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PERFORMANCE ANALYSIS OF FH-MFSK SYSTEM OVER MOBILE CHANNEL

by

Vinod K. Gupta

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the Degree of Master of Engineering

DEPARTMENT OF SYSTEMS AND COMPUTER ENGINEERING
CARLETON UNIVERSITY
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ABSTRACT

The performance of FH-MFSK over a mobile radio channel has been quantitatively analysed. For a particular address assignment scheme a model quantifying the effects of copattern interference, adjacent cell and mutual interferences among the users has been developed and evaluated.

For cellular application, the question of the reuse of a particular address in geographically separated areas has been considered in the presence of mobile channel impairments such as Rayleigh fading and lognormal shadowing.

FH-MFSK system performance has been evaluated under conditions of one or more above mentioned channel impairments for a given average probability of bit error rate of $10^{-3}$, considered to result in good quality digital speech communication.

It has been found that the number of simultaneous users is a function of copattern reuse distance. The number of users can be increased by increasing the copattern reuse distance (ref. Figures 6.5.4 and 6.5.5).

The results show that with a propagation loss exponent of four, the copattern reuse ratio for a FH-MFSK cellular mobile mobile system is 4.58, which is more than the DS/CDMA system for which it is between 3 and 2 for exponent loss of three and four respectively.

The worst interference is seen by the mobile in a cell corner, where it is equidistant from the three nearby copattern and adjacent cells. The distribution of users within a cell affects the system performance and results in spectrum efficiency of the system.
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\[ \gamma \]  
instantaneous signal to noise power ratio

\[ \gamma_0 \]  
average signal to noise power ratio

\[ W \]  
bandwidth in hertz

\[ T_c \]  
chip duration

\[ T_b \]  
duration of information

\[ G_T(t) \]  
rectangular pulse of duration \( T \)

\[ \omega \]  
frequency of carrier

\[ M \]  
number of simultaneous users

\[ L \]  
number of chips of duration \( r \)

\[ X_m \]  
message-word sent by user \( m \)

\[ Y_{m,t} \]  
address of user \( m \)

\[ P_D \]  
probability of deletion

\[ P_f \]  
probability of false alarm

\[ p(x) \]  
probability density function

\[ I_0(x) \]  
modified Bessel function

\[ \sigma \]  
standard deviation

\[ \beta_0 \]  
threshold level

\[ P_{EM} \]  
probability of error for a Mark

\[ P_{ES} \]  
probability of error for a Space

\[ Q(x, \alpha) \]  
Q-function

\[ m_d \]  
mean of lognormal shadowing

\[ a_4 \]  
fourth central moment

\[ P_u \]  
probability of farthest user within the distance \( < k \)

\[ a_n \]  
fourth moment of adjacent cell base station from user \( u \)

\[ P_i \]  
probability of insertion

\[ P_e \]  
probability of an entry in a correct row

\[ P_b \]  
probability of bit error

\[ \gamma_i \]  
average inter-cell interference to noise power ratio

\[ \gamma_a \]  
average adjacent cell interference to noise power ratio

\[ \gamma_{co} \]  
average cochannel interference to noise power ratio in a correct row of user \( u \)

\[ P_{di} \]  
probability of creating an entry in the spurious and correct row

\[ w_{di} \]  
where \( i = 1, 2, \ldots \)

\[ C \]  
square of the distance of the cochannel base station from user \( u \)
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Chapter 1

Introduction

1.1 Cellular Mobile Technology

A new communications technology called cellular mobile radio unfolding on the horizon of North America holds the key for meeting the challenge of increasing demand for mobile radio channels. The cellular technology will radically change the situation of spectral overcrowding which has prohibited the expansion of mobile radio service. This golden technology could generate billions of dollars of revenue and may prove to be the most significant change of the century. With the advent of cellular systems, the number of subscribers in a given system can be expected to expand dramatically over a period of time extending into the next decade. As the mobile subscriber population grows, the cellular technology will accommodate the growth in a way that is balanced, non-disruptive and incremental.

The Federal Communications Commission's decision in 1982, to accept applications for cellular system licences has erupted a competition among dozens of companies in the United States. The FCC has released 850 new channels at 800 MHz for private radio operators and 120 channels in the 900 MHz band for both public and private paging. The common carriers who had eight exclusive channels for paging (all either 35 MHz or 150 MHz) have been allocated 28 additional channels at 929 to 931 MHz. The 900 MHz channels were split as 40 for private operators, 40 for common carriers and 40 for reserve. The most important aspect has probably been the release of 250 channels at 800 MHz: 50 to industrial, land transportation users; 70 to public safety, special
emergency radio services; 50 to business radio users; and 80 channels to standard metropolitan statistical areas.

In Canada, the Federal Department of Communication has already issued a license to CANTEL and the committee is to make recommendations for local telephone companies and other bidders across the country.

The Canadian Cellular Radio System may probably need a total investment of 150 - million which may not be recovered for several years.

According to the proposal of Cellular Communication Canada Inc. of Montreal, the city of Toronto would probably have seven cells initially, which would expand to probably 20 cells within five years as the service expands. The downtown Toronto cell might have 30 channels in use at any time. 15 channels may be in use in a suburban cell. The downtown Toronto cell frequencies might be reused at Toronto Airport cell. Enhanced services such as conference calling between a mobile unit and two other parties, call forwarding, speed calling, an answering service, credit card charging and other 20 specialized services have also been planned for Canada.

The U.K. Cellular System would be the first nationally implemented system of its kind in the world. RACAL-MILLICOM LTD would provide national cellular radio telephone service in U.K.

The NORDIC Cellular System which covers a large part of the four Nordic countries (Norway, Sweden, Finland and Denmark) has also been selected by Spain, Saudi Arabia, Netherlands and Austria.

In a cellular mobile communication scheme, two basic methods are used for sharing the frequency band. The narrowband systems use a single carrier frequency for each pair of communicating units while the spread spectrum systems allow many users to share the same broad band
Introduction

and identified from each other by some address scheme [C N 83]. Figure 1.1.1 shows a general cellular radio configuration.

While narrowband frequency modulation systems may be used initially, the other modulation techniques such as single-side band amplitude modulation, narrowband digital modulation, and spread spectrum modulation may prove to be superior in the end. At present, the spread spectrum techniques are not common in civil applications and have relatively few regulations for them.

Currently, the spread spectrum is used in certain commercial satellite applications, where the FCC rules are not specific with respect to modulation techniques. There has been a mixed reaction to the use of spread spectrum in cellular mobile environment. The IEEE Committee on Communications and Information Policy (CCIP) in September, 1982 has submitted a policy statement on the spread spectrum issue which states the FCC need not, and should not at this time, foreclose further investigation of spread spectrum usage for civil applications [C N 83]. However, recently FCC has released some spectrum for commercial/experimental spread spectrum systems.

1.2 Choice of Spread Spectrum For Cellular Mobile Radio

Because of some unique features of spread spectrum system its use in cellular mobile radio has been proposed. There has been a range of attitude from strongly negative (GTE, AT&T and COMSAT) to positive (HEWLETT-PACKARD CO., RCA CORPO. AND GENERAL ELECTRIC CO.) [C N 83].

The spread spectrum system has many advantages over the narrowband systems which are as follows:

i) The spread spectrum can substantially improve the spectral efficiency of a cellular mobile system.
Figure 1.1.1 General cellular mobile radio system
Introduction

ii) In a spread spectrum cellular mobile radio system, the users can share the same bandwidth, and distinguished from each other by different addresses (patterns) assigned to them. The system can be accessed randomly anytime and the user need not wait for a free channel. Which means that there is no probability of blocked calls. However, if the design value of the system is exceeded there may be a graceful degradation of performance for all users [C N, 78].

However, we may design a system which will block the calls if the number of users in a cell exceeds the design value and thus maintaining a value of probability of error (e.g. $10^{-3}$ per) which will ensure satisfactory reception of information, thus, an acceptable system.

iii) Message privacy from a casual listener is another advantage of spread spectrum system.

iv) Channel switching and address changes are not required when the user moves from one cell to another because he is assigned a unique identification signal on a permanent basis.

v) Emergency messages, based on priority, for public safety vehicles, can be handled by the system without denying other users access to the system.

vi) The spread spectrum cellular mobile radio system can co-exist with its counterpart narrowband system without excessive mutual interference, using the same band twice.

vii) The hardware for all the users is identical except for filters associated with unique identification signal set.

viii) The spread spectrum system can operate satisfactorily even if the received signal is literally submerged in noise.

disadvantages of ss-cellular radio system

Though the ss-cellular radio system has significant advantages, there are some advantages too which should also be considered. For example, the start-up costs for such systems are probably much higher than the narrowband scheme which may be reduced to some extent as the technology
Introduction

matures. High-speed computing capability means the base station design must start with a mature system rather than grow as the demand builds. The power control to combat the near-far problem is another disadvantage. Though these are not difficult problems to solve, they certainly make the system more complex and expensive.

1.3 Spread Spectrum Cellular Mobile Radio

The spread spectrum cellular land-mobile radio has shown promise of being more spectrally efficient than the existing systems. As indicated by results the spread spectrum should be able to provide better-quality telecommunications service in a cellular environment in addition. A brief review of the system is given below. The users of spread spectrum system are distinguished by the addresses assigned to them. The available spectrum is divided into two parts, upstream (mobile-to-base) and downstream (base-to-mobile). The bandwidth for the two streams might be 20 MHz (one way). The severe effects of multipath propagation and fading distortions are much reduced with the large system bandwidth being larger than the coherence bandwidth of the mobile channel and a form of frequency diversity results. A power control strategy adjusts the transmitted power from each mobile unit in a cell such that equal power is received by the base station. The quality of the message is maintained by blocking calls exceeding the system design and denial to calls
Introduction
degradng the quality of signal excessively. Figure 1.3.2 shows a connecting system for a spread spectrum mobile radio [NET, 80].

1.4 Mobile-Radio Communications Environment

Radio signals transmitted from a mobile-radio base station suffer not only from free-space and terrestrial losses but also affected by various types of scattering and multipath phenomenon. The general topography of the terrain contributes to the terrestrial losses. In addition, multipath and scattering losses cause severe signal fading. Because in a mobile-radio environment the communication is at the ground level, the propagation between a base station and a mobile unit is mostly effected by the multipath fading. In a satellite-to-earth station and air-to-ground communication, the effects of multipath phenomenon are significant because the surrounding structures contribute negligibly small interference. In a mobile radio environment there is no direct line-of-sight path exists between a transmitter and a receiver because of obstructions. The received signal is a vector sum of various waves that arrive at the receiver by way of diffraction around and reflection from buildings. During deep fades the interruption between transmitter and receiver cause increased probability of error in the received message. Figure 1.4.1 shows the multipath phenomenon in an urban area. The short term amplitude variations of mobile signal due to fading can be accurately described by a Rayleigh Distribution.

\[ P(\gamma) = \gamma_0^{-1} \exp(-\gamma/\gamma_0) , \gamma \geq 0 \]  

(1)

where, \( P(\gamma) \) is the density function of \( \gamma \), \( \gamma_0 \) is the average signal-to-noise power ratio and \( \gamma \) is the instantaneous signal-to-noise power ratio. The other two important parameters describing the Rayleigh fading are the average fade duration and average cross over rate. Figure 1.4.2 shows a Rayleigh Fading envelope. Measurements show that the distance between the fades is 0.5 and 1 wavelength at the operating frequency. Whenever the signal falls below the threshold level an
Figure 1.3.1 Connecting system for spread spectrum mobile radio.
interruption in communication will occur and cause error in the received data. The error rate is proportional to the percentage of time the received signal is below threshold.
Introduction

The fading can be effectively reduced by using diversity techniques which are complex and expensive in terms of equipment. The other method to combat fading is to employ a modulation scheme which is not sensitive to the fluctuations of the signal amplitudes.

1.4.1 Diversity Transmission

In diversity technique, multiple transmission paths are used to combat fading. The multiple transmission paths or branches are independent of each other so that the failure of one branch has no effect on the other. Space diversity and time diversity are the two main techniques commonly employed in the VHF/UHF band. In space diversity technique, antenna spacings create multiple transmission paths. The received signal at all antennas is independent of each other and the probability of the signal being weak simultaneously at all antennas is low. The receiver switches to the branch receiving the strongest signal of all the other. In time diversity technique, the same signal (packet) is repeated a number of times and the probability of all transmissions hit by a deep fade is low. However, the redundancy in the transmission causes a low effective transmission rate.

1.4.2 Modulation Scheme

Frequency and phase modulation schemes are insensitive to the variations in the signal and hence the frequency shift keying (FSK) and phase shift keying (PSK) are commonly employed for data communication.

1.4.3 Error Control Schemes

Multipath fading introduces errors in the received data. An adequate error control schemes would reduce the probability of error to a low value and increase the reliability of the system. In data communication systems, an error detection and retransmission scheme is used by way of
adding appropriate redundancy such as addition of parity-check bits to detect and correct errors. The number of errors detected and corrected depend on the added redundancy.

1.5 FH-MFSK and FH-DPSK Spread Spectrum Systems

Of the three spread spectrum systems only two have been simulated to date; FH-MFSK (Frequency Hopped Multilevel Frequency Shift Keying) system conceived by Viterbi and developed at Bell Laboratories for land mobile radio with decoding scheme devised by Timor; and the FH-DPSK (Frequency Hopped Differential Phase Shift Keying) system of Cooper and Nettleton. The third spread spectrum scheme, a high data rate packet radio system using adaptations of the RAKE technique, has been built and tested by Stanford Research International (SRI) in the San Francisco Bay area which is not a cellular system. However, effectiveness of the spread spectrum technique to mobile radio has not been demonstrated by field trial as yet.

1.6 Thesis Objectives

In a spread spectrum cellular mobile radio system, the question of separation between the two cells where the same FH patterns are used remains unanswered [YUE, 83]. The main objective of this thesis is to calculate the cochannel reuse distance for a FH-MFSK Cellular Mobile Radio System where the same FH patterns can be used. This objective is accomplished as follows:

i) A quantitative evaluation of the effect of cochannel and adjacent cell interferences in a FH-MFSK cellular mobile radio has been carried out and necessary expressions developed for the purpose.

ii) Analysis, studies and computational results were obtained.

iii) The cochannel reuse distance for varying parameters has been calculated and FH patterns assigned in a FH-MFSK system.
1.7 Thesis Organization

In this thesis work, various expressions were developed and the effect of cochannel and adjacent cell interferences on the probability of bit error has been evaluated. The FH-MFSK cellular mobile radio system has been studied, analysed and computational results obtained. Based on the above results a FH pattern assignment scheme has been proposed for a cellular radio system using FH-MFSK transmission.

Chapter 2 and 3 review the narrowband cellular mobile radio and spread spectrum systems respectively, emphasizing major differences, their relative merits and demerits.

In chapter 4, generation and design of Frequency Hopping Patterns, FH-MFSK system operation and the effect of Rayleigh fading and lognormal-shadowing on FH-MFSK transmission is discussed.

In chapter 5, various expressions for cochannel, adjacent cell interferences for a general cellular structure using FH-MFSK technique with power control strategy and probability of error are developed.

In chapter 6, the computational results for cochannel, adjacent cell interferences and probability of error are presented, system performance analysed and FH Pattern assignment scheme proposed.

In chapter 7, the main conclusion of this research is presented with suggestion for future research in this and related areas.
Chapter 2

Cellular Mobile Radio System

2.1 Introduction

The purpose of cellular radio is to provide service to a large number of mobile radio customers and hence a very high subscriber capacity by intensive reuse of frequencies. The area to be served is divided into a number of cells. Each cell contains a base station which serves the mobile units with a number of frequency pairs. The radio channel set used in a cell is reused in another cell separated sufficiently to keep the cochannel interference at an acceptable level.

2.2 Cellular Geometry

A cellular system could be designed by dividing a large geographical area in various cells of shapes such as square, equilateral triangle, regular hexagonal shape etc. but for economic reasons a regular hexagonal shape is chosen. Similar shape of all cells helps to systemise the design and layout of cellular systems. A hexagonal layout requires fewer cells than the triangular or square cells with all other factors being equal. Therefore, a hexagonal layout requires fewer base stations
Figure 2.3.1 Geometric shapes for cellular system (a) regular hexagons (b) equilateral triangles (c) squares.

and hence less expensive than the other layouts. Figure 2.2.1 shows various geometric shapes for a cellular system.

The cochannel cells in a cellular system can be determined as follows:

Starting with any cell as a reference move i cells along any chain of hexagons; turn counterclockwise 60 degrees; move j cells along the chain that lies on this heading. Where i and j are called
shift-parameters which are predetermined in some manner [MAC 79]. The $j$th cell and the reference cell are called cochannel cells. Figure 2.2.2 shows an example of the cochannel cell determination.

The number of cells per cluster, $N$, can be given by

$$N = i^2 + ij + j^2$$

Let $R$ be the cell radius and $D$ be the distance between the centers of the nearest neighboring cochannel cells, the cochannel reuse ratio is given by $\frac{D}{R}$ which is related to the number of cells per cluster, $C$, as follows:

$$\frac{D}{R} = \sqrt{3N}$$

(hexagon)

The cochannel interference ratio determines the number of cells per cluster, $C$, can take only the selected values such as

$$N = 3, 4, 7, 9, 12, 13, \ldots$$

which is determined from $N = k + l^2 - kl$, where $k$ and $l$ range over positive integers.

The cochannel reuse ratio ($D/R$) has impact on the transmission quality and ultimate customer capacity of the system. $D/R$ ratio materially affects the cochannel statistics and hence influences the transmission quality. Cochannel reuse ratio determines the number of channels per channel-set and sets a limit on each site's traffic carrying capacity. The cochannel reuse ratio should be minimised for economy. Figure 2.2.3 shows a model for cochannel reuse.

2.3 Cellular Radio Concept

In a cellular network a number of base stations are distributed throughout the coverage area divided into cells of hexagonal shapes. The base station may use omnidirectional or directional antennas. The directional antennas are located at alternate corners of the cells with 120 degrees beamwidth to illuminate the adjacent cells. Each base station contains a group of low-power
transmitters/ receivers for communicating with the mobiles in its cell. The base stations are connected to the Mobile Telephone Switching Office (MTSO) which is connected to the national telephone network.

The MTSO controls the operation of a cellular mobile system. The mobile location must be known to the MTSO for hand off process which is initiated whenever a mobile crosses the boundary between two cells. A channel is reassigned to the mobile crossing into a cell from a set of channels available in that cell. A hand off process initiated during midconversation remains transparent to the user and takes approximately 50 ms. Figure 2.3.1 shows a hand off process which must be closely synchronized with the channel retuning process in the mobile unit. A subscriber operating
within its boundaries (the cell where the subscriber is primarily registered) is termed a home mobile
while outside his boundaries, he is called as a roamer.

![Diagram](image)

Figure 2.3.3 model for cochannel reuse

The subscriber equipment has a resident microprocessor and offers a number of options, such as called number display, push button dialling and recall of last number dialled. Other options may include on-hook dialling, automatic call queuing, storage and recall of 10 telephone numbers, electronic clock etc.

