Development of a Dynamic Simulation of an Automobile that Incorporates Four Wheel Steering and a Driver Model

by

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Abstract

In this thesis, the development of a four-wheel steering vehicle dynamics model and the implementation of a driver controller are detailed. The model is intended to be used for supporting research in the cybersecurity of conventional and autonomous road vehicles. It incorporates a 10 degree-of-freedom vehicle model and a driver controller capable of steering all four wheels for improved vehicle lateral dynamics performance. The vehicle model is developed as a clean-sheet design using the fundamental principles of vehicle dynamics engineering and implemented in the Matlab/Simulink computing environment. A suitable driver-controller model that meets the industry-standard requirements was adopted from the literature review and implemented in the model. The functionality of the model is demonstrated with established standard vehicle manoeuvres that were used as part of the validation process. It is also shown that the model compares favorably with an “equivalent” ADAMS/Car model. Experience with the model confirms that it is suitable for its intended purpose. This was evaluated by considering the effect of corrupt four wheel steering feedback signals and investigating the relative dynamic response of conventional and four wheel steering vehicles. The model shows moderate improvement in the lateral performance and stability of four-wheel steering vehicles compared to equivalent front-wheel only steering vehicles. The developed model is also shown to be easily adaptable to performing sensor integrity and cybersecurity research of four-wheel steering vehicles.
Table of Contents

Abstract ii

Table of Contents iii

List of Tables vi

List of Figures vii

Nomenclature xii

1 Introduction 1

1.1 Motivation .................................................. 1

1.2 Vehicle Model .............................................. 3

1.3 Literature review .......................................... 5

1.4 Driver Controllers .......................................... 11

1.5 Rear Wheel Steering ........................................ 13

1.6 Thesis Organization ......................................... 13

1.7 Contributions ............................................... 14

2 Vehicle Model 16

2.1 Vehicle-fixed Coordinate System ......................... 16

2.2 Bicycle Model .............................................. 18

2.3 Roll Model .................................................. 20
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3.1 Tire Model</td>
<td>23</td>
</tr>
<tr>
<td>2.3.2 Ackermann Steering</td>
<td>26</td>
</tr>
<tr>
<td>2.4 Full Dynamics Model</td>
<td>27</td>
</tr>
<tr>
<td>2.4.1 Wheel Rate</td>
<td>28</td>
</tr>
<tr>
<td>2.4.2 Vertical Degrees of Freedom</td>
<td>30</td>
</tr>
<tr>
<td>2.5 Simulink Model</td>
<td>33</td>
</tr>
<tr>
<td>2.6 Driver Model (Controller)</td>
<td>35</td>
</tr>
<tr>
<td>2.6.1 Front Wheel Steering Control</td>
<td>36</td>
</tr>
<tr>
<td>2.6.2 Throttle Control</td>
<td>40</td>
</tr>
<tr>
<td>2.7 Four Wheel Steering (4WS) Control Law</td>
<td>42</td>
</tr>
<tr>
<td>2.7.1 Yaw Rate Response Controller - 4WS</td>
<td>43</td>
</tr>
<tr>
<td>2.7.2 Speed Sensing Controller - 4WS</td>
<td>45</td>
</tr>
<tr>
<td>2.7.3 Rear Steering Linkage Dynamics</td>
<td>46</td>
</tr>
<tr>
<td>2.7.4 Vehicle Model Control Summary</td>
<td>51</td>
</tr>
<tr>
<td>3 Model Validation</td>
<td>53</td>
</tr>
<tr>
<td>3.1 Model Validation - Hand Calculations</td>
<td>55</td>
</tr>
<tr>
<td>3.2 Horizontal Plane Degrees of Freedom</td>
<td>56</td>
</tr>
<tr>
<td>3.2.1 Quarter Car Model</td>
<td>57</td>
</tr>
<tr>
<td>3.2.2 Pitch and Bounce Model</td>
<td>59</td>
</tr>
<tr>
<td>3.3 Model Validation - ADAMS/Car</td>
<td>61</td>
</tr>
<tr>
<td>3.3.1 Constant Radius Turn</td>
<td>64</td>
</tr>
<tr>
<td>3.3.2 Double Lane Change</td>
<td>68</td>
</tr>
<tr>
<td>4 Sample Results</td>
<td>74</td>
</tr>
<tr>
<td>4.1 Model Sophistication</td>
<td>74</td>
</tr>
<tr>
<td>4.2 Standard Manoeuvres</td>
<td>76</td>
</tr>
<tr>
<td>4.2.1 Double Lane Change - ISO 3888</td>
<td>77</td>
</tr>
</tbody>
</table>
# List of Tables

1.1 Summary of DOFs and dynamics, including the quantity of necessary states and the influence on the lateral vehicle dynamics. ........................................ 9

2.1 Pacejka tire parameters. ................................................................. 24

3.1 Summary of model validation checks. ................................................. 54

3.2 ADAMS/Car model specifications. .................................................. 55

3.3 Output from constant radius test. .................................................. 57

3.4 Numerical comparison between ADAMS/Car and Simulink results for the DLC manoeuvre. ............................................................. 73

4.1 Input parameters to the Simulink model for use in the DLC manoeuvre simulation. ................................................................. 75

4.2 Input parameters to the Simulink model for use for the ISO 4138 cornering manoeuvre. ............................................................. 86

4.3 Input parameters to the Simulink mode for ISO 7401 transient response manoeuvre. ............................................................. 89

4.4 Numerical comparison of lateral acceleration values for the FWS and 4WS models for the ISO 7401 transient response manoeuvre. .......... 91

4.5 Numerical comparison of yaw rate values for the FWS and 4WS models for the ISO 7401 transient response manoeuvre. ..................... 91

D.1 Input parameters to the Simulink model for the ISO 3888 manoeuvre. 111

D.2 Input parameters to the Simulink model for the ISO 4138 manoeuvre. 113
## List of Figures

1.1 The driver-vehicle-ground system. ........................................ 3
1.2 Schematic representation of 10-DOF car model. ...................... 6
1.3 Comparative responses for a step steer input, 14 DOF and 8 DOF models. .................................................. 8
1.4 A frequency analysis of \( a_y, \dot{\psi}, \beta, \) and \( \phi \) is shown. ................. 10
1.5 Geometric explanation of Pure-Pursuit Method. ...................... 12
2.1 SEA coordinate system definition ........................................ 17
2.2 Free Body Diagram of the bicycle model. ............................ 19
2.3 Relationship between the tire lateral forces and the slip angle. .... 20
2.4 FBD of sprung mass rolling about RC and unsprung mass ........... 21
2.5 Planar free-body diagram of vehicle .................................... 22
2.6 Tire lateral forces versus sideslip angle ............................... 25
2.7 Ackermann steering. ..................................................... 28
2.8 Quarter-car model free body diagram. ................................ 31
2.9 Half-car model free body diagram. ................................... 32
2.10 Simulink vehicle model showing input and output parameters. ... 34
2.11 Simulink vehicle model with driver controllers. .................... 36
2.12 Steady-state vehicle motion along a circular path of radius R ....... 39
2.13 Single-preview-point versus multiple-preview-point models ........ 40
2.14 Schematic representation of the control structure for RWS controller. 45
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.15</td>
<td>Speed sensing controller, 4WS</td>
<td>46</td>
</tr>
<tr>
<td>2.16</td>
<td>Schematic representation of the four bar linkage and wheels in an undisplaced position.</td>
<td>47</td>
</tr>
<tr>
<td>2.17</td>
<td>Schematic of the four bar linkage and wheels in a displaced position.</td>
<td>48</td>
</tr>
<tr>
<td>2.18</td>
<td>Integration of four bar linkage to RWS controller.</td>
<td>48</td>
</tr>
<tr>
<td>2.19</td>
<td>Four bar linkage actuation block diagram.</td>
<td>49</td>
</tr>
<tr>
<td>3.1</td>
<td>Solutions to the quarter car model, sprung and unsprung mass vertical translation.</td>
<td>59</td>
</tr>
<tr>
<td>3.2</td>
<td>Solutions to the half car model, body heave and pitch rotation.</td>
<td>62</td>
</tr>
<tr>
<td>3.3</td>
<td>Isometric wireframe view of the ADAMS/Car full vehicle assembly.</td>
<td>63</td>
</tr>
<tr>
<td>3.4</td>
<td>Front and rear wheel steering angles comparison of ADAMS and Simulink models for a constant radius cornering manoeuvre.</td>
<td>65</td>
</tr>
<tr>
<td>3.5</td>
<td>Trajectory comparison of ADAMS and Simulink models for a constant radius cornering manoeuvre.</td>
<td>66</td>
</tr>
<tr>
<td>3.6</td>
<td>Yaw rate and lateral acceleration comparison of ADAMS and Simulink models for a constant radius cornering manoeuvre.</td>
<td>67</td>
</tr>
<tr>
<td>3.7</td>
<td>Wheel slip angles comparison of ADAMS and Simulink models for a constant radius cornering manoeuvre.</td>
<td>68</td>
</tr>
<tr>
<td>3.8</td>
<td>Trajectory comparison of ADAMS and Simulink models for a DLC manoeuvre.</td>
<td>69</td>
</tr>
<tr>
<td>3.9</td>
<td>Yaw rate and lateral acceleration comparison of ADAMS and Simulink models for a DLC manoeuvre.</td>
<td>70</td>
</tr>
<tr>
<td>3.10</td>
<td>Front and rear wheel steering angles comparison of ADAMS and Simulink models for a DLC manoeuvre.</td>
<td>71</td>
</tr>
<tr>
<td>3.11</td>
<td>Wheel slip angles comparison of ADAMS and Simulink models for a DLC manoeuvre.</td>
<td>72</td>
</tr>
</tbody>
</table>
4.1 Trajectory comparison of 3-DOF, 4-DOF, and 10-DOF models, for the DLC manoeuvre at a vehicle speed of 17 m/s. ............................................. 76
4.2 Trajectory comparison of 3-DOF, 4-DOF, and 10-DOF models, for the DLC manoeuvre at a vehicle speed of 20 m/s. ............................................. 77
4.3 ISO 3888 track definition ............................................................. 78
4.4 Trajectory following performance of the 4WS controller comparison using different $l_d$ values. ............................................................. 78
4.5 Front wheel steering angle change with chosen $l_d$ for the ISO 3888 DLC manoeuvre. ................................................................. 79
4.6 Lateral acceleration change with chosen $l_d$ for the ISO 3888 DLC manoeuvre. ................................................................. 80
4.7 Path following accuracy for the FWS and 4WS models for the ISO 3888 DLC manoeuvre ........................................................ 81
4.8 Front steering angle comparison for the FWS and 4WS models for the ISO 3888 DLC manoeuvre .................................................. 82
4.9 Rear steering angle comparison for the FWS and 4WS models for the ISO 3888 DLC manoeuvre .................................................. 82
4.10 Yaw rate comparison for the FWS and 4WS models for the ISO 3888 DLC manoeuvre .................................................. 83
4.11 Lateral acceleration comparison for the FWS and 4WS models for the ISO 3888 DLC manoeuvre ............................................. 83
4.12 Curvature responses of neutral steer, understeer, and oversteer vehicles at a fixed steer angle. .................................................. 85
4.13 Constant radius path for ISO 4138 manoeuvre ............................................. 86
4.14 Longitudinal velocity, ISO 4138 manoeuvre. ............................................. 87
4.15 Front wheel steering angle, ISO 4138 manoeuvre. ............................................. 87
4.16 Input steering angle for use with the ISO 7401 transient response manoeuvre. .................................................. 88
4.17 Yaw rate comparison between FWS and 4WS models for the ISO 7401 transient response manoeuvre. ......................... 89
4.18 Lateral acceleration comparison between FWS and 4WS models for the ISO 7401 transient response manoeuvre. ............... 90
4.19 Pinion force which actuates the four-bar-linkage and turns the rear wheels. ............................................................... 93
4.20 Random noise signal used as perturbation to the RWS controller signal. ................................................................. 93
4.21 Pinion force after noise signal is added to the measured rear-wheel steer angle. ......................................................... 94
4.22 Measured rear wheel steering angle with and without the injected noise signal for the DLC manoeuvre. ......................... 95
A.1 Overall model (with controllers) Simulink block diagram. .......... 103
A.2 Vehicle model (without controllers) Simulink block diagram. ...... 104
B.1 Hub force versus wheel travel for the front wheels. .................. 105
B.2 Hub force versus wheel travel for the rear wheels. ................... 106
B.3 Roll centre versus Wheel travel for the front wheels. ............... 106
B.4 Roll centre versus Wheel travel for the rear wheels. ............... 107
C.1 Sideslip angle comparison of ADAMS and Simulink models, ISO 4138 manoeuvre. ..................................................... 108
C.2 Body roll angle comparison of ADAMS and Simulink models, ISO 4138 manoeuvre. ..................................................... 109
C.3 Sideslip angle comparison of the ADAMS and Simulink models for the ISO 3888 DLC manoeuvre. ................................. 109
C.4 Body roll angle comparison of the ADAMS and Simulink models for the ISO 3888 DLC manoeuvre. ................................. 110

x
D.1 Yaw rate comparison of 3-DOF, 4-DOF, and 10-DOF models for the DLC manoeuvre at a vehicle speed of 20 m/s. 112

D.2 Lateral acceleration comparison of 3-DOF, 4-DOF, and 10-DOF models for the DLC manoeuvre at a vehicle speed of 20 m/s. 113

D.3 Plots of yaw rate and lateral acceleration versus time, for ISO 4138 manoeuvre. 114
Nomenclature

$\alpha_i$ $i^{th}$ wheel slip angle, [rad]

$\delta_i$ $i^{th}$ wheel steering angle, [rad]

$\beta$ body slip angle, [rad]

$\phi$ body roll angle, [rad]

$\theta$ pitch rotation angle, [rad]

$\psi$ yaw rotation angle, [rad]

$F_{xi}$ $i^{th}$ wheel longitudinal/tractive force, [N]

$F_{yi}$ $i^{th}$ wheel lateral/cornering force, [N]

$F_{zi}$ $i^{th}$ wheel normal force, [N]

$C_{\alpha f}$ front wheel cornering stiffness, [N/rad]

$C_{\alpha r}$ rear wheel cornering stiffness, [N/rad]

$m$ total mass, [kg]

$m_s$ sprung mass, [kg]

$m_{ui}$ unsprung mass of $i^{th}$ wheel, [kg]

$W$ total weight, [N]

$W_f$ weight on front wheels, [N]

$W_r$ weight on rear wheels, [N]

$I_x$ longitudinal/roll axis moment of inertia, [kg m$^2$]

$I_y$ lateral/pitch axis moment of inertia, [kg m$^2$]
$I_z$  vertical/yaw axis moment of inertia, [kg m$^2$]

$l_1$  longitudinal distance from body CG to $i^{th}$ wheel hub, [m]

$b_i$  lateral distance from body CG to $i^{th}$ wheel hub, [m]

$l_f$  longitudinal distance from body CG to front wheel axle, [m]

$l_r$  longitudinal distance from body CG to rear wheel axle, [m]

$w$  track width, [m]

$l$  wheelbase, [m]

$F_{SL}$  combined spring and damping force of left suspension, [N]

$F_{SR}$  combined spring and damping force of right suspension, [N]

$d_{rc}$  distance from body CG to roll centre, [m]

$d_{pc}$  distance from body CG to pitch centre, [m]

$h_{rc}$  height of roll centre measured from ground, [m]

$h_{pc}$  height of pitch centre measured from ground, [m]

$k_{ti}$  $i^{th}$ tire stiffness, [N/m]

$k_{si}$  $i^{th}$ wheel suspension wheel rate, [N/m]

