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THE EFFECTS OF LOCAL GEOLOGY AND LIQUEFACTION ON SEISMIC GROUND MOTION

by

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B.Eng., Beijing Polytechnical University, 1982
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A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

Department of Civil and Environmental Engineering

The Doctor of Philosophy Program
in Civil and Environmental Engineering
is a joint program with the University of Ottawa,
administered by The Ottawa-Carleton Institute for Civil Engineering

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April 1997
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0-612-22178-4
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The Undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis

THE EFFECTS OF LOCAL GEOLOGY AND LIQUEFACTION ON SEISMIC GROUND MOTION

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ABSTRACT

Observations from major earthquakes show that local geology affects the seismic ground motion and the liquefaction potential of soil layers. Case studies also indicate that less damage happened to structures located on the sites where liquefaction was detected. However, numerical analyses to these phenomena have been seldom reported.

In this thesis, a stress-strain-liquefaction model is proposed based on the energy approach. The model involves the use of the non-linear constitutive relationship of soil and the modulus degradation curve determined from laboratory and field tests. A new boundary element formulation for visco-elastodynamic analysis in time domain is presented. The velocity and the acceleration terms in the dynamic equilibrium differential equation are approximated by an appropriate temporal discretization scheme. The differential equation is solved by the direct boundary element method in which the fundamental solution for elastostatic problems has been adopted. The domain integrals generated by the velocity and acceleration terms are transformed to boundary integrals by means of the dual reciprocity method. The new liquefaction model has been introduced into the boundary element formulation, to conduct the one-dimensional and two-dimensional liquefaction analysis.

One-dimensional liquefaction analysis has been conducted for the Port Island case. The computed motion agrees with the record of seismic motion. The analysis
shows that the ground motion with the period from 0.2 second to 2.5 second has been significantly reduced by the liquefaction of soil. This result explains the fact that less damage to most of the engineering structures occurs in liquefied zone.

A parametric study has been carried out on the effects of the two-dimensional valley on ground motion and liquefaction. It is found that for low-frequency motion, the maximum amplification occurs at the center of the valley. The amplification is significantly affected by two factors: the shear wave velocity of the fill material and the base material in the valley, and the depth and width of the valley. The liquefaction potential in an alluvial valley is generally higher than that in an evenly layered site. Liquefaction starts at the edges of the valley.

Based on the analyses, it is suggested that in a site with high liquefaction potential, pile foundation and structures with high resonant frequency will help to reduce the seismic damage to the structures. It should be noted that in earthquake zone, the existence of alluvial valleys may increase the seismic damage and the liquefaction potential.
ACKNOWLEDGEMENTS

The student is deeply grateful to Professor K. Tim Law of the Department of Civil and Environmental Engineering, Carleton University, for his continuous support and advice in the present study.

Special thanks are due to Dr. Jim Hunter, Energy, Mines and Resources, Professor E. Evgin, Department of Civil and Structural Engineering, University of Ottawa, Professor C. L. Tan, Department of Mechanical and Aerospace Engineering, Carleton University, Professor G. E. Bauer, Department of Civil and Environmental Engineering, Carleton University, Professor D. T. Lau, Department of Civil and Environmental Engineering, Carleton University, for their valuable comments and suggestions.

Sincere thanks to Professor C. L. Tan, Department of Mechanical and Aerospace Engineering, Carleton University, and Mr. G. X. Li, Ontario Hydro, for their generous consultation and help offered to the student during his study.

Last but not the least, the student wishes to express his sincere gratitude to his family for their unfailing support and constant encouragement during his Ph.D. studies.
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List of Symbols

$C_1$: converting factor between the liquefaction strength from triaxial test to the liquefaction strength in the field

$E$: Young’s modulus, $(Pa)$

$G$: shear modulus, $(Pa)$

$G_i$: shear modulus for initial loading, $(Pa)$

$G_s$: shear modulus for unloading–reloading, $(Pa)$

$G_{ij}$: element of $G$ matrix in BEM formulation

$H(t - \tau)$: Heaviside step function

$H_{ij}$: element of $H$ matrix in BEM formulation

$K_0$: coefficient of earth pressure at rest

$L$: length of the problem domain in the 1-D analysis

the number of internal nodes in the 2-D analysis

$N$: the number of boundary nodes

$N_f$: the number of loading cycles leads to liquefaction

$P$: average traction, $(Pa)$
\( P_{ij} \): traction field corresponding to the fundamental solution

\( T \): the total length of the time to be analyzed, \((s)\)

\( T_0 \): period, \((s)\)

\( U_{ij} \): fundamental solution

\( c_p \): pressure wave velocity, \((m/s)\)

\( c_s \): shear wave velocity, \((m/s)\)

\( f^{(k)} \): approximating function

\( n \): outside normal vector, dimensionless

\( n_i \): the \(i^{th}\) component of \(n\)

\( p_0 \): normalized confining pressure, dimensionless

\( p_{ij} \): traction, \((Pa)\)

\( \dot{p} \): the traction field corresponding to \(\dot{u}\), \((Pa)\)

\( r \): distance, \((m)\)

\( s \): a model parameter in the constitutive equation, dimensionless

\( t \): time, \((s)\)

\( u, u \): displacement, \((m)\)

\( \dot{u} \): a displacement field used in DRM formulation

\( \ddot{u} \): velocity, \((m/s)\)

\( \dddot{u} \): acceleration \((m/s^2)\)

\( x, x \): a point in the problem domain

\( x^{(A)} \): source point
$x^{(B)}$: boundary point

$x^{(k)}$: DRM collocation point

$\Delta t$: length of a time step in time discretization, (s)

$\Delta u$: excess pore-water pressure, (Pa)

$\Omega$: the problem domain

$\Gamma$: boundary of the problem domain

$\alpha$: model parameter in the liquefaction model, dimensionless

$\alpha_i^{(k)}$: approximating coefficient in DRM formulation

$\beta$: model parameter in the liquefaction model, dimensionless

$\delta_{ij}$: Kronecker delta

$\gamma$: shear strain

$\zeta$: damping factor, (N/m³)

$\mu$: damping coefficient, (Pa × s)

$\eta$: damping ratio

$\rho$: density, (kg/m³)

$\nu$: Poisson’s ratio

$\omega$: the frequency, (1/s)

$\tau$: shear stress, (Pa)

$\tau_{ult}$: ultimate shear stress, (Pa)

$\tau_f$: liquefaction strength, (Pa)

$\sigma$: stress, (Pa)
\( \sigma_1: \) the maximum principal stress, \((Pa)\)

\( \sigma_3: \) the minimum principal stress, \((Pa)\)

\( \lambda: \) a constant generated by time discretization

\( \xi: \) a constant generated by time discretization

\( \psi: \) a constant generated by time discretization
Chapter 1

INTRODUCTION

The estimation of site response is one of the most complex problems in engineering practice. First, ground stability should be examined to avoid damage caused by ground failure. Second, the intensity, duration, and frequency content of ground motion should be estimated, in order to provide the input seismic load for the dynamic analyses of structures.