2.3.1 Signalling and Call Processing

In cellular mobile telephony, two types of radio channels are made available viz. set-up channel and voice channel. The set-up channels are monitored by each idle mobile in the cell. They are used only for signalling purposes i.e. initiating or setting up phone calls by transmitting and receiving only data messages. The voice channel provides speech path for the subscribers and handle short burst of data blank and burst interleaved with the subscriber's conversation for handoff purposes.

An idle mobile scans the set-up channel frequencies over which paging message is transmitted
for call processing. A call from a fixed network transmits a telephone number of the called mobile as a paging message over (forward) set-up or paging channel throughout the service area. The mobile unit receives the paging message and identifies itself over the reverse set-up channel assigned to the cell.

![Diagram of a cellular mobile radio system](image)

**Figure 2.3.1 Signal strength measurement from adjacent base stations for handoff process**

A free voice channel is assigned to the mobile and command to retune to the assigned frequency is sent to the mobile transceiver over the set-up channel. The called mobile telephone rings and
call is established.

Similarly, the mobile dials the desired number which is stored in a register until the dialling is complete. Mobile subscriber then presses a send button which transmits a data message (identification of mobile and number of the called party) over the set-up channel. A free voice channel available within the cell is assigned to the call by MTSO and connection is established with the called party through the set-up channel as the mobile tunes to the voice channel.

When either party has hung-up, the mobile unit is directed over set-up channel to turn-off its transmitter and return to idle mode.

2.4 Narrowband Systems

The narrowband frequency modulation system is the most dominant type of mobile communication system of today. The other narrowband systems are single-side band amplitude modulation and digital modulation systems which are being considered for cellular system.

2.4.1 Narrowband Frequency Modulation

The FM capture effect is advantageously used to increase the radio spectrum efficiency in a narrowband FM spectrum. An FM receiver will capture to the stronger of the two signals at the same or slightly different frequencies. However, under conditions of fading and shadowing prevalent in mobile environment, the receiver may be captured by a more distant transmitter because the near transmitter happens to be located behind a large building or terrain irregularity. Under these circumstances the distant transmitter gives rise to high signal strength. The signal strength may be maintained by increasing the transmitted power by a fading margin of 20 dB or so during fades. The increase in transmitted power increases cochannel and adjacent channel interferences as well as demand on primary power source in mobile equipment.
2.4.3 Single-side Band Amplitude Modulation

As the spectral efficiency is mainly determined by the bandwidth of each channel, the SSB-AM can be advantageously used as it does not require as wide a bandwidth as frequency modulation. Frequency companding further reduces the bandwidth. Since speech does not tend to have high and low frequency components simultaneously, hence the spectrum can be folded and bandwidth reduced by about 60 percent at the expense of signal quality [C N, 83].

Signal-to-Noise ratio can be improved if amplitude companding is included in the system. SSB-AM, however, suffers from fading problem like narrowband FM. In addition, in case of SSB-AM there is no capture effect to suppress cochannel interference [C N 83]. Moreover, frequency stability requirement of SSB-AM is another problem, particularly in 900 MHz bands. The *tell* quality of speech, which the users are accustomed to, is hard to achieve in a system using amplitude or frequency companding. However, SSB-AM is being actively evaluated for its application in narrowband cellular systems.

2.4.3 Narrowband Digital Modulation

Digital-data transmission is an important part of a narrowband cellular mobile radio being needed on set-up channels for signalling purposes. Two basic approaches can be adopted to build data capability in a mobile communication network.

**base band modulation approach**

This approach uses baseband signals to modulate analog transmission e.g. *Mark* and *Space* tones can be used to modulate a standard FM voice radio. Communication telephone modems are used to transmit data. The base band approach is compatible with the existing system which is
the major advantage of this approach. However, it is limited by the FM channel width to data rates of the modem, typically 300 to 2400 bps.

digital modulation

In this approach of digital modulation the data rate can be chosen as per design requirement unlike the previous approach where the same is limited by the modem. Typically, in differentially coded biphase modulation, the phase shifts in a constant-frequency carrier encode the information.

However, in digital modulation the entire transmission fades together with deep fades lasting several data symbols. The repetition of data can reduce the effect of error bursts but this causes delays in the decoding process. Use of space diversity scheme and burst-error-correction coding techniques, however, reduce the effect of fading at the expense of complexity as discussed previously in chapter-1.
Chapter 3

Spread Spectrum

3.1 Introduction

The spread spectrum techniques were originally developed to foil jamming of military communications. The spread spectrum is currently in use in certain commercial satellite applications and its use in cellular mobile radio is under study. However, there has not been any demonstration of this technique applied to the mobile environment by field trial as yet.

The spread spectrum techniques are completely opposite in a sense of spectral occupancy to the conventional narrowband techniques. In spread spectrum, the channel bandwidth is expanded instead of limiting it resulting in transmitted bandwidth several orders of magnitude more than the signal bandwidth.

Before we take up an extensive study of a spread spectrum cellular mobile radio system, a brief review of the spread spectrum technique will be presented in this chapter.

3.2 Concepts of Spread Spectrum

In a spread spectrum, the power density to transmit a given information is much lower than the conventional transmissions. Figure 3.2.1 shows a relationship between the power density and bandwidth for the systems using conventional and spread spectrum techniques for a given constant power. As observed from the figure the narrowband transmission uses a bandwidth $B_n$ with a power density of $S_n$ to transmit a message. Since the power required to transmit this information
is constant, the bandwidth can be increased to \( B_x \) and power density reduced to \( S_x \) without affecting the message. Therefore, in a spread spectrum system the bandwidth \( B_x \gg B_n \) and \( S_x \ll S_n \). Since the transmitted power is constant in both the cases, we have

\[
\frac{B_x}{B_n} = \frac{S_n}{S_x}
\]

The signal-to-noise ratio at the receiver is affected by the noise in available bandwidth. In a spread spectrum system the received signal power is much smaller than the noise power because of large bandwidth, but in the receiver the bandwidth is compressed to its original (conventional) bandwidth and most of the interference power is spread in bandwidth by a complex processing of the signal. Therefore, the spread spectrum has the most effective and efficient interference rejection capabilities.

The spread spectrum technique is based on C.E. Shannon’s relationship in the form of channel capacity as follows:

\[
C = W\log_2(1 + \frac{S}{N})
\]

where

- \( C \) = capacity in bits per second
- \( W \) = bandwidth in Hertz
- \( S \) = signal power
- \( N \) = noise power

Rearranging equation (1)

\[
\frac{C}{W} = \log_2(1 + \frac{S}{N})
\]

\[
= 1.44\frac{S}{N}
\]

Since

\[
\log_2 x = \frac{\log x}{\log 2}
\]

\[
= 1.44\log x
\]

Therefore, equation (1) can be written as

\[
W = \frac{CN}{1.44S}
\]
Spread Spectrum

\[ B_s > B_n \text{ and } S_s < S_n \]

\[ B_s / B_n = S_s / S_n \]

Figure 3.2.1 Principle of spreading spectrum.
From equation (2) it is clear that by increasing bandwidth, a low information-error rate can be achieved for a given signal-to-noise ratio.

EXAMPLE: For an information rate of 4 kbps and $\text{SNR} = 0.01$, the required bandwidth can be given as

$$W = \frac{4 \times 10^3 \times 100}{1.44} = 2.777 \times 10^5 \text{Hz}$$

3.3 Techniques of Spreading Spectrum

There are several techniques to spread spectrum such as direct sequence, frequency hopping, chirping and many other hybrid combinations of modulation forms. A brief description of each technique is given below.

3.3.1 Direct Sequence

In a direct sequence or PN sequence modulation technique the binary data bit rate is multiplied by a binary PN sequence of higher symbol rate (or chip rate) to spread the spectrum. The coded signal is obtained by modulo-2 addition of a sequence of, say, $N$ chips to each of the data bits as shown in Figure 3.3.1.1.

Although there are many modulation techniques to modulate a carrier in spectrum spectrum but frequency and phase modulation schemes are common for data transmission being insensitive to the signal amplitude variations.

In BPSK [DIX 78] the phase of the carrier is shifted by $180^\circ$ for each level change in the NRZ. This means that a change of data value from 0 to 1 or 1 to 0 will provide $180^\circ$ phase shift in carrier signal. Figure 3.3.1.2 shows a block diagram of a direct sequence spread spectrum system.

A PN sequence is impressed upon a data signal and recovered at the demodulator. The synchronisation between the transmitted and received PN sequences is maintained during the
Spread Spectrum

Figure 3.3.1.1 Direct sequence waveforms.
Let $R$ bps be the information rate and $W$ is the channel bandwidth. In BPSK, the phase of the carrier is shifted pseudo-randomly in accordance with the pattern generated at a rate of $W$ times per second. The PN chip duration $T_c$ can be given as $\frac{1}{W}$. If $T_b$ denotes the duration of information $(1/R)$, the bandwidth expansion factor $r$ can be given by

$$r = \frac{W}{R} = \frac{T_b}{T_c}$$

where, $\frac{T_b}{T_c}$ is an integer and represents the number of chips per information bit $L_c$ and hence the number of phase shifts in transmitted signal during $T_b$.

If encoder generates a binary linear block code $(n,k)$ by collecting $k$ information bits at a time, then $n$ code elements can be transmitted in $kT_b$ seconds and the number of chips occurring during the period is given as $kL_c$. Therefore, the block length of the code $n = kL_c$ can be selected. In case of a binary convolution code of rate $\frac{k}{n}$, the number of chips 'n' remains the same.

Figure 3.3.3 shows various direct sequence waveforms.

### 3.3.3 Binary Direct-Sequence SSMA Communications

The binary DS SS modulation uses a baseband signal of the following form

$$y(t) = \sum_{i=-\infty}^{+\infty} y_i \phi(t - iT')$$

as a spectral-spreading signal. Where $y_i$ represents a periodic sequence $\{+1, -1\}$. The $\phi$ is a signal varying between 0 and $T'$. The signal $\phi$ can be chosen as

$$\phi(t) = G_T(t) = \begin{cases} 1, & 0 \leq t < T' \; ; \\ 0, & otherwise \end{cases}$$

The message binary data may be of the following form:

$$m(t) = \sum_{i=-\infty}^{+\infty} b_i G_T(t - it)$$
Figure 3.3.12 Direct sequence spread spectrum system.
Figure 3.2.13 Direct sequence waveforms.
where $G_T(t)$ represents a rectangular pulse of duration $T$. The baseband ss-signal is given by

$$u(t) = y(t) \cdot m(t)$$

The transmitted signal can be given by

$$s(t) = A u(t) \cos(\omega t + \beta)$$

substituting $u(t)$ in the above expression we get

$$s(t) = A y(t) m(t) \cos(\omega t + \beta)$$

(3)

where $\omega$ represents the frequency of the carrier and $\beta$ is is a phase angle of arbitrary value. Since there are $n$ such signals transmitted simultaneously from the binary DS SSMA system, the equation (3) can be written as

$$s_n(t) = A p_n(t) m_n(t) \cos(\omega t + \beta_n)$$

(4)

The transmitters are not phase synchronous with different propagation delays for different signals.

Figure 3.3.2.4 shows a model for binary direct-sequence SSMA communication system [PUR, CISM]. The received signal will be of the following form being adulterated by the channel noise (considering a Gaussian channel)

$$r(t) = N(t) + \sum_{n=1}^{k} s_n(t - \alpha_n)$$

(5)

where $N(t)$ is the time delay associated with the $n$-th signal and $N(t)$ is additive white Gaussian noise (thermal noise) with spectral density $\frac{N_0}{2}$. If we assume a correlation receiver (or a matched filter matched to the $k$-th signal), the output of the $k$-th receiver can be given by

$$X_k = \int_{0}^{T} r(t) a_k(t) \cos \omega t \, dt$$

(6)
The decision of the receiver will be based on $X_4$. If the receiver finds that $X_4 \geq 0$ then a positive pulse is assumed else a negative pulse is assumed to be sent.

3.4 Frequency Hopping

In a frequency hopped communication system the spreading of the spectrum is achieved by hopping (changing) the carrier signal at regular intervals. The transmitted signal is hopped to equally spaced frequencies across the available frequency band in a random sequence called hopping pattern. The choice of the pattern is such that the signal hops to each frequency only once within the pattern.

Figure 3.4.0.1 shows a block diagram of a frequency-hopped spread spectrum system. The information sequence (data) is modulated (FSK) and the resulting FSK signal is translated in frequency by an amount that is determined by the output sequence from the PN generator which in turn selects the discrete reference frequency that is synthesized by the frequency synthesizer. The mixer output is then transmitted over the channel.

At the receiver, the channel waveform is mixed with the output of the synthesizer which is controlled by an identical PN sequence generator synchronised with the received signal. The resultant signal is demodulated (FSK) and data recovered at the decoder output. Figure 3.4.0.2 shows the frequency hopping waveform.

3.4.1 Frequency-Hopped Multiple-Access Systems

Another form of simultaneous transmission using a communication channel is known as frequency-hopping multiple access (FHMA). Two forms of FHMA mobile radio systems for digitized speech have been simulated to date; Frequency-hopping Multilevel frequency shift keying
Spread Spectrum

Figure 3.3.4: General form of a FHMA transmitter
Spread Spectrum

FH-MFSK and Frequency-hopping Differential phase shift keying (FH-DPSK) developed and proposed by Goodman et al. and Cooper-Nettleton respectively.

Figure 3.4.1.1 shows a general form of FHMA transmitter. There are two branches in the transmitter, one for encoding and the other for addressing. Each user is assigned an L-word address, \( a_m = (a_{m,0}, a_{m,1}, ..., a_{m,L-1}) \), where \( m \) denotes the \( m \)th user. For a MFSK system, there are \( 2^k \) frequencies with each word of \( k \)-bits long. The data word \( \omega_m \) controls the modulator to produce the baseband waveform given by

\[
s_m(t) = \cos\left(\frac{2\pi nt}{r}\right)
\]

where \( n = 0, 1, ..., N - 1 \) and \( 0 \leq t < T \)

Defining a set of orthogonal waveforms over a time interval \((0, T)\) as

\[
v_{ml}(t) = \sqrt{2S} \phi(t - lr) \cos(2\pi f_0 + \frac{m}{r}t)
\]

for \( l = 0, 1, 2, ..., L - 1 \) and \( m = 0, 1, 2, ..., M - 1 \), where \( f_0 \) is an integral multiple of frequency separation \( \frac{1}{r} \), \( Lr = T \) and the rectangular pulse can be represented by

\[
\phi(t) = \begin{cases} 
1, & 0 \leq t < r; \\
0, & \text{otherwise}
\end{cases}
\]

The output of the frequency hopper is of the form

\[
v_m(t) = \sum_{l=0}^{L-1} \sqrt{2S} \phi(t - lr) \cos(2\pi f_0 + \frac{c_{ml}}{r}t), \quad 0 \leq t < T
\]

where \( c_{ml} = (n + a_l) \mod M \) and \( l = 0, 1, 2, ..., L - 1 \). Figure 3.4.1.2 shows a FH-MFSK receiver. The receiver consists of a frequency dehopper, energy detectors for \( M \) users, a combiner with or without hard-limiting. The address of the user is subtracted from the received signal at the frequency dehopper and the output applied to the energy detectors, each one with two matched filters as shown in Figure 3.4.1.3. The output \((X_m, l)\) of the matched filters are squared, summed and applied to the combiner which gives an output

\[
R_m = \sum_{l=0}^{L-1} X_{m,l}
\]
where $m = 0, 1, 2, ..., M - 1$
In linear combining scheme, the frequency corresponding to the largest $R_m$ is chosen and assumed to be the transmitted one. In hard-limited combining scheme, the decision is made by comparing the energy detector output against a threshold $\frac{E^*}{2E}$. The new decoding matrix $\tilde{X}_{n,t}$ thus formed can be given by

$$
\tilde{X}_{m,t} = \begin{cases} 
1, & \tilde{X}_{m,t} > \frac{E^*}{2E} \\
0, & \tilde{X}_{m,t} \leq \frac{E^*}{2E}
\end{cases}
$$

The row which contains the largest number of ones is chosen. If there are more than one row, the probability of each one being chosen as the correct one, is equal.

It has been shown [YUE,81] that for a large signal-to-noise ratio, the hard-limited combining can accommodate more users than with linear combining. Figure 3.4.1.4 shows a comparison between
Spread Spectrum

FHMA-MFSK system with hard-limited and linear combining for \(k=8\), \(M=256\), \(L=19\) and \(R=32.9\) kbps, \(W=20\) MHz. \(E_b/N_0\) denotes the energy per bit per one sided noise spectral density.

As the SNR decreases, the advantage of hard-limiting diminishes.

general

The frequency hopping does not offer instantaneous coverage of the broad signal band and, therefore, the hopping rate is usually selected to be either equal to the symbol rate (coded or uncoded) or faster than that. The fast-hopped signal has multiple hops per symbol and the frequency hop approximates spectrum spreading. In case of slow-hopped signal, the hopping is performed at a symbol rate. These two types of hoppings are distinguished from each other by an amount of time spent at each discrete frequency before changing or hopping to the next one. On time scale the two types are differentiated by the length of time necessary for the radio signal to propagate from the transmitter to the desired receiver or jammer.

slow-hopped signals

In slow frequency hopping, the carrier frequency remains constant for a much longer time durations than the propagation time e.g. of the order of several milliseconds (the propagation speed is roughly 1,000 ft per microsecond, and the corresponding time scale is of the order of 10 microseconds for several miles). This allows many data bits to be transmitted at each frequency. The transmitting and receiving system is less expensive and simpler than the fast hopping case.

Slow frequency hopping has the disadvantage of being followed by the jammer. The jammer can scan the signal frequency band by using search receiver and locate the transmission and concentrate his jamming power at the transmitting frequency.
Figure 3.4.1.3 Energy detector.
Figure 3.4.1.4 Comparison between FHMA-MFSK systems with hard-limited and linear combining.

[TUE 81]
The slow frequency hopping can be used in conjunction with frequency division multiplexing of various signals in the wide bandwidth range of the hop. A unique carrier frequency can be assigned to each channel in the overall bandwidth. The assignment of frequency is changed every time in a way that each channel will hop synchronously with others but for jammer would appear random. This will force the jammer to adopt some alternate techniques as it would be difficult for him to concentrate jamming power on a specific target signal which would appear to be lost among all others in the band.

**fast-hopped signals**

In fast frequency hopping, the signal is hopped to a new frequency before the jammer completes his measurements, retuning functions and interfere with the desired signals. The desired hopping rate can be determined by taking into account the time delays involved in the propagation of signal to the receiver and jammer, time delays in processing and tuning at jammer. The time delays provide a sort of **safe window** during which the signals are virtually immune to jamming. The so-called **safe window** is the time difference between the arrival of the direct signal and the arrival of interference and can be given by

\[ t_d = \frac{t_s + d_{st} + d_{jr} - d_{tr}}{c} \]

where

- \( t_d \) = time difference between the arrival of the direct signal and interference
- \( t_s \) = time required by the jammer for the processing of the signal, retuning and commencing transmission.
- \( d_{st} \) = distance from the transmitter to jammer.
- \( d_{jr} \) = distance from jammer to receiver.
- \( d_{tr} \) = distance from transmitter to receiver.
- \( c \) = velocity of light.