$c_{si}$  $i^{th}$ wheel suspension damping, [Ns/m]

$k_f$  combined stiffnesses of the front wheels, [N/m]

$k_r$  combined stiffnesses of the rear wheels, [N/m]

$z_s$  sprung mass vertical displacement, [m]

$z_{ui}$  $i^{th}$ unsprung mass vertical displacement, [m]

$\theta_x$  euler angle representing rotation about the x axis in XYZ rotation sequence, [rad]

$\theta_y$  euler angle representing rotation about the y axis in XYZ rotation sequence, [rad]

$\theta_z$  euler angle representing rotation about the z axis in XYZ rotation sequence, [rad]

$\omega_{n,zu}$  natural frequency of unsprung mass, bounce, [Hz]
\( \omega_{n,zs} \) natural frequency of sprung mass, bounce, [Hz]

\( v_x \) longitudinal velocity, [m/s]

\( v_y \) lateral velocity, [m/s]

\( a_x \) longitudinal acceleration, [m/s\(^2\)]

\( a_y \) lateral acceleration, [m/s\(^2\)]

\( u \) longitudinal velocity, [m/s]

\( l_d \) look ahead distance, [m]

\( o \) look ahead offset, [m]

\( R \) radius of curvature of path, [m]

\( e_i \) \( i^{th} \) preview point lateral offset, [m]

\( G_i \) \( i^{th} \) preview point lateral offset gain, [m]

\( a_{b,max} \) maximum braking deceleration

\( u_{max} \) maximum longitudinal velocity, [m/s]

\( \mu_x \) longitudinal tire-road friction coefficient

\( K_U \) longitudinal speed gain

\( e_U \) longitudinal speed control value error, [m/s]

\( r_{ref} \) reference yaw rate for RWS controller, [rad/s]

\( H_{r0} \) steady state yaw gain of regular FWS vehicle

\( \eta \) understeer gradient

\( \theta_1 \) independent DOF (output) of the four-bar linkage Lagrangian formulation, [rad]

\( Q_i \) generalized forces for the four-bar linkage, [N\cdotm]

\( T \) Total kinetic energy of system, [J]

\( U \) Total potential energy of system, [J]
# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOF</td>
<td>Degree Of Freedom</td>
</tr>
<tr>
<td>IRP</td>
<td>In Road Plane</td>
</tr>
<tr>
<td>OORP</td>
<td>Out Of Road Plane</td>
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<tr>
<td>SAE</td>
<td>Society of Automotive Engineers</td>
</tr>
<tr>
<td>CG</td>
<td>Centre of Gravity</td>
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<tr>
<td>FBD</td>
<td>Free Body Diagram</td>
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<td>FWS</td>
<td>Front Wheel Steering</td>
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<td>RWS</td>
<td>Rear Wheel Steering</td>
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<tr>
<td>4WS</td>
<td>Four Wheel Steering</td>
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<tr>
<td>ECU</td>
<td>Electronic Control Unit</td>
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<tr>
<td>SHACS</td>
<td>Self-Healing Auto Cyber Security System</td>
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<td>DLC</td>
<td>Double Lane Change</td>
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<tr>
<td>ISO</td>
<td>International Organisation for Standardization</td>
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<td>RPM</td>
<td>Revolutions Per Minute</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Motivation

Dynamic vehicle models play a vital role in the research and development of road vehicles ranging from passenger cars to articulated trucks. Vehicle models serve as virtual prototypes in the development phase, giving developers insight into the effect of design parameters on the vehicle’s response and performance. They provide a fast and cost-effective alternative to physical testing [1]. With simulation models, developers can test and optimize their designs, develop control algorithms, and perform several other functions. There are a number of applications where vehicle models are needed, for example in driving simulators, lap-time simulators, controller design for automated driving, and product development. Depending on the application, these models can be represented as point mass models, simple rigid body models, or can be extended to complex multibody systems.

Vertical dynamics models, such as the quarter-car and the half-car models, typically with 2 and 4 degrees-of-freedom, respectively, are commonly used to study vehicle suspension. These are “out-of-road plane” (OORP) planar models. The 3-DOF single track (bicycle) model, which assumes the vehicle chassis to be rigid, is
useful for studying the longitudinal and the lateral behaviour of a vehicle, and to de-
velop path tracking algorithms. In double track models, additional degrees of freedom
to describe the motion of the components of the wheel suspensions and the drivetrain
are required. These are “in-road plane” (IRP) planar models. A full dynamics model
can be seen as a combined model taking into account OORP and IRP motion [2].
For example, a nonlinear 10-DOF model is suitable for vehicle dynamics studies that
consider surge, sway, heave, roll, pitch, and yaw motion of the chassis, and the four
wheels vertical motions relative to the chassis.

This thesis describes the development and validation of a 10-DOF, double track,
reduced-order vehicle dynamics model that includes functionality for modelling four-
wheel steering (4WS). A path-following driver model able to mimic a real driver is
also developed. This allows for vehicle manoeuvres to be simulated for predefined
parameters and conditions. Using this simulation environment, critical, dangerous,
and costly manoeuvres can be simulated. Subsequent to this research work, the
model is intended to be used to research the cybersecurity and sensor integrity of
automobiles.

The thesis will further compare the performance of a front-wheel steering (FWS)
vehicle model with that of a 4WS one. The advantages of directly controlling the
steering angles of both the front and rear wheels is also investigated by running both
models through standard manoeuvres for passenger vehicles defined by the Interna-
tional Organization for Standardization (ISO).

The methodology used is to derive the differential equations of motion describing
the vehicle dynamics from basic principles. These equations are subsequently solved
numerically using Matlab/Simulink. Simulink is a Matlab add-on product which pro-
vides an interactive, graphical environment for modelling, simulating, and analyzing
dynamical systems.
1.2 Vehicle Model

The behaviour of a ground vehicle represents the results of the interactions among the driver, the vehicle, and the environment, as illustrated in Figure 1.1. A comprehensive model would have to take into account all these interactions.

![Diagram of the driver-vehicle-ground system](image)

Figure 1.1: The driver-vehicle-ground system [3].

The minimum system elements that compose a comprise vehicle model are:

- **Vehicle body/chassis.** The chassis is described as a rigid body with six degrees of freedom: roll, pitch, and yaw rotations and lateral, longitudinal, and heave displacements of the centre of mass.

- **Wheels and tires.** The tires are vital components for vehicle modelling since tires transmit forces from the ground to the vehicle and vice versa. These forces can be estimated through empirical methods, theoretically, or through complex
computational models (e.g., FEM tire models).

- **Powertrain.** The powertrain consists of the engine, clutch or torque converter, and transmission/final drive. A vehicle powertrain is a complicated system and has a great influence on the vehicle performance. However, it will not be investigated in the scope of this study, since the current focus is on vehicle handling.

- **Suspension.** For a full dynamics model such as a 10 or 14-DOF model, the transmission of forces from the wheels to the chassis is governed by the suspension. The forces transmitted are influenced by the kinematic and dynamic components of the suspension system including links, springs, and dampers.

- **Steering.** The steering system is responsible for providing direction to the front wheels of the vehicle and also controls the rear wheel steering angle if four wheel or rear-wheel steering is present on the vehicle. The most common steering mechanism is the rack and pinion gear setup. Power steering uses a hydraulic piston arrangement to reduce steering effort. Detailed steering models capturing the power steering mechanism will not be included in this study.

- **Brakes.** The braking system in a passenger car is a mechanical one that is actuated by a hydraulic subsystem. The brake force applied by the driver on the brake pedal is first converted into hydraulic pressure, which is then transferred to the final disc. Most modern cars also have power assistance to reduce the effort needed to apply the brakes. In Reference [4], the model and the working of a passenger car brake system is described.

Each of the systems listed above is a complex one that has to be studied independently and as part of the whole vehicle assembly. However, for the purposes of this study, only the net effect on the ride dynamics of the vehicle is of interest; therefore,
the systems are treated with varying degrees of complexity based on their relevance
to the problem at hand. It is revealed later that inputs to the model developed are
the brake pedal, the accelerator (throttle) positions, and the steering wheel angle.
The environment acts on the vehicle through predefined environmental conditions,
such as side and head wind, the coefficient of road adhesion, road inclination, and
road profile.

1.3 Literature review

Vehicle dynamics is a broad research area which can be split into three major branches:
ride quality, handling, and performance. The model developed for the purposes of this
study is concerned primarily with vehicle handling. A dynamic, multibody model of a
four-wheel ground vehicle, including roll and pitch dynamics, is formulated based on
the Newton-Euler modelling approach. The model is derived based on well-established
theory on vehicle dynamics found in a number of references [3, 5, 6].

The mathematical model of vehicle dynamics is usually described by a set of first
order differential equations representing the rate of change of the system state vari-
ables with respect to time. However, the results generated by numerical simulations
are only approximations and their accuracy is dependent on the exactness of the mod-
els and the reliability of the system data; great care has to be put into the modelling
of these systems. A number of different models can be found in literature, with the
complexity being determined by the nature of the application or the study. These
include: 3-DOF planar models such as the bicycle model [2], 4-DOF roll model [7],
7-DOF model [8], as well as 10-DOF (see Figure 1.2) and 14-DOF full dynamics
models [2]. These models are broadly classified as follows:

- Vertical dynamics models;
• Longitudinal and lateral dynamics models; and

• Full dynamics models.

Figure 1.2: Schematic representation of 10-DOF car model.

A lot of engineering research has been undertaken in order to understand and improve vehicle dynamics. Significant amount of research has been conducted to improve passenger ride comfort and driving safety. Active vehicle control systems; such as active suspension control, torque vectoring systems and anti-lock braking systems; have been developed. These applications necessitated the development of vehicle models that are specialized for the longitudinal, lateral, or vertical dynamics, or a combination of these aforementioned types [9].

To begin studying the effects of active rear wheel steering on vehicle handling, an advanced vehicle dynamics model is required. The basic single track “bicycle model” provides a good foundation for understanding vehicle dynamics; however, it is a linear
model, an assumption which is valid up to lateral accelerations of approximately 4 m/s² [10]. Beyond that, the model must be upgraded to include vehicle roll and suspension characteristics and a non-linear tire model more suited to higher speeds.

Shim et al. [11] proposed a 14-DOF suspended vehicle model for prediction of vehicle roll behaviour in the development of active/passive roll control systems. This model includes the dynamics of the roll centre and wheel spin in addition to chassis displacements. An 8-DOF vehicle model that gives good correlation with the 14-DOF model is also presented in Reference [11]. It is shown that the 14-DOF vehicle model, which considers the suspension at each corner, has the same benefits of an 8-DOF vehicle model, with the additional capabilities of predicting vehicle pitch and heave motions [11]. It is hence demonstrated that by making certain simplifying modelling assumptions, a lower-order model often may be used instead, which will match the higher order one reasonably accurately [11]. However, care must be taken in establishing their suitability for the desired application. This conclusion is summarized in Figure 1.3, which compares the results for a step steer manoeuvre performed with the two models.
Figure 1.3: Comparative responses – step steer – 14 DOF model and 8 DOF model [11].

Henning et al. [9] compared the utility of two vehicle models in the study of vehicle lateral dynamics. A 14-DOF lumped mass model with simplified suspension modelling and a complex tire model, and a simplified 3-DOF model with a focus on lateral dynamics applications with a simple tire model are compared. The lumped mass model consists of 6 DOFs which completely describe the motion of the vehicle body and 2 DOFs each at the wheels, capturing rotation of the wheels and vertical motion of the unsprung wheel mass. See Table 1.1. Using the results of a sensitivity analysis, a reduced driving dynamics model with a focus on lateral vehicle dynamics was presented. The 3-DOF reduced order model only considers DOFs which are critical to the investigation of lateral dynamics. In the end only the longitudinal, lateral and yaw DOFs are left. What is interesting is that the authors acknowledge
Table 1.1: Summary of DOFs and dynamics, including the quantity of necessary states and the influence on the lateral vehicle dynamics [9].

<table>
<thead>
<tr>
<th>Variable</th>
<th>Name</th>
<th>Influence</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_x$</td>
<td>Longitudinal DOF</td>
<td>High</td>
<td>2</td>
</tr>
<tr>
<td>$v_y$</td>
<td>Lateral DOF</td>
<td>Highest</td>
<td>2</td>
</tr>
<tr>
<td>$v_z$</td>
<td>Vertical DOF</td>
<td>low</td>
<td>2</td>
</tr>
<tr>
<td>$k_\phi$</td>
<td>Roll DOF</td>
<td>High</td>
<td>2</td>
</tr>
<tr>
<td>$k_\theta$</td>
<td>Pitch DOF</td>
<td>High</td>
<td>2</td>
</tr>
<tr>
<td>$k_\psi$</td>
<td>Yaw DOF</td>
<td>Highest</td>
<td>2</td>
</tr>
<tr>
<td>$k_{T_{i,z}}$</td>
<td>Wheel vertical DOF</td>
<td>Lowest</td>
<td>8</td>
</tr>
<tr>
<td>$k_{T_{i,\omega y}}$</td>
<td>Wheel rotational DOF</td>
<td>High</td>
<td>4</td>
</tr>
<tr>
<td>$k_{\kappa_{i,dyn}}$</td>
<td>Longitudinal tyre dynamics</td>
<td>low</td>
<td>4</td>
</tr>
<tr>
<td>$k_{T_{vy,i,dyn}}$</td>
<td>Lateral tyre dynamics</td>
<td>High</td>
<td>4</td>
</tr>
</tbody>
</table>

the effect of pitch and roll on lateral dynamics through load transfer.

The authors of the study in Reference [9] use an Audi A7 test vehicle equipped with precise measuring units in order to validate the models in the time and frequency domains. A sine sweep manoeuvre with a steering wheel excitation frequency from 0.2 Hz to 2.5 Hz at a velocity of 100 km/h is used for validation, see Figure 1.4.

The conclusion is that “Although the reduced model was excessively simplified, the validation results are excellent for static and dynamic manoeuvres at steering wheel excitation frequencies up to 1 Hz” [9].
Figure 1.4: A frequency analysis of $a_y$, $\dot{\psi}$, $\beta$, and $\phi$ is shown [9].

The models discussed so far are all low-order models derived with various assumptions and have limitations on their applicability. There are other more detailed vehicle models with as many as 38-DOF [12], which consider actuating mechanisms, or multibody dynamics models available within commercial software packages with more than 100-DOF and finite-element-method (FEM) models with 1000+ DOF. The degree of detail required in the model is determined by the application.
1.4 Driver Controllers

From a control systems perspective, the driver and the vehicle can be modelled as a control loop, where the driver acts as a controller that is responsible for the stability of the plant (vehicle). The steering controller is typically a “path following problem” where the controller monitors the ideal path ahead of the vehicle and outputs the steering wheel displacement which will yield an effective path following. Several driver models based on preview of the road ahead were reviewed by Sharp et al. [13]. The driver is represented as an optimal preview controller, constructing a path error function by previewing the road over a known preview distance, and minimizing a weighted integral of squares of differences between the previewed path points and the corresponding estimated lateral positions of the vehicle over the preview distance. The parameters used to describe the ideal path are: the path curvature $k$, the tangent angle $\psi$, and the co-ordinates $x$ and $y$ measured from the axis reference system fixed in space. Single or multi-point preview models are widely used in the literature [14]. The pure pursuit method [15] and variations of it are among the most common approaches to the path tracking problem for mobile robots, Figure 1.5.
The Stanley method [16] was the path tracking approach used by Stanford University’s autonomous vehicle entry in the DARPA Grand Challenge. The Stanley method is a closed-loop model which calculates the steering angle as a nonlinear feedback function of the cross track error (measured from the centre of the front axle to the nearest path point), for which exponential convergence can be shown [15].