Reports on earthquake damage show that many cases of ground failure were caused by the liquefaction of sandy soils. Therefore, the estimation of liquefaction potential is an important part of site evaluation.

Liquefaction potential is affected by a number of factors, including soil properties, groundwater table, overburden pressure, the intensity and duration of earthquake shaking, and, as reported by some researchers (Wang, 1981, Wang and Law, 1994), the local geology.

The evaluation of liquefaction potential includes "macroscopic evaluation" and "microscopic evaluation". The macroscopic evaluation is based on the historical records of liquefaction, the earthquake intensity, and the kind of the liquefiable
soil layer(s) (Wang et al., 1991), while the microscopic evaluation is based on the comparison of the liquefaction strength of soil and the earthquake induced stress in a soil element.

For important projects such as nuclear power stations, the liquefaction potential for the site should be carefully examined to ensure the integrity of the structures. In this case, the microscopic evaluation of liquefaction potential should be carried out along with the macroscopic evaluation.

In microscopic evaluation, the determination of liquefaction strength and the computation of seismically induced stress are needed. Therefore, two main parts are involved in the evaluation: a liquefaction model to relate the soil parameters to the liquefaction strength, and a computational method to determine the seismic force acting on the soil mass.

Many kinds of liquefaction models have been developed in the last 30 years. Simple models like the traditional one proposed by Seed (1966) are easy to use, while complicated ones are able to consider more factors under different situations.

In liquefaction analysis, one-dimensional (1-D) analysis is widely used in computing the seismic stress. In 1-D analysis it is assumed that the ground surface and the soil layers are horizontal, and the seismic wave propagates vertically from the bedrock to the ground surface. One-dimensional analyses have been used for many years, and been proven to be successful in many cases (Seed et al., 1991).

Case studies show that local geological conditions, such as irregularity of soil layers and bedrock, will affect the seismic ground motion (Aki, 1988, Finn, 1991, EQE, 1995), and therefore the liquefaction potential of soil (Wang, 1981, Huang
et al., 1981). There is no established relationship between the local geological conditions and the ground motion, and no appropriate factor has been found to describe the characteristics of the irregularity of the local geology. Therefore, in the microscopic evaluation, two-dimensional or three-dimensional numerical analysis is required in the assessment of the seismic induced force. The numerical analysis is expensive in terms of necessary input and computer time.

It is also found that liquefaction in subsoil will alter the intensity and the frequency components of the earthquake ground motion, which determines the seismic load acting on the structures. This effect has been noticed for a long time (Aki, 1988, Chen, 1990, Wang and Law, 1994), but it still cannot be estimated quantitatively to determine the ground motion before and after the liquefaction. To find the response of the liquefied site, numerical analysis is needed.

The subject of this thesis is to study the effects of local geology and soil liquefaction on seismic ground motion. In order to carry out the study, new numerical tools will be developed.

In this thesis, a new liquefaction model will be developed for the efficient evaluation of pore-pressure build-up and the softening of soil. In developing this model, the energy approach for liquefaction potential evaluation will be introduced into the numerical analysis. Compared with existing elastoplastic models, the new model is simpler in form, it requires less computing time, and the model parameters can be easily obtained.

A new boundary element formulation will also be developed. This formulation is developed by first discretizing the governing equation of visco-elastodynamic problem in time domain. A non-homogeneous governing differential equation is
obtained from the time discretization. By applying the dual reciprocity boundary element method, a pure-boundary solution to the non-homogeneous governing differential equation is obtained. The main feature of this formulation is that it can solve visco-elastodynamic problems in time domain. Therefore, the non-linear properties of a soil subjected to a strong earthquake motion can be studied, and the time-dependent soil behavior, like the pore-pressure build-up during the development of liquefaction, can be considered.

Based on the new developed liquefaction model and the boundary element formulation, a boundary element method for one-dimensional liquefaction analysis is obtained. By means of this method, the subsequent effect of liquefaction on the characteristics of earthquake ground motion, such as its intensity and frequency content, are examined. The results from the analyses are compared with the records from a major earthquake.

To evaluate the effects of local geology on ground motion and soil liquefaction, the new-developed boundary element formulation is applied to the two-dimensional analysis of alluvial valleys. A parametric study will be carried out to evaluate the effects of geometric and physical factors of an alluvial valley. The factors that have significant effects on ground motion will be identified, and their effects will be evaluated.
Chapter 2

LITERATURE REVIEW

The study of seismic liquefaction of the soil can be traced back to the 1960s (Seed and Lee, 1966. Seed and Idress, 1969). In the last thirty years, the factors that control soil liquefaction have been investigated by means of laboratory tests, in situ tests, and case studies. Some approaches to the assessment of liquefaction have been developed based on the investigation (Seed and Idress, 1969, Law et al., 1991). Mainly because of the difficulties in determining the seismic loading acting on the soil layer, the assessment is limited to the simplest one dimensional analysis, which represents a horizontal ground with an underlain sand layer of even thickness.

The development in numerical analysis enables us to study seismic liquefaction under some complex conditions. The numerical approach to liquefaction potential estimation includes two parts:

1. the computation of seismic load acting on the soil mass, and,

2. the modeling of the behavior of liquefiable soil.
Numerical methods and computer programs for one-dimensional site response analysis, that give the seismic induced forces in the soil, were developed and tested in 1970s (Seed and Idress, 1971, Seed, 1979). Some two or three dimensional analyses have been carried out in solving the soil-structure interaction problems or site response problems (Lysmer, 1974, Luco et al., 1990). On the other hand, different types of liquefaction models have been proposed (Matasovic and Vucetic, 1993, Ramsamoog and Alwash, 1990). By combining the seismic response analysis model and the liquefaction model, numerical tools can be obtained to simulate the seismically induced liquefaction of soils (Blázquez et al., 1980, Gu et al., 1993).

Observations show that some factors significantly affect the ground motion and the liquefaction potential, but not all these factors have been well studied because of the lack of appropriate numerical tools. One of them, the local geology, will be discussed.