The fast frequency hopping time at each signal frequency should be less than \( t_d \) so that the
interference would arrive after the receiver has hopped to the next frequency and thus rejecting jammer's effort. The fast frequency hop rate in this case is \( \frac{1}{f_s} \) hops per second.

However, the jammer might intend to jam only a fraction of the total hopped band so as to interfere with a sufficient number of hops to exceed the desired bit error rate. To overcome this problem, powerful error correcting codes are used which allow information from the unjammed frequencies. In a communication system the error correcting codes can be used to protect signals against external noise, intentional or unintentional jamming.

However, in fast frequency hopping, the penalty paid in dividing a signal into many frequency-hopped elements is that the energy from these differential elements are combined non-coherently and thus a form of non-coherent combining loss occurs at the demodulator.

The processing gain of the frequency hopping system (for contiguous channel) can be given by

\[
\text{Processing gain} = \frac{\text{RF Bandwidth}}{\text{Information rate}}.
\]

If the channels are not contiguous, the processing gain can be given by

\[
\text{Processing gain} = \text{the number of frequency choices}.
\]

EXAMPLE: If we have 2000 frequency choices, the processing gain available is 33 dB.

3.5 Time Hopping

In time hopping technique of modulation, the transmitter is keyed on and off pseudo-randomly in accordance with the code sequence. The time hopping signal is usually employed in combination with the frequency hopping. The change in frequency may be only at one/zero transitions in the code sequence.

In time division multiplexing the time hopping may be employed to reduce interference. Figure 3.5.1 shows a block diagram of a simple time hopping system. The time hopping may also be useful
Figure 3.5.1 Block diagram of a time hopping system.

... in ranging, multiple access etc.
3.6 Chirp system

In a chirp system (pulsed FM), the frequency of the RF signal is varied in some known way during each pulse period. This modulation technique does not employ coding but a wider bandwidth to realize processing gain. Figures 3.6.1 and 3.6.2 show typical waveforms for a chirp system.

A matched filter is usually employed to receive chirp signals which is matched to the angular rate of change of the transmitted frequency-swept signal.

The main application of the chirp technique is in radar where significant amount of power can be saved by its use. However, it can also be used in communications. It is expected that future radar systems will employ chirp techniques to a greater extent than past, because of the large reduction in size and improvement in performance that has been achieved recently because of surface wave, delay lines and other types of dispersive elements.

3.7 HYBRID SYSTEMS

The hybrid spread spectrum techniques offer certain advantages over the frequency hopping and direct sequence techniques. The following hybrid spread spectrum signals are most often used.

FH/DS systems

In FH/DS (frequency-hopped direct-sequence) form of hybrid system, the centre frequency of the direct-sequence modulated signal hops periodically in accordance with the frequency hopping
pattern. Figure 3.7.1 shows a frequency spectrum of hybrid FH/DS system. The carrier frequency of the FH/DS system is hopped and not constant as in DS system. The code sequence generator controls the frequency synthesizer to program its hopping pattern and the balanced modulator for direct modulation. Various reasons for using hybrid FH/DS signals are to extend spectrum spreading capability, for multiplexing, for multiple access and discrete address.

TH/DS system

In a Time-Hopping/Direct-Sequence system, the time-division multiplexing is added to a direct sequence which is simply on-off switching and control of the direct sequence signal.

3.8 Time-Frequency Hopping

In a time-frequency system, the code sequence determines the transmitted frequency and the transmission bit rate both. For mobile radio the time-frequency hopping system is main choice. A unique set of time- and frequency- coded waveforms are assigned to each mobile unit for transmission and reception. With large time- bandwidth products of these coded waveforms, the interfering signals are suppressed easily [LEE] and [CN 78].
transmitted chirp signal

\[ f_2 - f_1 = \delta f \]

Figure 3.6.1 Transmitted chirp signal.
Figure 3.6.2 Typical waveform for a chirp system.
Figure 3.7.1 Frequency spectrum of a hybrid FH/DS system.
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Chapter 4

FH-MFSK CELLULAR RADIO SYSTEM

4.1 Introduction

Viterbi[VIT79] proposed a Frequency-Hopped Multilevel Frequency Shift Keying (FH-MFSK) spread spectrum system to provide mobile telephone service to a large number of customers. This system has been further studied by Goodman, Henry and Prabhu [GHP 80]. In FH-MFSK system all users employ the full system bandwidth simultaneously in every cell and every mobile. Therefore the cluster size is one and there is no difference between the system bandwidth and user bandwidth. However, each user is assigned a distinct tone sequence called address or hopping pattern to distinguish him/her/it from the other users. Each address is a carrier signal spread over the entire time interval of each code word. The FH-FSK is resistant to frequency selective fading because the address is spread in frequency in such a way that the tones are spread more than the coherence bandwidth of the channel.

Mutual interference among the users within a cell limits the performance of FH-MFSK system. The mutual interference can be reduced to an acceptable level by properly designing the set of frequency-hopping patterns [T E 80]. Transmission impairment affect the system performance and increase the probability of error when many users communicate simultaneously. With perfect transmission and taking only mutual interference the number of simultaneous users upto 209 can be accommodated at an average bit error rate of $10^{-3}$ [G P 80]. The Rayleigh fading and lognormal shadowing degrades the system performance which brings down the number of simultaneous users upto 110 with standard deviation of 6 dB and normalized area mean of 20 dB. Further degradation of FH-FSK may be due to synchronization problems.
4.2 Generation of Hopping Patterns

In a frequency-hopped communication system the spreading of the spectrum is achieved by hopping the frequency of a carrier signal at regular time-span across the desired frequency band. The hopping or changing of the carrier to these frequencies is done in a random sequence called hopping pattern. Each frequency within the pattern is hopped exactly once by the carrier signal. In a FH system a conventional binary or multitone frequency-shift keying modulation technique is used for transmitting data. For example, in binary FSK, a carrier signal (hopped to frequency \( f_s \)) is shifted to \( f_s + \Delta f \) and \( f_s - \Delta f \) frequencies by mark and space respectively. This generates a small frequency band or slot around the nominal carrier signal \( f_s \). The carrier signal is hopped from slot-to-slot under the control of hopping pattern.

A fast hopping rate in access of data rate may be used such that during the transmission of a single message bit or symbol several frequency hops occur. This technique will defeat a jammer's attempt to measure signal frequency and tune to that particular portion of the band to jam it. However, if the hopping rate exceeds a few tens of thousands of hops per second, it may create difficulties in the design of frequency synthesizers.

At the receiver, the frequency dehopper (a wideband mixer) multiplies the incoming signal with a local oscillator output which in turn is controlled by a hopping pattern similar to the one used at transmitter. If there is a synchronism between the incoming hopping pattern (in received signal) and a locally generated hopping pattern, the mixer output will give a FSK signal which can be demodulated by a conventional FSK receiver.

In FH Multi-access system the signal generated by a transmitter occupies only one frequency slot at any time. The remaining slots may be occupied by other signals during that time. If synchronism is maintained among all the hopping signals, the hopping patterns may be chosen such that the various signals are in different frequency slots at any time. However, such coordinated
slot-to-slot hopping is difficult to achieve. In an synchronous system there is a possibility of two or more signals to hop simultaneously to the same frequency-slot known as 'hit'. The interference due to 'hits' cause errors in the received signal. The use of error correcting codes while designing hopping patterns would reduce the probability of error in the demodulation process.

4.2.1 Design of Hopping Patterns

The approach in designing the frequency hopping patterns should include the minimization of the number of coincidences (hits) between the sequences originating from different users so as to reduce the mutual interference. The objective can be achieved by ascertaining that (i) each pattern is different from its time-shifted version (ii) a large constant difference exists between two hopping patterns and (iii) a weak cross-correlation exists between patterns and allocation of one or more slots to two different users.

Assume each user is assigned a unique address vector,

\[ h_m = (h_{m1}, h_{m2}, \ldots, h_{mL}) \]  

where, \( h_m \in GF(Q) \) represents the frequency channel occupied by the address at time slot \( i \). \( GF(Q) \) denotes the finite field (Galois field) of \( Q \) elements. Appendix-A presents some elementary concepts of algebra for finite fields. If \( Q \) is a prime number, the rules of addition and multiplication in \( GF(Q) \) are defined by modulo-\( Q \) arithmetic. The output of a modulo - \( 2^k \) adder is a sequence \( Z_m \) formed by the operation

\[ Z_m = h_m + X_m \cdot 1 \]  

which corresponds to the address \( h_m \) shifted cyclically \( X_m \) steps in signal matrix. The notation \( 1 \) is a vector

\[ 1 = (1, 1, \ldots, 1) \]

For \( Q \) not a prime number, different rule of addition is applied for transforming \( h_m \) into \( Z_m \).
FH-MFSK Cellular Radio System

The received signal matrix is scanned by the receiver for active chips. The message \( X_m \) appearing as a complete row in the matrix can be recovered by the receivers by subtracting the address \( h_m \) from the received signal.

Synchronous System

Consider a synchronous system in which the signals are aligned in time for all users. The mutual interference will result between the two transmitted signals if the same time-frequency slot is occupied by them in the signal matrix.

A set of \( Q \) patterns can be generated as follows [EIN 80]: Let

\[
h_m = (\gamma_m, \gamma_m \beta, \gamma_m \beta^2, \ldots, \gamma_m \beta^{L-1})
\]

where \( \gamma_m \) is an element of GF\((Q)\) assigned to user \( m \) and \( \beta \) is a GF\((Q)\) primitive element.

Let \( Z_1 \) and \( Z_2 \) be two sequences generated by two different patterns \( h_1 \) and \( h_2 \):

\[
Z_1 = h_1 + x_1.1 \\
Z_2 = h_2 + x_2.1
\]

At position \( j \) the symbols are

\[
Z_{1j} = h_{1j} + x_1 = \gamma_1 \beta^{j-1} + x_1 \\
Z_{2j} = h_{2j} + x_2 = \gamma_2 \beta^{j-1} + x_2
\]

If \( Z_{1j} = Z_{2j} \) we have,

\[
(\gamma_1 - \gamma_2) \beta^{j-1} = x_1 - x_2 = 0
\]

Consider symbols \( Z_{1i} \) and \( Z_{2i} \) at position \( i \neq j \), we have

\[
Z_{1i} - Z_{2i} = (\gamma_1 - \gamma_2) \beta^{j-1} + x_1 - x_2
\]
Substituting (6) in (7) we get,

\[ Z_{1i} - Z_{2i} = (\gamma_i - \gamma_j)(\beta^{i-j} - \beta^{i-j}) \]  

(8)

Since \( h_1 \neq h_2 \), therefore, \((\gamma_i - \gamma_j) \neq 0 \) and \((\beta^{i-j} - \beta^{j-i}) \neq 0 \) for \( i \) and \( j \leq Q - 1 \).

From (8) we find that for an arbitrary \( i \neq j \) and appropriate choice of \( \beta \), the coincidence of two transmitted vectors with different patterns will be in one chip. The messages will cause coincidence at position \( j \) for any pattern symbols \( h_1 \) and \( h_2 \). The maximum value of \( L \) will be \( Q - 1 \) and the patterns designed be \( Q \).

EXAMPLE-1

Let \( L=4 \) and \( Q=7 \) (prime number), \( \beta=3 \) (primitive element) [EIN 80].

With \( \gamma_m = m \), where \( m = 0, 1, 2 \ldots Q - 1 \), the following patterns can be generated [APP-A]

\[
\begin{align*}
0 & \quad 0 & \quad 0 & \quad 0 \\
1 & \quad 3 & \quad 2 & \quad 6 \\
2 & \quad 6 & \quad 4 & \quad 5 \\
h = 3 & \quad 2 & \quad 6 & \quad 4 \\
4 & \quad 5 & \quad 1 & \quad 3 \\
5 & \quad 1 & \quad 3 & \quad 2 \\
6 & \quad 4 & \quad 5 & \quad 1 \\
\end{align*}
\]

Figure 4.2.1 depicts the address \( h=(2,6,4,5) \) with message \( x=3 \). The transmitted signal will be \( Z_2 = (2,6,4,5) + (3,3,3,3) = (5,2,0,1) \).

EXAMPLE-2

Let \( Q=8 \) and \( L=4 \), where \( Q = 2^k \)

The choice of such \( Q \) is advantageous as \( k \) bits of information is conveyed by each message and with binary logic the operations in \( GF(2^k) \) can be readily carried out. Refer to APP-A for \( Q \) equal to the power of a prime number.
Let $\beta^0, \beta^1, \beta^2, \ldots, \beta^6 = 1, 2, 4, 3, 6, 7, 5$ (Table A).

The set of patterns can be obtained as follows:

\[
\begin{align*}
0 & \quad 0 \quad 0 \quad 0 \\
1 & \quad 2 \quad 4 \quad 3 \\
2 & \quad 4 \quad 3 \quad 6 \\
3 & \quad 6 \quad 7 \quad 5 \\
\vdots & \quad \vdots & \quad \vdots & \quad \vdots \\
4 & \quad 3 \quad 5 \quad 7 \\
5 & \quad 1 \quad 2 \quad 4 \\
6 & \quad 7 \quad 5 \quad 1 \\
7 & \quad 5 \quad 1 \quad 2
\end{align*}
\]

For the pattern $h_5 = (5, 1, 2, 4)$ and message $x = 2$, the transmitted signal will be $z_1 = (7, 3, 0, 6)$.

Though the rules of addition in this example are different than the previous one but it has no effect on the capability of the system. The receiver detects the chips separately and the pattern is subtracted to recover the message.

Asynchronous System

In actual practice, synchronisation among all users is difficult to achieve. The pattern generation method described above can not be used for asynchronous case as the patterns generated by (3) are cyclic shifts or shortened cyclic shifts of each other. The transmitted sequences from different users which are time-shifted will cause interference and have a high tendency for coincidence. In this case any user can receive other user's message by a cyclic shift.
The patterns for asynchronous system can be generated [EIN 80] as follows: The transmitted sequence $Z$ can be obtained by a new rule

$$Z_m = z_m + \gamma_m \cdot 1$$

for minimum asynchronous interference. Where $\beta = (1, \beta^1, \beta^2, \ldots, \beta^{L-1})$, $z_m \in GF(Q)$ and $\gamma_m \in GF(Q)$ represents the pattern of user $m$. Let $Z^1$ and $Z^2$ represent the transmitted sequences from two users

$$Z^1 = z_1 + \gamma_1 \cdot 1$$
$$Z^2 = z_2 + \gamma_2 \cdot 1$$  \hspace{1cm} (10)$$

Considering the interference between $Z^1$ and $Z^2$ shifted $k$ steps to the left. The symbol in position $j$ of $Z^1$ is given as

$$Z^1_j = X_1 \beta^{j-k} + \gamma_1$$  \hspace{1cm} (11)$$

The symbol in $Z^2$ that can interfere with $Z^1_j$ is given as

$$Z^2_{j+k} = X_2 \beta^{j+k-k} + \gamma_2$$  \hspace{1cm} (12)$$

For interference to exist

$$Z^1_j - Z^2_{j+k} = X_1 \beta^{j-k} - X_2 \beta^{j+k-k} + \gamma_1 - \gamma_2 = 0$$  \hspace{1cm} (13)$$

For an arbitrary position $i \neq j$ the two symbols $y^1_i$ and $y^2_{i+k}$ will have the difference

$$y^1_i - y^2_{i+k} = X_1 \beta^{i-k} - X_2 \beta^{i+k-k} + \gamma_1 - \gamma_2$$  \hspace{1cm} (14)$$

Combining (13) and (14), we get

$$y^1_i - y^2_{i+k} = (1 - \beta^{i-j})(\gamma_1 - \gamma_2) \neq 0$$  \hspace{1cm} (15)$$
Equation (15) shows that there will not be a coincidence for more than one position for any cyclic shift between the two sequences from different users. Since the transmitted signals are unsynchronized, the undesired signal may occur during a part of time-interval of the message time.

The basic difference between (2) and (9) is that in case 1 the Q patterns constitute a shortened cyclic code while in case 2 the Q possible messages of a particular user form a cyclic set.

**EXAMPLE-3**

Let L=4 and Q=8

The pattern set consists of vectors $h_1 = (0, 0, 0, 0), h_2 = (1, 1, 1, 1)$ etc [from (9)] For $\beta = 2$, vector $\beta = (\beta^0, \beta^1, \beta^2, \beta^3) = (1, 2, 4, 3)$.

For multiplication of $\beta$ by $x$, we can express $x$ as a power of $\beta$ e.g. $x = 3$ is equal to $\beta^3$

$\beta^3 \beta = \beta^3 (\beta^0, \beta^1, \beta^2, \beta^3) = (\beta^3, \beta^4, \beta^5, \beta^6) = (3, 6, 7, 5)$. Refer to Table A in APP - A. The transmitted sequence $Z$ can be formed by adding pattern to the message $x$ as shown in Example 2.

For example, the pattern $h_4 = (3, 3, 3, 3)$ will give $Z_4 = (3, 6, 7, 5) + (3, 3, 3, 3) = (0, 5, 4, 8)$.

In an asynchronous system operating according to equation (9), the receiver will detect each chip in the signal matrix and the address $\gamma$ is subtracted from all chips. It then multiplies the first column by $\beta^0 = 1$, the next by $\beta^{-1}$, the third by $\beta^{-2}$ and so on. This process will transform the vector $x. \beta$ into $x.1$ which in turn will appear as a complete row in the matrix at position $x$. 


4.3 System Operation

Transmitter

Figure 4.3.1 shows a simplified block diagram of a FII-MFSK transmitter [GHF 80].

Assume that each user sends his message symbols in blocks of k bits at a rate of R bps. Therefore available orthogonal frequencies for the system are $2^k$, each of duration $\tau$ seconds ($1, 2, 2^2, \ldots, 2^k$) and a user, say, $m$ sends a message-word $X_m$ to the buffer. If there is only one user then the message transmission requires only one chip duration ($\tau$). When the number of simultaneous users is more than one ($M > 1$), more than one chip ($L > 1$) will be required for simultaneous communication.
using frequency hopping techniques, so that individual transmissions could be properly decoded. In frequency hopping technique each user is assigned a unique frequency hopping sequence; for example user $m$ will have the sequence, $h_{m1}, h_{m2}, \ldots, h_{mL}$ ($1 \leq h_{mi} \leq 2^k$) called address vector which distinguishes his message from other users. A user $m$ sends frequency $(X_m + h_{mi})$ modulo $-2^k$ during chip1, $(X_m + h_{m2})$ modulo $-2^k$ during chip2, \ldots and $(X_m + h_{mk})$ modulo $-2^k$ during chip-L and thus all of his $k$ bits corresponding to $X_m$ are transmitted. The message word $X_m$ modulates the address vertically in the transmitted matrix as shown in Figure 4.3.2. The output of the modulo $-2^k$ adder is a sequence $Z_{m,i}$ which is formed every $T/L$ second as follows

$$Z_{m,i} = Y_{m,i} \oplus X_m$$

where $\oplus$ denotes modulo $-2^k$ addition. The sequence $Z_{m,i}$ selects one of the available $2^k$ orthogonal frequencies from the tone generator and the signal is transmitted accordingly.