The vehicle longitudinal motion is specified separately. At the simplest level, the vehicle forward velocity is set equal to a constant value or made a function of the position along the intended path; i.e., a function of the path curvature. The torque applied to the wheels is determined by the representation of the engine/drivetrain and braking systems which are not detailed in this study. Only the net effect, which is a tractive force $F_x$ (+ve for acceleration, -ve for braking) at the tire/ground interface is considered. It is important to note that the lateral and the longitudinal vehicle controls are treated as being completely uncoupled [13].
1.5 Rear Wheel Steering

The advantage provided by including rear wheel steering is improving vehicle handling characteristics, primarily lateral stability and manoeuvrability [17]. At higher velocities, the overshoot in yaw velocity (“fishtailing”) when cornering is undesirable and leads to an increased workload for the driver to maintain stability. This can be mitigated with rear wheel steering. At lower velocities, a reduction in turn radius achieved by steering the rear wheels improves manoeuvrability. This is particularly useful for turning in tight spaces such as when parking.

There has been ongoing research into active rear wheel steering since the 1980’s, with car makers such as Nissan, Honda, and BMW all having some of their passenger cars with an implementation of a rear wheel steering system; however, the technology was popularised by the 1987 Honda Prelude [18]. Renewed interest in 4WS is related to the industry goal of improving safety. Recent steer-by-wire technology allows for more cost-effective and space-efficient implementation [19].

Initial research papers focus on feedforward control aiming to minimize the sideslip angle of the vehicle. A wide variety of controllers have been proposed, and an early overview can be found in [10]. More recent papers use a reference model to describe the desired steering response, as described in [20] as an example. In the Honda prelude, the rear wheel steering mechanism was mechanically linked to the front through a gear setup. However, modern four wheel steering vehicles employ a steer-by-wire setup [21].

1.6 Thesis Organization

Chapter 2 contains a detailed derivation of the vehicle model, driver controller, and the rear-wheel steering implementation. Mathematical models are presented in
order of increasing complexity, starting from the basic planar 3-DOF bicycle model to the final 10-DOF full dynamics multibody system. The development of a driver controller which simulates a driver is described; and a detailed look at the steering and throttle/braking control will be discussed. In Chapter 3, a series of calculations will be carried out to ensure that the model is behaving as expected. The model will also be validated by comparing it against an equivalent model developed with an industry-standard code, ADAMS/Car, which will be the benchmark used to validate the model. A complete vehicle model provided by MSC Software will be used as the vehicle for comparison. MSC Software is the company which developed and maintains ADAMS/Car. In Chapter 4, certain selected manoeuvres are performed to evaluate the performance of the model and controllers, and sample results are evaluated as part of the validation process. Finally, concluding statements are made in Chapter 5.

1.7 Contributions

The main contributions of this thesis include:

- Development of a series of vehicle dynamics models ranging from simple to complex and assessing their relative performance for vehicle handling applications.

- Including a suitable driver model thereby allowing the vehicle model to follow prescribed paths without the need for a human driver in the loop or prescribed vehicle control inputs.

- Including a four-bar-mechanism-based dynamic model of a rear wheel steering actuation such that the option for four wheel steering vehicles exists with the resulting simulation.

- Validating the developed models using a combination of analytical and simulation methods; where, for simulation, the developed model results were compared
against the industry standard ADAMS/Car modelling software.

• Using the developed models to perform preliminary assessment of the relative performance of front and four wheel steering vehicles for ISO standard handling tests; and the potential impact of feedback signal corruption in a four-wheel steering control system.
Chapter 2

Vehicle Model

The different vehicle models formulated as part of this study will be derived in detail in this chapter. Starting from the planar 3-DOF single track bicycle model, the basic model is extended to a double track model. The 4-DOF roll model adds roll dynamics to the model, thereby making it an OORP model. Finally, body pitch dynamics and the individual wheel vertical displacements are considered for the complete 10-DOF full dynamics model.

2.1 Vehicle-fixed Coordinate System

To describe the position and orientation of the vehicle in space, a vehicle-fixed reference frame \([O_v, x_v, y_v, z_v]\) will be introduced. The body-fixed reference point is located at the vehicle centre of gravity, \(\text{CG}\). The Newton-Euler modelling approach is used to derive the equations of motion for all the models; using a right-handed Cartesian coordinate system as defined by SEA J670 [22]. This represents an orthogonal system of axes that determines the sense or orientation of the various vehicle motions, i.e. longitudinal (x), lateral (y), and vertical (z) translations, and roll \((\phi)\), pitch \((\theta)\), and yaw \((\psi)\) rotations; as illustrated in Figure 2.1, such that:

1. The positive x axis extends forward in the direction of travel of the vehicle
which has a forward positive velocity and acceleration, $v_x$ and $a_x$, respectively;

2. The positive $y$ axis extends outwards to the right-hand side of the vehicle with positive lateral velocity and acceleration, $v_y$ and $a_y$ respectively, in that direction;

3. The positive $z$ axis extends downwards through the floor of the vehicle with positive vertical velocity and acceleration, $v_z$ and $a_z$ respectively; and the negative $z$ axis extends upwards through the roof;

4. Positive roll ($\phi$) corresponds to the vehicle leaning out of a left-hand turn (rolling clockwise to the right); and negative roll to its leaning out of a right-hand turn (rolling anticlockwise to the left);

5. Positive pitch ($\theta$) corresponds to a “nose-up” pitch; and negative pitch to a “nose-down” pitch; and

6. Positive yaw ($\psi$) corresponds to a turn to the right and negative yaw to a turn to the left.
The orientation of the vehicle-fixed coordinate system with respect to an Earth-Fixed (inertial) system $[O_E, x_E, y_E, z_E]$ is uniquely determined by a set of Euler angles. The transformation from one set of coordinates to another is detailed in [5] and will be further explored in Section 2.4.

### 2.2 Bicycle Model

The bicycle model is a 3-DOF single track linear model constrained to the road plane. It considers translational motion of the vehicle centre of gravity in the longitudinal $x$ and lateral $y$ directions as well as yaw rotation around its vertical $z$ axis. All heave, rolling, and pitching motion is neglected. It is a simple model useful for preliminary studies into the lateral dynamics of a vehicle, such as the understeer behaviour, which is critical for vehicle stability control. The front and rear axles are reduced to single wheels along the vehicle longitudinal centreline. Steer angles and tire slip angles for the left and right tires are likewise combined into effective values representing the two.

Figure 2.2 shows the free body diagram (FBD) for the bicycle model. Longitudinal (tractive and braking) forces acting at the wheel/ground contact patch are denoted by $F_x$. For generality, it is assumed that tractive forces can be generated by the front wheels, rear wheels, or all wheels. Lateral (cornering) forces, $F_y$, are approximated using a simple linear model whereby they are assumed to be proportional to the slip angles generated at the contact patch, see Figure 2.3. The constant of proportionality is defined by the cornering stiffness $C_\alpha$. This greatly-idealized vehicle model allows the investigation of the fundamental driving dynamic relationships within the lateral acceleration limit of around 4 m/s$^2$. The resulting rigid-body dynamic equations of motion of the vehicle in the $xy$-plane are,
\[
\begin{bmatrix}
\dot{v}_x \\
\dot{v}_y \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
0 & \frac{2C_{\alpha_f} \delta_f + 2C_{\alpha_r} \delta_r}{mv_x} & v_y + \frac{2C_{\alpha_f} \delta_f l_f - 2C_{\alpha_r} \delta_r l_r}{mv_x} \\
0 & -\frac{2C_{\alpha_f} \delta_f + 2C_{\alpha_r} \delta_r}{mv_x} & -v_x + \frac{2l_f C_{\alpha_f} - 2l_r C_{\alpha_r}}{mv_x} \\
0 & -\frac{2l_f C_{\alpha_f} - 2l_r C_{\alpha_r}}{I_z v_x} & -\frac{2l_f^2 C_{\alpha_f} + 2l_r^2 C_{\alpha_r}}{I_z v_x}
\end{bmatrix} \begin{bmatrix}
v_x \\
v_y \\
\dot{\psi}
\end{bmatrix}
\]

(2.1)

where \( v_x \) is the vehicle forward speed, \( v_y \) is the vehicle lateral speed, \( \dot{\psi} \) is the yaw rate, \( m \) is the vehicle mass, \( I_z \) is the yaw moment of inertia about the CG, \( C_{\alpha_f} \) and \( C_{\alpha_r} \) are the front and rear cornering stiffnesses (per tire). Angles \( \delta_f \) and \( \delta_r \) are the front and rear wheel steer angles respectively; and \( l_f \) and \( l_r \) are the distances from the vehicle CG to the front and rear axles, respectively.

Equation 2.1 is derived by considering force balances in the longitudinal \( x \) and
2.3 Roll Model

The roll model is a double track, 4-DOF model. It adds a single out-of-road-plane DOF to the bicycle model, thereby increasing its complexity. The rolling action of the vehicle chassis causes a lateral load transfer resulting in uneven vertical forces on the left and right hand sides of the vehicle. The model is derived by considering the vehicle as two lumped sprung and unsprung masses. As a reaction to the roll torque, which results from the lateral acceleration $a_y$ of the chassis, forces $F_{SL}$ and $F_{SR}$ are the combined forces due to the springs and dampers that form the suspension. These forces act to create moments opposite to the roll motion of the vehicle.
The sprung mass is assumed to pivot about an imaginary point called the “roll centre”, an instantaneous point about which the body rolls as seen from the FBD in Figure 2.4, and is determined from the geometry of the suspension [24]. The stationary forces $R_z$ and $R_y$ of the body rolling motion are transferred to the axles via virtual rotational joints at $\text{RC}$.

Figure 2.4: FBD of sprung mass rolling about RC and unsprung mass [7].

Figure 2.5 shows the planar free body diagram of a two track vehicle viewed from the top, i.e. positive $z$-axis pointing into the page. The vehicle (local) coordinate frame is fixed at the CG and all the forces acting at the tire/road contact patch are shown. Subscripts 1 through 4 will be used throughout the thesis to refer to the wheels. Index 1 corresponds to the front-left wheel, and wheels are labelled clockwise as viewed from above. In a similar manner to the bicycle model, the 3 planar equations of motion are,

$$a_x = \dot{v}_x = \frac{\sum_{i=1}^{4} F_{xi} \cos \delta_i - \sum_{i=1}^{4} F_{yi} \sin \delta_i}{m} - \dot{\psi} v_y$$ (2.2)
\[ a_y = \dot{v}_y = \frac{\sum_{i=1}^{4} F_{y_i} \cos \delta_i + \sum_{i=1}^{4} F_{x_i} \sin \delta_i}{m} - \dot{\psi} v_x \] (2.3)

\[ \ddot{\psi} = \left[ F_{x1} l_1 \sin \delta_1 + F_{y1} l_1 \cos \delta_1 + F_{x2} l_2 \sin \delta_2 + F_{y2} l_2 \cos \delta_2 + F_{x1} b_1 \cos \delta_1 - F_{y1} b_1 \sin \delta_1 \\
- F_{x2} b_2 \cos \delta_2 + F_{y2} b_2 \sin \delta_2 - F_{x3} l_1 \sin \delta_1 - F_{y3} l_1 \cos \delta_1 - F_{x4} l_2 \sin \delta_2 - F_{y4} l_2 \cos \delta_2 \\
+ F_{x3} b_3 \cos \delta_1 - F_{y3} b_3 \sin \delta_1 - F_{x4} b_4 \cos \delta_2 + F_{y4} b_4 \sin \delta_2 \right] / I_z \] (2.4)

Figure 2.5: Planar free body diagram of vehicle showing tire forces.

Equations 2.2 through 2.4 are derived by considering the force and moment balances along or about their respective axes. They are upgraded to account for the four wheels instead of two. In addition to the extra degree of freedom, there are two other points of divergence from the bicycle model: the consideration of two front and rear wheels and the calculation of cornering forces using a more sophisticated tire model. The latter is treated in more detail in Section 2.3.1.
The roll equation of motion is formulated by considering the moments generated by the suspension forces, sprung mass inertial forces due to acceleration, and forces due to lateral load transfer at the wheels about the roll centre. The resulting roll equation is

$$\ddot{\phi} = \frac{1}{I_x} \left[ m_s g d_{rc} \sin(\phi) + m_s a_y d_{rc} \cos(\phi) - C_\phi \dot{\phi} - K_\phi \dot{\phi} \right]$$  \hspace{1cm} (2.5)

where $K_\phi$ and $C_\phi$ are the effective rotational spring stiffness and damping coefficient of the suspension.

### 2.3.1 Tire Model

At moderate to high vehicle speeds, cornering is affected by the centrifugal force acting on the vehicle. All tires must develop slip angles and resulting cornering forces to generate the required lateral acceleration to turn the vehicle [7]. Therefore, it is critical to choose an appropriate tire model. Several models exist having varying degrees of complexity. A simple linear model is acceptable for low speeds; however, at higher speeds a more accurate model is required. Pacejka’s Magic Formula tire model [25] is widely used in models requiring a higher degree of accuracy.

The Magic Formula tire model calculates lateral cornering forces $F_y$ on a tire as a function of the slip angle and the normal force $F_z$ acting on the tire. Careful selection of parameters results in excellent agreement with empirical data [3]. The tire lateral force is given by the function:

$$F_y = D \sin \left[ C \tan^{-1} \left\{ B_y \alpha - E (B_y \alpha - \tan^{-1}(B_y \alpha)) \right\} \right]$$  \hspace{1cm} (2.6)

where $C = (2/\pi) \sin^{-1}(y_s/D)$ is the shape factor, $D = a_1 F_z^2 + a_2 F_z$ is the peak factor, and $E = a_6 F_z^2 + a_7 F_z + a_8$ is the curvature factor. The coefficient $B_y$ is called the stiffness factor, and it is derived from the product $B_y CD = a_3 \sin(a_4 \tan^{-1}(a_5 F_z))$
which corresponds to the slope of the curve at the origin, representing the cornering stiffness of the tire. The physical significance of the terms can be found in Reference [3].

The parameters \(a_0, a_1, ..., a_8\) are constants defined for each tire. Table 2.1 lists sample parameter values taken from a reference by Pacejka in 1989 [25]. These are the values used for simulation results presented in this thesis.

Table 2.1: Pacejka tire parameters [25].

\[
\begin{align*}
  a_0 &= 0 & a_3 &= 1078 & a_6 &= 0 \\
  a_1 &= -22.1 & a_4 &= 1.82 & a_7 &= -0.354 \\
  a_2 &= 1011 & a_5 &= 0.208 & a_8 &= 0.707
\end{align*}
\]

Figure 2.6 depicts the tire lateral force versus its slip angle for various normal loads. The tire cornering stiffness, \(C_\alpha\), and peak cornering force, \(F_y\), are altered with changing normal loads; however, a point is reached where increasing the tire normal force results in little change in lateral force produced.

The slip angles, \(\alpha_i\), at each wheel are approximated by:

\[
\begin{align*}
  \alpha_{1,2} &= \delta_{1,2} + \frac{v_y + l_1 \dot{\psi}}{v_x} \\
  \alpha_{3,4} &= \delta_{3,4} - \frac{v_y - l_2 \dot{\psi}}{v_x}
\end{align*}
\] (2.7)

Normal forces \(F_z\) at the tires comprise the static weight of the vehicle, longitudinal load transfer, and lateral load transfer due to the actual vehicle manoeuvre.
For longitudinal load transfer, the normal forces at each of the front and rear tires are half of \(W_f\) and \(W_r\) respectively, where \(W_f\) and \(W_r\) are the total axle loads:

\[
W_f = \frac{l_2}{l} W - \frac{h}{l} \left( \sum F_x \right) \quad (2.9)
\]

\[
W_r = \frac{l_1}{l} W + \frac{h}{l} \left( \sum F_x \right) \quad (2.10)
\]

The first terms in these equations are the static loads distributed between the front and rear axles while the second term is the dynamic component due to longitudinal acceleration (or deceleration). Lengths \(l_1\) and \(l_2\) are the distances from the CG to the front and rear axles respectively, and \(W\) is the total weight of the vehicle. The second terms are the longitudinal load transfer terms, \(\sum F_x\) represents the net "propulsive force" acting on the vehicle in the longitudinal direction, which include;
tractive forces (acceleration and braking), aerodynamic forces (drag), and rolling resistance.