The effect of soil liquefaction on ground motion has been observed during major earthquakes. It is found that the characteristics of ground motion are significantly altered by the liquefaction of soil layers. This effect will be investigated in detail in the following by discussing cases of major earthquakes.

In the following discussion, the existing literature on the effects of soil liquefaction on ground motion, and on the effects of local geology on liquefaction potential, will also be summarized.
2.1 CASES OF EARTHQUAKES

2.1.1 Niigata Earthquake, 1964

An earthquake which occurred on June 16, 1964, caused extensive damage in Niigata City, a major city on the Japan Sea coast of the island of Honshu. The magnitude of this earthquake is M7.5 (Japan National Committee on Earthquake Engineering, 1965).

During this earthquake, the damage to structures was mainly caused by subsidence, uneven settlement or cracking due to the liquefaction of sandy ground. Two-third of the damaged buildings just settled, tilted or overturned as a whole without appreciable damage in their superstructures(Ohsaki, 1965, Kawakami and Asada, 1966).

Investigations indicated that the extent of damage greatly depended upon the geology of the ground on which the structure stood. The most serious damage was encountered on some river sites. Ohsaki(1966) investigated the distribution of damage and the soil condition in Niigata City. The distribution of damaged buildings is shown in Figure 2.1. The damage was measured by the tilt of a building due to liquefaction in the subsoil. It is found that heavy damage (angle of tilt greater than 2.5°) was concentrated in some areas close to Shinano River. Figure 2.2 is a soil profile in Tsusen River District, Niigata City. The profile shows that the heavily damaged zone is located on an alluvial valley filled with loose sand ($N < 10$).

Mishina and Sato(1966) reported that in Tsusen River district, nearly all the structures were heavily damaged due to the rupture of ground caused by lique-
faction. The soil profiles show that the heavily damaged zone was located on an alluvial valley again.

It is found from the above observations that severe liquefaction failure occurred in alluvial valleys. One reason, as mentioned in all the references, is the existence of loose sand ($N < 10$) within the depth of 5-15m. The other possible reason is the amplification effect of the valley, as will be discussed later in Section 2.3.

Another interesting phenomenon observed in Niigata earthquake is that little damage to the structural elements was found in the sites where liquefaction occurred. A well-known example is the No.4 apartment building at Kawagishi-cho of Niigata City. The building was completely overturned but no crack, not even a hairline crack, was detected in its superstructure. Doors and windows worked quite well just as usual even in the completely overturned state (Ohsaka, 1966).

### 2.1.2 Tangshan Earthquake, 1976

Tangshan Earthquake, which is known as the worst natural disaster in this century, occurred on July 28, 1976. The magnitude was M7.8. More than 242,000 people were killed in this earthquake.

Soil liquefaction was found in many sites during Tangshan Earthquake. Some of the sites were close to the epicenter, while some others were more than two hundreds kilometers away from the epicenter.

Huang et al. (1981) reported the liquefaction failures in Tianjin area, which is about 100 km from Tangshan. During the earthquake, the seismic intensity in this area was VII to XI (corresponding to horizontal ground acceleration 0.15g to
0.30g). There are seven ancient rivers and forty-five underground pits located in this area. Sand boils and ground cracks caused by liquefaction were found in all these sites. In the ancient rivers, liquefaction caused sand boils and cracks at the center of the river beds, and lateral ground movements or landslides at the river banks. In the underground pits, liquefaction resulted in sand boils and cracks on the ground surface. Figure 2.3 shows the distribution of liquefied sites and the locations of the ancient river courses. It can be found that extensive liquefaction occurs more frequently near old river courses. It is also found that at the sites where the subsoil is evenly layered, liquefaction caused only even settlement, with little damage to foundations and structures. At the sites with unevenly layered subsoils, the foundations, including pile foundations, were badly damaged by large differential settlements and lateral movements caused by liquefaction.

Yao and Wang (1981) reported that in Xiji-Langfu area, about one hundred and fifty kilometers from the epicenter, numerous ground cracks and sand boils occurred. This area is located in the alluvial delta of Chaobai River, which changed its course several times in history. The river left three main ancient river beds in this area. All the liquefied sites are located at the old river beds, river flats, and river bends, especially at the intersections of old rivers.

Wang (1981) reported liquefaction occurrences in Hai river and Dou river district during Tangshan Earthquake. Three characteristics had been found in this area:

1. On basically level ground, sand boiling was obvious on the convex bank of a river, whereas on the concave bank there was hardly any sand boiling at all.

2. On both sides of a straight river course or on sites far from the river, sand
boiling is apparently weakened, even invisible.

3. On sites surrounded by several river bends (skirt of river bends), macroscopic liquefaction tends to be more severe.

In general, the investigation of Tangshan Earthquake shows that liquefaction potential is high in the old river beds, skirt of river bends, or alluvial valleys. This phenomenon indicates that certain local geology condition, such as the existence of alluvial valleys, will affect the liquefaction potential. The observation also shows that in the areas where liquefaction was found, the intensity varies from VII to XI (Modified Mercali Scale). Liquefaction was seldom found in high intensity areas (with intensity of X or XI). A possible reason is that when liquefaction occurs, the intensity of ground motion will be reduced by the liquefied soil layer, that is, high intensity is possible to be observed only when there is no liquefaction.

2.1.3 Loma Prieta Earthquake, 1989

Seed et al. (1991) reported liquefaction failures in San Francisco area during the 1989 Loma Prieta Earthquake. Liquefaction damage occurred in areas where the subsoils are likely to be irregularly layered, such as lagoon, bay, or seashore. The most significant liquefaction damage was found in the Marina District.

The subsoil in Marina District can be grouped into four categories: hydraulic fill, artificial fill, Strawberry Island (modern beach) deposits, and dune sand (native soil). The hydraulic fill was placed into the lagoon enclosed by a seawall, in 1912, to create land for the 1915 Panama Pacific Exposition. The fill material, consisted primarily of fine, silty sand, was pumped in hydraulic suspension into the enclosed
lagoon and allowed to settle. This procedure resulted in a loose, saturated fill. The zonation of this site (Rosidi and Wigginton, 1991), and the soil profiles (O’Rourke et al., 1991), are shown in Figure 2.4 and Figure 2.5.

During the Loma Prieta Earthquake, liquefaction was found in the hydraulic fill, the artificial fill, and the modern beach, but was significantly more severe and extensive in the hydraulic fill zones. One of the reasons, as mentioned by O’Rourke et al. (1991), is the low SPT value of the fill, which shows that the material itself is easy to liquefy when subjected to seismic loading. Furthermore, it is worthy to note that the subsoil condition is complex: The hydraulically filled lagoon is softer than the surrounding soils, and the site is located on the bay mud in different thickness. Ground motion amplification has also been reported. The amplification effect of the basin filled with soft soil would be one of the reasons for the extensive liquefaction in this area.