![Figure 4.3.2 Signal matrix for transmitter](image-url)
Receiver

At the receiver of user $m$, all the frequencies including the one sent by other users will be received during a particular chip time. Figure 4.3.3 shows a simplified block diagram of a FH-MFSK receiver [GHP 80].

![Block Diagram](image)

**Figure 4.3.3 Simplified block diagram of FH-MFSK Receiver**

The received signal is detected and the $k$-bit codeword is recovered after modulo $-2^k$ subtraction process. The address is subtracted from the received signal during each chip time $T/1$, as follows:

$$X_m = Z_{m,i} \oplus Y_{m,i}$$
where \( Z_{m,t} \) is the detected signal sequence, \( Y_{m,t} \) is the address and \( X_m \) is the received message word. Therefore, a complete row of correct entries is detected at the detection matrix. However, the detection errors cause spurious entries in the detection matrix which increases the probability of errors in the system. Figure 4.3.4 shows the signal matrices for a receiver.

Due to multipath propagation, noise and mutual interference errors will be introduced in the detection matrix. A false alarm condition will exist if a tone is detected though none was sent by the transmitter. Similarly, a receiver may miss a tone at the detection matrix which was sent by the transmitter, a condition called miss. A false alarm or miss cause errors at the receiver. A majority decision criteria may be used to minimise the probability of bit errors in which a code word associated with a row containing the greatest number of entries is chosen as the most probable message. The probability of bit error depends on factors such as miss probability, \( P_D \), false alarm.
probability, \( P_r \), number of simultaneous users \( (M) \), message block-size \( (k) \) and bandwidth \( (W) \)

4.4 Rayleigh Fading and Lognormal-Shadowing Channel

In land mobile environment the signal arrives at the receiver by way reflection, refraction and scattering from and around buildings, grounds and other obstacles. As the vehicles move through the service area, the received signal envelope fluctuates rapidly due to multipath propagation and wave interference and is a vector sum of many waves of random amplitudes, phases and time delays. In most urban and suburban areas it is well known that the fluctuating signal envelope has a Rayleigh probability density function [HAN 77]. The local mean of the Rayleigh distributed signal envelope is nearly constant over a few tens of wavelength but varies due to obstacles and terrain over large distances. In a Rayleigh environment the errors tend to occur in bursts and the bit error rate is higher than that of the non-fading channels where the bit errors are usually considered to be independent and randomly distributed. The bit error rate in a fading channel decreases logarithmically with signal-to-noise ratio. Therefore, multipath fading is a major source of problem in a mobile environment.

The Rayleigh probability density function, \( p(x) \), adopted as a model for rapid fading in a radio channel is written as

\[
p(x) = \frac{1}{z_0} \exp\left(-\frac{x}{z_0}\right)
\]

where \( x \) is the instantaneous envelope level and \( z_0 \) is mean power. It can be shown that the instantaneous power of a Rayleigh distributed signal amplitude has exponential distribution.

The received signal in a mobile environment is characterized by rapid Rayleigh fading with a slowly varying mean signal strength. Slow or long-term variations of the mean signal strength are attributed to shadowing caused by terrain and obstruction variations. Figure 4.4.1 shows a typical received signal envelope in fading environment. Shadowing effects cause relatively slow variations
in the short-term mean of the signal level and is represented by [HAN 77] the lognormal probability density function.

\[ p(x_0) = \frac{1}{\sqrt{2\pi}x_0} \exp\left(-\frac{1}{2\sigma^2} \left[ \ln\left(\frac{x_0}{x_0}\right) \right]^2 \right) \]

where \(\sigma\) and \(x_0\) are the standard deviation and median value of \(x_0\) respectively.

4.5 Performance of FH-MFSK SS-Receiver

The Rayleigh fading and lognormal-shadowing degrades the system performance.
by increasing the number of simultaneous users in a cell. The probability of error depends on factors such as false alarm, miss probability, message block size, number of users and available bandwidth [GHP 80].

In our analysis we assume perfect synchronization between all users in all cells, randomly chosen address vectors and non-coherent detection for base-to-mobile unit transmission within each square in signal matrix of a FH-MFSK receiver is non-coherent on-off keying and the signal is assumed to be contaminated at the receiver input with additive Gaussian noise. At the output of a receiver filter, the signal is given by

\[
S(t) = \begin{cases} 
  u(t) \cos \omega_0 t, & \text{if signal present in row} \\
  0, & \text{if signal not present in row} 
\end{cases}
\]

The narrowband Gaussian noise can be represented by

\[
n(t) = R_n [Z(t) \exp(j\omega_0 t)]
\]

\[
Z(t) = x(t) + j y(t)
\]

where \(x(t)\) and \(y(t)\) are quadrature components of \(Z(t)\), or

\[
n(t) = x(t) \cos \omega_0 t - y(t) \sin \omega_0 t
\]

Consider a Mark is transmitted, then the receiver filter output is given by

\[
e(t) = S(t) + n(t) = [u(t) + x(t)] \cos \omega_0 t - y(t) \sin \omega_0 t
\]

This signal is applied to the detection circuit and a decision is made whether a Mark or Space is present. The instantaneous envelope at the output of the band pass filter is written as

\[
u(t) = \sqrt{|u(t) + x(t)|^2 + y^2(t)}
\]

The distribution of the envelope can be given by [SCH 68]

\[
p(u) = \frac{v}{N} \exp\left(-\frac{v^2 + u^2}{2N}\right) \left|\frac{e^1}{N}\right|
\]

(1)
where \( N \) represents the mean square value (mean power) of the output noise, \( u \) is the signal envelope and \( I_0(z) \) is the modified Bessel function of the first kind and zeroth order and is defined by

\[
I_0(z) = \frac{1}{2\pi} \int_0^{2\pi} e^{r \cos \theta} \, dr
\]

The distribution of the signal envelope and noise is shown in Figure 4.5.1. For an on-off keying the Mark-Space decision is based on comparing the instantaneous envelope, at the sampling instant, \( u(t) \) against some threshold.

Let \( \beta \) denote the threshold level. The decision criteria for Mark or Space received is given by

\[
u(t) \geq \beta \quad \text{Mark} \quad \text{(at the end of chip)}
\]
The probability of error for a Mark transmitted is given by

\[
P_{eM} = \text{Prob}[v < \beta] = \int_{\beta}^{\infty} p(v)dv = 1 - \int_{\beta}^{\infty} p(v)dv = 1 - \int_{\beta}^{\infty} \frac{v}{N} \exp\left(\frac{-v^2 + u^2}{2N}\right) |I_0(u)\frac{uv}{N}|dv = 1 - Q(\sqrt{2\gamma}, \beta_0)
\]

where

\[
\beta_0 = \frac{\beta}{\sqrt{N}}
\]

is the threshold level normalized to the rms noise

\[
\gamma = \frac{u^2}{2N}
\]

is the signal-to-noise ratio (SNR) at the sampling instant in the filter output. \(Q(a,b)\) is the Q function and is given by

\[
Q(a,b) = \int_{b}^{\infty} \exp\left[-\frac{a^2 + x^2}{2}\right] I_0(ax) x dx
\]

With no intersymbol interference, the probability of error for a space transmitted (no signal) can be given by

\[
P_{eS} = \frac{u}{N} \int_{\beta}^{\infty} \exp\left(-\frac{u^2}{2N}\right) du = Q(0, \beta_0) = \exp\left[-\frac{\beta^2}{2}\right]
\]

In a Rayleigh fading and lognormal-shadowing environment the probability density function of the received signal envelope can be found as follows:

The Rayleigh fading of the envelope \(u\) relative to the local mean \(\bar{u}\) is given by the density function [MG 82].

\[
P(u/\bar{u}) = \frac{\pi u}{2\bar{u}^3} \exp\left[-\frac{\pi u^2}{4\bar{u}^2}\right]
\]
where \( \mu = \langle u > \) The lognormal variation of the local mean about the area mean \( m_d \) can be represented by the density function

\[
P(u_d) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left[-\frac{(u_d - m_d)^2}{2\sigma^2}\right]
\]  

where \( m_d = \langle \bar{u}_d > \)

\( \bar{u}_d, m_d \) and \( \sigma \) are expressed in dB and \( \sigma \) is the standard deviation, typically between 5 and 12 dB in urban area.

\[
m_d = \frac{4.10^{m_d/10}}{2\pi N}
\]

is the normalized area mean with respect to noise power spectrum density \( N \). For a Rayleigh fading and lognormal-shadowing the probability density function of the signal envelope can be found by directly combining (4.5.4) and (4.5.5).

\[
P\left(\frac{u}{\bar{u}_d}\right) = P\left(\frac{u}{\bar{u}}\right)
\]

(4.5.6)

The probability density function of the signal envelope with Rayleigh fading and lognormal-shadowing is

\[
P(u) = \int_{-\infty}^{\infty} P(u/\bar{u}_d)P(\bar{u}_d) d\bar{u}_d
\]

\[
= \sqrt{\frac{\pi}{8\sigma^2}} \int_{-\infty}^{\infty} \frac{u}{\bar{u}_d^2} \exp\left[-\frac{ru^2}{4\bar{u}_d^2}\right] \exp\left[-\frac{(u_d - m_d)^2}{2\sigma^2}\right] d\bar{u}_d
\]

Since

\[
\bar{u}_d = 20 \log \bar{u}
\]

is the mean of Rayleigh distribution and

\[
\bar{u} = 10^{\frac{\mu}{10}}
\]

the equation(7) can be written as

\[
P(u) = \sqrt{\frac{\pi}{8\sigma^2}} \int_{-\infty}^{\infty} \frac{u}{10^{\frac{\mu}{10}}} \exp\left[-\frac{ru^2}{4 \times 10^{\frac{\mu}{10}}\bar{u}_d^{0.5}}\right] \exp\left[-\frac{(u_d - m_d)^2}{2\sigma^2}\right] d\bar{u}_d
\]

(4.5.8)
The probability density function of signal-to-noise ratio $\gamma = \frac{u^2}{2N}$ is derived as below

$$d\gamma = \frac{du}{N},$$

$$du = \frac{N}{\gamma} d\gamma,$$

$$P(\gamma) = P(u) du,$$

$$P(\gamma) = P(u) \frac{du}{\gamma},$$

$$P(\gamma) = \frac{N}{\gamma} P(u),$$

$$u = \sqrt{2N\gamma},$$

$$\bar{u}_d = 20 \log \bar{u},$$

$$\bar{u} = 10^{-\frac{\bar{u}_d}{10}},$$

$$\bar{u}_d = 10^{-\frac{\bar{u}}{10}}$$

substituting equation (8) in (10), we get

$$P(\gamma) = \left[ \sqrt{\frac{\pi}{8}} \int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2} N \gamma}}{\sqrt{2N\gamma}} \exp\left[ -\frac{\bar{u}_d - m_d}{2\sigma^2} \right] \right] \frac{N}{\sqrt{2N\gamma}} \bar{u}_d$$

From equation (12) and (13), we get

$$\bar{u}_d = 20 \left( \frac{\log \bar{u}}{\log_{10} 10} \right)$$

$$d\bar{u}_d = \frac{20}{\log_{10} 10} \frac{1}{\bar{u}} du$$

and

$$\bar{\gamma}_0 = \frac{4\bar{u}_d}{2\pi N}$$

is the mean signal-to-noise ratio over Rayleigh fading.

$$\bar{u} = \frac{\sqrt{2\pi N \bar{\gamma}_0}}{4}$$

$$d\bar{u} = \frac{2\pi N}{8} \frac{4}{\sqrt{2\pi N \bar{\gamma}_0}} d\bar{\gamma}_0$$

substituting (17) and (18) in (16), we get

$$d\bar{u}_d = \frac{1}{\bar{\gamma}_0} \frac{10}{\log_{10} 10} d\bar{\gamma}_0$$
From equation (15), we get

$$\exp\left[\frac{-\pi (2N\gamma)}{4 \times 10^{14}}\right] = \exp\left[\frac{-\gamma}{\gamma_0}\right]$$

(4.5.20)

Again using equation (15), we get

$$\exp\left[\frac{-(u_d - m_d)^2}{2\sigma^2}\right]
= \exp\left[\frac{-1}{2\sigma^2} (20 \log \bar{u} - m_d)^2\right]
= \exp\left[\frac{-1}{2\sigma^2} \left[10 \log \left(\frac{\bar{u}^2}{2\pi N}\right) + \frac{4}{2\pi N} - m_d\right]^2\right]
+ 10 \log \frac{m_d}{m_d} - 10 \log \frac{4}{2\pi N} - m_d^2\right]
= \exp\left[\frac{-1}{2\sigma^2} \left[10 \log \left(\frac{\gamma_0}{m_d}\right) + 10 \log \frac{4}{2\pi N} + m_d - 10 \log \frac{4}{2\pi N} - m_d\right]^2\right]
= \exp\left[\frac{-1}{2\sigma^2} (10 \log \frac{\gamma_0}{m_d})^2\right]$$

(4.5.21)

substitution of (19), (20), (21) in (15) and simplification will give the probability density function of $\gamma$ as

$$P(\gamma) = \frac{10}{\log_{10}\sqrt{2\pi \sigma^2}} \int_{\gamma_0}^{\infty} \frac{1}{\gamma_0} \exp\left[\frac{-\gamma}{\gamma_0}\right] - \exp\left[\frac{-1}{2\sigma^2} (10 \log \frac{\gamma_0}{m_d})^2\right] d\gamma_0$$

(4.5.22)

The probability of error for false alarm, ($P_F$) and miss probability, ($P_D$) may be obtained by averaging $P_{SM}$ and $P_{SS}$ over $\gamma$ [M G 83].

$$P_F = \int_{0}^{\infty} P_{SS} P(\gamma) d\gamma = \exp\left[\frac{-\beta_0^2}{2}\right]$$

(4.5.23)

and

$$P_D = \int_{0}^{\infty} P_{SM} P(\gamma) d\gamma
= 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[\frac{-\beta_0^2}{2(1 + m_d 10^{N})}\right] \cdot \exp\left(\frac{-y^2}{2}\right) dy$$

(4.5.24)

If there is no shadowing ($\sigma = 0$) the probability density function of $u$ is a Rayleigh distribution with a mean $m_d$ in dB.

$$\gamma_0 = m_d$$
Therefore, equation (23) and (24) may be written as

\[ P_F = \exp \left( -\frac{\beta^2}{2} \right) \] (4.5.25)

\[ P_D = 1 - \exp \left( -\frac{\beta^2}{2(1 + \gamma_0)} \right) \] (4.5.26)
Chapter 5

Interference In FH-MFSK Cellular Mobile Radio System

5.1 Introduction

Consider a small set of FH patterns. The design objective for a FH-MFSK cellular radio system is to conserve the hopping patterns by reusing them in areas that are geographically located as close to each other as possible. However, the reuse distance depends on the amount of cochannel (copattern) interference. Moreover, the number of simultaneous users that could be handled at a specific bit error rate tends to decrease with more cells operating nearby. The power control strategy reduces adjacent cell interference significantly. In addition to cochannel and adjacent cell interferences, the mutual interference among the users within a cell increases the probability of error in FH-MFSK systems. Mutual interference is the result of number of coincidences (hits) between address sequences of the users. It can be reduced to an acceptable level by properly designing the set of frequency-hopping patterns.

5.2 Adjacent Cell Interference

In a cellular mobile radio system, there are many adjacent cells arranged to cover the service area. All these cells interfere with one another to some degree and reduce the spectral efficiency of the system. However, in an urban mobile radio environment, the signal power falls off approximately as the inverse of the fourth power of the distance \( \frac{1}{r^4} \) away from the transmitter, therefore, only nearby cells cause significant interference.
The worst interference is that seen by a mobile in a cell corner, where it is equidistance from three nearby base stations. The interference in this case is four times higher than that of an isolated cell.

It has been shown [V G, 83] that a power control strategy could reduce the adjacent cell interference significantly. Analysis of the system employing no power control strategy can support a less number of simultaneous users than the one with power control strategy for an average bit error probability of less than $1 \times 10^{-3}$. Power control also helps to reduce the average power transmitted from a base [V G, 83].

5.3 Distribution of Users Within a Cell

A certain distribution of users within a cell might lead to a better performance of the system than the other. Uniform distribution of users within a cell results in a 32% reduction in spectral efficiency compared to an isolated cell. An approximate knowledge of the distribution of users within a cell allows the optimization of the receiver threshold with respect to the distance from base station. With this optimization, at probability of error $< 10^{-3}$, each cell could accommodate a minimum of 115 users, the exact figure being dependent on the users distribution [V G, 83]. For example, when the users are beta (3.5) distributed with respect to the normalized distance, about 140 users can be accommodated in each cell at probability of error $< 10^3$. Moreover, for asynchronous FH system, the number of users and the distribution of users could greatly influence the probability of error.
5.4 Adjacent Cell Interference In a Cellular System With Power Control

A general cellular configuration is shown in Figure 5.4.1. To calculate the adjacent cell interference, we shall consider only the six adjacent cells since the contribution of interference power from other cells due to the users with different hopping patterns is negligibly small.

Consider a user u moving along a line in a seven-cell general cellular system. Let $kR$ be the distance of the user u from the base station x, where $0.2 < k < 0.9$ and $R$ be the distance...
of the corner from the base $x$. The distances of the six neighbourly cell centers $\sqrt{D, R} = 1, 2, \ldots$ from the receiver $u$ can be given by

\[
\begin{align*}
D_1 &= (1.5 - k)^2 + 0.75 \\
D_2 &= k^2 + 3 \\
D_3 &= (1.5 + k)^2 + 0.75 \\
D_4 &= (1.5 + k)^2 + 0.75 \\
D_5 &= k^2 + 3 \\
D_6 &= (1.5 - k)^2 + 0.75
\end{align*}
\]

(5.41) (5.42) (5.43) (5.44) (5.45) (5.46)

5.5 Probability of Error Calculations For FH-MFSK System

Let $\gamma$ be the average signal-to-interference-noise ratio in the correct row of user $u$ due to the users in the same cell, $\gamma_1$ be the average interference to noise power ratio in the spurious row of user $u$ due to the users in the same cell and $\gamma_2$ be the average interference to noise power ratio in the receiver of user $u$ due to the users in the adjacent cells $1, 2, \ldots 6$.

SIJOR Calculations (base to mobile case)

Assume a perfect synchronization between all users in all the cells and the transmission is considered to be from base to mobile. As discussed in Chapter-4, a complete row of correct entries is detected at the detection matrix for a time-frequency slot $(l, q)$ where $l \in (1, 2, \ldots)$ and $q \in (0, 1, 2, \ldots 2^k - 1)$. The base station will transmit $q$th tone corresponding to the $l$th slot. Since we are assuming a power control strategy within each cell, the base station determines the distance of the farthest user requiring the frequency tone and the transmitted power in the tone is adjusted accordingly so that the farthest user receives a fixed average signal-to-interference-noise ratio $\gamma$ dB. The probability of more than four users in a cell requiring the transmission of the same tone in the same slot is negligibly small [V G, 83 and APP-H]. Therefore four users will decide the transmitted power.
As stated earlier the distribution of users within a cell determines the optimum threshold in
the receiver and hence a certain distribution of users may result in a better performance of the
system than the other.