For lateral load transfer, roll dynamics has to be considered. A detailed derivation can be found in Lambert [7]. Additional loads $\Delta F_{zf}$ and $\Delta F_{zr}$ are developed at the front and rear axles, respectively, due to the lateral acceleration of the sprung mass. The total normal loads at each of the four wheels are then:

\[
F_{z1} = \frac{W_1}{2} - \Delta F_{zf}
\]
\[
F_{z2} = \frac{W_1}{2} + \Delta F_{zf}
\]
\[
F_{z3} = \frac{W_2}{2} - \Delta F_{zr}
\]
\[
F_{z4} = \frac{W_2}{2} + \Delta F_{zr}
\]

The tires are modelled as vertical springs with high stiffnesses; hence, the total normal loads at each wheel can also be calculated as

\[
F_{zi} = k_{ti} \cdot z_{ui}
\] (2.11)

where $k_{ti}$ is the $i^{th}$ tire stiffness and $z_{ui}$ is the $i^{th}$ difference between the unsprung mass deflection from equilibrium and terrain elevation. This approach is employed in subsequent sections.

### 2.3.2 Ackermann Steering

At low speeds, there is a simple kinematic relationship between the direction of motion of the vehicle and the steering wheel angle. The prime consideration in the design of the steering system is minimum tire scrub during cornering [3]. The Ackermann
geometry takes into consideration that the outer wheel has to travel longer than the inner wheel during a turn. To satisfy this requirement, the wheels should follow curved paths with different radii originating from a common turn centre as shown in Figure 2.7.

\[ \cot \delta_o - \cot \delta_i = \frac{w}{l} \]  \hspace{1cm} (2.12)

where subscripts \(i\) and \(o\) refer to the inner and outer wheels, \(w\) is the track width, and \(l\) is the wheelbase. The steering geometry that satisfies Equation 2.12 is usually referred to as the Ackermann steering geometry.

For a 4WS vehicle travelling at low speed, the rear wheels turn in the opposite direction to the front wheels. This will have the effect of shortening the turn centre for a tighter radius turn.

At moderate and high speeds, cornering dynamics are governed by the balancing of centrifugal force and lateral tire force. Lateral force and yaw moment dynamic equations determine the slip angles corresponding to the tire forces.

For a front wheel steering vehicle, the idea of Ackermann geometry is to rotate the inner wheel slightly sharper than the outer wheel to reduce tire slippage when cornering. This is achieved by aligning the centre of rotation of both front wheels with that of the rear axle, as shown in Figure 2.7. Most practical steering linkages attempt to achieve Ackermann geometry though this is never completely achieved with a mechanical linkage.

2.4 Full Dynamics Model

The full dynamics model adds another 6 degrees of freedom to the Roll model derived in Section 2.3. These are the heave of the chassis (sprung mass), pitching motion of the chassis about the lateral axis passing through the CG, and the four
individual relative vertical wheel displacements. Hence the vehicle has 10 DOFs. The body/chassis is modelled as being rigid, with body-fixed coordinates, \(x-y-z\), attached at the CG and aligned in the principal directions. The transmission of forces from the unsprung masses (wheels / suspension) to the sprung mass (chassis) is governed by the suspension design. The suspension is modelled as a linear spring and damper system at the interface of each wheel. The forces transmitted are influenced by the kinematic and dynamic response of the suspension system.

### 2.4.1 Wheel Rate

Figure 2.8 illustrates the Quarter-car model. It is the most minimal representation required to describe the vertical movement of a vehicle, namely the sprung mass motion and the unsprung mass motion. For a quarter vehicle model, the sprung mass represents approximately a quarter of the mass of the vehicle body, which includes all parts supported by the suspension system. Meanwhile, the unsprung mass comprises the masses of all parts of a single wheel station that are acted on by the suspension
To derive the equations of motion for the roll, pitch, and heave DOFs, the suspension forces have to be determined at each corner of the sprung mass. This approach to modelling the roll DOF should be contrasted with the roll model from Section 2.3. Previously the transmission of forces through the suspension was treated using roll stiffness and damping coefficients. With the full dynamics model, suspension forces are determined from the relative vertical displacements of the wheels to the body, which is a more intuitive approach. However, the results are the same.

To calculate suspension forces, the wheel rate $k_s$ is required. The wheel rate is a measure of the effective stiffness of the suspension system measured at the wheel centre. It is the slope of a graph plotting wheel hub force versus wheel travel in the vertical direction expressed in Newtons per metre or N/m, see Appendix B. The primary contributor to the wheel rate is the suspension spring. The terms $k_{si}$ defined in this thesis will refer to the wheel rates rather than the spring stiffnesses. This distinction is important as the suspension forces dictate the vertical DOFs as will be seen in the next section.

Due to the rotational motion of the sprung mass, the calculation of suspension forces becomes more involved. The vertical displacements $\Delta z_{si}$ at each corner of the sprung mass must be evaluated in the global frame. The following sequence is performed to calculate suspension forces:

1. The respective position vectors from the vehicle CG to the sprung mass/suspension attachment points, in the local vehicle frame are determined.

2. Using the XYZ sequence of Euler rotations defined by the vehicle orientation, the position vector is transformed from the vehicle body frame to the inertial frame.
\[
\begin{bmatrix}
    l_i \\
    b_i \\
    \Delta z_{si}
\end{bmatrix}_G = [T_{lg}]
\begin{bmatrix}
    l_i \\
    -b_i \\
    0
\end{bmatrix}_L
\]

where \(l_i\) and \(b_i\) are the longitudinal and lateral distances of the suspension attachment points measured from the CG. The subscripts \(G\) and \(L\) denote the global and local frames, respectively. The local to global transformation matrix \([T_{lg}]\) is presented in Section 2.5.

3. Once this new position is obtained, the vertical component is the extension of the spring due to rotation of the sprung mass, which is used to calculate the suspension force \(F_{si}\) as,

\[
F_{si} = k_{si}(z_s - \Delta z_{si} - z_{ui}) + c_{si}(\dot{z}_s - \Delta \dot{z}_{si} - \dot{z}_{ui})
\]  

(2.13)

### 2.4.2 Vertical Degrees of Freedom

Based on the free body diagram in Figure 2.4, and fully derived by Patil [2], the roll equation of motion about the roll centre is derived and then solved for the roll angular acceleration.

\[
\ddot{\phi} = \frac{F_{s1}b_1 - F_{s2}b_2 - F_{s3}b_3 - F_{s4}b_4 + \sum_{i=0}^{4} (F_{yi} \cos \delta_i) h_{rc} + m_s a_y d_{rc} + (I_y - I_z) \dot{\theta} \dot{\psi}}{I_x + m_s d_{rc}^2}
\]

(2.14)
Equation 2.14 varies from Equation 2.4 in the way the suspension forces are accounted for. Henceforth Equation 2.14 is the one considered for the development of the model.

The pitch motion is represented by rotation about a pitch axis in an analogous manner to roll motion. As before, the Euler equation of motion is written about the pitch centre to solve for the pitch acceleration, considering the forces causing moments about the lateral y axis. The FBD in Figure 2.9 shows the suspension forces considered when deriving the equation.

\[
\ddot{\theta} = \frac{(F_{s1} + F_{s2}) l_1 - (F_{s3} + F_{s4}) l_2 + \sum_{i=0}^{4} (F_{xi} \cos \delta_i) h_{pc} + m_s a_x d_{pc} + (I_z - I_x) \dot{\psi} \phi}{I_y + m_s d_{pc}^2} \tag{2.15}
\]
In Equations 2.14 and 2.15; $d_{rc}$ is the distance between the CG and the roll centre; $d_{pc}$ is the distance between the CG and the pitch centre; and $I_x$, $I_y$, and $I_z$ are the principal moments of inertia of the sprung mass $m_s$.

Note that the parallel axis theorem is applied when computing moments of inertia $I_x$ and $I_y$ as moments are not taken about the CG but instead about the roll and pitch centres, respectively.

At each corner of the vehicle, the wheel assembly is modelled as an unsprung mass connected to the chassis (sprung) mass through the suspension. Equation 2.16 is solved for the vertical accelerations $\ddot{z}_{ui}$ of the $i^{th}$ unsprung mass at each corner,

$$\ddot{z}_{ui} = \frac{k_{si} (z_s - \Delta z_{si} - z_{ui}) + c_{s1} (\dot{z}_{s} - \dot{\Delta z}_{si} - \dot{z}_{ui}) - k_{ti} (z_{ui}) - m_{ui} \cdot g}{m_{ui}}$$

(2.16)
The wheel heave displacements, $z_{ui}$, are obtained by looking at the suspension and tire forces acting at the interface, shown in Figure 2.8. Here, the tires have been modelled as springs having a stiffness, $k_t$.

The final DOF is the vertical motion (heave) of the sprung mass. The equation of motion is obtained by considering Newton’s second law in the vertical direction such that,

$$
\ddot{z}_s = \frac{-\sum_{i=1}^{4} (F_{si}) - m_s \cdot g}{m_s}
$$

From the above Equation 2.17, the vertical acceleration of the sprung mass is determined, which when integrated twice gives the vertical displacement of the sprung mass.

2.5 Simulink Model

Figure 2.10 shows the setup of the Simulink vehicle model. The inputs to the vehicle model are the front and rear wheel steering angles ($\delta_f, \delta_r$) and the tractive forces $F_{xi}$, where $i = 1$ to 4. The output is a variety of variables including the position of the vehicle, orientation, longitudinal velocity, yaw velocity, tire slip angles, and cornering forces. Additional outputs can be provided, as needed.

The key steps taken in solving the equations in Matlab/Simulink are as follows (the model structure and the code are provided in Appendix A).

1. The set of differential equations derived for each DOF are solved for the linear and angular accelerations, which in turn are integrated numerically to yield the translational and angular velocities ($v_x, v_y, v_z, \dot{\phi}, \dot{\theta}, \dot{\psi}$) in the vehicle body-fixed frame.
2. The time derivatives of the Euler angles, $\dot{\theta}_x$, $\dot{\theta}_y$, and $\dot{\theta}_z$, are related to the angular velocities as follows,

\[
\begin{bmatrix}
\dot{\theta}_x \\
\dot{\theta}_y \\
\dot{\theta}_z
\end{bmatrix} = \begin{bmatrix}
\frac{\cos(\theta_z) \dot{\phi} - \sin(\theta_z) \dot{\theta}}{\cos \theta} \\
\sin(\theta_z) \dot{\phi} + \cos(\theta_z) \dot{\theta} \\
\sin(\theta_z) \dot{\phi} - \cos(\theta_z) \dot{\theta} \tan(\theta_y) + \dot{\psi}
\end{bmatrix}
\] (2.18)

where Euler angles $\theta_x$, $\theta_y$, and $\theta_z$ define the vehicle orientation in the inertial frame relative to the local vehicle frame and vice versa.

3. Once the Euler angle derivatives are obtained, they are numerically integrated to obtain the new orientation of the body in the inertial frame $(\theta_x, \theta_y, \theta_z)$.

4. Euler angles are used to define the orientation of the vehicle in the global (inertial) frame of reference at any given point in time. The $x-y-z$ frame (vehicle body fixed) is rotated according to the sequence: first about the $x$-axis by an angle $\theta_x$; then about the new $y$-axis by an angle $\theta_y$, then about the newest $z$-axis by an angle $\theta_z$. This results in the following local to global transformation.
Expanding the terms of the equation gives

\[
[T_{lg}] = [T_{\theta z}][T_{\theta y}][T_{\theta x}]
\]

The transformation matrix is used to determine the vehicle orientation in space in an inertial reference frame. The translational velocities obtained upon integration of Equations 2.2 and 2.3 in Step 1 are transformed using this matrix, then integrated twice to obtain the displacement of the vehicle. The orientation, \( \psi \), of the vehicle is obtained by integrating the yaw acceleration obtained from Equation 2.4 twice.

### 2.6 Driver Model (Controller)

The vehicle dynamic model described in Section 2.4 responds to the following inputs: acceleration, braking, and steering. To simulate real driver behaviour, a driver controller is necessary. This controller is required to determine suitable vehicle inputs to the vehicle from road geometry and advisory speeds. Interaction with other vehicles is beyond the intended scope of this model.
When approaching a curve, a typical driver will try to match the vehicle’s longitudinal speed to the maximum speed which they believe the vehicle will safely negotiate the turn. As they reach the corner, they turn the steering wheel to match the heading of the road. It is on this basis that a path-following driver model is developed. It consists of steering (heading) control and speed (throttle/braking) control.

The driver and vehicle are modelled as a closed-loop system, where the driver acts as a controller for the plant, which is the vehicle. The variables to be controlled are the difference between desired and actual speeds as well as the difference between desired and actual headings. Figure 2.11 shows the control structure of the two related controllers. It should be noted that this is a front-wheel-steering (FWS) controller only. It is later extended to a four-wheel-steering (4WS) controller in Section 2.7.

![Figure 2.11: Simulink vehicle model with driver controllers.](image)

### 2.6.1 Front Wheel Steering Control

The control algorithm that determines the front wheel steering angle is based on work carried out by Jalali et al [28]. They developed a path-tracking controller
which continuously “looks ahead” to vary the steering angle and keep the vehicle on the desired path. Only the most important aspects of the controller are highlighted here, whereas readers interested in its derivation are referred to Reference [28].

The steering angle required $\delta_{ss}$ to keep a vehicle on a circular path when in steady state motion is a function of: the look ahead distance $l_d$, look ahead offset $o$ (distance between the look ahead point and the point on the curve closest to it), distance between the $l_d$ and the centre of the curve $h$, vehicle velocity, and vehicle parameters, such as the cornering stiffness $C_\alpha$, that have been defined previously.

