 tap STFT is $\lambda_{opt}=1-0.4/M=0.9943$). The ERLE curves are displayed for the different algorithms in Figure 72 to 85.

The ES-based algorithms were just marginally better in terms of final converged ERLE and tracking ability than the best NLMS or LMS. The reason for this is that only the 70 tap FIR adaptive model was used to give the undermodelling. When the tap number is increased to an appropriate range (tap number > 300), the superiority of ES-based algorithms is demonstrated (see section 5.2.1).

Surprisingly, when the forgetting factor was chosen to be 0.9970 or 0.9943, the performance of SFTF algorithm is much better than the corresponding RLS algorithm (comparing Figure 80 and 84, Figure 81 and 85). Theoretically [58], the error level of the SFTF algorithm coincides with that of the conventional RLS algorithm, as shown for forgetting factors set to 1.0 and 0.9995 (see Figure 78 and 82, Figure 79 and 83).

Good tracking performance with RLS type algorithms can only be achieved by setting the forgetting factor below 1 (see Figure 78 to 85). However, the SFTF algorithm forgetting factor is bounded between 1 and 1-(1/2M) for stability [58], where M is the filter order. For a 70 tap filter, this works out to a bound between 0.9929 and 1.0 (see Table 17). When a filter size is up to 750, the lower bound, $\lambda_{min}=0.9993$, is not far below 1. But from expe-
rience in simulations performed for this thesis, the forgetting factor should not be set too close to the lower bound. The effect of these restrictions on the allowable forgetting factor implies that the RLS type algorithms appear unsuitable for large order filtering applications where good tracking ability is required.

**TABLE 17** Minimum forgetting factor of different tap numbers for the SFTF algorithm.

<table>
<thead>
<tr>
<th># of Taps (M)</th>
<th>70</th>
<th>230</th>
<th>375</th>
<th>750</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{min}}=1-(1/2M)$</td>
<td>0.9929</td>
<td>0.9978</td>
<td>0.9987</td>
<td>0.9991</td>
<td>0.9997</td>
</tr>
</tbody>
</table>

Next, the effect of the initialization constant $\mu$ on the initial convergence speed should be addressed. Simulations were run for a 100 tap SFTF with a forgetting factor of 0.9995, and initialization constant $\mu=1, 10$. The reason for increasing the tap number to 100 is just to get a better comparison. The ERLE curves are displayed in Figure 86 for $\mu=1$ and Figure 87 for $\mu=10$. The initial transient is decreased by increasing the initialization constant[58].

Simulations were run for a 230 tap NLMS ($\alpha=0.3$), ES_NLMS($\alpha(n)=1.2E-0.005n$), and RLS (with three different forgetting factors $\lambda=1.0, 0.9995, 0.9980$). The ERLE curves are displayed in Figure 88 to 92. (The maximum tap number for FIR RLS adaptive filtering is 230 to give the least undermodelling.) Simulation results demonstrated that RLS family algorithms have poorer tracking ability than simpler algorithms such as LMS family algorithms.

From the simulations performed in this section, it has been shown that for large filter orders and nonstationary environments, LMS type algorithms will give better overall performance than RLS type algorithms.
Set One: 1. 70 Tap NLMS

**FIGURE 72** ERLE performance for 70 tap NLMS, \( \alpha = 0.1 \), for NP1003.TIM and NR1003.TIM nonstationary conditions.

**FIGURE 73** ERLE performance for 70 tap NLMS, \( \alpha = 1.0 \), for NP1003.TIM and NR1003.TIM nonstationary conditions.
II. 70 Tap ES_NLMS

FIGURE 74  ERLE performance for 70 tap ES_NLMS, $\alpha_p=1.2E-0.005n$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
III. 70 Tap LMS

**FIGURE 75** ERLE performance for 70 tap LMS, $\mu=0.001$, for NP1003.TIM and NR1003.TIM nonstationary conditions.

**FIGURE 76** ERLE performance for 70 tap LMS, $\mu=0.014$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
IV. 70 Tap ES_LMS

FIGURE 77 ERLE performance for 70 tap ES_LMS, $\mu_S=0.03E-0.005n$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
V. 70 Tap RLS

FIGURE 78 ERLE performance for 70 tap RLS, $\lambda=1.0$, for NP1003.TIM and NR1003.TIM nonstationary conditions.