Distribution of the farthest user in a sample space

Assume the users are uniformly distributed in a cell with respect to distance from the base
station. Let X be the distance of the farthest user which lies between \( b_1 \) and \( b_2 \). Also assume that
\( X \) is equally likely to lie in any small subinterval, no matter where that subinterval lies within
\((b_1, b_2)\). The probability density function for the uniform distribution is given by

\[
f(x) = \frac{1}{b_2 - b_1}, \quad b_2 \leq x \leq b_1
\]  

(5.5.1)

If we consider any subinterval \((a_1, a_2)\) contained entirely within \((b_1, b_2)\) we have

\[
P(b_1 \leq X \leq l) = \int_{b_1}^{l} f(x) \, dx
= \int_{b_1}^{l} \frac{1}{b_2 - b_1} \, dx
= \left[ \frac{l - b_1}{b_2 - b_1} \right]
\]  

(5.5.2)

Consider an n sample users space and \( l_n \) be the distance of the farthest user creating \( (1, j) \)^{th}
entry in the transmitted matrix having a probability density function \( f(x) \). Let \( x_{(1)}, x_{(2)}, \ldots, x_{(n)} \)
be the order statistics of this sample. The cumulative distribution function of the n sample user
space is given by

\[
F_n(k) = \int_{b_1}^{k} f_s(t) \, dt
\]  

(5.5.3)

where \( k \) is the normalised distance of the user from base to the corner of the cell. The farthest
distance of a user in the sample \( x_{(n)} \) is a random variable defined by [51B, 73]

\[
x_{(n)} = \max(x_1, x_2, \ldots, x_n) = G(x_1, x_2, \ldots, x_n)
\]
Therefore
\[
P_n = P[b_1 \leq x_1(n) \leq k] \\
= P[b_1 \leq G \leq k] \\
= P[b_1 \leq \max(z_1, z_2, \ldots, z_n) \leq k] \\
= P[(b_1 \leq z_1 \leq k), (b_1 \leq z_2 \leq k), \ldots, (b_1 \leq z_n \leq k)] \\
= P[b_1 \leq z_1 \leq k]P[b_1 \leq z_2 \leq k]P[b_1 \leq z_n \leq k] \\
\] (5.5.4)

Substituting (2) in (4) we get
\[
P_n = \frac{1}{b_2 - b_1} \left[ \frac{k - b_1}{b_2 - b_1} \right] \left[ \frac{k - b_1}{b_2 - b_1} \right] \\
P_n = \left[ \frac{k - b_1}{b_2 - b_1} \right]^n \\
P_n = 1 - P_n \\
\] (5.5.5)

where \( P_n \) denotes the probability of the farthest user within the distance < \( k \) and \( P_n \), the farthest user outside the distance \( k \) and \( 0.5 \leq b_1 \leq b_2 \leq 0.9 \).

Differentiating equation (5) with respect to \( k \), we obtain the density function of the farthest distance of the user in the sample.
\[
f_n = \frac{n(k - b_1)^{n-1}}{(b_2 - b_1)^n} \\
\] (5.5.7)

In general, for the farthest user \( (k_1 \leq l < k) \), the probability density function is given by
\[
f_n = \frac{n(l - b_1)^{n-1}}{(b_2 - b_1)^n} \\
\] (5.5.8)

The fourth central moment which is an indication of the maximum number of farthest users outside the distance \( k \), can be given by
\[
a_4 = \int_{b_1}^{b_2} \frac{n(m(l - b_1))^{n-1} dl}{(b_2 - b_1)^n} \\
\]

Solving, we get
\[
a_4 = \left\{ \frac{n}{n+4} [b_2 - b_1]^4 + b_1^4 \\
+ 6b_1^2(b_2 - b_1)^2 \frac{n}{n+2} \\
+ 4b_1^3(b_2 - b_1)^1 \frac{n}{n+3} \right\} \\
\] (5.5.9)
Assume \( M \) is the number of users in each cell, each with an address that is a sequence of \( L \) \( k \)-bit words. Each word of an address is selected independently of other and each user's transmitted codewords are equally likely over \( 2^k \) possibilities and independent of each other. For a given message block size of \( k \)-bits per user, the system has \( Q = 2^k \) orthogonal sinusoidal waveforms. Consider a time-frequency slot \((j, l)\), where \( j \) is the spurious row and \( l \) is the time-slot of the detection matrix. If we have only one interfering user, the probability of that user not sending a tone to \((j, l)\) position is

\[
(1 - 2^{-k})
\]

The probability that none of the \((M-1)\) interfering users send a tone to \(j, l\) position is

\[
(1 - 2^{-k})^{M-1}
\]

or,

\[
(1 - p_r)^{M-1}
\]

where,

\[
p_r = 2^{-k}
\]

and let, \(10 \times a\) represents the average signal-to-noise ratio in dB. Therefore, the average signal-to-interference ratio at the receiver of user \( u \) due to \((M-1)\) interfering users is given by

\[
\gamma_l = 10^a (1 - p_r)^{M-1}
\]

Assume \( n \) be the number of users requiring the transmission of same tone in the same slot. The binomial distribution of the \( n \) users requiring the same tone out of \((M-1)\) interfering users is given by

\[
\binom{M-1}{n} p_r^n (1 - p_r)^{M-1-n}
\]
The probability distribution of the farthest users requiring the same tone in the same slot within a distance is given by

\[
\binom{M-1}{n} p^\nu (1 - p_r)^{M-1-n} p_n^n
\]

Since the probability of more than four farthest users requiring the same tone in the same slot of correct row is negligibly small [V G, 83 and APP-H], the average signal-to-noise ratio \( \gamma_2 \) at the receiver is given by

\[
\gamma_2 = 10^\nu \sum_{n=1}^{4} \binom{M-1}{n} p^\nu (1 - p_r)^{M-1-n} p_n^n
\]  

(5.5.12)

Similarly, the average signal-to-noise ratio \( \gamma_3 \), due to the farthest users outside the above distance, is given by

\[
\gamma_3 = 10^\nu \sum_{n=1}^{4} \binom{M-1}{n} p^\nu (1 - p_r)^{M-1-n} p_n^n \frac{\alpha_4}{k^4}
\]  

(5.5.13)

Assume the signal strength varies inversely as the fourth power of the distance. Therefore, the total average signal-to-noise ratio \( \gamma \), at the receiver of the user \( u \) is

\[
\gamma = 10^\nu ((1 - p_r)^{M-1} + \sum_{n=1}^{4} \binom{M-1}{n} p^\nu (1 - p_r)^{M-1-n} [p_n + \frac{\alpha_4}{k^4}])
\]  

(5.5.14)

**Average Interference to Noise Ratio within a cell, \( \gamma_i \)**

The average interference-to-noise ratio within a cell, \( \gamma_i \), in the spurious row of user \( u \), due to the users in the same cell is given by

\[
\gamma_i = 10^\nu \sum_{n=1}^{4} \binom{M-1}{n} p^\nu (1 - p_r)^{M-1-n} \frac{\alpha_4}{k^4}
\]  

(5.5.15)

**Average Adjacent-Cell Interference-to-Noise Ratio, \( \gamma_a \)**

The average adjacent-cell interference-to-noise ratio, \( \gamma_a \), due to the users of all the six adjacent cells is given by

\[
\gamma_a = 10^\nu \sum_{i=1}^{6} \sum_{n=1}^{4} \binom{M-1}{n} p^\nu (1 - p_r)^{M-1-n} \alpha_a
\]  

(5.5.18)
where \( a_n \) is the fourth moment of the adjacent cell base station distance, \( \sqrt{D,R} \) from the user \( u \) and is given by

\[
a_n = \int_{b_1}^{b_2} \left( \frac{r}{D_i} \right)^2 \frac{n}{(b_2 - b_1)^n} (l - b_1)^{n-1} \, dz
\]

or

\[
a_n = \frac{a_4}{D_i^4}
\]

(5.5.17)

5.6 Probability of Error Calculations

The false alarm condition, \( P_f \), occurs due to the noise and multipath propagation as they influence the detection matrix by detecting a tone which has not been transmitted. As shown in Chapter-4, the probability of false alarm for Rayleigh fading environment with no shadowing, \((\sigma = 0)\), is given as

\[
P_f = \exp\left(-\frac{\beta^2}{2}\right)
\]

(5.6.1)

The probability of creating an entry in the spurious and correct row is given as follows: [APP-E].
\[ P_{d1} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a)} \right] \quad (5.6.2) \]
\[ P_{d2} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + 15\gamma_a)} \right] \quad (5.6.3) \]
\[ P_{d3} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + 2\gamma_a)} \right] \quad (5.6.4) \]
\[ P_{d4} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + 15\gamma_a)} \right] \quad (5.6.5) \]
\[ P_{d5} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a)} \right] \quad (5.6.6) \]
\[ P_{d6} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + 6\gamma_a)} \right] \quad (5.6.7) \]
\[ P_{d7} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a)} \right] \quad (5.6.8) \]
\[ P_{d8} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a + 6\gamma_a)} \right] \quad (5.6.9) \]
\[ P_{d9} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a + 15\gamma_a)} \right] \quad (5.6.10) \]
\[ P_{d10} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a + 20\gamma_a)} \right] \quad (5.6.11) \]
\[ P_{d11} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a + 15\gamma_a)} \right] \quad (5.6.12) \]
\[ P_{d12} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a + 6\gamma_a)} \right] \quad (5.6.13) \]
\[ P_{d13} = \exp\left[ -\beta_0^2 \frac{1}{2(1 + \gamma_a + \gamma_a)} \right] \quad (5.6.14) \]

The probability of a tone transmitted to position \((j, 1)\) by \((M-1)\) interferers within the cell is
\[ P_{u0} = 1 - (1 - 2^{-k})^{M-1} \quad (5.6.15) \]

The probability of a tone transmitted to position \((j, l)\) by the interferers from the adjacent cells is
\[ P_{u1} = 1 - (1 - 2^{-k})^{M} \quad (5.6.16) \]
\[ \overline{P}_{u0} = 1 - P_{u0} \quad (5.6.17) \]
\[ \overline{P}_{u1} = 1 - P_{u1} \quad (5.6.18) \]
Therefore, the overall insertion probability due to all interferers for the position \((j, l)\) is

\[
P_i = \bar{P}_u P_u^a P_l + 6\bar{P}_u P_u P_u^a P_d + 15\bar{P}_u P_u^2 P_u^a P_d + 20\bar{P}_u P_u^3 P_u^a P_d + 15\bar{P}_u P_u^4 P_u^a P_d + 6\bar{P}_u P_u^5 P_u^a P_d + \bar{P}_u P_u^6 P_d + P_u P_u^5 P_e + 6P_u P_u P_u^5 P_d + 15P_u P_u^2 P_u^5 P_d + 20P_u P_u^3 P_u^5 P_d + 15P_u P_u^4 P_u^5 P_d + 6P_u P_u^5 P_u^5 P_d + P_u P_u^6 P_d 
\]  

(5.6.19)

Similarly, the probability of \(i\) entries in the correct row can be given by

\[
P_{d14} = \exp\left[\frac{-\beta_5^2}{2(1 + \gamma)}\right] 
\]  

(5.6.20)

\[
P_{d15} = \exp\left[\frac{-\beta_5^2}{2(1 + \gamma + 5\gamma_a)}\right] 
\]  

(5.6.21)

\[
P_{d16} = \exp\left[\frac{-\beta_5^2}{2(1 + \gamma + 15\gamma_a)}\right] 
\]  

(5.6.22)

\[
P_{d17} = \exp\left[\frac{-\beta_5^2}{2(1 + \gamma + 20\gamma_a)}\right] 
\]  

(5.6.23)

\[
P_{d18} = \exp\left[\frac{-\beta_5^2}{2(1 + \gamma + 15\gamma_a)}\right] 
\]  

(5.6.24)

\[
P_{d19} = \exp\left[\frac{-\beta_5^2}{2(1 + \gamma + 6\gamma_a)}\right] 
\]  

(5.6.25)

\[
P_{d20} = \exp\left[\frac{-\beta_5^2}{2(1 + \gamma + 7\gamma_a)}\right] 
\]  

(5.6.26)
The probability of an entry in the correct row of user \( u \) is

\[
P_e = P_{u1}^0 P_{d14} + 5 P_{u1} P_{u1}^3 P_{d15} + 15 P_{u1} P_{u1}^4 P_{d16} + 20 P_{u1} P_{u1}^5 P_{d17} + 15 P_{u1} P_{u1}^2 P_{d18} + 5 P_{u1} P_{u1} P_{d19} + P_{u1} P_{d20}
\]  

(5.6.27)

Therefore, the probability of \( s \) entries in the correct row is

\[
P_e(s) = \binom{L}{s} P_e^s (1 - P_e)^{L-s}
\]  

(5.6.28)

Following [APP-B] for the error rate formulas, we can calculate the probability of bit error, \( P_b \), due to six adjacent cells in a general cellular structure.

### 5.7 Cochannel Interference in FH-MFSK Cellular System

For a large coverage area the available FH patterns can be reused such that the distance between the cochannel cells is sufficient enough to keep the cochannel interference to an acceptable level. The reuse of FH patterns increases the number of channels per unit area of a cellular system.

The probability of cochannel interference due to all cochannel cells surrounding the base station and operating in a Rayleigh fading with no shadowing environment can be evaluated quantitatively as follows:

If \( n \) is the number of active cochannels and \( a \) represents the channel state active ('1') or idle ('0'), the probability of finding the active cochannel can be given by Bernoulli distribution

\[
P(a) = P^a (1 - P)^{1-a} \quad [MG \ 82]
\]

(5.7.29)

where, \( a = 0, 1 \). Consider first type of cochannels, the number of active cochannels is given by,

\[
n = \sum_{i=1}^{6} a_i
\]
where the channels are assumed to be identical and independent

The binomial probability function for the six channels is given as

$$P(n) = \binom{6}{n} P^n (1 - P)^{6-n}$$  \hspace{1cm} (5.7.30)

where, \(n = 0, 1, 6\)

Let \(M\) be the number of users in each cell and \(B_k\) is the probability of blocking under overload condition, then (30) can be written as

$$P(n) = \binom{6}{n} (B_k) \hat{g} (1 - B_k \hat{g})^{6-n}$$  \hspace{1cm} (5.7.31)

Let \(r_i, \bar{C}, R, i = 1, 2, 6, \) be the distance of the cochannel base station from the receiver \(i,\) therefore, the average cochannel interference due to first type of cochannel cells is given by

$$\gamma_{ee} = 10^r \sum_{i=1}^{6} \sum_{n=0}^{6} \binom{6}{n} (B_k) \hat{g} (1 - B_k \hat{g})^{6-n} \frac{\alpha_i}{\bar{C}}$$  \hspace{1cm} (5.7.32)

However, the probability of more than than four cochannel being active at the same time is negligibly small [APP-D], therefore

$$\gamma_{ee} = 10^r \sum_{i=1}^{6} \sum_{n=1}^{5} \binom{6}{n} (B_k) \hat{g} (1 - B_k \hat{g})^{6-n} \frac{\alpha_i}{\bar{C}}$$

$$\gamma_{ee} = 10^r \sum_{i=1}^{6} \sum_{n=1}^{5} \binom{6}{n} (B_k) \hat{g} (1 - B_k \hat{g})^{6-n} \alpha_{ee}$$  \hspace{1cm} (5.7.33)

where, \(\alpha_{ee} = \frac{\alpha_i}{\bar{C}}\)

Probability of Error Calculations With Adjacent and Cochannel Cell Interferences

The probability of error with adjacent and cochannel cell interferences can be calculated as follows:
The false alarm condition is

\[ P_f = \exp\left(\frac{-\beta_0^2}{2}\right) \]  

(5.7.34)

The probabilities of creating an entry in the spurious row of the detection matrix conditioned on the fact that different combinations of the base stations transmit the tone corresponding to that entry [APP-E-F-G] (assume the cochannel and adjacent cell transmissions are independent of each other), is given by

\[ P_{d1} = \exp\left(\frac{-\beta_0^2}{2(1 + 6\gamma_a)}\right) \]  

(5.7.35)

\[ P_{d2} = \exp\left(\frac{-\beta_0^2}{2(1 + 15\gamma_a)}\right) \]  

(5.7.36)

\[ P_{d3} = \exp\left(\frac{-\beta_0^2}{2(1 + 20\gamma_a)}\right) \]  

(5.7.37)

\[ P_{d4} = \exp\left(\frac{-\beta_0^2}{2(1 + 15\gamma_a)}\right) \]  

(5.7.38)

\[ P_{d5} = \exp\left(\frac{-\beta_0^2}{2(1 + 6\gamma_a)}\right) \]  

(5.7.39)

\[ P_{d6} = \exp\left(\frac{-\beta_0^2}{2(1 + \gamma_a)}\right) \]  

(5.7.40)

\[ P_{d7} = \exp\left(\frac{-\beta_0^2}{2(1 + 6\gamma_a)}\right) \]  

(5.7.41)

\[ P_{d8} = \exp\left(\frac{-\beta_0^2}{2(1 + 15\gamma_a)}\right) \]  

(5.7.42)

\[ P_{d9} = \exp\left(\frac{-\beta_0^2}{2(1 + 20\gamma_a)}\right) \]  

(5.7.43)

\[ P_{d10} = \exp\left(\frac{-\beta_0^2}{2(1 + 15\gamma_a)}\right) \]  

(5.7.44)

\[ P_{d11} = \exp\left(\frac{-\beta_0^2}{2(1 + 6\gamma_a)}\right) \]  

(5.7.45)

\[ P_{d12} = \exp\left(\frac{-\beta_0^2}{2(1 + \gamma_a)}\right) \]  