When the left hand side of Equation 2.1 is set to $\{0\}$, the steady state values of the velocities and wheel steering angle can be solved for, resulting in the steady state yaw rate and wheel steer angle

$$\dot{\psi}_{ss} = \frac{u}{\sqrt{R^2 - T^2}}$$  \hspace{1cm} (2.20)

where

$$T = l_r - \frac{l_fm_{cg}u^2}{C_{ar}(l_f + l_r)}$$  \hspace{1cm} (2.21)

and

$$\delta_{ss} = \frac{1}{\sqrt{R^2 - T^2}} \left( l_f + l_r - \frac{m_{cg}u^2(l_f C_{af} - l_r C_{ar})}{(l_f + l_r)C_{af}C_{ar}} \right)$$  \hspace{1cm} (2.22)

and according to the geometry defined in Figure 2.12, $h_{ss}$ and $o_{ss}$ are defined as follows

$$h_{ss} = \sqrt{l_d^2 + R^2 + 2l_dT}$$  \hspace{1cm} (2.23)

and
Finally, from Equations 2.22 and 2.24, the ratio between the desired steering input $\delta_{ss}$ and the look-ahead offset $o_{ss}$ is calculated as follows

$$o_{ss} = \sqrt{l_d^2 + R^2 + 2l_dT} - R \quad (2.24)$$

At this point, two important assumptions are made by the authors of Reference [28] in order to simplify Equation 2.25. First, using Taylor’s expansion:

$$\forall x, \epsilon \in \mathbb{R}, x > 0 : \text{if } \frac{\epsilon}{x} \ll 1 \Rightarrow \sqrt{x + \epsilon} = \sqrt{x} + \frac{\epsilon}{2\sqrt{x}}$$

and assuming that $\frac{|l_d(l_d+2T)|}{R} \ll 1$, Equation 2.25 can be rewritten as follows

$$\frac{\delta_{ss}}{o_{ss}} = \frac{1}{\sqrt{R^2 - T^2}} \left( \frac{l_f + l_r - \frac{m_{cg}u^2(l_fC_{\alpha f} - l_rC_{\alpha r})}{(l_f + l_r)C_{\alpha f}C_{\alpha r}}}{\sqrt{l_d^2 + R^2 + 2l_dT} - R} \right) \quad (2.26)$$

Next, by assuming that $\frac{|T|}{R} \ll 1$ and, thus, $\sqrt{1 - \frac{T^2}{R^2}} \approx 1$, Equation 2.26 can be further simplified as follows

$$\frac{\delta_{ss}}{o_{ss}} = \frac{2 \left( l_f + l_r - \frac{m_{cg}u^2(l_fC_{\alpha f} - l_rC_{\alpha r})}{(l_f + l_r)C_{\alpha f}C_{\alpha r}} \right)}{l_d(l_d + 2T)} \quad (2.27)$$
Equation 2.27 indicates that the steering angle required to keep the vehicle on a circular path when in steady-state motion is a function of the look-ahead offset $\alpha_{ss}$, the vehicle longitudinal velocity $u$, the look-ahead distance $l_d$, and tire cornering stiffnesses.

Next, a look-ahead distance is defined, that can be constant or a function of vehicle speed and the driver's reaction time, such that

$$l_d(u) = d_{const} + t_{driver} u(t) \quad (2.28)$$

where $d_{const}$ is a constant distance even at lower speeds, $t_{driver}$ is the reaction time of the driver, and $u(t)$ is the longitudinal velocity.

Research has shown that using multiple look-ahead distances leads to a more robust controller; hence, the $l_d$ is split into five equidistant points called preview points, as illustrated by Figure 2.13. The lateral offset of each preview point from the desired
path is calculated; where these distances are measured along a line that is perpendicular to the optical lever. A new look ahead offset distance is then defined as the weighed sum of all the lateral offsets.

$$o(t) = \sum_{i=1}^{5} (G_i e_i(t))$$

These offsets can be considered position errors. The gains $G_i$ are chosen in an ad hoc fashion based on trial and error, not on any formal optimization scheme [28]. This results in the final driver model

$$\delta(t) = \frac{2 \left( l_f + l_r - \frac{m_{cg} u^2 (l_f C_{\alpha f} - l_r C_{\alpha r})}{(l_f + l_r) C_{\alpha f} C_{\alpha r}} \right)}{l_d(t) \left( l_d(t) + 2(l_r - \frac{a m_{cg} u^2}{C_{\alpha r} (l_f + l_r)}) \right) \sum_{i=1}^{5} (G_i e_i)}$$ (2.29)

Figure 2.13: Single-preview-point versus multiple-preview-point models [28].

The path tracking performance of the controller is demonstrated in Section 4.2.1 where it is applied in a double lane change manoeuvre.

2.6.2 Throttle Control

The first task of the driver model is to ensure that the vehicle’s longitudinal speed is appropriate when negotiating a curve. Therefore, a maximum speed is calculated
which is a function of the vehicle’s lateral acceleration, actual speed, and the curvature of the road.

The controller requires the curvature of the road to be defined. It takes in as inputs the road profile coordinates \((x_i, y_i)\), where \(i\) ranges from 1 to the number of points along the road, and the curvature. A suitable preview distance, which allows the driver to reduce the speed of the vehicle accordingly, is calculated as

\[
s_p = \frac{u^2}{2a_{b,\text{max}} \cdot \mu_x}
\]  

(2.30)

where \(a_{b,\text{max}}\) is the maximum braking deceleration, which is another characteristic parameter of the vehicle and it shows its ability to utilize the maximum longitudinal forces acting on the tires; and \(\mu_x\) is the longitudinal tire-road friction coefficient. The value of \(a_{b,\text{max}}\) would be larger for vehicles employing anti-lock braking systems.

Preview points are defined along the path to be followed starting from the vehicle CG. The curvature at each of the points is calculated and the largest curvature \(k_t\) is used to calculate maximum speed as

\[
 u_{\text{max}} = \left( \frac{a_{b,\text{max}} \mu_x}{k_t} \right)^2
\]  

(2.31)

An upper limit can be imposed on the calculated speed such that when curvature is negligibly small the speed will not exceed a certain limit, i.e., when driving in a straight line, \(k = 0\) and the nominal speed limit is imposed. The calculated speed is used at each instant in time and is compared with the actual vehicle speed and an error is computed. This error is multiplied by a longitudinal speed control gain, \(K_U\), to calculate the longitudinal speed control value error \(e_U\).
\[ e_U = K_U(u_{\text{max}} - u) \] (2.32)

A positive value of the error corresponds to the position of the accelerator pedal whereas a negative value corresponds to the position of the brake pedal. A value of 1 or -1 corresponds to full throttle or brakes, respectively. The torque applied to the wheels is dependant on the engine/drivetrain and braking system which are not modelled in this study. The net effect, however, is represented by the tractive forces \( F_{xi} \) at the wheels.

### 2.7 Four Wheel Steering (4WS) Control Law

Adding rear-wheel steering to a car helps improve a car’s handling capabilities. At higher speeds, the rear wheels turn in the same direction as the front wheels. This can help improve stability by decreasing the amount of yaw the car experiences and the corresponding tire sideslip angles. Counter-phase steering can be employed at lower speeds to improve manoeuvrability by reducing the vehicle turn radius. This is useful when parking in tight spaces for example [29]. The cut-off speed below which counter-phase steering is applied is given by

\[ v = \sqrt{\frac{C_{a\tau}r}{ml_f}} \] (2.33)

Consensus does not yet exist on the optimal use of 4WS. As an example, Veldhuizen states that “Shibahata et al. [9] conclude that it is not very attractive to steer the rear wheels at a large angle opposed to the front wheels, since it makes the rear end of the vehicle stick out further towards the outside of the curve. Whitehead [10] in turn reports that in parallel parking the improved manoeuvrability is not desirable. Therefore, improving the high speed handling quality is recognized as the
main purpose of 4WS systems whereas the low speed manoeuvrability improvement is hardly relevant.” [20].

To span the range of possible applications of 4WS, two controllers will be considered. A “yaw rate response” controller suitable for addressing high speed handling and a “speed sensing” controller that is also applicable at lower speeds.

2.7.1 Yaw Rate Response Controller - 4WS

This rear wheel steering control scheme includes a reference model which determines the desired yaw rate response depending on the front wheel steering angle $\delta_f$ and vehicle forward velocity $V$. This reference yaw rate $r_{ref}$ is the solution of the first-order differential equation

$$\dot{r}_{ref} = -\frac{1}{\tau} r_{ref} + \frac{H_{r0}}{\tau} \delta_f$$

(2.34)

where $H_{r0}$ is the steady-state yaw gain of a regular front-wheel steer vehicle and is given by

$$H_{r0}(v_x) = \frac{v_x}{l + \frac{\eta v_x^2}{g}}$$

(2.35)

An equivalent time constant $\tau_r$, used in Equation 2.34, is defined by the ratio between the steady-state yaw rate and the derivative of the step response of yaw rate at $t = 0$ [30],

$$\tau_r(v_x) = \frac{I_z}{l_1 C_f} H_{r0}$$

(2.36)

Other values for the time constant and the steady state yaw gain can also be chosen. However, this choice is reasonable as it is based on the yaw rate response of the bicycle
model. The understeer coefficient, $\eta$, is given by

$$\eta = \left( \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right)$$

(2.37)

The actual yaw rate of the vehicle is fed back and subtracted from this reference yaw rate, and the resulting error is the control variable for the controller which calculates the steering angle of the rear wheels as an output [10], which is shown in Figure 2.14. The transfer function for this controller $K(s)$, developed by Veldhuizen [20] and based on the transfer function of the extended 3-DOF bicycle model (reference model) is given by

$$K(s) = -0.17 \left( \frac{(60 \times 2 \times \pi)^2}{s^2 + 2 \times 0.5 \times 60 \times 2 \times \pi \times s + (60 \times 2 \times \pi)^2} \right)$$

(2.38)

This controller is used in this study. It is designed for a vehicle speed of 120 km/h; however, it can be tuned for lower velocities and still provides satisfactory results. The effect of this controller is demonstrated subsequently in Chapter 4, where the step steering response test in Section 4.2.3 uses $v_x=100$ km/h and the results are compared in Figure 4.16 where the effect of adding rear wheel steering on the yaw rate is observed as the vehicle is performing the manoeuvre.

Notably, the steady-state yaw rate will approximate the steady-state yaw rate of the front wheel steering (FWS) vehicle; and it is only the transient response that is different, as the overshoot is significantly reduced.
2.7.2 Speed Sensing Controller - 4WS

The controller described in Section 2.7.1 performs well at high speeds. However, if the rear wheels are always steered in the same direction as the front ones, the vehicle’s minimum turning radius always increases. It may be desirable to steer the rear wheels in an opposite direction to the front ones for improved manoeuvrability, depending on operating conditions.

Sato et al. [29] proposed the following feed-forward 4WS control technique for a vehicle described by the linear bicycle model

\[ k = \frac{\delta_r}{\delta_f} = \frac{-l_2 + \frac{m_1}{C_{\alpha,r}} l_1 u^2}{l_1 + \frac{m_2}{C_{\alpha,f}} l_2 u^2} \]  \hspace{1cm} (2.39)

where \( C_{\alpha,f} \) and \( C_{\alpha,r} \) are the front and rear cornering stiffnesses respectively, \( l \) is the wheelbase, and \( u \) is the longitudinal speed.

The rear wheels are steered at a steering angle ratio \( k \) relative to the front wheels. It can be seen that at lower speeds, \( \delta_r \) will have an opposite sign to \( \delta_f \); and the opposite is true at higher speeds. The ratio \( k \) is plotted as a function of speed in Figure 2.16.

The objective of this controller is to control the vehicle in such a way as to keep the
centreline of the body tangential to the turning path, thereby minimizing the delay in the lateral acceleration response with respect to the yaw rate response.

![Diagram of speed sensing controller, 4WS](image)

Figure 2.15: Speed sensing controller, 4WS [29].

Once the rear wheel steer angle has been determined, the question of how this is achieved is addressed. A drive by wire system is employed in production models employing four-wheel steering, such as the Lexus GS sedan [21]. Sensors which measure the steering wheel angle send signals to an electro-mechanical actuator that provides the actuating force to turn the rear wheels.

### 2.7.3 Rear Steering Linkage Dynamics

Section 2.7.1 discusses the control logic, i.e., how the desired rear wheel steering angles are calculated for improved lateral stability. The actual actuation mechanism used to achieve this is discussed in this section. A planar four-bar linkage has been used as a steering mechanism in early implementations [31]. Since its inception,
this simple structure has been extensively applied in different types of road vehicles. The Ackermann steering linkage, discussed in Section 2.3.2, is approximated as a planar four-bar mechanism. All real steering mechanisms are complex spatial linkages and the parameters defining their geometry are quite numerous. However, from a kinematic standpoint, the approximation is valid [32]. Figures 2.16 and 2.17 shows a schematic diagram of a typical Ackermann four-bar steering linkage in a displaced and undisplaced configuration, respectively.

![Figure 2.16: Schematic representation of the four bar linkage and wheels in an undisplaced position.](image)

In a traditional mechanical setup, the input motion from the driver at the steering wheel is transmitted via a steering column and gear box to the Ackermann linkage, which steers the front wheels. A car equipped with a steer-by-wire system controls direction of the wheels through electric motors which are actuated by ECUs monitoring the steering wheel inputs from the driver. Rear wheel steering was initially achieved through a mechanical system of linkages connecting the front wheels to the rear wheels. The latest systems use electromechanical connections to a compact rear
Figure 2.17: Schematic of the four bar linkage and wheels in a displaced position. The rear wheels can be actuated individually or through a rack-and-pinion mechanism as with the Lexus GS sedan. The latter approach is adopted in this study.

Figure 2.18: Integration of four bar linkage to RWS controller.

The desired rear wheel steering angle, derived in Section 2.7, is achieved using a PID controller. The controller minimizes the error between the actual and desired...
angles by adjusting the pinion force at the steering rack. Figure 2.18 shows how the four bar linkage integrates with the RWS controller. It takes forces at the contact patch of the wheels \( F_1 \) and \( F_2 \) and desired steering angle as input, and uses this information to calculate the value of \( \theta_1 \). The output, which is the actual steering angle, is the difference between \( \theta_1 \) and a reference \( \theta_{1,0} \) value that is dependent on the geometry of the mechanism. Figure 2.19 shows the inner workings of the RWS controller.

![Four bar linkage actuation block diagram](image)

Figure 2.19: Four bar linkage actuation block diagram.

The four-bar-linkage is a 1-DOF problem whose solution is readily available in literature [33]. The following equation is a second-order nonlinear differential equation which describes the motion of the four-bar-linkage

\[
\Psi \ddot{\theta}_1 + \Phi \dot{\theta}_1 - \mu \dot{\theta}_1^2 + \nu = Q_i
\]  

(2.40)

where the generalized forces \( Q_i \) comprise moments about the origin ‘o’ generated by the tire forces at the contact patch and the force on the steering rack exerted by the pinion, \( F_p \).
\( Q_i = \sum_{i=1}^{2} F_i \cdot \frac{\partial r_{i/A}}{\partial \theta_1} \)  

(2.41)

The \( \Psi \) and \( \Phi \) terms in Equation 2.41 factor in the inertial properties of the linkage and are defined as

\[
\Psi = I_1 A_1^2 + I_2 A_2^2 + I_3 A_3^2 + m_3 B_3
\]

(2.42)

and

\[
\Phi = 2 \left( I_1 A_1 \dot{A}_1 + I_2 A_2 \dot{A}_2 + I_3 A_3 \dot{A}_3 + \frac{1}{2} m_3 \dot{B}_3 \right)
\]

(2.43)

where the \( I_i \) terms are the moments of inertia of the links and \( m_3 \) is the mass of the link B-C. The terms \( A_1, A_2, A_3, \) and \( B_3 \) are defined as

\[
\begin{align*}
\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \end{bmatrix} &= \begin{bmatrix} 1 \\ \frac{L_1 \sin(\theta_1 + \theta_3)}{L_2 \sin(\theta_2 + \theta_1)} \\ \frac{L_1 \sin(\theta_2 - \theta_1)}{L_3 \sin(\theta_2 + \theta_1)} \\ \end{bmatrix} \\
B_3 &= L_1^2 + e_3^2 A_3^2 - 2e_3 L_1 A_3 \cos(\theta_1 + \theta_3)
\end{align*}
\]

(2.44)

(2.45)

The \( \mu \) term in Equation 2.40 factors in the derivative of the total kinetic energy of the system with respect to the independent DOF \( \theta_1 \), i.e.

\[
\frac{\partial T}{\partial \theta_1} = \mu \dot{\theta}_1^2
\]

(2.46)

where

\[
\mu = I_1 A_1 \frac{\partial A_1}{\partial \theta_1} + I_2 A_2 \frac{\partial A_2}{\partial \theta_1} + I_3 A_3 \frac{\partial A_3}{\partial \theta_1} + \frac{1}{2} m_3 \frac{\partial B_3}{\partial \theta_1}
\]

(2.47)
and finally the $\nu$ term in Equation 2.40 factors in the derivative of the total potential energy of the system with respect to the independent DOF $\theta_1$, i.e.

$$\frac{\partial U}{\partial \theta_1} = \nu = m_1 g e_1 \cos \theta_1 + m_2 g e_2 \cos \theta_2 \frac{\partial \theta_2}{\partial \theta_1} + m_3 g \left( L_1 \cos \theta_1 - e_3 \cos \theta_3 \frac{\partial \theta_3}{\partial \theta_1} \right)$$ (2.48)

Equation 2.40 was derived using the Lagrangian formulation. The equation is solved for the independent DOF $\theta_1$, and angles $\theta_2$ and $\theta_3$ are geometrically constrained to $\theta_1$.