2.1.4 Kushiro-oki Earthquake, 1993

In the 1993 Kushiro-oki Earthquake of Richter magnitude 7.8, simultaneous recording of earthquake motions was obtained at the ground surface and at the depth of 77 meters (Iai et al., 1995). The records are shown in Figure 2.6.

By examining the acceleration time histories in the NS direction, a significant change in the frequency component of the ground surface record after 30 seconds is found. Most of the high frequency motions are filtered out and instead cyclic motion with a period of about 1.5 seconds becomes predominant.

In contrast to the significant change in ground motion during the main shock,
the record from an aftershock with a magnitude of 4.9 is shown in Figure 2.7. The records show no significant change in the frequency components of the motion at the ground surface (Iai et al., 1995).

This phenomenon can be explained as follows: During the main shock, strong seismic motion caused liquefaction in the sand layer at the time around the 30th second in the record. The liquefied soil changed the frequency components of the subsequent ground motion. During the aftershock, the seismic shaking was not strong enough to cause a significant build-up of pore water pressure to liquefy the soil. Therefore, the frequency components were not altered.

2.1.5 The Hyogoken-Nanbu(Kobe) Earthquake, 1995

The Hyogoken-Nanbu(Kobe) earthquake occurred at 5:46 a.m. local time on 17 January 1995. The Richter Magnitude of the earthquake was estimated to be 7.2. The duration of strong shaking is 10 to 15 seconds (Oba, 1995).

This earthquake is among the most severe natural disasters in this century. Nearly 5,500 people were killed; about 35,000 were injured; 180,000 buildings were destroyed or badly damaged, 300,000 people were rendered homeless on the night of the earthquake. The economic loss is the worst in history. Damage was reported over an area within 100 km from the epicenter, including the cities of Kobe, Osaka, and Kyoto. The estimated direct damage is US$147 billion (EQE, 1995).

It is found that in the Kobe earthquake, heavy damage concentrated in a banded area along the coastline, with approximately 20 km in length and 1 km in width. Extensive soil liquefaction was found in these areas as well. In down-
town Kobe, liquefaction induced settlement less than 50 centimeters was observed. While along the coastline, the settlement increased to as much as 3 meters. In Port Island and Rokko Island, massive sand spreading and lateral movements of retaining walls were reported. The possible reasons for the concentration of damage include the existence of soft reclaimed soil, and the amplification effect of the topographic irregularity (Oba, 1995, Shinozaki, 1995).

The city of Kobe is located in a narrow plain area between Mount Rokko in the North, and Osaka Bay in the South. The southern side of Mount Rokko has steep-like slopes. The granite bedrock of Mount Rokko is exposed only 3 km from the coastline. While in Port Island, which is about 1 km from the coastline, bedrock is not found until the depth of 83 m. The urban area of Kobe consists of an alluvial fan, a coastal plain, and reclaimed lands. The soils in the alluvial fan are mainly gravel and sand. The reclaimed lands include areas at the waterfront, the Rokko Island and the Port Island. The soil material for the reclaimed land is decomposed granite soils.

Figure 2.8 shows the cross section of Kobe area.

It is noted from the damage investigation that soil liquefaction changes the ground motion. Figure 2.9 shows an extensively liquefied site in the Kobe port area. The ground was badly damaged by cracks and sand spreads. Lateral movement and settlement can be found in the photo as well, but the tanks built on this site suffered no damage. The tanks are believed to be supported by pile foundations (EQE, 1995). At another site where no liquefaction was found, tanks were found to have fallen off their support Figure 2.10). Also, at the Wangan Expressway, liquefaction caused the loss of bearing capacity of a foundation of bridge abutment,
while the seismic shaking did not cause any damage to the structural elements of the bridge (Figure 2.11).

Figure 2.12 is the photo of a shop in Meriken Wharf Chuuou-ku, Kobe. It shows that soil liquefaction caused heavy ground failure with the sidewalk being cracked and rugged. In contrast, the building suffered little damage (ERC, 1995).

Another case is two identical elevated sections of the bullet train railway in Kobe. About one kilometer length of the elevated structures collapsed extensively in an area where there was no sign of liquefaction. The adjoining section which was of almost equal length, suffered hardly any damage. There, soil liquefaction was readily observed.

2.1.6 Summary

Reports on earthquake damage show that the seismic ground motion is affected by local geology. Extensive damage has been found in mountain areas, river sites, and alluvial valleys.

It is also found that liquefaction occurs frequently at river sites, filled sites, or lagoons. The well-accepted reasons for the higher liquefaction potential in these areas are the existence of the recently deposited loose sand layers and the high ground water table. However, it is worthy to note that in all these sites, buried valleys or basins were found. The seismic waves would be reflected and scattered within the soil mass. Therefore, the seismic energy dissipation in these sites would be different from that in a regularly layered site. To study the effects of local geology on soil liquefaction, detailed site investigation is needed, and two or three
dimensional numerical analysis is required.

When liquefaction occurs, the seismic response of the site will be changed. The liquefied soil layer, because of its significant low shear modulus, will "absorb" the upward propagation of the shear wave. Therefore, the response of a site underlain by sandy soil layers will depend highly on the liquefaction potential of the sandy soil.

The effects of liquefaction and the effects of local site condition on ground motion have been noticed in the last decade. Related research works based on case studies, field tests, and numerical analyses have also been reported.

2.2 DUAL EFFECTS OF LIQUEFACTION

In all the above-mentioned earthquakes, structural damage caused by vibration was seldom reported on sites where liquefaction occurred. This phenomenon can be explained by the "dual effects of liquefaction", as mentioned by Wang and Law(1994):

"Liquefaction is a failure of ground soil which may cause ground settlement, slide and subsidence. However, as soon as it occurs, some new phenomena may be produced, which may reduce the capacity of ground motion. These phenomena are described in the following:

1. The liquefied layer acts as a liquid medium which can isolate seismic shear waves transmitted from the solid bedrock.

2. The seismic energy transmitted to the liquefied layer will be immediately
absorbed and exhausted by way of producing sand boils or landslides. The residual seismic energy is largely weakened and its power of further damage is reduced.

3. From any kind of dynamic soil tests in the laboratory (for example, dynamic triaxial test), it can be easily noted that before liquefaction occurs, the soil is softened and can no longer take on and transmit any significant stress. This reduction of stress level being transmitted by the liquefied soil mass helps to lower damage potential."