(5.7.46)
\text{Inference in FH-MFSK Cellular Mobile Radio System}

\begin{align*}
P_{d_{13}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + 6\gamma_a + 7\gamma_e)}\right] \quad (5.7.47) \\
P_{d_{14}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + 15\gamma_a + 7\gamma_e)}\right] \quad (5.7.48) \\
P_{d_{15}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + 20\gamma_a + 7\gamma_e)}\right] \quad (5.7.49) \\
P_{d_{16}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + 15\gamma_a + 7\gamma_e)}\right] \quad (5.7.50) \\
P_{d_{17}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + 5\gamma_a + 7\gamma_e)}\right] \quad (5.7.51) \\
P_{d_{18}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 7\gamma_e)}\right] \quad (5.7.52)
\end{align*}

\begin{align*}
P_{d_{19}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a)}\right] \quad (5.7.53) \\
P_{d_{20}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 6\gamma_e)}\right] \quad (5.7.54) \\
P_{d_{21}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 15\gamma_e)}\right] \quad (5.7.55) \\
P_{d_{22}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 20\gamma_e)}\right] \quad (5.7.56) \\
P_{d_{23}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 15\gamma_e)}\right] \quad (5.7.57) \\
P_{d_{24}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 6\gamma_e)}\right] \quad (5.7.58) \\
P_{d_{25}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 7\gamma_e)}\right] \quad (5.7.59)
\end{align*}

\begin{align*}
P_{d_{26}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 6\gamma_e)}\right] \quad (5.7.60) \\
P_{d_{27}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 15\gamma_e)}\right] \quad (5.7.61) \\
P_{d_{28}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 20\gamma_e)}\right] \quad (5.7.62) \\
P_{d_{29}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 15\gamma_e)}\right] \quad (5.7.63) \\
P_{d_{30}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 6\gamma_e)}\right] \quad (5.7.64) \\
P_{d_{31}} &= \exp\left[-\beta_6^2 \frac{1}{2(1 + \gamma_a + 7\gamma_e)}\right] \quad (5.7.65)
\end{align*}
\[ P_{432} = \exp\left[ \frac{-\beta_0^2}{2(1 + \gamma_i + \gamma_a + \gamma_{ae})} \right] \]  
(5.7.66)

\[ P_{433} = \exp\left[ \frac{-\beta_0^2}{2(1 + \gamma_i + 15(\gamma_a + \gamma_{ae}))} \right] \]  
(5.7.67)

\[ P_{434} = \exp\left[ \frac{-\beta_0^2}{2(1 + \gamma_i + 20(\gamma_a + \gamma_{ae}))} \right] \]  
(5.7.68)

\[ P_{435} = \exp\left[ \frac{-\beta_0^2}{2(1 + \gamma_i + 15(\gamma_a + \gamma_{ae}))} \right] \]  
(5.7.69)

\[ P_{436} = \exp\left[ \frac{-\beta_0^2}{2(1 + \gamma_i + 6(\gamma_a + \gamma_{ae}))} \right] \]  
(5.7.70)

\[ P_{437} = \exp\left[ \frac{-\beta_0^2}{2(1 + \gamma_i + (\gamma_a + \gamma_{ae}))} \right] \]  
(5.7.71)

The probability of a tone transmitted by (M-1) users to position \( j, l \) within the cell is

\[ P_{u_j} = 1 - (1 - 2^{-k})^{M-1} \]

The probability of none of the adjacent cell interferer sends a tone to \( j, l \) position is

\[ P_{ua} = 1 - (1 - 2^{-k})^M \]

The probability of none of the cochannel interferers sends a tone to \( j, l \) position is

\[ P_{uc} = 1 - (1 - 2^{-k})^M \]

and

\[ P_{u_j} = 1 - P_{u_j} \]

\[ P_{ua} = 1 - P_{ua} \]

\[ P_{uc} = 1 - P_{uc} \]

The probability of insertion or the unconditional probability of creating an entry in the spurious row, \( P_i \), is given by

\[
P_i = P_{ui} P_{ua} P_{uc} P_f \\
+ 6P_{ui} P_{ua} P_{uc} P_{d1} P_{d1} \\
+ 15P_{ui} P_{ua} P_{uc} P_{d2} P_{d2} \\
+ 20P_{ui} P_{ua} P_{uc} P_{d3} P_{d3} \\
+ 15P_{ui} P_{ua} P_{uc} P_{d4} P_{d4} \\
+ 6P_{ui} P_{ua} P_{uc} P_{d5} P_{d5} \\
+ P_{ui} P_{ua} P_{uc} P_{d6} P_{d6}
\]
Similarly, the probability of an entry in correct row based on the fact that different combinations
Figure 5.5.2: Average cochannel interference to noise power ratio vs distance from the base station.

REF: Eqn. (S.7.32)
Similarly, the probability of an entry in correct row based on the fact that different combinations
of base stations transmit the tone corresponding to that entry is

\[
\begin{align*}
P_{d30} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma)}\right] \\
P_{d30} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 5\gamma_a)}\right] \\
P_{d40} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 15\gamma_a)}\right] \\
P_{d41} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 20\gamma_a)}\right] \\
P_{d42} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 15\gamma_a)}\right] \\
P_{d43} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 6\gamma_a)}\right] \\
P_{d44} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + \gamma_a)}\right] \\
P_{d45} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 6\gamma_a)}\right] \\
P_{d46} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 15\gamma_a)}\right] \\
P_{d47} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 20\gamma_a)}\right] \\
P_{d48} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 15\gamma_a)}\right] \\
P_{d49} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 6\gamma_a)}\right] \\
P_{d50} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + \gamma_a)}\right] \\
P_{d51} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 6(\gamma_a + \gamma_{\text{res}}))}\right] \\
P_{d52} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 15(\gamma_a + \gamma_{\text{res}}))}\right] \\
P_{d53} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 20(\gamma_a + \gamma_{\text{res}}))}\right] \\
P_{d54} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 15(\gamma_a + \gamma_{\text{res}}))}\right] \\
P_{d55} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + 6(\gamma_a + \gamma_{\text{res}}))}\right] \\
P_{d56} &= \exp\left[\frac{-\beta_0^2}{2(1 + \gamma + (\gamma_a + \gamma_{\text{res}}))}\right]
\end{align*}
\]
The probability of an entry in the correct row of user u is given by

\[ P_e = \mathcal{P}_n^4 \mathcal{P}_n^4 \mathcal{P}_{d4} \]
\[ + 6 \mathcal{P}_n^4 \mathcal{P}_n^4 \mathcal{P}_{d4} \mathcal{P}_{d40} \]
\[ + 15 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_n^1 \mathcal{P}_{d10} \]
\[ + 20 \mathcal{P}_n^4 \mathcal{P}_n^3 \mathcal{P}_{d41} \mathcal{P}_{d41} \]
\[ + 15 \mathcal{P}_n^4 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_{d42} \]
\[ + 6 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_n^1 \mathcal{P}_{d43} \]
\[ + \mathcal{P}_n^4 \mathcal{P}_n^4 \mathcal{P}_{d44} \]
\[ + 6 \mathcal{P}_n^4 \mathcal{P}_n^3 \mathcal{P}_{d45} \mathcal{P}_{d45} \]
\[ + 15 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_n^1 \mathcal{P}_{d46} \]
\[ + 20 \mathcal{P}_n^4 \mathcal{P}_n^3 \mathcal{P}_{d47} \mathcal{P}_{d47} \]
\[ + 15 \mathcal{P}_n^4 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_{d48} \]
\[ + 6 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_n^1 \mathcal{P}_{d49} \]
\[ + \mathcal{P}_n^4 \mathcal{P}_n^4 \mathcal{P}_{d50} \]
\[ + 6 \mathcal{P}_n^4 \mathcal{P}_n^3 \mathcal{P}_{d51} \mathcal{P}_{d51} \]
\[ + 15 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_n^1 \mathcal{P}_{d52} \]
\[ + 20 \mathcal{P}_n^4 \mathcal{P}_n^3 \mathcal{P}_{d53} \mathcal{P}_{d53} \]
\[ + 15 \mathcal{P}_n^4 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_{d54} \]
\[ - 6 \mathcal{P}_n^4 \mathcal{P}_n^2 \mathcal{P}_n^1 \mathcal{P}_{d55} \]
\[ + \mathcal{P}_n^4 \mathcal{P}_n^4 \mathcal{P}_{d56} \]

Therefore, the probability of i entries in the correct row is given by [G H P, 80]

\[ P_e(i) = \binom{L}{i} \mathcal{P}_n^i (1 - \mathcal{P}_n)^{L-i} \] (5.7.93)

Using Error Rate Formula [G H P, 80], the probability of bit-error can be calculated as given in APP-B.

\[ P_{b_{e}} \geq \sum_{i=1}^{L} P_e(i) [P(i, 0) + \frac{1}{2} P(i, 1)] \] (5.7.94)
Chapter 6

Computational Results and Performance Analysis of FH-MFSK System

6.1 Introduction

In this chapter, the performance analysis of a FH-MFSK Cellular System has been done on the basis of the computational results using expressions derived in Chapter 5 for adjacent cell, cochannel cell and incell interferences and probability of bit error. We assume perfect synchronisation between the users in all the cells and random address assignment for a user in a cell. The mobile transmission is considered to be from base-to-mobile where near perfect synchronisation is feasible. In addition, a power control strategy is employed by the system which is exclusive to base-to-mobile transmission. Finally, a FH-pattern assignment scheme has been proposed for a FH-MFSK Cellular Mobile Radio System based on the computational results and performance analysis.

general

We consider a case where the degradation of the system performance is due to the adjacent cell, cochannel and incell interferences. Assume one-way (base to mobile) voice transmission is carried out digitally and pertains to the following possible mobile radio system parameter:
\[ W = \text{total spread bandwidth in hertz} \]
\[ R = \text{bit rate (assumed to be same for all users)} \]
\[ P_b = \text{bit error rate} \]
\[ K = \text{bits per code word} \]
\[ M = \text{simultaneous users} \]
\[ L = \text{transmitted tones per code word} \]
\[ Q = \text{available orthogonal sinusoids each of duration } r \text{ seconds} \]
\[ T = \text{word duration} \]
\[ N = \text{number of channels} \]

Consider a FH-MFSK cellular mobile radio system with the following system parameters.

\[ W = 20 \text{ MHz} \]
\[ R = 10 \text{ kbps} \]
\[ K = 11 \text{ bits per code word} \]

Therefore,

\[ Q = 2^4 \]
\[ = 2^{11} \]
\[ = 2048 \]

\[ L = \frac{K}{rR} \]
\[ = \frac{KW}{QR} \]
\[ \approx 11 \]

\[ r = \frac{Q}{W} \]
\[ = 102.4 \mu s \]

\[ N = Wr \]
\[ = 2048 \]

In a real multiple access user communication system, we shall be concerned not with the bit error rate averaged over all users, but with the worst bit error rate suffered by any one user. If the bit error rate is too high, the user will not receive satisfactory service.

6.2 Evaluation of SINR with Power Control Strategy

The power control is generally considered to be essential for the efficient performance of a cellular spread-spectrum land-mobile radio schemes. As we are considering the case of down stream
transmission (base-to-mobile), a mobile which is closer to the base station will not experience any interference from outside its cell; but a mobile near a cell corner will receive nearly three times the level of interference because of its equidistance from two adjacent cells and its own base station. In addition, the cochannel interference experienced by a mobile at the corner is more than that near to its own base station. Therefore, without power control, some mobiles near the cell corner might be incapacitated during the periods of relatively high traffic.

Figure 6.2.1 shows a general cellular structure employed in a mobile radio system. The computational results for the average Signal-to-interference+Noise ratio versus the distance of the user from the base station, k, in the correct row of the users due to the users in the same cell has been plotted as shown in Figure 6.2.2 for M=100, 150 and 240.
Figure 5.2.2 Average signal-to-interference+noise power ratio vs distance from the base station.

Eqn. (5.5.19)
The results show that because of the power control strategy adopted for the system, the average signal-to-noise power ratio at the receiver of the farthest user is maintained at about 25 dB for which the system has been designed from halfway onwards to the cell corner.

6.3 Evaluation of Adjacent cell Interference

We consider only the six nearest cell as shown in Figure 6.3.1 for evaluating the average adjacent cell interference to power ratio due to a tone from base 1 to 6 using distinct address for each user. The other cells (using distinct address for each user) contribute negligibly small interference power to the user in cell '0'.

The computational results for the average interference to noise power ratio due to the cells 1 to 6 in the receiver of user u versus k are shown in Figure 6.3.2. The results indicate that the average interference to noise power ratio due to the adjacent cells increases as the mobile moves from its own base station to the cell corner being equidistant from the its own base station and the two adjacent cells. For s=2.5, the adjacent cell interference to noise power ratio, \( \eta_a \), is calculated to be <20 dB for \( M=100, 150 \) and 240.

6.4 Evaluation of Incell Interference

The performance of a FH-MFSK System is limited by the mutual interference among the users. The transmission errors could occur even if all the transmitted tones are perfectly detected because of the simultaneous transmission from several users. The mutual interference is the result of the number of coincidences (hits) between address sequences of the user.

The computational results shown in Figure 6.4.1 indicate that the in-cell interference to noise power ratio in the spurious row of the user u due to the users in the same cell decreases towards the cell corner. For s=2.5, M=100, 150 and 240, it is calculated to be <19 dB near the corner.
6.5 Evaluation of Cochannel Interference & FH-Pattern Assignment

We consider a set of FH-copatterns reassigned in Tier-2 which forms the cochannel ring of six cells as shown in Figure 6.5.1. The computational results shown in Figure 6.5.2 indicate that the cochannel interference to noise power ratio increases towards the cell corner being closer to the three nearest cochannel cells.

However, when a set of FH-copatterns is assigned to Tier-3 as shown in Figure 6.5.3, the cochannel or copattern interference is reduced compared to the value obtained for Tier-2.
Figure 6.3.7: Average interference to noise ratio as distance from the base station.
REP - Dyn (5.5.16 and 5.6.17)
Figure 8.11: Average cell interference vs. range vs. distance from the base station.

Ref.: (7.5.15)
Therefore, as we increase the separation between two similar set of FH-patterns in a cellular network the cochannel (copattern) interference reduces which in turn reduces the probability of bit error and thus more number of users can be accommodated in a cell. Figure 6.5.4 and 6.5.5 show the probability of bit error-

-versus-the number of users curves when the FH-patterns are reassigned in cochannel Tier-2 and Tier-3 respectively. We assume that the optimization of the threshold with respect to the distance is done by an assumption which allows a maximum of 130 users in a cell at $P_b < 10^{-3}$. 

Figure 6.5.1 FH-patterns assigned in cochannel Tier-2.
Figure 5.5: Average cochannel interference to noise power ratio vs. distance from the base station.

REF: Eqn. (5.7-33)
Figure 6.5 2(a) Average cochannel interference to noise power ratio vs distance from the base station

REF - Eqn. (5.7.35)
Figure 6.5.3 FH-patterns assigned in Tier-2.

From the results it is observed that the number of users decreases from 120 to 125 if the FH-patterns are assigned in Tier-2 instead of Tier-3. This decreases the spectral efficiency of the system from 6.5% to 6.25% and increases the probability of error.

Figure 6.5.6 and 6.5.7 show the variation of the probability of bit error versus SNR for Tier-2 and Tier-3 respectively. The results indicate that...
Figure 9.5.4: Probability of bit error versus number of users

Ref.: Eqn. (5.7.94)
Figure 8.5.5 Probability of bit error versus number of users.

REF - Eqs. (5.7.94)
Figure 8.5.5: Probability of bit error versus signal-to-noise ratio.

REF - Eqn. (5.5.16) and (5.7.94)
the probability of error increases towards the cell corner due to the adjacent and cochannel cell interferences.

6.5 Conclusion

The results show that a set of FH-patterns in a FH-MFSK cellular System may be reused in Tier-3 which will result in accommodating more number of users in a cell and will increase the spectral efficiency of the system. To evaluate the spectral efficiency, we assume that the one-way voice transmission is carried out digitally at a rate of $R=10$ kbps for each user and at an acceptable average bit error rate of $10^{-3}$. Let $M$ be the number of users served by the system simultaneously, and $W$ is the one-way bandwidth, then the spectral efficiency $\eta$ is given by

$$\eta = \frac{MR}{W}$$

The results show that for the FH-patterns assigned in Tier-3 the spectral efficiency of the system is estimated to be

$$\eta = \frac{130 \times 10 \times 10^2}{20 \times 10^5} = 0.065$$

The number of users for Tier-3 and Tier-2 is calculated to be 130 and 125 (Figure 6.5.5/6.5.4) respectively.

Cochannel Reuse Ratio

The number of cells per cluster, $N$, is given by

$$N = i^2 + ij + j^2$$

where $i$ and $j$ are called the shift parameters (ref 2.2). For Tier-3 FH-pattern assignment scheme as proposed, $N = 7$. 
If $R$ be the cell radius and $D$ be the distance between the centers of the nearest neighbouring cochannel cells, the cochannel reuse ratio is given by

$$\frac{D}{R} = \sqrt{3} N$$

The cochannel reuse ratio for the Tier-3 assignment scheme is estimated to be 4.58.

The cochannel reuse ratio determines the number of channels per channel-set and sets limit on each site’s traffic carrying capacity.

Figure 6.5.8 shows a FH-pattern assignment scheme proposed for FH-MFSK cellular mobile radio environment.

![Diagram of FH-pattern assignment](image)

$i=1, j=2$

Figure 6.5.8 Assignment of FH-patterns in a FH-MFSK Cellular Mobile Radio Network.
Chapter 7

Conclusions & Recommendations for Future Research

7.1 Conclusions

The research and study reported in this thesis attempted to achieve three objectives:

i) A quantitative evaluation of the effect of cochannel, adjacent channel and mutual interferences on the probability of bit error and number of simultaneous users in a FH-MFSK cellular mobile radio environment. Necessary expressions were developed for the purpose.

ii) Analysis and evaluation of the computational results was done.

iii) Based on the above research and study an assignment of frequency hopping patterns in a FH-MFSK cellular mobile radio environment was proposed.

In Chapter 2 and 3, a brief review of narrowband cellular mobile radio and spread spectrum systems was given with emphasis on major differences, their relative merits and demerits.

The spread spectrum is currently in use in certain commercial satellite applications and its use in cellular mobile environment is under study and research. There has not been any field demonstration of this technique applied to cellular network. However, Hewlett-Packard Labs. have developed and tested a spread spectrum radio communication system intended for indoor use. The system operates at a data rate of 100 kbps over a range of 1000 meters with a power of 50 mw using BPSK modulation technique and center frequency of 1.5 GHz. Two receivers were constructed including a surface Acoustic Wave (SAW) based receiver.
Of the three spread spectrum systems, only two have been simulated to date; FH-MFSK system conceived by Viterbi and developed by Bell Labs. and FH-DPSK system of Cooper and Nettleton. The third spread spectrum scheme, a high data rate packet radio system using adaptations of the RAKE technique, has been built and tested by Stanford Research International (SRI) in the San Francisco Bay area which is not a cellular system.

In Chapter-4, the generation and design of Frequency hopping patterns, FH-MFSK system operation and effect of Rayleigh fading and lognormal shadowing on transmission were discussed. With perfect transmission and taking only mutual interference among the simultaneous users, up to 200 simultaneous users can be accommodated at an average bit error rate of $10^{-3}$. However, the Rayleigh fading and lognormal shadowing degrades the system performance which brings down the number of simultaneous users to 110 with a standard deviation of 5 dB.

The results in this thesis show that in a Rayleigh fading with no lognormal shadowing, the number of simultaneous users up to 130 at an average probability of bit error rate of $10^{-3}$, can be accommodated in each cell using a set of 2048 FH-patterns.

In chapter-5, the necessary expressions for the evaluation of the cochannel, adjacent channel and mutual interferences and probability of error were developed by considering a small set of FH-patterns.

The reuse of FH-patterns in a cellular environment depends on the amount of cochannel interference. Moreover, the number of simultaneous users that could be handled at a specific bit error rate tends to decrease with more cells operating nearby than a single cell. The power control strategy reduces the adjacent cell interference significantly. The mutual interference among simultaneous users within a cell increases the probability of error in a FH-MFSK system.