The complete mathematical derivation of the equation is a lengthy one and will not be reproduced here. A detailed derivation is presented in Reference [33].

### 2.7.4 Vehicle Model Control Summary

At this point the development of the driver controller for a 4WS vehicle is complete. This controller itself is made up of three parts: the front wheel steering controller, the rear wheel steering controller, and the throttle/brake controller. The following initial conditions/tuning parameters are to be considered when testing the controller:

1. Initial conditions: An initial position $(x, y, z)$ of the vehicle in space, initial orientation $\psi$, and an initial longitudinal velocity $V_{x0}$.

2. Traction force: This is calculated by the controller based on the vehicle speed and curvature of the road as outlined in Section 2.6.2. However, if the velocity profile is known beforehand, then it can also be user defined as will be demonstrated in Section 4.2.2.

3. An appropriate look-ahead distance $l_d$: this tuning parameter affects the performance of the steering controller.
4. Gains $G_i$, for $i = 1$ to $5$, for the steering controller. These are tuning parameters which affect the performance of the front-wheel steering controller.

5. P, I, and D gain coefficients of the PID controller for the four bar linkage.
Chapter 3

Model Validation

Ideally, to validate the model, an instrumented test vehicle (physical prototype), adapted to incorporate 4WS would be driven on a test track, and the results from the Simulink simulations would be compared to the experimental results obtained for this vehicle. However, the scope of this study limits the validation exercise to computer aided simulation tools. The tool chosen for the validation (see Section 3.3) is the MSC ADAMS/Car simulation software, which is an industry-standard commercial vehicle dynamics computer-aided-engineering tool. The next section attempts to validate the simulation results of the Simulink model against hand-calculated values obtained from vehicle dynamics theory. Table 3.1 provides a summary of the tests performed. Following that, the Simulink model is validated by comparing against results generated using ADAMS/Car.
Table 3.1: Summary of model validation checks.

<table>
<thead>
<tr>
<th>Test</th>
<th>Description</th>
<th>Check</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In-road-plane DOFs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal translation</td>
<td>Straight line motion satisfies the constant velocity motion equation.</td>
<td>$x(t) = x_0 + vt$</td>
<td>3.2</td>
</tr>
<tr>
<td>Lateral acceleration</td>
<td>At steady state, the summation of tire forces must balance centrifugal forces caused by the lateral acceleration of the sprung mass.</td>
<td>$ma_y = \sum_{i=1}^{4} F_{y,i}$</td>
<td>3.2</td>
</tr>
<tr>
<td>Yaw rotation</td>
<td>At steady state the total moments caused by the cornering forces about the vertical axis through the CG must equal 0.</td>
<td>$I_z \ddot{\psi} = \sum_{i=1}^{4} F_{y,i} l_i = 0$</td>
<td>3.2</td>
</tr>
<tr>
<td>Body slip angle</td>
<td>For a constant radius turn. At steady state, the body slip angle should be constant.</td>
<td>$\beta = \frac{v_y}{v_x} = \text{const}$</td>
<td>3.3.1</td>
</tr>
<tr>
<td><strong>Out-of-plane DOFs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bounce (sprung mass)</td>
<td>The undamped natural frequency and vertical displacement amplitude of the sprung mass at equilibrium.</td>
<td>Solution to half car model.</td>
<td>3.2.2</td>
</tr>
<tr>
<td>Pitch (sprung mass)</td>
<td>The undamped natural frequency and amplitude of the pitch angle at equilibrium.</td>
<td>Solution to half car model.</td>
<td>3.2.2</td>
</tr>
<tr>
<td>Suspension forces</td>
<td>At steady state the suspension (spring) forces must equal the sprung weight of the vehicle.</td>
<td>$\sum_{i=1}^{4} F_{s,i} = W_{sprung}$</td>
<td>3.2.1</td>
</tr>
<tr>
<td>Tire forces</td>
<td>At steady state the normal forces on the tires at the tire/road contact must equal the total weight of the vehicle.</td>
<td>$\sum_{i=1}^{4} F_{z,i} = W_{total}$</td>
<td>3.2.1</td>
</tr>
<tr>
<td>Roll</td>
<td>During a constant radius turn manoeuvre the body roll angle should remain constant.</td>
<td>$\phi = \text{const}$</td>
<td>3.3.1</td>
</tr>
<tr>
<td>Bounce (unsprung mass)</td>
<td>The undamped natural frequency and vertical displacement amplitude of the unsprung mass at equilibrium.</td>
<td>Solution to quarter car model.</td>
<td>3.2.1</td>
</tr>
</tbody>
</table>
Table 3.2 lists the MSC ADAMS/Car model parameters/specifications used to validate the Simulink model. These values are used throughout the remainder of the chapter and when further results are discussed in Chapter 4.

<table>
<thead>
<tr>
<th>Parameter (units)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total mass (kg)</td>
<td>1792</td>
</tr>
<tr>
<td>Unsprung masses (front/rear) (kg)</td>
<td>99/84</td>
</tr>
<tr>
<td>Wheelbase (m)</td>
<td>2.72</td>
</tr>
<tr>
<td>Trackwidth (front/rear) (m)</td>
<td>1.55/1.56</td>
</tr>
<tr>
<td>CG height (m)</td>
<td>0.54</td>
</tr>
<tr>
<td>Distance of CG from front axle (m)</td>
<td>1.127</td>
</tr>
<tr>
<td>Roll centre height (front and rear) (m)</td>
<td>0.2335/0.2335</td>
</tr>
<tr>
<td>Pitch centre height (front and rear) (m)</td>
<td>0.2/0.2</td>
</tr>
<tr>
<td>Principal Moment of Inertia about z axis (kg.m$^2$)</td>
<td>1187</td>
</tr>
<tr>
<td>Principal Moment of inertia measured about x axis (kg.m$^2$)</td>
<td>233</td>
</tr>
<tr>
<td>Principal Moment of Inertia about pitch y axis (kg.m$^2$)</td>
<td>1086</td>
</tr>
<tr>
<td>Suspension stiffness, individual wheels (front/rear) (N/m)</td>
<td>265000/52500</td>
</tr>
<tr>
<td>Suspension damping, individual wheels (front/rear) (N s/m)</td>
<td>4500/11207</td>
</tr>
<tr>
<td>Tire stiffnesses, individual tires (front/rear) (N/m)</td>
<td>200000/200000</td>
</tr>
<tr>
<td>Cornering stiffnesses, individual wheels (front/rear) (N/m)</td>
<td>123000/115000</td>
</tr>
</tbody>
</table>

3.1 Model Validation - Hand Calculations

As part of the model validation, it is necessary to carry out elementary checks to gain confidence in the accuracy of the model. These are seemingly trivial, yet critical checks which solve for the DOFs at known states, such as equilibrium. Hand calculations are performed to solve for the various DOFs of the model and these values are compared with the output of the Simulink model.
3.2 Horizontal Plane Degrees of Freedom

The first test is to confirm that motion with constant velocity satisfies the constant velocity kinematic equation

$$x(t) = x_0 + vt$$

(3.1)

The second test is a constant radius cornering test which gives two results: at steady state, the total of the tire cornering forces must equal the centrifugal forces caused by the lateral acceleration of the sprung mass and the total moments caused by the cornering forces about the vertical axis through the CG must balance.

For the first test, it is sufficient to simply run the model in a straight line at a constant velocity and note the distance travelled after $t$ seconds. For the constant radius cornering test, the model is run at 10 m/s, and with a front steering angle of $\delta_{1,2} = 0.1$ rad. For the test to pass, the following equations must be satisfied

$$ma_y = \sum_{i=1}^{4} F_{y,i}$$

(3.2)

and

$$I_\psi \ddot{\psi} = \sum_{i=1}^{4} F_{y,i}l_i = 0$$

(3.3)

The results in Table 3.3 show steady state values extracted from Simulink after running the test. The vehicle total and sprung masses are 1792 kg and 1609 kg respectively. The lateral acceleration is 2.98 m/s$^2$ and the yaw rate is 0 rad/s$^2$. Plugging the numbers into Equations 3.2 and 3.3 shows that the tests pass.
3.2.1 Quarter Car Model

At equilibrium, normal loads at the wheels must equal the total weight of the vehicle. Likewise, the suspension loads must equal the sprung mass of the vehicle. By examining the numbers in Table 3.3, it is shown that

\[ \sum_{i=1}^{4} F_{zi} = 17.6 \text{kN} \]  
(3.4)

and

\[ \sum_{i=1}^{4} F_{si} = 15.8 \text{kN} \]  
(3.5)

Table 3.3: Output from constant radius test.

<table>
<thead>
<tr>
<th>Wheel</th>
<th>( F_y ) (N)</th>
<th>( F_z ) (N)</th>
<th>( F_s ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1513</td>
<td>3935</td>
<td>3449</td>
</tr>
<tr>
<td>2</td>
<td>1303</td>
<td>6168</td>
<td>5682</td>
</tr>
<tr>
<td>3</td>
<td>949</td>
<td>3334</td>
<td>2922</td>
</tr>
<tr>
<td>4</td>
<td>1032</td>
<td>4145</td>
<td>3733</td>
</tr>
<tr>
<td>Total</td>
<td>4797</td>
<td>17582</td>
<td>15856</td>
</tr>
</tbody>
</table>

while \( W_{total} \) and \( W_{sprung} \) are 17.6 kN and 15.8 kN respectively.

If the front left wheel is isolated and considered alone, then the system of equations describing the undamped dynamics of the sprung and unsprung masses is given by

\[
\begin{bmatrix}
  m_s & 0 \\
  0 & m_{us}
\end{bmatrix}
\begin{bmatrix}
  \dddot{z}_s \\
  \dddot{z}_{u1}
\end{bmatrix} +
\begin{bmatrix}
  k_s & -k_s \\
  -k_s & (k_s + k_l)
\end{bmatrix}
\begin{bmatrix}
  \dot{z}_s \\
  \dot{z}_{u1}
\end{bmatrix} =
\begin{bmatrix}
  m_s g \\
  m_{u1} g
\end{bmatrix} \tag{3.6}
\]
This is known as a quarter car model as shown in Figure 2.8. The model has 2 DOF. It is possible to numerically integrate Equation 3.6 to obtain the vertical displacement and velocity time histories of the vehicle motion as well as the evaluated accelerations, suspension travel data, and suspension and tire forces. The steady state solution of the equation is the displacement of the masses at equilibrium.

Equation 3.6 may be written more compactly as

\[
[M] \{\ddot{z}\} + [K] \{z\} = \{F(t)\} \tag{3.7}
\]

At equilibrium the \{\ddot{z}\} vector is \{0\} and the forcing function, \{F(t)\}, is the vector of body weights within the system. Substituting values and solving for the vertical displacements at equilibrium yields; \(z_s = 0.04\) m and \(z_{us} = 0.022\) m. The numbers agree with the Simulink model, as can be seen from the output in Figures 3.1a and 3.1b.

To determine the natural frequencies (principal modes) of the two-degree-of-freedom system, the free vibration of the system is considered. The equations of motion of the free vibration problem are obtained by setting the right-hand sides of Equations 3.6 or 3.7 to \{0\}. The general form of the solution is known to be

\[
\begin{bmatrix}
z_s \\
z_{u1}
\end{bmatrix}
= 
\begin{bmatrix}
Z_s \\
Z_{u1}
\end{bmatrix}
\cos \omega_n t \tag{3.8}
\]

where \(\omega_n\) is the natural frequency of the system. With this assumed solution, Equation 3.6 can be solved for the vibration modes. Substituting the vehicle parameters in Table 3.2 yields

\[
\omega_{n, zu1} = 98.9\ \text{s}^{-1} \ (15.74\ \text{Hz}) \quad \text{and} \quad \omega_{n, zs} = 16.5\ \text{s}^{-1} \ (2.63\ \text{Hz})
\]
The results can be confirmed graphically when compared to the output of the Simulink model in Figures 3.1a and 3.1b. It is noted that the natural frequency of the unsprung mass is six times higher than that of the sprung mass.

This result is particularly important in studies related to ride quality. The wide separation of the natural frequencies of the sprung and unsprung mass has a significant implication on the vibration isolation characteristics of the suspension system. A more detailed treatment of the natural frequencies and mode shapes of the quarter car model, and their physical significance can be found in [3].

![Graphs showing sprung and unsprung mass translation](image)

(a) Sprung mass translation  (b) Unsprung mass translation

Figure 3.1: Solutions to the quarter car model, sprung and unsprung mass vertical translation.

### 3.2.2 Pitch and Bounce Model

The pitch and bounce model is a 2-DOF half-car model, illustrated in Figure 2.9. It considers the up and down translational motion (heave) and the angular motion (pitch) of the vehicle body. In general, the pitch and bounce motions are coupled, and
an impulse at the front or rear wheel excites both motions. To obtain the natural frequencies for the coupled bounce and pitch motions, the free vibration (or the principal modes of vibration) of the system is considered [3]. However, to simplify the calculations, the model will be assumed to be symmetric (uncoupled), i.e. suspension properties at the front and rear are equal and the centre of gravity lies halfway along the wheelbase of the car. This assumption means that

$$k_f l_1 = k_r l_2$$  \hspace{1cm} (3.9)

where $k_f$ and $k_r$ are the combined stiffness of the suspension and tire and called the front and rear ride rates, respectively.

For free vibration, the equation of motion for bounce is

$$m_s \ddot{z} + k_f(z - l_1 \theta) + k_r(z + l_2 \theta) = 0$$  \hspace{1cm} (3.10)

and the equation of motion for pitch is

$$I_y \ddot{\theta} - k_f l_1(z - l_1 \theta) + k_r l_2(z + l_2 \theta) = 0$$  \hspace{1cm} (3.11)

For this particular case, the bounce and pitch equations would be completely uncoupled since $k_f l_1 = k_r l_2$. This reduces the equations to

$$\ddot{z} + D_1 z = 0$$  \hspace{1cm} (3.12)

and

$$\ddot{\theta} + D_3 \theta = 0$$  \hspace{1cm} (3.13)

where
\[ D_1 = \frac{k_f + k_r}{m_s} \]  

and

\[ D_3 = \frac{l_1^2 k_f + l_2^2 k_r}{r_y^2 m_s} \]  

where \( r_y \) is the radius of gyration.

Equations 3.14 and 3.15 are solved for the natural frequencies \( \omega_{n,\text{bounce}} = \sqrt{D_1} \) and \( \omega_{n,\text{pitch}} = \sqrt{D_3} \) respectively. The model specifications are provided in Table 3.2. Substituting these values into the equations, the following results are obtained.

\[ \omega_{n,\text{bounce}} = 16.83 \text{ s}^{-1} \ (2.68 \text{ Hz}) \quad \text{and} \quad \omega_{n,\text{pitch}} = 23.8 \text{ s}^{-1} \ (3.79 \text{ Hz}) \].

As expected, the bounce frequency is approximately the same as what was obtained with the quarter car model, since there is no coupling between the two modes. The difference can be explained by round-off error when calculating the radius of gyration used in Equation 3.15.