FIGURE 79 ERLE performance for 70 tap RLS, $\lambda=0.9995$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
FIGURE 80 ERLE performance for 70 tap RLS, $\lambda=0.997$, for NP1003.TIM and NR1003.TIM nonstationary conditions.

FIGURE 81 ERLE performance for 70 tap RLS, $\lambda=0.9943$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
VI. 70 Tap SFTF

FIGURE 82 ERLE performance for 70 tap SFTF, $\lambda=1.0$, $\mu=10$, for NP1003.TIM and NR1003.TIM nonstationary conditions.

FIGURE 83 ERLE performance for 70 tap SFTF, $\lambda=0.9995$, $\mu=10$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
FIGURE 84 ERLE performance for 70 tap SFTF, $\lambda=0.997$, $\mu=10$, for NP1003.TIM and NR1003.TIM nonstationary conditions.

FIGURE 85 ERLE performance for 70 tap SFTF, $\lambda=0.9943$, $\mu=10$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
Set Two: 100 Tap SFTF

**FIGURE 86** ERLE performance for 100 tap SFTF, $\lambda=0.9995$, $\mu=1$, for NP1003.TIM and NR1003.TIM nonstationary conditions.

**FIGURE 87** ERLE performance for 100 tap SFTF, $\lambda=0.9995$, $\mu=10$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
Set Three: I. 230 Tap NLMS

FIGURE 88 ERLE performance for 230 tap NLMS, \( \alpha=0.3 \), for NP1003.TIM and NR1003.TIM nonstationary conditions.

II. 230 Tap ES_NLMS

FIGURE 89 ERLE performance for 230 tap ES_NLMS, \( \alpha(n)=1.2E-0.005n \), for NP1003.TIM and NR1003.TIM nonstationary conditions.
III. 230 Tap RLS

FIGURE 90 ERLE performance for 230 tap RLS, \( \lambda=1.0 \), for NP1003.TIM and NR1003.TIM nonstationary conditions.

FIGURE 91 ERLE performance for 230 tap RLS, \( \lambda=0.9995 \), for NP1003.TIM and NR1003.TIM nonstationary conditions.
FIGURE 92 ERLE performance for 230 tap RLS, $\lambda=0.9980$, for NP1003.TIM and NR1003.TIM nonstationary conditions.
CHAPTER 6

CONCLUSIONS

6.1 Discussion of Results

This thesis presented a study of the dynamic behavior of the acoustic echo cancellation in hands-free communications using digital signal processing approaches. The results show the impact of several environmental conditions on the achievable cancellation in an acoustic echo cancellation system. The results also show the impact a nonstationarity in the room environment has on canceller performance. The LMS-based algorithms with an exponential decaying step size/step gain have been investigated to improve tracking performance during nonstationary periods.

Room noise and system non-linearities have been shown to impose an asymptote on the achievable ERLE as the number of taps in the adaptive filter is increased. With room noise as the asymptote, the maximum achievable ERLE is equivalent to the (signal+noise) to noise ratio of the primary signal. For a fixed room noise environment, it was shown that system non-linearities (such as loudspeaker distortion) have a large impact on the achievable ERLE. For short adaptive filter lengths, the uncancellation echoes remaining in the error signal may take precedence as the limit on achievable ERLE.