The worst interference is that seen by the mobile in a cell corner, where it is equidistant from three nearby cochannel and adjacent cells. The distribution of users within a cell affects
the system performance. Uniform distribution of users within a cell results in a 32\% reduction in spectral efficiency compared to an isolated cell.

In Chapter 6, the performance of a FH-MFSK cellular mobile radio system has been evaluated. The results show that a minimum of 130 simultaneous users can be accommodated within a cell at a probability of bit error rate of $10^{-3}$ or less. With a propagation loss exponent of four, the reuse factor was calculated to be 4.58. The experience from DS/CDMA system is that the reuse factor is between three and two for propagation loss exponent of three and four respectively based on the AWGN interference model.

However, the reuse factor for FH-MFSK system is larger as the Rayleigh fading, mutual interference among the users within a cell was taken into account.

The number of simultaneous users will further reduce if lognormal shadowing is taken into account. The spectral efficiency of the system was found to be 8.5\% for a set of 2048 FH-patterns.

7.2 Recommendation for Future Research

The research reported in this thesis is a step forward in answering the question of separation between two cells where the same FH-patterns can be reused [YUE 83] in a FH-MFSK cellular mobile radio environment.

The other areas requiring research and study in a spread spectrum cellular environment are suggested as follows:

i) More realistic performance evaluation of a spread spectrum cellular system is required by taking lognormal fading, ignition noise of the mobile and doppler effects into consideration which would further degrade the system performance.

ii) The effects of imperfect power control strategy on adjacent cell and cochannel cell interferences need investigations.
iii) The distribution of users within a cell affects the system performance, therefore, an optimum model for the distribution of the users in an urban, suburban and rural areas may be studied and investigated.

iv) Compatibility of the a-cellular system with the conventional narrowband system may further be investigated in the light of the recent developments in the narrowband technology.

v) Finally, the cost of a a-cellular system needs attention. The hardware of a a-cellular system is very expensive because of fast hopping synthesizers and the MFSK receiver is essentially an FFT processor. The use of Surface Acoustic Wave (SAW) technology can be further explored while designing a fast hopping synthesizer which may be less expensive and within the reach of common mobile users.
PROGRAM FOR CALCULATING S:N04, INW, IMA, INCO & PB

REAL PPC(20),NFL,M,INW,INCO,PS(20),C(5),PI1
REAL X,NM,INTER1,INTER2,INTER3,INTER4,INTER
REAL ERR,ERDD,IMA,IMAD,INCOD,INOD,D(6),NIMA
INTEGER II,L,N,FAC1,FAC3,T,IC,IN,FAC2,ID
INTEGER KX,KM,KW,KM2,TT,FAC33,FAC740

READ*,N
BM=0.01
BL=0.05
DIFF=0.86
L=11
K=0.2
DO 110 T=1.5
     S=2.5
     CONST=(K-B1)/DIFF
     D(1)=(1.5-K)**2+0.75
     D(2)=K**2+3.0
     D(3)=(1.5-K)**2+0.75
     D(4)=(1.5-K)**2+0.75
     D(5)=K**2+3.0
     D(6)=(1.5-K)**2+0.75

     G(1)=(4.5-K)**2+0.75
     G(2)=(1.5-K)**2+16.75
     G(3)=(3.0-K)**2+12.0
     G(4)=(4.5-K)**2+0.75
     G(5)=(1.5-K)**2+18.75
     G(6)=(3.0-K)**2+12.0
     B=12.0
     DO 100 KX=1.9

PRINT*,"S","B","K"
PRINT*,"I","B","K"
PR=1.0/2048.0
PROBAR=1.0-PR
S10=(10.0)**S
SUM1=S
SUM2=0.0
SUM3=0.0
DO 40 M=1,4
     PM=CONST*M
     PROBAR=1.0-PM

TERM1=(M/FLOAT(M+4))**(DIFF+4)*(B1**4)
TERM2=6.0*B1*B1*DIFF*DIFF*(M/FLOAT(M+2))
TERM3=4.0*B1*B1*B1*DIFF**2*(M/FLOAT(M+1))
TERM4=4.0*B1*DIFF*DIFF**2*(M/FLOAT(M+3))
Program

MFL=TERM1+TERM2+TERM3+TERM4

DO 30 ID=1,6
  MINA=MFL/(D(ID)*D(ID))
  SUMIC=0.0
  DO 20 IC=1,6
    SUMIN=0.0
    DO 10 IN=1,4
      CALL CHOOSE(8,IN,FACT3)
      TTERM1=FACT3
      TTERM2=EX**(IN/FLOAT(M))
      TTERM3=1.0-EX**(1.0/FLOAT(M))
      TTERM4=TERM3**(8-IN)
      TTERM=TTERM1*TTERM2*TTERM3*TTERM4
    END
    SUMIN=SUMIN+TTERM
    CONTINUE
  END
  MIN=MFL/(C(IC)*C(IC))
  SUMIC=SUMIC+(SUMIN-MIN)
  CONTINUE
  INCO=510+SUMIC

  X=(FM+((PMBAR*MFL)/(K**4)))
  TEMP=PM**N
  CALL CHOOSE(M-1,W,FACT1)
  CALL CHOOSE(M,W,FACT2)
  TERMS=FACT1*(TEMP)*((PMBAR**((M-1)-W))
  TERMS=FACT2*(TEMP)*((PMBAR**((M-N))-MINA
  SUM1=SUM1+TERMS*X
  SUM2=SUM2+TERMS*/(MFL/(K**4))
  SUM3=SUM3+TERMS

30 CONTINUE
40 CONTINUE

SINOR=(PMBAR**(CM-1))*SUM1*810
INW=SUM2*810
INA=SUM3*810

PRINT,'SINOR','INW','INCO','INA','INWDB'

PRINT,'SINOR','INW','INCO','INA','INWDB'
PRINT*, 'INA=', INA
INCDB=(10.0)*ALOG10(INCO)
IMADB=(10.0)*ALOG10(INA)
SNRDB=(10.0)*ALOG10(SNR)
INWDB=(10.0)*ALOG10(INW)

PRINT*, 'INCDB=', INCDB, 'IMADB=', IMADB, 'INWDB=', INWDB
PRINT*, 'SNRDB=', SNRDB

B2=(B*2)
PF=1.0/EXP(B2/2.0)
P1=1.0/EXP(B2/(2.0*(1.0+6*INCO)))
P2=1.0/EXP(B2/(2.0*(1.0+15*INCO)))
P3=1.0/EXP(B2/(2.0*(1.0+20*INCO)))
P4=1.0/EXP(B2/(2.0*(1.0+15*INCO)))
P5=1.0/EXP(B2/(2.0*(1.0+6*INCO)))
P6=1.0/EXP(B2/(2.0*(1.0+20*INCO)))
P7=1.0/EXP(B2/(2.0*(1.0+6*INA)))
P8=1.0/EXP(B2/(2.0*(1.0+15*INA)))
P9=1.0/EXP(B2/(2.0*(1.0+20*INA)))
P10=1.0/EXP(B2/(2.0*(1.0+15*INA)))
P11=1.0/EXP(B2/(2.0*(1.0+6*INA)))
P12=1.0/EXP(B2/(2.0*(1.0+INA)))
P13=1.0/EXP(B2/(2.0*(1.0+6*(INA+INCO))))
P14=1.0/EXP(B2/(2.0*(1.0+15*(INA+INCO))))
P15=1.0/EXP(B2/(2.0*(1.0+20*(INA+INCO))))
P16=1.0/EXP(B2/(2.0*(1.0+15*(INA+INCO))))
P17=1.0/EXP(B2/(2.0*(1.0+6*(INA+INCO))))
P18=1.0/EXP(B2/(2.0*(1.0+INA+INCO))))
P19=1.0/EXP(B2/(2.0*(1.0+INW)))
P20=1.0/EXP(B2/(2.0*(1.0+INW+6*INCO)))
P21=1.0/EXP(B2/(2.0*(1.0+INW+15*INCO)))
P22=1.0/EXP(B2/(2.0*(1.0+INW+20*INCO)))
P23=1.0/EXP(B2/(2.0*(1.0+INW+15*INCO)))
P24=1.0/EXP(B2/(2.0*(1.0+INW+6*INCO)))
P25=1.0/EXP(B2/(2.0*(1.0+INW+INCO)))
P26=1.0/EXP(B2/(2.0*(1.0+INW+INCO)))
P27=1.0/EXP(B2/(2.0*(1.0+INW+6*INA)))
P28=1.0/EXP(B2/(2.0*(1.0+INW+15*INA)))
P29=1.0/EXP(B2/(2.0*(1.0+INW+15*INA)))
P30=1.0/EXP(B2/(2.0*(1.0+INW+6*INA)))
PD31 = 0 \cdot \text{EXP}(B2/(2 \cdot 0\cdot (1 \cdot \text{INW} \cdot \text{INA})))

PD32 = 0 \cdot \text{EXP}(B2/(2 \cdot 0\cdot (1 \cdot \text{INW} \cdot \text{INA} \cdot \text{INC})))

PD33 = 0 \cdot \text{EXP}(B2/(2 \cdot 0\cdot (1 \cdot \text{INW} \cdot \text{INA} \cdot \text{INC})))

PD34 = 0 \cdot \text{EXP}(B2/(2 \cdot 0\cdot (1 \cdot \text{INW} \cdot \text{INA} \cdot \text{INC})))

PD35 = 0 \cdot \text{EXP}(B2/(2 \cdot 0\cdot (1 \cdot \text{INW} \cdot \text{INA} \cdot \text{INC})))

PD36 = 0 \cdot \text{EXP}(B2/(2 \cdot 0\cdot (1 \cdot \text{INW} \cdot \text{INA} \cdot \text{INC})))

PD37 = 0 \cdot \text{EXP}(B2/(2 \cdot 0\cdot (1 \cdot \text{INW} \cdot \text{INA} \cdot \text{INC})))

PUW = 0 - (\text{PRBAR} \cdot (M - 1))

PUA = 0 - (\text{PRBAR} \cdot M)

PUC = 0 - (\text{PRBAR} \cdot M)

PUWBAR = 0 - PUW

PUABAR = 0 - PUA

PUCBAR = 0 - PUC

TERM51 = PUWBAR \cdot PF \cdot (PUABAR \cdot 5) \cdot (PUCBAR \cdot 6)

TERM52 = 6 \cdot PUWBAR \cdot (PUABAR \cdot 6) \cdot PUC \cdot (PUCBAR \cdot 6) \cdot PD1

TERM53 = 15 \cdot PUWBAR \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 2) \cdot (PUCBAR \cdot 4) \cdot PD2

TERM54 = 20 \cdot PUWBAR \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 3) \cdot (PUCBAR \cdot 3) \cdot PD3

TERM55 = 15 \cdot PUWBAR \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 4) \cdot (PUCBAR \cdot 2) \cdot PD4

TERM56 = 6 \cdot PUWBAR \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 5) \cdot PUCBAR \cdot PD5

TERM57 = PUWBAR \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 6) \cdot PD6

TERM58 = 6 \cdot PUWBAR \cdot PUA \cdot (PUABAR \cdot 5) \cdot (PUCBAR \cdot 6) \cdot PD7

TERM59 = 15 \cdot PUWBAR \cdot (PUABAR \cdot 4) \cdot (PUCBAR \cdot 6) \cdot PD8

TERM60 = 20 \cdot PUWBAR \cdot (PUABAR \cdot 3) \cdot (PUCBAR \cdot 6) \cdot PD9

TERM61 = 15 \cdot PUWBAR \cdot (PUABAR \cdot 6) \cdot (PUCBAR \cdot 2) \cdot PD10

TERM62 = 5 \cdot PUWBAR \cdot (PUABAR \cdot 6) \cdot PUABAR \cdot (PUCBAR \cdot 6) \cdot PD11

TERM63 = PUWBAR \cdot (PUABAR \cdot 5) \cdot (PUCBAR \cdot 6) \cdot PD12

TERM64 = 6 \cdot PUWBAR \cdot PUA \cdot (PUABAR \cdot 6) \cdot PUC \cdot (PUCBAR \cdot 5) \cdot PD13

TERM65 = 15 \cdot PUWBAR \cdot (PUABAR \cdot 2) \cdot (PUCBAR \cdot 4) \cdot (PUC \cdot 2) \cdot (PUCBAR \cdot 4)

1 \cdot PD14

TERM66 = 20 \cdot PUWBAR \cdot (PUABAR \cdot 3) \cdot (PUCBAR \cdot 3) \cdot PD15

TERM67 = 15 \cdot PUWBAR \cdot (PUABAR \cdot 4) \cdot (PUCBAR \cdot 2) \cdot (PUC \cdot 4) \cdot (PUCBAR \cdot 2)

1 \cdot PD16

TERM68 = 6 \cdot PUWBAR \cdot (PUABAR \cdot 5) \cdot PUABAR \cdot (PUCBAR \cdot 5) \cdot PUCBAR \cdot PD17

TERM69 = PUWBAR \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 6) \cdot PD18

TERM70 = PUW \cdot (PUABAR \cdot 6) \cdot (PUCBAR \cdot 6) \cdot PD19

TERM71 = 6 \cdot PUW \cdot (PUABAR \cdot 6) \cdot PUC \cdot (PUCBAR \cdot 6) \cdot PD20

TERM72 = 15 \cdot PUW \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 2) \cdot (PUCBAR \cdot 4) \cdot PD21

TERM73 = 20 \cdot PUW \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 3) \cdot (PUCBAR \cdot 3) \cdot PD22

TERM74 = 15 \cdot PUW \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 4) \cdot (PUCBAR \cdot 2) \cdot PD23

TERM75 = 6 \cdot PUW \cdot (PUABAR \cdot 6) \cdot (PUCBAR \cdot 6) \cdot PD24

TERM76 = PUW \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 6) \cdot PD25

TERM77 = 6 \cdot PUW \cdot (PUABAR \cdot 6) \cdot (PUCBAR \cdot 6) \cdot PD26

TERM78 = 15 \cdot PUW \cdot (PUABAR \cdot 6) \cdot (PUC \cdot 2) \cdot (PUCBAR \cdot 6) \cdot PD27

TERM79 = 20 \cdot PUW \cdot (PUABAR \cdot 6) \cdot (PUCBAR \cdot 3) \cdot (PUCBAR \cdot 6) \cdot PD28

TERM80 = 15 \cdot PUW \cdot (PUABAR \cdot 4) \cdot (PUCBAR \cdot 6) \cdot PD29

TERM81 = 6 \cdot PUW \cdot (PUABAR \cdot 5) \cdot PUABAR \cdot (PUCBAR \cdot 6) \cdot PD30

TERM82 = PUW \cdot (PUABAR \cdot 6) \cdot (PUCBAR \cdot 6) \cdot PD31

TERM83 = 6 \cdot PUW \cdot PUA \cdot (PUABAR \cdot 6) \cdot PUC \cdot (PUCBAR \cdot 6) \cdot PD32

TERM84 = 15 \cdot PUW \cdot (PUABAR \cdot 5) \cdot (PUCBAR \cdot 6) \cdot (PUCBAR \cdot 4) \cdot PD33

TERM85 = 20 \cdot PUW \cdot (PUABAR \cdot 3) \cdot (PUCBAR \cdot 6) \cdot (PUCBAR \cdot 3) \cdot PD34

TERM86 = 15 \cdot PUW \cdot (PUABAR \cdot 4) \cdot (PUCBAR \cdot 2) \cdot (PUCBAR \cdot 6) \cdot PD35
TERM87 = 6 * PUW * (PUA**6) * PUBAR * (PUC**6) * PUCBAR * PD36
TERM88 = PUW * (PUA**6) * (PUC**6) * PD37

TERMA = TERM51 + TERM52 + TERM53 + TERM54 + TERM55 + TERM56 + TERM57 + TERM58
TERM89 = TERM59 + TERM60 + TERM61 + TERM62 + TERM63 + TERM64 + TERM65 + TERM66
TERMC = TERM71 + TERM72 + TERM73 + TERM74
TERM75 = TERM76 + TERM77 + TERM78 + TERM79 + TERM80 + TERM81 + TERM82
TERM8E = TERM83 + TERM84 + TERM85 + TERM86 + TERM87 + TERM88

PI = TERMA + TERMS + TERMC + TERM7 + TERME

PIBAR = (1 - PI)

PD38 = 1 / EXP(B2 / (2 * (1 + SNR)))
PD39 = 1 / EXP(B2 / (2 * (1 + SNR + 6 * INCO)))
PD40 = 1 / EXP(B2 / (2 * (1 + SNR + 15 * INCO)))
PD41 = 1 / EXP(B2 / (2 * (1 + SNR + 20 * INCO)))
PD42 = 1 / EXP(B2 / (2 * (1 + SNR + 15 * INCO)))
PD43 = 1 / EXP(B2 / (2 * (1 + SNR + 6 * INCO)))
PD44 = 1 / EXP(B2 / (2 * (1 + SNR + INCO)))

PD45 = 1 / EXP(B2 / (2 * (1 + SNR + 6 * INA)))
PD46 = 1 / EXP(B2 / (2 * (1 + SNR + 15 * INA)))
PD47 = 1 / EXP(B2 / (2 * (1 + SNR + 20 * INA)))
PD48 = 1 / EXP(B2 / (2 * (1 + SNR + 15 * INA)))
PD49 = 1 / EXP(B2 / (2 * (1 + SNR + 6 * INA)))
PD50 = 1 / EXP(B2 / (2 * (1 + SNR + INA)))

PD51 = 1 / EXP(B2 / (2 * (1 + SNR + 6 * (INN + INCO))))
PD52 = 1 / EXP(B2 / (2 * (1 + SNR + 15 * (INN + INCO))))
PD53 = 1 / EXP(B2 / (2 * (1 + SNR + 20 * (INN + INCO))))
PD54 = 1 / EXP(B2 / (2 * (1 + SNR + 15 * (INN + INCO))))
PD55 = 1 / EXP(B2 / (2 * (1 + SNR + 6 * (INN + INCO))))
PD56 = 1 / EXP(B2 / (2 * (1 + SNR + 6 * (INN + INCO))))

TERM91 = (PUAAB**6) * (PUCBAR**6) * PD38
TERM92 = 6 * (PUAAB**6) * PUCB * (PUCBAR**6) * PD39
TERM93 = 15 * (PUAAB**6) * (PUC**2) * (PUCBAR**4) * PD40
TERM94 = 20 * (PUAAB**6) * (PUC**3) * PD41
TERM95 = 15 * (PUAAB**6) * (PUC**4) * (PUCBAR**2) * PD42
TERM96 = 6 * (PUAAB**6) * (PUC**5) * PUCBAR * PD43
TERM97 = (PUAAB**6) * (PUC**6) * PD44
TERM98 = 6 * PUAB * (PUAAB**6) * (PUCBAR**6) * PD45
TERM99 = 15 * (PUA**2) * (PUAAB**4) * (PUCBAR**6) * PD46
TERM100 = 20 * (PUA**3) * (PUAAB**3) * (PUCBAR**6) * PD47
TERM101 = 15 * (PUA**4) * (PUAAB**2) * (PUCBAR**6) * PD48
TERM102 = 6 * (PUA**6) * PUAB * (PUCBAR**6) * PD49
TERM103 = (PUA**6) * (PUCBAR**6) * PD50
TERM104 = 6 * PUAB * (PUAAB**6) * PUC * (PUCBAR**6) * PD51
TERM105 = 15 * (PUA**2) * (PUAAB**4) * (PUC**2) * (PUCBAR**4) * PD52
TERM106 = 20 * (PUA**3) * (PUAAB**3) * (PUC**3) * (PUCBAR**3) * PD53
TERM107 = 15 * (PUA**4) * (PUAAB**2) * (PUC**4) * (PUCBAR**2) * PD54
TERM108 = 6 * (PUA**6) * PUAB * (PUC**6) * PUCBAR * PD55
TERM109 = (PUA**6) * (PUC**6) * PD56
TERM1=TERM91+TERM92+TERM93+TERM94+TERM95+TERM96+TERM97
TERM2=TERM98+TERM99+TERM100+TERM101+TERM102+TERM103
TERM3=TERM104+TERM105+TERM106+TERM107+TERM108+TERM109

PC=TERM1*TERM2+TERM3
PCBAR=1.0-PC

CALL CHOOSE(L,II,FACT40)
PPC(II)=FACT40*(PC**II)*((PCBAR**(L-III))
CONTINUE

CALL CHOOSE(L,TT,FACT33)
PS(TT)=FACT33*(PI**TT)*((PIBAR**(L-TT))
CONTINUE

PS(20)=PIBAR**L

TERM30=1.0/((2.0+PIBAR)**L)
TERM50=(PS(20)**256)+(0.5*255.0+PS(1)**(PS(20)**254))
SUM50=L+PC*(PCBAR**(L-1))*TERM50

DO 90 WW=2,L
SUM20=PS(20)
IM1=WW-1
DO 70 MM=1,IM1
SUM20=SUM20+PS(MM)
CONTINUE

PIO=SUM20**255
SUM30=PS(20)
DO 80 MM=1,IM1
SUM30=SUM30+PS(MM)
CONTINUE

PII=(SUM30**254)*PS(WW)**255.0
SUM50=SUM50+(PPC(WW)**(PIO+(0.5*PII)))
CONTINUE

PB=TERM30*(1.0-SUM50)

PRINT*, 'PB=', PB
PRINT*

B=B+1.0
CONTINUE

K=K+0.1
CONTINUE
STOP
END

SUBROUTINE CHOOSE(M,M,FACT)
INTEGER M,M,FACT,I

FACT=1
DO 10 I=1,N
   FACT=FACT*(M-N+I)/I
10   CONTINUE
RETURN
END
APPENDIX-A

A few elementary concepts of algebra for finite fields are presented in this appendix [EIN, 80].