The record of ground motion taken from a liquefied site during the 1964 Niigata Earthquake (Japan National Committee on Earthquake Engineering, 1965) is a good example for demonstrating the effect of liquefaction to the ground motion (Figure 2.13. This time-acceleration history can be divided into three parts: The first part is from 0 to 7 second, short-period component can be found in the record. In the last part of the record (10-30 second), when the soil layer liquefied, the ground motion is dominated by low-frequency motion. Between these two parts (7 to 10 second), the record shows that the high-frequency component fades from the motion, indicating the development of liquefaction.

The ground acceleration time history obtained from the Kobe Earthquake also shows the effect of liquefaction on ground motion. Figure 2.15 is the acceleration record at Port Island Array Site in NS direction. Massive soil liquefaction was found in this site. The soil profile (Figure 2.14) shows that the liquefiable layer is the 19 m thick reclaimed soil.

It can be found that the peak acceleration at ground surface is lower than that in the depth of 16 m and 32 m. This de-amplification is believed to be the
result of soil liquefaction (Oka, 1995). It is also found that at the ground surface, high-frequency components have been filtered out in the horizontal motion, while in the record of vertical motion, the de-amplification and the change in frequency components are not found.

The above two examples show that the existence of liquefied soil layer changes the characteristics of the horizontal ground motion. This effect can be explained as follows: the high frequency motion has been filtered out by the liquefied layer, which has a very low shear modulus and relatively high damping ratio. On the other hand, the propagation of the pressure wave is determined by the bulk modulus of the soil, which is not reduced by liquefaction, therefore the high frequency components in the vertical motion, which results from the pressure wave motion, are not reduced by the liquefied soil layer.

This explains why the structures located on liquefied sites suffered less damage: At sites far from the epicenter, it is believed the damage to the structures is caused by the horizontal ground motion. At the liquefied sites, the horizontal ground motion in the frequency ranges from 2 Hz to 4 Hz was reduced by the occurrence of soil liquefaction, and the resonant frequencies of many types of buildings fall into this frequency range.

It should be noted that, although liquefaction will reduce the intensity of ground motion, it may also cause serious foundation failure, as mentioned by Huang et al. (1985), and shown in Figure 2.11. Therefore, proper design of the footings (for example, using pile foundation, as shown in Figure 2.10), will be necessary for sites with high liquefaction potential.
2.3 AMPLIFICATION BY ALLUVIAL VALLEYS

It has been observed from the above cases, and from other reports (Aki, 1988, Papageorgiou, 1991, Finn, 1991) that in sediment-filled valleys, the ground motion is amplified. This amplification is caused by the interface between bedrock and sediment. The interface may generate surface waves and trap body waves in the alluvium.

The combined effects of surface topography and sediment-filled valleys on site response are given in Table 2.1 (Finn, 1991). From this table, it can be found that the “shape ratio”, which is defined as the ratio of the width and the depth of the valley, is an important factor in determining the ground response.

2.3.1 Case Study

An interesting case study showing the amplification effect of alluvial valley is the investigation of the damage in Kirovakan during the 1988 Armenia Earthquake (Yegian et al., 1994).

Kirovakan is located in a narrow valley, surrounded by high mountains. Stiff clays or sandy alluvia were found overlying the bedrock, which is Basalt or Andesito-Basalt. The thickness of the stiff clay layer varies greatly. Figure 2.16 shows the map of Kirovakan. The city can be divided into five zones based on the soil profiles and damage statistics. Figure 2.17- 2.19 shows the cross sections of the five zones and the damage statistics.
Observations showed that about 98% of the collapsed buildings in Kirovakan were located within a very small region (only a few city blocks in zone 2). The soil profile of this region shows that the bedrock is in the shape of a conical bowl, and the bowl is filled with stiff clays with a maximum depth of about 150 m. In zone 1, where about 100 m of very dense gravely sandy alluvium was found, the damage is also significant. In zones 3, 4, and 5, where the alluvium is thinner than 30 m, buildings performed reasonably well, with no collapse and very little damage.

One-dimensional soil amplification analysis has been carried out for this region. The analysis shows that for zone 1 and zone 2, the spectral acceleration is in the periods ranging from 0.25 to 0.4 second, which is slightly higher than that for zone 3, 4, and 5. The authors concluded that these differences in the spectral accelerations are not large enough to explain the very significant disparity of damage statistics between these zones.

To explain the significant differences in damage between regions, the authors conducted an approximate analysis using a simple geometric 3-D solution developed by Sanchez-Sesma et al. (1988). At the middle of the valley where most of the damaged buildings were located, the Fourier amplification ratios for 3-D and 1-D analyses were compared. It is found that the 3-D amplification exceeds the 1-D amplification substantially over most of the period range of the buildings (0.25 to 0.4 second) in zone 2 of Kirovakan. The authors believed that this result can explain the enormous localized damage to buildings.

It is worthy to note that the clay layer in zone 2 has a high shear modulus. The shear wave velocity of the clay from 20 m to 150 m ranges from 450 to 910 m/s. This site is considered a good one if the amplification effect of the alluvial
valley is not taken into account.

Oba (1995) discussed the amplification effect of local geology in the 1995 Kobe Earthquake. Because ground motion records were not obtained at suitable points on the Rokko mountains and adjacent plains, records from the Ikoma mountains and Osaka plain are used in his discussion.

Point P in Figure 2.20 is located on the Osaka Plain, 1.85 km away from Point U, which is located adjacent to the mountain. Acceleration histories recorded at Points P and U are shown in Figure 2.21. The peak accelerations at Point U are 110 gal in the north-south direction (NS), 104 gal in the east-west direction (EW), and 55 gal in the vertical direction (UD). Peak accelerations at Point P are approximately 1.4 times larger than those at Point U in the horizontal direction and about 1.8 times larger in the vertical direction. It is believed that the ground motion was amplified in the plain near the mountains.

Shinozaki (1995) also discussed the concentration of damage in the banded area during the Kobe Earthquake. Irregularity of topography is believed to be the main reason for the amplification of ground motions. To support this conclusion, a test of vibration amplification on sites with topographic irregularities was reported.

The test was conducted by putting a vibrating foundation on a 320 m x 220 m cut-and-fill site, and measuring the ground motion at different points. Figure 2.22 shows ground motions recorded in the EW direction at points along the line BC. The ground motions are normalized by the amplitude of the forcing motion applied to the foundation at point B. The figure was plotted for various excitation frequencies. It can be observed that the ground acceleration at the site with thinner fill is larger than that with thicker fill, and the motion is greatly amplified in the fill
soils near the boundary between cut (hard) and fill (soft) materials.