To improve tracking under nonstationary room impulse response conditions, application of an ES-based (exponential step) algorithm was proposed. The study of the room impulse response and statistics of the variation in the room impulse response indicated that the
room impulse response attenuates exponentially, and the variation in the impulse response attenuates by the same exponential ratio [51]. In this algorithm, the traditional scalar step size $\mu$ (or step gain $\alpha$) was replaced by an exponentially decaying $\mu$-matrix (or corresponding $\alpha$-matrix). This ES-based algorithm has the same computational load as the conventional LMS type algorithm. Simulations show its excellent tracking ability of severe nonstationarities (object moving 1 foot above the hands-free phoneset), while preserving good cancellation under stationary conditions. This approach is very promising and fairly simple, i.e., the use of ES_LMS and ES_NLMS algorithms. Preliminary simulations on the windowed ES-based algorithms were conducted.

The principle of an adaptive step size (VS-based) was extended to NLMS and ES-based algorithms. VS-based algorithms increase the adaptive gain factor (or step size in the context of LMS algorithm) under conditions of high mean-squared error output (such as during a room nonstationarity), and reduce the adaptive gain factor (or step size) under conditions of low mean-squared error output. It is hoped that this approach can provide a fast tracking ability in addition to gain good steady-state performance. Although unfortunately, simulation results in this thesis didn’t show that this approach provides performance to be as good as the regular ES-based algorithm.

6.2 Recommendation for Further Work

As a result of the investigations reported in this thesis, the following areas are suggested for further research:
6.2.1 Variable Gain Schemes

A detailed study of the nature of VS-based algorithms might provide some insight into a more careful selection of parameters to be used. It is still hoped that an improvement can be obtained by using the VS-based technique.

6.2.2 Techniques such as Sparsely Located Taps and Multirate Processing

Techniques for reducing the computational complexity of adaptive filters impact on the performance of the system. Sparsely located taps could maintain the convergence rate, but at the expense of steady-state performance. Margo, Etter and Carlson provided encouraging results in [95]. Aimed at reducing the complexity and increasing the convergence speed, a broad class of multirate systems was analyzed [32][35]. Further investigations may be beneficial.

6.2.3 Subband Structures

Adaptive filtering in subbands is a relatively new technique in the acoustic echo cancellation. This technique generally allows computational savings as well as better convergence behavior [84]. The enhancement should be further investigated.

6.2.4 Real Time Implementation

The speech signal under various conditions should be tested. Implementation of the adaptive echo canceller in real time will allow for investigations of the performance of the canceller in the presence of speech to be evaluated and tested more easily.
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APPENDICES

Appendix A - Convergence Condition for the ES_LMS Algorithm Derivation

The derivation of the convergence condition for the ES_LMS algorithm follows the derivation for the LMS algorithm of Haykin [4].

**ES_LMS algorithm:**

\[ W(n + 1) = W(n) + \mu e(n) X(n) \]  \hspace{1cm} (42)

where \( \mu \) is the diagonal step size matrix, \( \mu = \text{diag}(\mu_0, \mu_1, \ldots, \mu_{M-1}) \). Each step size \( \mu_i = \mu_0 D^i \), \( D \times i, i = 0, 1, \ldots, M-1 \), where \( D \) is the decay factor.

Convergence conditions of the ES_LMS algorithm in the mean square is

\[ 0 < \mu_0 < \frac{2}{\sum_{i=0}^{M-1} \lambda_i E - Di} \]  \hspace{1cm} (43)

where \( \lambda_0 > \lambda_1 > \ldots > \lambda_{M-1} \).

We define the weight-error vector for the ES_LMS algorithm as follows:

\[ \varepsilon(n) = W(n) - W_0 \]  \hspace{1cm} (44)

where \( W_0 \) denotes the optimum Wiener solution for the tap-weight vector.
Substracting \( w_0 \) from both sides of Equation 42, and using the definition of Equation 44 to eliminate \( w(n) \) from the correlation term on the right side of Equation 42, we get

\[
\varepsilon(n + 1) = [I - \mu X(n) X^H(n)] \varepsilon(n) + \mu X(n) e^*_0(n) \tag{45}
\]

where \( I \) is the identity matrix, and \( e_0(n) \) is the estimation error produced in the optimum Wiener solution.