A finite field $GF(Q)$ (Galois Field) is a set of $Q$ elements with defined rules for multiplication (and division), addition (and subtraction).

A finite field has unique zero element and a unit element. The zero element has the property $a + 0 = a$ and for the unit element $a1 = a$ for all $a \in GF(Q)$. Let the elements of $GF(Q)$ be denoted by the integers $0, 1, 2, ..., Q - 1$ with $0$ (zero element) and $1$ (unit element). A fundamental result of algebra is that there exist finite fields only for $Q$ equal to a prime or the power of a prime number $p$, i.e., $Q = p^n$. This means that $Q = 2, 3, 4, 5, 7, 8, 9, 11, 13, etc.$ are permissible but there is no finite field with, for instance, $10$ elements. All fields with $Q$ elements are isomorphic which means that they differ only in the way the elements are named.

$Q$ equal to a prime number

When $Q$ is equal to a prime number, the rules of addition and multiplication in $GF(Q)$ are defined by modulo $Q$ arithmetic. This means that the sum or product between two elements is defined as this operation in the usual algebra of integer numbers with the results reduced modulo $Q$ (i.e., equal to the remainder after dividing by $Q$). Example: Let $Q=7$. We will have $2 + 3 = 5$, $1 + 4 = 5$, $4 \cdot 3 = 5 \equiv 12 \mod 7$, $2 + 5 = 0 \equiv 7 \mod 7$.

A non-zero element $a \in GF(Q)$ is said to be of (multiplicative) order $N$ if $N$ is the lowest non-zero integer such that $a^N = 1$. Since $a^0$ is equal to a non-zero element and there are $Q - 1$. An element with $N=Q-1$ is called a primitive element. Examples: In $GF(7)$, the element $a = 2$ has the powers $a^0, a^1, a^2, ..., = 1, 2, 4, 1, 2, ...$. The order of $a = 2$ is thus $N = 3$. The element $a = 3$ is a primitive element, i.e., of order $N = 6$. Therefore, the powers of a primitive element are all non-zero elements of a finite field.
Q equal to the power of a prime number

When Q is a power of a prime number, the addition in a field with $Q = p^k$ are represented as $p$-ary numbers or vectors of length k. Consider $p = 2$ (for binary circuitry) case. For example if $Q = 2^3 = 8$ the elements are expressed as three-digit binary numbers: $1=001$, $3=011$, $4=100$, etc. which makes $0, 1, ... 7$ the octal representation of these numbers.

Addition can be defined as mod $p$ addition of component $s$. For $p = 2$, the binary addition is: $1+0=1$ and $1+1=0$, which gives $3+1=011+001=010=2$, $5+4=101+100=001=1$, etc.

For multiplication in $GF(p^k)$, the k-tuple is transformed into polynomials in $s$ of degree k-1 by letting the first digits be the coefficient of $s^k - 1$, the second digit the coefficient of $s^{k-1} - 1$, etc.

The triplet (111) corresponds to $s^2 + s + 1$, (011) to $s + 1$, and so on. Addition and multiplication of polynomials is defined as in ordinary algebra using the mod-p rule for the coefficients. The multiplication rule of $GF(p^k)$ is polynomial multiplication modulo an irreducible polynomial $P(x)$ of degree k. A polynomial is irreducible polynomial if it is not possible to factor it into a product of polynomials of lower degree. It thus has the features of a prime number in the algebra of polynomials.

Example: Consider $p = 2$ and $k=3$. The polynomial $P(x) = x^3 + x + 1$ is irreducible. The multiplication of $5=(101)$ and $3=(011)$ gives $x^2 + 1)(x + 1) = (x^2) = (x^3 + x^2 + x + 1)modP(x)$.

Thus we have $5.3=(100)=4$ in $GF(8)$. The element $\delta = (x) = (010)$ is primitive having the table of powers shown in Table-I. For multiplication, a list of nonzero elements is expressed as the power of a primitive element. For example, $5 = \delta^8$, and $3 = \delta^3$, will give $5.3 = \delta^{8+3} = \delta^9 = \delta^7.\delta^2 = \delta^2 = 4$. 
<table>
<thead>
<tr>
<th>$b^n$</th>
<th>Binary $\text{binary}$</th>
<th>Octal $\text{octal}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^0$</td>
<td>1</td>
<td>001</td>
</tr>
<tr>
<td>$b^1$</td>
<td>$z$</td>
<td>010</td>
</tr>
<tr>
<td>$b^2$</td>
<td>$z^2$</td>
<td>100</td>
</tr>
<tr>
<td>$b^3$</td>
<td>$z^3 = z + 1$</td>
<td>011</td>
</tr>
<tr>
<td>$b^4$</td>
<td>$z^2 + z$</td>
<td>110</td>
</tr>
<tr>
<td>$b^5$</td>
<td>$z^3 + z^2 = z^2 + z + 1$</td>
<td>111</td>
</tr>
<tr>
<td>$b^6$</td>
<td>$z^3 + z + z = z^2 + 1$</td>
<td>101</td>
</tr>
<tr>
<td>$b^7$</td>
<td>$z^3 + z = 1$</td>
<td>001</td>
</tr>
</tbody>
</table>
APPENDIX-B

A table for error rate formulas as given by [G H P, 80] is presented below.

Table-B: ERROR RATE FORMULAS Probability of insertion due to interference:

\[ p = \left[ 1 - (1 - 2^{-k})^M \right] - 1(1 - p_D) \]

Probability of insertion due to interference or false alarm:

\[ p_I = p + p_F - pp_F \]

Probability of \( m \) entries in a spurious row:

\[ P_s(m) = \binom{L}{m} p_I^{L-m} \]

Probability that no unwanted row has as many as \( n \) entries:

\[ P(n, 0) = \sum_{m=0}^{n-1} P_s(m) \] \( n > 0 \)

Probability that \( n \) is the maximum number of entries in an unwanted row and only one unwanted row has \( n \) entries:

\[ P(n, 1) = (2^k - 1) P_s(n) \sum_{m=0}^{n-1} P_s(m) \]

where \( n = 1, 2, \ldots, L \).

Probability of \( i \) entries in the correct row:

\[ P_s(i) = \binom{L}{i} (1 - p_D)^i p_D^{L-i} \]

Upper bound on bit error rate:

\[ P_b = \frac{2^{k-1}}{2^k-1} \left[ 1 - \sum_{i=1}^{L} P_s(i) \left( P(i, 0) + \frac{1}{2} P(i, 1) \right) \right] \]
APPENDIX-C

The probability of more than four users transmitting the same frequency tone in an i-th time-slot is negligibly small, and neglecting these probabilities in the calculations of power ratios \( \tau_a, \tau_i, \) and \( \tau_i \) does not introduce any error. Please refer to [V G 83] for the proof which is reproduced below.

Consider \( M \) to be between 50 to 200 and calculate

\[
P = \binom{M}{n} p_r^{n-1} - p_r^{M-n}
\]

where \( p_r = 2^{-a} = 2^{-3} \) and \( \alpha_a \) as in chapter-5. Whereas \( \alpha_a \) slowly increases towards a constant value as \( n \to \infty \) the value of \( P \) decreases towards zero for large \( n \), and hence the product also goes towards zero for large \( n \). Table-C shows \( \alpha_a \) and the product \( P \times \alpha_a \) determines the contribution to \( \tau_a, \tau_i, \tau_{ii} \), and since this value is very small for \( n \geq 4 \) compared to values for \( n \approx 1, 2 \), we are justified in neglecting the terms beyond \( n = 4 \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \alpha_a )</th>
<th>( P \times \alpha_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( M=50 )</td>
</tr>
<tr>
<td>1</td>
<td>0.1389</td>
<td>0.0223</td>
</tr>
<tr>
<td>2</td>
<td>0.2208</td>
<td>0.0035</td>
</tr>
<tr>
<td>3</td>
<td>0.2919</td>
<td>2.9 \times 10^{-4}</td>
</tr>
<tr>
<td>4</td>
<td>0.3387</td>
<td>1.7 \times 10^{-5}</td>
</tr>
<tr>
<td>5</td>
<td>0.3748</td>
<td>1 \times 10^{-6}</td>
</tr>
<tr>
<td>6</td>
<td>0.4035</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>0.4269</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0.4463</td>
<td>-</td>
</tr>
</tbody>
</table>
The probability of more than 4 cochannel cells to be active at the same time is negligibly small. Calculating $P'$ and $\alpha_4$ as in APPENDIX-C, we shall find that the value of $P'$ decreases towards zero for large value of $n$ and the $\alpha_4$ increases attaining a constant value. The product of $P'$ and $\alpha_4$ is very small for $n \geq 4$ and hence the probability of more than 4 cochannels being active at the same time is negligibly small. Table-D shows $\alpha_4$ and the product $P' \times \alpha_4$.

### TABLE - D

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\alpha_4$</th>
<th>$P' \times \alpha_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1389</td>
<td>0.055</td>
</tr>
<tr>
<td>2</td>
<td>0.2288</td>
<td>0.045</td>
</tr>
<tr>
<td>3</td>
<td>0.2919</td>
<td>0.015</td>
</tr>
<tr>
<td>4</td>
<td>0.3387</td>
<td>0.0027</td>
</tr>
<tr>
<td>5</td>
<td>0.3748</td>
<td>2.4 $10^{-4}$</td>
</tr>
<tr>
<td>6</td>
<td>0.4035</td>
<td>8.6 $10^{-6}$</td>
</tr>
</tbody>
</table>

$N = \text{number of active cochannels}$

$$P' = \binom{\theta}{n} p_r^n (1 - p_r)^{\theta - n}$$
APPENDIX-E

Consider a seven cell structure. Let '0' denotes the condition that the base station is not transmitting a tone to the user u in '0' cell and '1' denotes the condition that the base station is transmitting a tone to the user u in '0' cell. For a seven cell cellular structure, the total possibilities for the base stations to transmit tones to user u are \(2^7 = 128\) which is given in Table-E as follows:

Table-E

<table>
<thead>
<tr>
<th>S.No.</th>
<th>(P_B/P_D)</th>
<th>CELL0</th>
<th>CELL1</th>
<th>CELL2</th>
<th>CELL3</th>
<th>CELL4</th>
<th>CELL5</th>
<th>CELL6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(P_B)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>(P_D1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(P_D2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>63</td>
<td>(P_D63)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>64</td>
<td>(P_D64)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>126</td>
<td>(P_D126)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>127</td>
<td>(P_D127)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

CASE I: Only six adjacent cells are active.

The number of combinations of six adjacent-cell base stations taken r stations at a time,
transmitting tones to the user $u$ is given by the binomial coefficient

$$C(6, r) = \binom{6}{r}$$  \hspace{1cm} (1)

where $r = 1, 2, \ldots, 6$.

Therefore, the average adjacent-cell interference-to-noise power ratio, $\gamma_0$, from the six adjacent cells, transmitting tones to user $u$ in '0' cell is given by

$$\left(\binom{6}{r}\right) \gamma_0$$  \hspace{1cm} (2)

The false alarm condition is said to exist if the detection matrix detects a tone which has not been transmitted by a base station. As proved in Chapter-4, the probability of false alarm for $\sigma = 0$ is given by

$$P_f = \exp[-\frac{\beta_0^2}{2}]$$  \hspace{1cm} (3)

The probability of creating an entry at the detection matrix conditioned on the fact that the base station transmits a tone corresponding to that entry is [V G 85].

$$P_d = \exp\left[-\frac{\beta_0^2}{2(1 + \gamma_0)}\right]$$  \hspace{1cm} (4)

where

$$\gamma_0 = \text{average signal to noise ratio or average interference to noise ratio}$$

$$\beta_0 = \text{threshold level normalized to the rms noise}$$

Figure 1 shows a seven-cell cluster of a general cellular system.

Therefore, for the CASE-1, the probability of creating an entry in the spurious row of the detection matrix of user $u$ is given by

$$P_{di} = \exp\left[-\frac{\beta_0^2}{2(1 + \binom{6}{r}\gamma_0)}\right]$$  \hspace{1cm} (5)
where \( i = 1, 2, \ldots, 6 \) CASE-II: Only '0' cell is active.

The probability of creating an entry in the spurious row of detection matrix of user \( u \) is given by

\[
P_{d0} = \exp\left[\frac{-\beta_0^2}{2[1 + \gamma_i]}\right]
\]

(6)

where \( \gamma_i = \text{average in-cell interference to noise power ratio} \)

CASE-III: When all the seven cells are active, the probability of creating an entry in the spurious row of the detection matrix of the user \( u \) is given by

\[
P_{dj} = \exp\left[\frac{-\beta_0^2}{2[1 + \frac{6}{j}(\gamma_i + \gamma_s)]}\right]
\]

(7)

where \( j = 1, 2, \ldots, 6 \) Next, consider the correct row of the user \( u \). Proceeding along similar lines, the probability of an entry in the correct row of user \( u \) for

CASE-A: '0' cell transmitting a tone to the correct row:

\[
P_{d0} = \exp\left[\frac{-\beta_0^2}{2[1 + \gamma]}\right]
\]

(8)

CASE-B: All cells seven-cells transmitting tones to the correct row:

\[
P_{dt} = \exp\left[\frac{-\beta_0^2}{2[1 + \frac{6}{j}(\gamma + \gamma_s)]}\right]
\]

(9)

where \( t = 0, 1, \ldots, 6 \).
APPENDIX-F

Probability of error calculations with six cochannel and six adjacent cells active

When six cochannel and six adjacent cells are assumed to be active, the various miss probabilities \( P_d \)s are given as follows:

CASE-I: Only six cochannel cells are sending tones to the user \( u \) in cell '0' with different combinations.

\[
P_{d|c} = \exp\left[-\frac{\beta_0^2}{2[1 + (\frac{6}{5})\gamma_{co}]^2}\right]
\]

where \( \gamma_{co} \) = average cochannel interference - to - noise power ratio

CASE-II: Only six adjacent cells are transmitting tones to the user.

\[
P_{d|a} = \exp\left[-\frac{\beta_0^2}{2[1 + (\frac{6}{5})\gamma_a]^2}\right]
\]

CASE-III: Six cochannel and six adjacent cells are transmitting tones to the user \( u \).

\[
P_{d|c+a} = \exp\left[-\frac{\beta_0^2}{2[1 + (\frac{6}{5})^2(\gamma_{co} + \gamma_a)]^2}\right]
\]

CASE-IV: Only '0' cell transmits a tone to the user.

\[
P_{d|l} = \exp\left[-\frac{\beta_0^2}{2[1 + \gamma_l]^2}\right]
\]

CASE-V: Only '0' cell and six cochannel cells transmit tones to the user \( u \).

\[
P_{d|c|l} = \exp\left[-\frac{\beta_0^2}{2[1 + \gamma_l + (\frac{6}{5})\gamma_{co}]^2}\right]
\]

CASE-VI: Only '0' cell and six adjacent cells transmit tones to the user.

\[
P_{d|a|l} = \exp\left[-\frac{\beta_0^2}{2[1 + \gamma_l + (\frac{6}{5})\gamma_a]^2}\right]
\]

Similarly, various probabilities of creating an entry in the correct row can be derived and using these expressions and [APP-B], the probability of bit error can be calculated.
APPENDIX-G

Calculation of base station distance, $\sqrt{C}$,

**Tier-2**

\[
C_1 = (1.5 - k)^2 + 8.75 \\
C_2 = (1.5 + k)^2 + 8.75 \\
C_3 = (3.0 + k)^2 \\
C_4 = (1.5 + k)^2 + 8.75 \\
C_5 = (1.5 - k)^2 + 8.75 \\
C_6 = (3.0 - k)^2
\]

**Tier-3**

\[
C_1 = (3.0 - k)^2 + 12.0 \\
C_2 = (1.5 + k)^2 + 18.75 \\
C_3 = (4.5 + k)^2 + 0.75 \\
C_4 = (3.0 + k)^2 + 12.0 \\
C_5 = (1.5 - k)^2 + 18.75 \\
C_6 = (4.5 - k)^2 + 0.75
\]

**Tier-4**

\[
C_1 = (4.5 - k)^2 + 18.75 \\
C_2 = (1.5 + k)^2 + 36.75 \\
C_3 = (6.0 + k)^2 + 3.0 \\
C_4 = (4.5 + k)^2 + 18.75 \\
C_5 = (1.5 - k)^2 + 36.75 \\
C_6 = (6.0 - k)^2 + 3.0
\]
REFERENCES