Once again, comparison with the output of the Simulink model in Figures 3.2a and 3.2b shows good agreement. A more generalized and detailed solution of the derivation of the natural frequencies of the half car model, and their physical significance, can be found in [3].

### 3.3 Model Validation - ADAMS/Car

The model validation is carried out using ADAMS, a multibody dynamics and motion analysis software package widely used in both academia and industry.
ADAMS/Car is a specialized environment for modelling vehicles using the ADAMS solver. It allows for the creation and the analysis of virtual prototypes of vehicle subsystems, much like one would analyze the physical prototypes.

ADAMS/Car provides vehicle templates which include all necessary geometry, constraints, forces, and measurements necessary to define the physical dimensions and the physics of the car. Subsystems allow the user to change the parametric data of the template to suit the specific vehicle being modelled. One or several subsystems can be grouped together to form assemblies. Standard assemblies in ADAMS/Car are, for example, the suspension assembly or full-vehicle assembly. Figure 3.3 shows a screenshot of the fully-assembled ADAMS/Car model used to validate the Simulink model in this research. It corresponds to a front-wheel drive four-wheel-steering sedan.
Figure 3.3: Isometric wireframe view of the ADAMS/Car full vehicle assembly.

The objective of this section is to verify that the Matlab/Simulink model compares favorably with an ‘equivalent’ ADAMS/Car model. In order to achieve this, it is first necessary to make sure that both the vehicle’s inertial properties and physical dimensions match. This can be achieved either by building an ADAMS model with the desired specifications or adjusting the Simulink model specifications to match an existing ADAMS/Car model. The latter approach was adopted because the ADAMS/Car software package comes with existing representative full vehicle assemblies.

For the purposes of this study, an existing full car vehicle assembly was modified to include rear wheel steering. This was done by replacing the regular steering subsystem with a four-wheel-steering subsystem provided by MSC Software support. ADAMS/Car comes with pre-defined manoeuvres. By running the vehicle through these standard manoeuvres a benchmark can be established with which the performance of the Simulink model can be compared. The manoeuvres that were used for
model validation are an open-loop constant radius cornering manoeuvre and a closed-loop double lane change. This chapter focusses on how the Simulink model compares with the ADAMS/Car results. In Chapter 4, the manoeuvres and their implications will be discussed in further detail.

3.3.1 Constant Radius Turn

The constant radius turning manoeuvre is an open-loop test. In this test the vehicle is run at a constant speed of 10 m/s (36 km/h), and the wheel steering angle is quickly increased from 0 to 0.1 rad (5.73°) and held constant, Figure 3.4a. The results of interest are the trajectory (Figure 3.5), yaw rate (Figure 3.6a), lateral acceleration (Figure 3.6b), and slip angles of the front and rear wheels (Figure 3.7).
Figure 3.4: Front and rear wheel steering angles comparison of ADAMS and Simulink models for a constant radius cornering manoeuvre.

The maximum deviation of the Simulink model from the ADAMS model is 4 m at t=30 s. Which is 4% of the turn diameter of 100 m. The main cause for this
deviation is that the Simulink model is not strictly running at constant speed. Due to the yaw rate, the velocity gradually falls by about 10% from 10 m/s to 9.2 m/s after 30 s. The other reason for the deviation in the results is a difference in the control logic used between the two models for the steering of the rear wheels. The inner detailed workings of the ADAMS/Car rear wheel steering model are unknown, i.e., how the rear wheel steering angle varies with front wheel steering angle, velocity, and yaw rate of the vehicle. How closely the results match will also depend on how the rear wheel steering controller of the Simulink model is tuned.

Figure 3.5: Trajectory comparison of ADAMS and Simulink models for a constant radius cornering manoeuvre.
Figure 3.6: Yaw rate and lateral acceleration comparison of ADAMS and Simulink models for a constant radius cornering manoeuvre.

Appendix C provides additional plots which compare the body sideslip angle and
the roll angle between the ADAMS/Car and Simulink models for the constant radius cornering manoeuvre.

Figure 3.7: Wheel slip angles comparison of ADAMS and Simulink models for a constant radius cornering manoeuvre.

3.3.2 Double Lane Change

The double lane change (DLC) is a closed-loop manoeuvre with a feedback control system. The manoeuvre is discussed in greater detail in Section 4.2.1. It is used for the validation of the Simulink model as it is a pre-defined manoeuvre that comes
with the ADAMS/Car package.

Once again the same parameters are compared as with the previous test. The results plotted in Figures 3.8 through 3.11 show that the results compare favourably.

Figure 3.8: Trajectory comparison of ADAMS and Simulink models for a DLC manoeuvre.
Figure 3.9: Yaw rate and lateral acceleration comparison of ADAMS and Simulink models for a DLC manoeuvre.
Once more, the most evident reason for the deviation between the two models is the implementation of both the front and rear wheel steering control logic. Different
Figure 3.11: Wheel slip angles comparison of ADAMS and Simulink models for a DLC manoeuvre.
rear wheel steering angles will result in different turn radii.

Appendix C provides additional plots which compare the body sideslip angle and roll angle between the ADAMS/Car and Simulink models for the DLC manoeuvre.

Table 3.4 compares the values of key results from the DLC manoeuvre. It compares the maximum values and the times at which they occur for the front and rear wheel steering angles, lateral acceleration, yaw rate, sideslip angles and body roll angle. The ADAMS/Car model is used as the reference model. The high percentage errors only occur at the peaks, indicating that the front wheel steering controller for the ADAMS model responds more to a curvature in the track. A possible cause would be use of a smaller $l_d$ value.

Table 3.4: Numerical comparison between ADAMS/Car and Simulink results for the DLC manoeuvre.

<table>
<thead>
<tr>
<th></th>
<th>ADAMS/Car</th>
<th>Simulink</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_f$ (rad)</td>
<td>4.51</td>
<td>-0.17</td>
<td>4.51</td>
</tr>
<tr>
<td>$\delta_r$ (rad)</td>
<td>4.51</td>
<td>-0.076</td>
<td>4.51</td>
</tr>
<tr>
<td>$a_y$ (m/s$^2$)</td>
<td>4.51</td>
<td>-6.9</td>
<td>4.43</td>
</tr>
<tr>
<td>$\dot{\psi}$ (rad/s)</td>
<td>4.65</td>
<td>-0.411</td>
<td>4.43</td>
</tr>
<tr>
<td>$\beta$ (rad)</td>
<td>4.6</td>
<td>-0.045</td>
<td>4.6</td>
</tr>
<tr>
<td>$\phi$ (rad)</td>
<td>4.56</td>
<td>-0.035</td>
<td>4.47</td>
</tr>
</tbody>
</table>

From this preliminary exercise, it can be concluded with a fair degree of confidence that the 10-DOF model developed in Chapter 2 is suitable for studying aspects of vehicle handling that are relevant to the overall objectives of the current project.
Chapter 4

Sample Results

In the previous chapter, the Simulink model developed in Chapter 2 was validated using an established and industry-recognized multibody-dynamics solver. This chapter will attempt to compare the performance of a standard front-wheel steering (FWS) vehicle and a four-wheel-steering (4WS) vehicle as a means for exploring the suitability of the developed model for its intended application. The next few sections will demonstrate the improved vehicle handling characteristics (if any) of the latter model. To this end, the models will be run through a set of standard manoeuvres, found in the literature, to ensure that the resulting behaviour is consistent with expected results, and to better understand the response of four-wheel steering vehicles.

4.1 Model Sophistication

In Chapter 2, the vehicle model was developed starting from the relatively simple 3-DOF bicycle model, and progressing to a 4-DOF roll model, and finally to the 10-DOF model which was validated in Chapter 3. Before discussing the results of running the model through a set of manoeuvres, an important question is investigated: “What is the degree of sophistication of a vehicle model required for the purposes of this study, i.e., improvement of lateral dynamics by incorporation of
rear-wheel steering?” By running a double lane change (DLC) manoeuvre, discussed in detail in Section 4.2.1, the performance of the various models is compared. The DLC is a closed-loop manoeuvre whereby the driver tries to avoid an obstacle by changing lanes at constant velocity.

Table 4.1 provides the input parameters required to run each simulation; recalling that the parameters are described at the end of Chapter 2. Using these parameters and running all three models produces the results in Figure 4.1, whereas increasing the speed of the simulation from 17 m/s to 20 m/s produces the results in Figure 4.2. The graphs show very good agreement between all three models at lower speeds. As the speed is increased, however, it is evident that the 4-DOF and 10-DOF models follow the path with a fair degree of accuracy; while a noticeable difference exists with the 3-DOF model.

Table 4.1: Input parameters to the Simulink model for use in the DLC manoeuvre simulation.

<table>
<thead>
<tr>
<th>Initial position</th>
<th>$x_0 = 0$ m, $y_0 = 0$ m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial orientation</td>
<td>$\psi = 0$ rad</td>
</tr>
<tr>
<td>Initial longitudinal velocity</td>
<td>$v_{x_0} = 17$ m/s</td>
</tr>
<tr>
<td>Traction forces</td>
<td>$F_{x_i} = 0$ N on each wheel</td>
</tr>
<tr>
<td>Look-ahead distance</td>
<td>$l_d = 9$ m</td>
</tr>
<tr>
<td>Controller gains</td>
<td>$G_i = (0.4, 1.2, 2.0, 1.6, 0.8)$ for $i = 1$ to $5$</td>
</tr>
</tbody>
</table>

From this exercise, it can be concluded that for constant velocity applications where only the IRP DOFs are of interest, the 4-DOF model gives a sufficient degree of accuracy. This conclusion only applies to studies involving lateral dynamics. For studies which involve OORP DOFs, the 10-DOF model is still required as it is a more comprehensive model. An example of such a study is ride comfort (vibration)
over a non-uniform road or when there is a sufficiently large acceleration/deceleration for pitch rotation to affect performance. Additional plots which compare the yaw rates and lateral accelerations are provided in Appendix D. The results presented in the next sections will be based on the 10-DOF model.

![Trajectory Comparison](image)

Figure 4.1: Trajectory comparison of 3-DOF, 4-DOF, and 10-DOF models, for the DLC manoeuvre at a vehicle speed of 17 m/s.

### 4.2 Standard Manoeuvres

The International Organization for Standardization (ISO) defines a few standard tests for passenger vehicles and light trucks. These tests are important for vehicle characterization. Three tests were considered in this study. They are: the Double Lane Change (ISO 3888) [34], Steady State Cornering (ISO 4138) [35], and Lateral Transient Response/Step Steer (ISO 7401) [36].

The vehicle properties used for simulating these manoeuvres are the same as those
4.2.1 Double Lane Change - ISO 3888

The double lane change (DLC) manoeuvre is a closed-loop test to determine the road-holding ability of passenger cars. It tests the lateral acceleration limit of the vehicle under severe lateral load transfer [34]. The ISO 3888 Standard specifies the dimensions of the track, see Figure 4.3. It is applicable to passenger vehicles as defined by ISO 3833 and light commercial vehicles up to a gross mass of 3.5 tonnes.

The simulation input parameters for this manoeuvre are listed in Table 4.1. It is performed at an entry speed of 20 m/s (72 km/h). Figure 4.4 shows how the 10-DOF, 4WS driver model is able perform this manoeuvre. At this speed it takes 8 s to complete the manoeuvre. The main parameter which affects the performance is
the chosen look ahead distance as discussed in Section 2.7.1. The results show that an $l_d$ of 7 m will more closely follow the track than an $l_d$ of 9 m or 12 m. This is because a larger $l_d$ will result in a larger weighted error function, $(2.24)$. 

Figures 4.5 and 4.6 show how the front wheel steering angle and lateral acceleration
vary with the chosen $l_d$, respectively. Both graphs show the same trend, i.e., the lesser the $l_d$ the higher the front steering angle amplitude and the higher the lateral acceleration. This is to be expected since using a smaller $l_d$ means the “driver” is making tighter turns, which in turn causes larger lateral accelerations on the vehicle. Therefore, choosing an appropriate $l_d$ is probably a compromise between how accurately the model follows the trajectory and the induced lateral accelerations, which is a concern at higher speeds. An $l_d$ of 9 m will be used to generate the plots in subsequent sections. The test is a completely subjective one, as the standard does not give any recommendations on the entry speeds or lateral acceleration limits of the vehicle.

Figure 4.5: Front wheel steering angle change with chosen $l_d$ for the ISO 3888 DLC manoeuvre.
Figure 4.6: Lateral acceleration change with chosen $l_d$ for the ISO 3888 DLC manoeuvre.

Figures 4.7 through 4.10 compare results between vehicles equipped with rear wheel steering and the equivalent front-wheel steering only vehicle. As expected, the addition of rear-wheel steering reduces the ‘severity’ of the manoeuvre, i.e., leads to lower lateral accelerations. These results are very important as they demonstrate the utility of adding rear wheel steering. The characteristics of the 4WS plots can be adjusted by tuning the gains of both the front and rear wheel steering controllers.
Figure 4.7: Path following accuracy for the FWS and 4WS models for the ISO 3888 DLC manoeuvre.

Figure 4.8 shows that the front-wheel steering angle is greater for the 4WS configuration than for the FWS configuration. This is due to a swaying effect caused by addition of rear-wheel steering. In the limit whereby the rear-wheel steer angle equals the front-wheel steer angle, the yaw rate reduces to 0, i.e, the vehicle will sway sideways with a constant velocity, instead of turning. Therefore, the effect of adding rear-wheel steering is reducing the yaw rate, as seen in Figure 4.10 (and lateral acceleration, as seen in Figure 4.11). This implies that for a 4WS vehicle to achieve the same yaw rotation and maintain the trajectory, it has to compensate by increasing the front-wheel steering angle.
Figure 4.8: Front steering angle comparison for the FWS and 4WS models for the ISO 3888 DLC manoeuvre.

Figure 4.9: Rear steering angle comparison for the FWS and 4WS models for the ISO 3888 DLC manoeuvre.
Figure 4.10: Yaw rate comparison for the FWS and 4WS models for the ISO 3888 DLC manoeuvre.

Figure 4.11: Lateral acceleration comparison for the FWS and 4WS models for the ISO 3888 DLC manoeuvre.
4.2.2 Steady State Cornering - ISO 4138

ISO 4138 specifies open-loop test methods for determining the steady-state circular driving behaviour of passenger cars as defined in ISO 3833. The open-loop manoeuvres included in these methods are not representative of real driving conditions, but are nevertheless useful for obtaining measures of vehicle steady-state behaviour resulting from specific types of control inputs under closely-controlled test conditions.

This manoeuvre reveals the vehicle under/over-steer behaviour, roll angle at steady state, steering as a function of lateral acceleration, and sideslip as a function of lateral acceleration. To obtain a desired set of steady-state equilibrium conditions for speed, steering-wheel angle, and turn radius, it is possible to hold one of them constant, vary a second, and measure the third [37]. A constant radius test method (in which speed is varied and steering wheel angle is measured) is applied here to demonstrate understeer behaviour of the car. The steady state cornering manoeuvre is used as a reference for values in several other test manoeuvres such as a sine with dwell and continuous sinusoidal input (ISO 6401) [37].

The understeer coefficient of the vehicle, calculated using Equation 2.37, is 0.02. For this test, the turning radius is held constant while steadily increasing the velocity. Since the vehicle is understeer, i.e. \( \eta > 0 \), “when it is accelerated in a constant radius turn, the driver must increase the steer angle. In other words, when it is accelerated with the steering wheel fixed, the turning radius increases” [3], as illustrated in Figure 4.12.
Figure 4.12: Curvature responses of neutral steer, understeer, and oversteer vehicles at a fixed steer angle with increasing vehicle speed. [3].