\[
e_0(n) = d(n) - W_0^H(n) X(n) \tag{46}
\]

Using \( K(n) \) to denote the correlation matrix at time \( n \), we have, by definition

\[
K(n) = E[\varepsilon(n) \varepsilon^H(n)] \tag{47}
\]

To follow the same procedure as described in [4], \( K(n+1) \) is derived as

\[
K(n + 1) = K(n) - \mu [RK(n) + K(n) R] + \mu \mu^T R_{tr} [RK(n)] + \mu \mu^T RK(n) R + \mu \mu^T J_{min} R \tag{48}
\]

where \( R \) denotes the correlation matrix of the tap-weight vector \( X(n) \).

\[
R = E[X(n) X^H(n)]
\]

\( J_{min} \) is the minimum mean squared error produced by the Wiener solution

\[
J_{min} = E[e_0(n) e^*_0(n)] = E[|e_0(n)|^2]
\]
Let \( q_0, q_1, \ldots, q_{M-1} \) be the eigenvectors corresponding to the distinct eigenvalues \( \lambda_0, \lambda_1, \ldots, \lambda_{M-1} \) of the \( M \times M \) matrix

\[
Q = [q_0, q_1, \ldots, q_{M-1}]
\]

where

\[
q_i^H q_j = 1, \quad i = j
\]

\[0, \quad i \neq j\]

Define the \( M \times M \) diagonal matrix

\[
\Lambda = \text{diag}(\lambda_0, \lambda_1, \ldots, \lambda_{M-1})
\]

Then, we get the unitary similarity transformation:

\[
Q^H R Q = \Lambda
\] (49)

Furthermore, let

\[
Q^H K(n) Q = Z(n)
\] (50)

Using the transformations described by Equation 49 and 50, we may rewrite the recursive equation Equation 48 as follows:

\[
Z(n + 1) = Z(n) - \mu \left[ \Lambda Z(n) + Z(n) \Lambda \right] + \mu \mu^T \text{tr} \left[ \Lambda X(n) \right] \\
+ \mu \mu^T \Lambda Z(n) \Lambda + \mu \mu^T J_{mn} \Lambda
\] (51)
Examine the diagonal terms of the Equation 51, the $z_i$ decouple from the off-diagonal terms, and we have

$$
z_i(n+1) = z_i(n) - 2\mu_i\lambda_i z_i(n) + \mu_i^2\lambda_i \sum_{j=0}^{M-1} \lambda_j z_j(n)
+ \mu_i^2\lambda_i^2 z_i(n) + \mu_i^2 J_{min} \lambda_i, \quad i = 0, 1, \ldots, M - 1
$$  \hspace{1cm} (52)

Define the $M$-by-$1$ vector $z(n)$ and $\lambda$ as follows, respectively

$$
z(n) = [z_0(n), z_1(n), \ldots, z_{M-1}(n)]^T
$$  \hspace{1cm} (53)

$$
\lambda = [\lambda_0, \lambda_1, \ldots, \lambda_{M-1}]^T
$$  \hspace{1cm} (54)

Then we may rewrite Equation 52 in matrix form as

$$
z(n+1) = Bz(n) + \mu \mu^T J_{min} \lambda
$$  \hspace{1cm} (55)

where $B$ is an $M$-by-$M$ matrix with elements

$$
b_{ij} = (1 - \mu_i\lambda_i)^2 + \mu_i^2\lambda_i^2, \quad i = j
$$

$$
\mu_i^2\lambda_i\lambda_j, \quad i \neq j
$$  \hspace{1cm} (56)

From Equation 56, we see that the matrix $B$ is the sum of a diagonal matrix (with positive values real values for all of its elements) and the outer product $\mu \mu^T \lambda \lambda^T$. The matrix $B$ is therefore real, positive and symmetric.