From the above discussion, it could be concluded that the amplification effect of alluvial valley is an important factor that affects the seismic ground motion, and therefore the liquefaction potential of a site.

2.3.2 Numerical Analysis

For the estimation of ground motion amplification in an alluvial valley, as discussed earlier, two-dimensional or three-dimensional analysis is needed. In such an analysis, numerical methods must be employed because theoretical solutions can only be found for the simplest cases.

Bouden et al. (1990) studied the ground motion amplification by 2-D valleys. The numerical method employed in the study is a hybrid method which combines the finite element method and the boundary element method. The dynamic problem is solved in frequency domain. Cylindrical valleys filled with layered pure-elastic soil material are considered.

It is found from the analyses that the location of the maximum amplification depends on the type of the incident waves (P, SV, or SH) and its angle of incidence, as well as the component of the displacement field being observed. In the case of a layered semicircular valley with either vertically incident P wave or SH wave, the maximum amplification of horizontal displacement is found near the edges. With incident SV wave, the maximum amplification of horizontal displacements is at the center of the valley (Figure 2.23).

Sanchez-Sesma et al. (1988) studied the response of 2-D alluvial valleys for
incident SH waves by means of an approximate solution. Significant amplification of horizontal motion is found near the edges of the valley.

The amplification effects of a 3-D alluvial valley has been studied by means of boundary element method (Mossessian and Dravinski, 1990). The analysis is carried out in frequency domain. Unevenly layered soils with pure-elastic constitutive relationship is assumed. Incident P, SV, SH and surface waves are considered.

The analyses show that the amplification effects are highly dependent upon the shape of the basin and the type of incident wave under consideration. On comparison with the 2-D cases, it is found that strong coupling between P/SV and SH waves occurred in 3-D cases.

Luco et al.(1990) also studied the 3-D response of a cylindrical canyon in a layered half-space. Their analysis shows that the direction of incidence of the excitation is one of the controlling factors to both the amplitude and the duration of the motion within and near the canyon. Therefore, 2-D analysis may not be sufficient to give a complete description of the real response of canyons.

Different results have been reported by Papageorgiou (1991). When studying the Caracas Earthquake of July 29, 1967, analyses show that the highest amplification in the spectral acceleration is found at the edges, while the highest amplification in the spectral velocity occurs in the middle of the valley (Figure 2.24). The damage investigation also shows that the heavily damaged zone falls in the middle of the valley, where four high-rise buildings collapsed (indicated by arrows in Figure 2.24).

In general, the amplification effects of alluvial valleys have been found in both
the case studies and the numerical analyses. The location and the magnitude of the maximum amplification are dependent upon the dimension and shape of the valley, the type of the incident wave, and the component of motion being observed.

2.4 LIQUEFACTION MODELS

For the numerical analysis of soil liquefaction, a mathematical model is needed to describe the stress-strain-liquefaction behavior of the liquefiable soil layers. This model should be able to describe the procedure of pore-pressure build-up during the cyclic loading.

Most existing models are constructed by introducing a pore-water pressure build-up function into a non-linear elastic or elastoplastic constitutive model (Ramsamoog and Alwash, 1990, Matasovic and Vucetic, 1993). In these models, the pore-pressure build up functions are defined by a “stress approach”, in which the excess pore pressure is estimated by dynamic effective stress analysis. This estimation requires material constants for describing the relationship between the applied shear stress and the plastic volumetric strain. These constants are obtained from special tests that need considerable care to conduct. Therefore, the application of the models is limited.

The other way of computing the pore-pressure build-up is by the “energy approach”. It has been found that for saturated granular soils, a unique relationship exists between the accumulated energy dissipation and the pore-water pressure generated during the application of dynamic load (Nemat-Nasser and Shokook, 1979, Law et.al, 1990, Figueroa and Dahisaria, 1991). The dissipated energy in a loading cycle is defined as the area enclosed by the hysteresis loop (Figure 2.25).
The pore-pressure-energy dissipation relationship is found from cyclic shear tests.

2.5 NUMERICAL ANALYSIS

2.5.1 The Problem

The dynamic response of an alluvial valley can be represented by a non-homogeneous half space subjected to an incident wave field.

The source of the seismic wave is the epicenter of the earthquake. If the field is far from the epicenter, it can be assumed that the propagation of the seismic wave is vertical, because of the refraction of waves in the bedrock. Therefore, for far-field problems, it is assumed that the source is located in the bedrock, which is an elastic body with a high elastic modulus.

In many cases, soft soil layers exist in the alluvial valleys. This kind of soil has low elastic modulus, so the dynamic strain in the soil will be high when subjected to strong seismic motion. The strain level may range from 0.001 to 0.01. At this strain level, the soil will display a significant non-linear behavior in its stress-strain relationship.

2.5.2 Finite Element Method

The finite element method (FEM) has been widely used in the dynamic analyses of soils and structures since the early seventies. With the rapid development of computer technology, the most advanced FEM programs can solve many complex problems such as the seismic responses of foundation-dam-water system or long-

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span bridges.

The FLUSH program developed in University of California at Berkeley is one of the well-known finite element programs for dynamic analysis of soil-structure interaction. This program performs a pseudo 3-D analysis in frequency domain. The visco-elastic stress-strain relationship is used to model the foundation soil (Lysmer et al., 1975). FLUSH is considered a good finite element program in dynamic analysis for its ability to solve many kinds of engineering problems with acceptable accuracy and a relatively high efficiency. FLUSH can be used for the analysis of two-dimensional site response when the soil behaves in a non-linear manner.

The finite element method is one of the most powerful numerical tools in the analysis of site response or liquefaction problems. It has some unique advantages, such as the straightforward handling of soil inhomogeneities, the well-demonstrated ability for nonlinear analysis which makes it possible to model the liquefaction properties of sand, and the availability of a wide range of software for various applications.

The finite element method also has some inherent limitations in solving dynamic problems. The first limitation is that the FEM requires a spatial discretization of the solution domain which is considered as a continual medium in the seismic response analysis. In other words, FEM models the continuum with infinite degrees of freedom (DOF) by a finite element mesh with a finite DOF. In the analysis of seismic wave propagation, the spatial discretization will introduce a distortion in the solution domain. This distortion includes dispersion and oscillations. For a given finite element mesh, there exists a cutoff frequency, which is determined by the size of the elements and the shear wave velocity.
When the elastic wave propagates in the mesh, two types of error appear (Belytschko and Mullen, 1977):

1. The wave form will be distorted by the effect of dispersion. This distortion becomes higher when the frequency of the wave gets closer to the cutoff frequency.