Table 4.2 lists the input parameters for this manoeuvre. The ISO standard recommends a minimum test radius of 30 metres to be used and the test may be conducted up to a maximum lateral acceleration of 6 m/s\(^2\). The simulation is conducted for 30 seconds on a constant radius track having \( r = 40 \text{ m} \), shown in Figure 4.13. The speed profile and change in front and rear wheel steer angles are shown in Figures 4.14 and 4.15 respectively. The results confirm the expectations for an understeer vehicle; as the velocity increases, the magnitude of the steering angle has to increase to keep the vehicle on the constant-radius track.

Lateral acceleration \( a_y \) and yaw rate \( \dot{\psi} \) both increase as longitudinal velocity increases. Plots of \( a_y \) and \( \dot{\psi} \) as a function of time for the steady state cornering manoeuvre are provided in Appendix D.2.
Table 4.2: Input parameters to the Simulink model for use for the ISO 4138 cornering manoeuvre.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial position</td>
<td>$x_0 = 0$ m, $y_0 = -40$ m.</td>
</tr>
<tr>
<td>Initial orientation</td>
<td>$\psi = 0$ rad</td>
</tr>
<tr>
<td>Initial longitudinal velocity</td>
<td>$v_{xo} = 1$ m/s</td>
</tr>
<tr>
<td>Traction forces</td>
<td>$F_{xi} = {600, 600, 0, 0}$ N</td>
</tr>
<tr>
<td>Look-ahead distance</td>
<td>$l_d = 9$ m</td>
</tr>
<tr>
<td>Controller gains</td>
<td>$G_i = (0.4, 1.2, 2.0, 1.6, 0.8)$, for $i = 1$ to $5$</td>
</tr>
</tbody>
</table>

Figure 4.13: Constant radius path for ISO 4138 manoeuvre.
Figure 4.14: Longitudinal velocity, ISO 4138 manoeuvre.

Figure 4.15: Front wheel steering angle, ISO 4138 manoeuvre.
4.2.3 Lateral Transient Response (Step Steer) - ISO 7401

ISO 7401 provides the standard for an open loop test method for characterizing the lateral, transient response behavior of road vehicles. It applies to passenger cars and light trucks, as defined in ISO 3833 [38]. It gives the transient response to a step steering input, including response times and overshoots. Results of interest are the transient lateral acceleration and yaw rates in the time domain.

The standard recommends a vehicle speed of 100 km/h. The front wheel steering angle profile in Figure 4.16 is generated such that a maximum steady-state lateral acceleration of 4 m/s² is achieved. This angle is the input steering angle for the manoeuvre. The other parameters are listed in Table 4.3.

![Figure 4.16: Input steering angle for use with the ISO 7401 transient response manoeuvre.](image)

Figures 4.17 and 4.18 show how the yaw rate and lateral accelerations vary with time.
Table 4.3: Input parameters to the Simulink mode for ISO 7401 transient response manoeuvre.

<table>
<thead>
<tr>
<th>Initial position</th>
<th>$x_0 = 0$ m, $y_0 = 0$ m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial orientation</td>
<td>$\psi = 0$</td>
</tr>
<tr>
<td>Initial longitudinal velocity</td>
<td>$v_{xo} = 27.8$ m/s</td>
</tr>
<tr>
<td>Traction forces</td>
<td>$F_{xi} = {0, 0, 0, 0}$ N</td>
</tr>
<tr>
<td>Look-ahead distance</td>
<td>$l_d = 9$ m</td>
</tr>
<tr>
<td>Controller gains</td>
<td>$G_i = (0.4, 1.2, 2.0, 1.6, 0.8)$, for $i = 1$ to $5$</td>
</tr>
</tbody>
</table>

Both results show a similar trend. The transient behaviour shows that addition of rear wheel steering reduces the overshoot, while the steady-state values are about the same for both the FWS and 4WS configurations.

Figure 4.17: Yaw rate comparison between FWS and 4WS models for the ISO 7401 transient response manoeuvre.
Tables 4.4 and 4.5 summarise the results of the ISO 7401 manoeuvre for the lateral acceleration and yaw rate responses, respectively.

The ISO 7401 standard defines the response time ($T_{ay}$) as the time taken to reach 90% of the steady state value. Considering the lateral acceleration response, shown in Figure 4.18, 90% of 4.21 m/s$^2$ is 3.79 m/s$^2$ and this was reached at $t = 3.19$ s such that $T_{ay} = 0.19$s. The peak response time ($T_{ay,max}$) is the time to reach the maximum response value which for centripetal/lateral acceleration is 3.31 s such that $T_{ay,max} = 0.31$ s.

Considering the yaw rate response, shown in Figure 4.17, 90% of 0.151 rad/s is 0.136 rad/s and this was reached at $t = 3.19$ s such that $T_{\dot{\psi}} = 0.25$ s. The peak response time ($T_{\dot{\psi},max}$) for yaw rate is 3.32 s such that $T_{\dot{\psi},max} = 0.32$ s.
Comparing the difference in percentage overshoot of both responses shows the moderate advantage offered by including rear wheel steering to the lateral dynamics of the vehicle.

Table 4.4: Numerical comparison of lateral acceleration values for the FWS and 4WS models for the ISO 7401 transient response manoeuvre.

<table>
<thead>
<tr>
<th></th>
<th>FWS</th>
<th>4WS</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{y,ss}$ (m/s$^2$)</td>
<td>3.19</td>
<td>4.09</td>
<td>3.0</td>
</tr>
<tr>
<td>$a_{y,max}$ (m/s$^2$)</td>
<td>3.31</td>
<td>4.86</td>
<td>-7.4</td>
</tr>
<tr>
<td>% overshoot</td>
<td>3.31</td>
<td>19</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.5: Numerical comparison of yaw rate values for the FWS and 4WS models for the ISO 7401 transient response manoeuvre.

<table>
<thead>
<tr>
<th></th>
<th>FWS</th>
<th>4WS</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{\psi}_{ss}$ (rad/s)</td>
<td>3.19</td>
<td>0.147</td>
<td>3.0</td>
</tr>
<tr>
<td>$\dot{\psi}_{max}$ (rad/s)</td>
<td>3.31</td>
<td>0.173</td>
<td>-7.0</td>
</tr>
<tr>
<td>% overshoot</td>
<td>3.31</td>
<td>18</td>
<td>-</td>
</tr>
</tbody>
</table>

4.3 Rear Wheel Steering Performance

In the preceding section, the effect of adding rear wheel steering was demonstrated by running the model through a few manoeuvres and comparing results between FWS and 4WS models. For the ISO 3888 manoeuvre, the rear-wheel steer angle is shown in Figure 4.9. To achieve this angle, the required pinion force exerted by the
4WS controller through the four-bar linkage is shown in Figure 4.19.

The rear wheel steering mechanism employs a steer-by-wire architecture whereby the rear wheel steering angle (signal) is measured, compared with the RWS controller output, and corrected by the actuated four-bar linkage. Therefore, it is important to see how the model is able to respond to a disturbance to this signal. The disturbance could be in the form of noise or deliberate tampering. This section demonstrates how the controller will counteract any potential change in this signal to keep the vehicle on track. A randomly-generated noise signal will be added to the output (measured) signal of the controller, $\delta_r$, and the pinion force required to counteract the noise will be observed.

The Simulink block *Band-Limited White Noise* creates a noise profile as in Figure 4.20. This is added to the output of the four-bar linkage block to simulate a disturbance. The White Noise block generates normally-distributed random numbers that are suitable for use in continuous systems. The magnitude of the noise signal is chosen to be about the same as the measured rear wheel steer angle without noise.

The addition of noise to the measured rear wheel steering signal increases the magnitude of the rear wheel steer angle. This means that the pinion force required to correct the angle so that it matches the desired angle (from the RWS controller) will increase, as shown in Figure 4.21.
Figure 4.19: Pinion force which actuates the four-bar-linkage and turns the rear wheels.

Figure 4.20: Random noise signal used as perturbation to the RWS controller signal.
Figure 4.21: Pinion force after noise signal is added to the measured rear-wheel steer angle.

Figure 4.22, shows how the rear wheel steering angle compares, with and without noise. From this, it can be concluded that the pinion forces act to keep the vehicle on track even when the signal is somewhat disturbed.
Figure 4.22: Measured rear wheel steering angle with and without the injected noise signal for the DLC manoeuvre.

It should be noted that this considered a specific case of signal corruption. Other signal faults, such as from an external source, in the case of autonomously controlled vehicles, could have more detrimental effects on vehicle performance.
Chapter 5

Conclusion and Recommendations

In this thesis, the development of a 10-DOF multi-body dynamics vehicle model was presented. The model is intended to be used for further research into the cybersecurity of road vehicles. Starting from a less complicated 3-DOF bicycle model, mathematical equations representing the dynamics of a vehicle were developed from basic principles. The complexity of the model was sequentially increased to a 4-DOF roll model, and finally a 10-DOF multi-dynamics model. A driver model which simulates a real driver was developed. Finally, rear-wheel steering capability was incorporated into the model. The resulting model was implemented in the Matlab/Simulink programming environment and validated by comparing it with an equivalent ADAMS/Car model. The modular vehicle model can be easily extended for further development.

The primary investigation carried out with the model is the investigation of added benefit of incorporating rear wheel steering to the lateral dynamics of a vehicle.

The model was run through several standard manoeuvres published by the ISO and the results obtained suggest that the model performs satisfactorily. The results conclude that the addition of four-wheel steering to the model improves the lateral dynamics of the vehicle. The main variables emphasized on in this thesis were the yaw rate and the lateral acceleration. Further, the flexible method for specifying
manoeuvres, the autonomy provided by the driver model, and the ability to output a wide variety of vehicle kinematics and dynamic parameters as a function of time, make this model well suited for supporting the vehicle cyber-security research for which it is intended. It is demonstrated that the model is able to self correct, through the rear wheel steering controller, following a specific perturbation to its rear-wheel steering signal.

For accurate path tracking, and to benefit from the lateral stability improvements of the rear wheel steering, the controllers have to be tuned depending on the vehicle characteristics (physical dimensions and inertial properties), the manoeuvre, and the test track.

### 5.1 Future Work

In the future, few recommendations are made to improve the fidelity of the model. The steering subsystem can be improved by setting a steering ratio between the wheels and steering wheel. This will make the input to the model in terms of the steering wheel angle, which is more intuitive. The drivetrain can be revised such that the tractive forces are calculated based on engine RPM, gear selection, and throttle position. A model which calculates the rolling resistance of the tires should be used when calculating total longitudinal forces. Anti-roll bars can be included in the model to reduce body roll. The roll centre is assumed fixed in this model; whereas in reality it is a function of the compression of the suspension. This can be implemented by mapping the roll centre height to the suspension compression and interpolating. These are all improvements that can be made to the model without changing its core architecture or functionality.
In terms of the validation of the model, the preliminary tests described in Chapter 4 demonstrate that the vehicle behaves reasonably well, i.e., it follows the expected trends. However, the validation tests conducted are by no means exhaustive and more tests should be carried out to ensure the robustness of the model. More standard and non-standard test procedures were published by Karlsson [37]. It would be beneficial to benchmark the developed model against these tests as well.
List of References


Appendix A

Simulink Model

Figure A.1 shows the structure of the overall model, i.e., vehicle model and the controllers. There are two primary controllers, the steering controller and the throttle controller. Additionally, the steering controller can itself be considered to be composed of three controllers; front-wheel steering controller and a rear wheel steering controller.

Figure A.1: Overall model (with controllers) Simulink block diagram.
A.1 Vehicle Model

Figure A.2 shows the inner workings of the vehicle model alone without controllers.

Figure A.2: Vehicle model (without controllers) Simulink block diagram.
Appendix B

Wheel Rate and Roll Centre

The following graphs provide the front and rear wheel rates used in the vehicle model. It is obtained from a “Parallel wheel travel” simulation on ADAMS/Car. The wheel rate is the slope of the curve when wheel travel is 0.

Figure B.1: Hub force versus wheel travel for the front wheels.
Figure B.2: Hub force versus wheel travel for the rear wheels.

Figures B.3 and B.3 show the front and rear roll centre of the vehicle respectively. The roll centre varies with compression of the suspension springs, however it is approximated as the value when wheel travel is 0.

Figure B.3: Roll centre versus Wheel travel for the front wheels.
Figure B.4: Roll centre versus Wheel travel for the rear wheels.
Appendix C

Additional Model Validation Plots

The following graphs provide additional plots from the Simulink model validation exercise in Section 3.3. Figures C.1 and C.2 compare the vehicle CG sideslip angle and the body roll angles of the ADAMS/Car and Simulink models respectively, for the constant radius turn manoeuvre.

Figure C.1: Sideslip angle comparison of ADAMS and Simulink models, ISO 4138 manoeuvre.
Figure C.2: Body roll angle comparison of ADAMS and Simulink models, ISO 4138 manoeuvre.

Figures C.3 and C.4 compare the vehicle CG sideslip angle and the body roll angles of the ADAMS/Car and Simulink models respectively, for the DLC manoeuvre.

Figure C.3: Sideslip angle comparison of the ADAMS and Simulink models for the ISO 3888 DLC manoeuvre.
Figure C.4: Body roll angle comparison of the ADAMS and Simulink models for the ISO 3888 DLC manoeuvre.
Appendix D

Model Comparison Plots - 3-DOF versus 4-DOF versus 10-DOF models

D.1 Additional Results - ISO 3888 Manoeuvre

Table D.1 provides the input (initial conditions/tuning parameters) for the ISO 3888 manoeuvre. Figures D.1 and D.2 show how the yaw rate compare for the 3-DOF, 4-DOF and 10-DOF models.

Table D.1: Input parameters to the Simulink model for the ISO 3888 manoeuvre.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial position</td>
<td>$x_0 = 0$ m, $y_0 = 40$ m.</td>
</tr>
<tr>
<td>Initial orientation</td>
<td>$\psi = 0$</td>
</tr>
<tr>
<td>Initial longitudinal velocity</td>
<td>$v_{x_0} = 17$ m/s</td>
</tr>
<tr>
<td>Traction forces</td>
<td>$F_{x_i} = 0$ N each wheel</td>
</tr>
<tr>
<td>Look-ahead distance</td>
<td>$l_d = 9$ m</td>
</tr>
<tr>
<td>Controller gains</td>
<td>$G_i = (0.1, 0.3, 0.5, 0.4, 0.2)$, for $i = 1$ to 5</td>
</tr>
</tbody>
</table>
Figure D.1: Yaw rate comparison of 3-DOF, 4-DOF, and 10-DOF models for the DLC manoeuvre at a vehicle speed of 20 m/s.

**D.2 Additional Results - ISO 4138 Manoeuvre**

Table D.2 provides the input (initial conditions/tuning parameters) for the ISO 4138 manoeuvre. Figures D.3a and D.3b show plots of the longitudinal velocity and steering angle respectively for the manoeuvre.
Figure D.2: Lateral acceleration comparison of 3-DOF, 4-DOF, and 10-DOF models for the DLC manoeuvre at a vehicle speed of 20 m/s.

Table D.2: Input parameters to the Simulink model for the ISO 4138 manoeuvre.

- Initial position $x_0 = 0$ m, $y_0 = -40$ m.
- Initial orientation $\psi = 0$
- Initial longitudinal velocity $v_{xo} = 1$ m/s
- Traction forces $F_{xi} = \{600, 600, 0, 0\}$ N
- Look-ahead distance $l_d = 9$ m
- Controller gains $G_i = (0.4, 1.2, 2.0, 1.6, 0.8)$, for $i = 1$ to $5$
Figure D.3: Plots of yaw rate and lateral acceleration versus time, for ISO 4138 manoeuvre.