Let $g$ be an eigenvector of matrix $B$, associated with eigenvalue $\epsilon$. Then, by definition, we
have

\[ Bg = cg \]  \hspace{1cm} (57)

or, equivalently,

\[ \sum_{j=0}^{M-1} b_{ij} g_j = cg_i \quad i = 0, 1, \ldots, M - 1 \]  \hspace{1cm} (58)

where the \( g_i \) are the elements of the eigenvector \( g \). Using Equation 56, for the elements of matrix \( B \) in Equation 58, we get

\[ (1 - \mu_i \lambda_i)^2 g_i + \mu_i^2 \lambda_i \sum_{j=0}^{M-1} \lambda_j g_j = cg_i \quad i = 0, 1, \ldots, M - 1 \]  \hspace{1cm} (59)

Solving Equation 59 for \( g_i \), we may thus write

\[ g_i = \frac{\mu_i^2 \lambda_i}{c - (1 - \mu_i \lambda_i)^2} \sum_{j=0}^{M-1} \lambda_j g_j \quad i = 0, 1, \ldots, M - 1 \]  \hspace{1cm} (60)

Employing the results derived previously that \( B \) is a positive matrix since all of its elements are positive, we may use Perron's theorem [4] which states that

If \( B \) is a positive square matrix, there is a unique eigenvalue of \( B \), which has the largest magnitude. This eigenvalue is positive and of multiplicity 1, and its associated eigenvector consists entirely of positive elements.
Accordingly, we may associate a positive eigenvector (i.e., a vector consisting entirely of positive elements) with the special eigenvalue of matrix $B$ that has the largest magnitude. Thus setting the eigenvalue $\lambda$ equal to 1 in Equation 60, letting $\mu_{\text{crit}}$ denote the critical value of the step size parameter that corresponds to this limiting condition of stability, and then simplifying, we get

$$g_i = \frac{\mu_{\text{crit}}}{2 - \mu_{\text{crit}} \lambda_i} \sum_{j=0}^{M-1} \lambda_j g_j \quad i = 0, 1, \ldots, M - 1$$  \hspace{1cm} (61)

From Equation 61 we see that $g_i$ is positive for all $i$, if and only if,

$$\mu_{\text{crit}} < \frac{2}{\lambda_{\text{max}}}$$  \hspace{1cm} (62)

where $\lambda_{\text{max}}$ is the largest eigenvalue of the correlation matrix $R$. Moreover, multiply both sides of Equation 61 by $\lambda$, and then summing over all integer values of $i$ from 0 to $M-1$. We get

$$\sum_{i=0}^{M-1} \frac{\mu_{\text{crit}} \lambda_i}{2 - \mu_{\text{crit}} \lambda_i} = 1$$  \hspace{1cm} (63)

With the actual value of $\mu$, required to be positive and less than the critical value $\mu_{\text{crit}}$, we get the convergence conditions of the ES_LMS algorithm in the mean square (using the fact that the mean-squared error $J(n)$ decays to 0, if and only if, the eigenvalues of matrix $B$ are less than 1 in magnitude):
\[ 0 < \mu < \frac{2}{\lambda_{\text{max}}} \]  \quad \quad (64)

\[ \sum_{t=0}^{M-1} \frac{\mu \lambda_t}{2 - \mu \lambda_t} < 1 \]  \quad \quad (65)

Assuming that \( \mu \lambda_t \ll 2 \), which is quite reasonable, we may simplify the condition of Equation 65 as follows:

\[ 0 < \sum_{t=0}^{M-1} \mu \lambda_t < 2 \]  \quad \quad (66)

Rewrite the equation for each step size \( \mu_i = \mu_0 E - D_i \). From Equation 66, we could get

\[ 0 < \mu_0 < \frac{2}{\sum_{t=0}^{M-1} \lambda_t E - D_i} \]

where \( \lambda_0 > \lambda_1 > \ldots > \lambda_{M-1} \).
Appendix B - Convergence Condition for the ES_NLMS Algorithm Derivation

**ES_NLMS algorithm:**

\[ W(n + 1) = W(n) + \alpha e(n) X(n) \]  \hspace{1cm} (67)

where \( \alpha \) is the diagonal step gain matrix, \( \alpha = \text{diag}(\alpha_0, \alpha_1, \ldots, \alpha_{M-1}) \), each step gain \( \alpha_i = \alpha_0 e^{-(\text{decay\_factor})i} \), \( i = 0, 1, \ldots, M-1 \).