2. Any input motion with the frequency higher than the cutoff frequency will result in the spurious oscillations near the source instead of the wave propagation in the mesh. Due to the effect of the cutoff frequency, the maximum size of elements is limited by the highest frequency used in the computation, and the wave velocity of the material.

The existence of a cutoff frequency results in a limitation in the size of elements. Lower shear modulus and higher frequency in the analysis require smaller elements. When liquefaction is considered, the shear modulus of liquefied soil is extremely low, therefore small elements need to be used, the spatial discretization may result in a huge amount of input data and high computational cost. For two dimensional site response or liquefaction analyses, the region under consideration may be as large as several hundreds meters long, up to fifty meters deep. The corresponding finite element mesh may have up to $10^5$ degrees of freedom (DOF).

The second limitation of FEM is that the finite element mesh always represents a finite region. When a semi-infinite region is considered in a wave propagation problem, artificial boundaries must be used in order to satisfy the radiation condition. The use of artificial boundaries will introduce an error to the finite element.
2.5.3 Boundary Element Method

In the last quarter of century, the appearance and evolution of the boundary element method (BEM) are among the most important advances in the development of seismic response analysis. Now BEM has been widely used in solving 2-D or 3-D site response problems because it presents some attractive advantages over FEM (Dineva et al., 1991, Cao et al., 1990, Luco et al., 1990, Mossessian and Dravin- ski, 1990a, b).

The first advantage is that BEM requires only a boundary discretization. As a result, the dimensions of the problem are reduced by one, that is, 2-D problem is reduced to 1-D, and a 3-D problem is reduced to 2-D. The total number of DOF of a BEM model is much less than that of a FEM mesh for the same problem, and a true 3-D analysis becomes possible.

The second advantage is the use of the fundamental solution, which is an accurate solution of the governing differential equation in the domain of interest. Therefore high accuracy can be obtained. Moreover, in BEM the radiation condition is automatically satisfied if the appropriate fundamental solution is used. This is a distinct advantage for seismic site response analysis which deals with a semi-infinite region.

The boundary element method also has its limitations. For BEM, it is difficult, and sometimes impossible, to deal with inhomogeneous or non-linear material properties, because of using superposition of the fundamental solution over the whole solution domain.
Steady State Wave Propagation Problem

The governing differential equation for an elastic wave propagation problem is:

\[ \nabla^2 u - \frac{1}{c^2} \ddot{u} = 0 \]  \hspace{1cm} (2.1)

where \( c \) is the pressure wave velocity or shear wave velocity, \( u = u(x_1, x_2, x_3) \) is the displacement vector, and \( x_1, x_2, \) and \( x_3 \) are the coordinates.

Various forms of fundamental solutions are available for solving Equation 2.1 in frequency domain (Manolis and Beskos, 1981, Mossessian and Dravinski, 1990a, b, Tanaka and Matsumoto, 1989).

When a linear viscoelastic medium is considered, Equation 2.1 becomes:

\[ \nabla^2 u - \zeta \ddot{u} - \frac{1}{c^2} \dddot{u} = 0 \]  \hspace{1cm} (2.2)

where \( \zeta \) is the viscous damping factor of the medium. A boundary integral solution for Equation 2.2 can be found in frequency domain by substituting the elastic modulus of the medium with the complex modulus (Kobayashi and Kawakami, 1985).

A frequency domain solution is easy to obtain. When the concept of complex modulus is adopted, the damping of the material can be considered. The frequency domain solution has been applied to many kinds of site response problems, including the analysis of the amplification effect of alluvial valleys (Mossessian and Dravinski, 1990a).
However, this method cannot be used when the material properties change with time. This will happen, for example, in the liquefaction analysis. The reason is that the frequency domain solution is the superposition of a series of harmonic responses of the elastic body for the whole time history. The use of superposition is only valid for linear problems, and, the use of harmonic responses for the whole time history makes the formulation not able to represent the real response of the system, because the material properties change with time.

In time domain analysis, the reaction of the site is computed for each small time step, based on the results obtained in the previous time step. Therefore, it is possible to carry out a non-linear analysis in the time domain as it has been done in the static analysis. For the analysis of the soil liquefaction problem, a time domain analysis is required.

**Transient Elastodynamics Problem**

The governing equation of an elastodynamics problem is given as:

\[ G u_{j,ii} + \frac{G}{1 - 2\nu} u_{i,ij} + b_j = \rho \ddot{u}_j \quad (2.3) \]

where, \( G \) is the shear modulus, \( \nu \) is Poisson's ratio, \( \rho \) is the mass density, and \( b_j \) is the body force. In a dynamic problem, Equation 2.3 is accompanied by initial conditions:

\[ u_i(x, 0) = u_i|_{t=0} \quad \dot{u}_i(x, 0) = \dot{u}_i|_{t=0} \quad (2.4) \]
and the boundary conditions:

\[ u_i(x, t) = U_i, \ x \in \Gamma_1, \quad p_i(x, t) = \sigma_{ij}n_j = P_i, \ x \in \Gamma_2 \] (2.5)

The fundamental solution for a 2-D problem can be given as (Manolis and Beskos, 1988):

\[
G_{ij}(x, t, x^{(A)}) = \frac{1}{2\pi \rho} \left\{ \left( \frac{2t^2 - \frac{r^2}{c_p^2}}{t^2 - \frac{r^2}{c_p^2}} \right)^{1/2} H \left( t - \frac{r}{c_p} \right) - \frac{2t^2 - \frac{r^2}{c_s^2}}{t^2 - \frac{r^2}{c_s^2}} \right)^{1/2} H \left( t - \frac{r}{c_s} \right) \right\} \frac{r_{ij}}{r^4} + \left( t^2 - \frac{r^2}{c_p^2} \right)^{1/2} H \left( t - \frac{r}{c_p} \right) - \left( t^2 - \frac{r^2}{c_s^2} \right)^{1/2} H \left( t - \frac{r}{c_s} \right) \right\} \frac{\delta_{ij}}{r^2} + (2.6)
\]

where \( H \) is the Heaviside step function, which gives:

\[
H(t - \tau) = \begin{cases} 
1 & t - \tau > 0 \\
0 & t - \tau < 0 
\end{cases} (2.7)
\]

\( \delta_{ij} \) is defined as:

\[
\delta_{ij} = \begin{cases} 
1 & i = j \\
0 & i \neq j 
\end{cases} (2.8)
\]

\( c_s \) and \( c_p \) are shear wave velocity and pressure wave velocity, respectively, \( r \) is the distance from the source point to the field point.