Convergence conditions of the ES_NLMS algorithm in the mean square is

\[ 0 < \alpha_0 < 2 \]  \hspace{1cm} (68)

We define the weight-error vector for the ES_NLMS algorithm as follows:

\[ e(n) = W(n) - W_0 \]  \hspace{1cm} (69)

where \( W_0 \) denotes the optimum Wiener solution for the tap-weight vector.

Subtracting \( W_0 \) from both sides of Equation 67, and using the definition of Equation 69 to eliminate \( W(n) \) from the correlation term on the right side of Equation 67, we get

\[ e(n + 1) = \left[ I - \frac{\alpha}{\|X(n)\|^2} X(n) X^H(n) \right] e(n) + \frac{\alpha}{\|X(n)\|^2} X(n) e_0^*(n) \]  \hspace{1cm} (70)

where \( I \) is the identity matrix, and \( e_0(n) \) is the estimation error produced in the optimum Wiener solution.
\[ e_0(n) = d(n) - W_0^H(n) X(n) \]  

(71)

Here, following conditions are assumed:

5. \( E[x(i)x(j)] \) equals to 1 for \( i = j \), and equals to 0 for \( i \neq j \).

6. \( \varepsilon(n) \) and \( X(n) \) are independent

7. \( E[x(n)/\|X(n)\|^2] \equiv E[x(n)]/E[\|X(n)\|^2] \)

Then, the mean-square of the i-th component of \( \varepsilon(n) \) is derived as

\[
E[\varepsilon_i(n + 1)^2] = b_i(n + 1)^2 \\
= b_i(n)^2 - 2\frac{\alpha_i}{M} h_i(n)^2 + \left(\frac{\alpha_i}{M}\right)^2 \sum_{j=0}^{2M-1} h_j(n)^2 + \frac{\alpha_i^2}{M} J_{min} 
\]

(72)

where \( b_i(n)^2 = E[\varepsilon(n)]^2 \), \( J_{min} \) is the minimum mean squared error produced by the Wiener solution

\[ J_{min} = E[\varepsilon_0(n)e_0^*(n)] = E[|e_0(n)|^2] \]

Then we may rewrite Equation 72 in matrix form as

\[
b(n + 1) = Qb(n) + \frac{J_{min}}{M^2} \alpha\alpha^H
\]

(73)

where \( b(n) = [b_0(n), b_1(n), ..., b_{M-1}(n)]^T \), \( Q \) is an M-by-M matrix with elements \( q_{ij} \)

\[
q_{ij} = 1 - 2\frac{\alpha_i}{M} + \left(\frac{\alpha_i}{M}\right)^2, \quad i = j \\
\left(\frac{\alpha_i}{M}\right)^2, \quad i \neq j
\]

(74)
From Equation 74, we see that the matrix $Q$ is positive and symmetric.

Let $g$ be an eigenvector of matrix $Q$, associated with eigenvalue $c$. Then, by definition, we have

$$Qg = cg$$  \hspace{1cm} (75)

or, equivalently,

$$\sum_{j=0}^{M-1} q_{ij} g_j = cg_i \quad i = 0, 1, \ldots, M - 1$$  \hspace{1cm} (76)

where the $g_i$ are the elements of the eigenvector $g$. Using Equation 74, for the elements of matrix $Q$ in Equation 76, we get

$$
(1 - \frac{2\alpha_i}{M}) g_i + \left( \frac{\alpha_i}{M} \right) \sum_{j=0}^{2M-1} g_j = cg_i \quad i = 0, 1, \ldots, M - 1
$$  \hspace{1cm} (77)

Solving Equation 77 for $g_i$, we may thus write

$$g_i = \frac{\alpha_i^2}{c - 1 + \frac{\sum_{j=0}^{M-1} g_j}{M}} \quad i = 0, 1, \ldots, M - 1$$  \hspace{1cm} (78)

Employing the results derived previously that $Q$ is a positive matrix since all of its elements are positive, we may use Perron's theorem [4].
Accordingly, we may associate a positive eigenvector (i.e., a vector consisting entirely of positive elements) with the special eigenvalue of matrix $Q$ that has the largest magnitude. Thus setting the eigenvalue $c$ equal to 1 in Equation 78, letting $\alpha_{\text{crit}}$ denote the critical value of the step gain parameter that corresponds to this limiting condition of stability, and then simplifying, we get

$$g_i = \frac{\alpha_{\text{crit}}}{2M} \sum_{j=0}^{M-1} g_j \quad i = 0, 1, \ldots, M-1 \quad (79)$$

Moreover, summing over all integer values of $i$ from 0 to $M-1$, we get

$$\sum_{i=0}^{M-1} g_i = \frac{\alpha_{\text{crit}}}{2} \sum_{j=0}^{M-1} g_j \quad (80)$$

$$\alpha_{\text{crit}} = 2 \quad (81)$$

Using the fact that the mean-squared error $J(n)$ decays to 0, if and only if, the eigenvalues of matrix $Q$ are less than 1 in magnitude, with the actual value of $\alpha$ required to be positive and less than the critical value $\alpha_{\text{crit}}$, we get the convergence conditions of the ES-LMS algorithm in the mean square:

$$0 < \alpha_i < 2 \quad (82)$$

or

$$0 < \alpha_0 < 2 \quad (83)$$
Appendix C - Long Term Non-Stationarity In Conference Room
Object Moving Above The Phoneset (1 Foot)

FIGURE 93  ERLE performance comparison between 1500 tap NLMS (solid line) with $\alpha=1.0$, and 1500 tap ES_NLMS (dashed line) with $\alpha(n)=1.2E-0.005n$ for NP714.TIM and NR714.TIM Nonstationary conditions.
Appendix D - Short Term Non-Stationarity In Anechoic Chamber
Object Moving Above The Phoneset (1 Foot)

FIGURE 94 ERLE performance for 750 tap NLMS with $\alpha=0.4$, for AP501.TIM and AR501.TIM. Nonstationary conditions.
FIGURE 95 ERLE performance for 750 tap ES_NLMS with $\alpha = 1.4E-0.005n$, for AP501.TIM and AR501.TIM, Nonstationary conditions.

FIGURE 96 ERLE performance for 750 tap NLMS with $\alpha = 0.4$, for AP502.TIM and AR502.TIM, Nonstationary conditions.
FIGURE 97 ERLE performance for 750 tap ES_NLMS with $\alpha=1.4E-0.005n$, for AP502.TIM and AR502.TIM, Nonstationary conditions.
Appendix E - Short Term Non-Stationarity In Anechoic Chamber
Person Walking By (1 Foot Away)
Phoneset Placed Horizontally On The Foam

FIGURE 98 ERLE performance for 750 tap NLMS with \( \alpha = 0.4 \), for AP701.TIM and AR701.TIM, Nonstationary conditions.
FIGURE 99  ERLE performance for 750 tap NLMS with $\alpha(n)=1.4E-0.005n$, for AP701.TIM and AR701.TIM, Nonstationary conditions.
Appendix F - Short Term Non-Stationarity In Anechoic Chamber
Person Walking By (1 Foot Away)
Phoneset Placed 45° From Horizontal On The Foam

FIGURE 100 ERLE performance for 750 tap NLMS with $\alpha=0.4$, for AP702.TIM and AR702.TIM. Nonstationary conditions.
FIGURE 101 ERLE performance for 750 tap NLMS with $\alpha=1.4E-0.005n$, for AP702.TIM and AR702.TIM, Nonstationary conditions.