The above is the time domain solution to the pure elastodynamic problem. As mentioned in Section 2.5.1, the non-linear behavior of soils should be considered in the analysis of a site subjected to strong seismic shaking.
The boundary element solutions for simple non-linear cases such as viscoelastic problems have been given. Some of these solutions contain non-boundary integration, which will result in a loss of efficiency in the computation.

**Time Domain Solution for Viscoelastic Problem**

Tanaka and Matsumoto (1989) proposed a BEM approach to analyze transient elastodynamic problems with damping. In this approach, the acceleration and velocity terms are approximated by time discretization, then the fundamental solution for general Laplace equation is applied to obtain the boundary integral solution. The derivation of BEM formulation for 1-D problem has been presented as an example (Tanaka and Matsumoto, 1989).

1. Time discretization by finite difference scheme

   The governing differential equation for 1-D dynamic problem is:

   \[
   c^2 \frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\zeta}{\rho} \frac{\partial u(x,t)}{\partial t} - \frac{\partial^2 u(x,t)}{\partial t^2} = 0
   \]  

   (2.9)

   where, \( c \) is the pressure or shear wave velocity, and \( \zeta \) is the damping factor.

   Denoting by \( \Delta t \) the infinitesimal time interval, one can approximate the velocity and the acceleration at time \( t + \Delta t \) as:

   \[
   \frac{\partial u(x,t + \Delta t)}{\partial t} \approx \frac{1}{\Delta t} [u(x,t + \Delta t) - u(x,t)]
   \]  

   (2.10)

   \[
   \frac{\partial^2 u(x,t + \Delta t)}{\partial t^2} \approx \frac{1}{(\Delta t)^2} [u(x,t + \Delta t) - 2u(x,t) - u(x,t - \Delta t)]
   \]  

   (2.11)
Substitution of Equation 2.10 and Equation 2.11 into Equation 2.9 results in

$$\frac{\partial^2 u(x, t + \Delta t)}{\partial t^2} - s^2 u(x, t + \Delta t) = -f(x, t, t - \Delta t)$$  \hspace{1cm} (2.12)

where

$$s = \sqrt{\frac{\zeta \Delta t}{\rho} + 1}$$  \hspace{1cm} (2.13)

and $f(x, t, t - \Delta t)$ is a function calculated from the displacements at previous time steps. In BEM formulation, it can be considered as a body force term.

2. Boundary integral solution

When the right hand side of Equation 2.12 equals zero,

$$\frac{\partial^2 u(x, t + \Delta t)}{\partial t^2} - s^2 u(x, t + \Delta t) = 0$$  \hspace{1cm} (2.14)

the fundamental solution for Equation 2.14 is given by

$$u^* = \frac{1}{2s} e^{-s|x - x^{(A)}|}$$  \hspace{1cm} (2.15)

where $x^{(A)}$ is the source point.

Applying the above fundamental solution to Equation 2.12, and following the regular BEM approach, one obtains

$$u(x^{(A)}) = u^*(L, x^{(A)})q(L) - u^*(0, x^{(A)})q(0) - q^*(L, x^{(A)})u(L) + q^*(L, x^{(A)})u(L) + F(x^{(A)})$$  \hspace{1cm} (2.16)
where $L$ is the length of the domain, and $F$ is given by

$$F(x^{(A)}) = \int_0^L u^*(x, x^{(A)})f(x, t, t - \delta t)dx$$

(2.17)

Equation 2.16 is the boundary integral solution for Equation 2.9.

This approach can be extended to solve 2-D and 3-D problem if corresponding fundamental solutions are used. Because time discretization is carried out on the governing equation (Equation 2.9), the fundamental solution for Equation 2.2 is not required. This approach can be modified to solve other kinds of viscoelastic problems in which the damping of material is in some other form than the linear viscous damping. When this method is used, the differential equation after time discretization should be in the form $u''(x, t) - Ku(x, t) = f(x)$, where $K$ is a constant.

The problem with this method is that it requires a domain integral to deal with the non-homogeneous body force $f(x)$. The calculation of the domain integral results in the lost of efficiency and accuracy, and, this calculation cannot be carried out in infinite domain such as the semi-infinite soil layer.

**Dual Reciprocity Method**

Estimating domain integration in BEM formulation often implies an internal spatial discretization. This spatial discretization will considerably increase the amount of data needed to run the program, hence BEM loses some of its attraction in relation to FEM, especially when an infinite domain is involved. In order to avoid the domain integration, Nardini and Brebbia (1982, 1985) proposed a special boundary
integral approach for dynamic analysis.

For the 2-D steady state vibration problem, the differential equation for dynamic equilibrium is:

\[ \sigma_{ij,j} + \omega^2 \rho u_i = 0 \quad i, j = 1, 2 \]  \hspace{1cm} (2.18)

Following the direct BEM approach, one obtains the boundary integral equation:

\[ c_{ij}(x^{(A)})u_j(x^{(A)}) = \int_{\Gamma} U_{ij}^*(x^{(A)}, x^{(B)})p_j(x^{(B)}) \, d\Gamma - \]
\[ - \int_{\Gamma} P_{ij}^*(x^{(A)}, x^{(B)})u_j(x^{(B)}) \, d\Gamma + \]
\[ + \omega^2 \rho \int_{\Omega} u_j(x)U_{ij}^*(x^{(A)}, x) \, d\Omega \]  \hspace{1cm} (2.19)

where \( x \) is a point in domain \( \Omega \), \( c_{ij}(x^{(A)}) \) is the coefficient which depends on the position of the source point \( x^{(A)} \), \( u_j(x) \) and \( p_j(x) \) are the unknown displacement and traction at point \( x \) in domain \( \Omega \), \( U_{ij}^*(x^{(A)}, x) \) is the fundamental solution to the 2-D elastostatic problem, which represents the displacement at point \( x \) in direction \( j \), caused by a unit force acting at point \( x^{(A)} \) in \( i \) direction. \( P_{ij}^*(x^{(A)}, x) \) is the traction corresponding to \( U_{ij}(x^{(A)}, x) \), and \( x^{(B)} \) is a point at the boundary.

This equation has a domain integration which contains the unknown displacement \( u_j(x) \). In order to formulate the problem in terms of the boundary integrals only, the unknown displacement is approximated by the summation of a set of functions \( f^{(k)}(x) \) multiplied by constants \( \alpha_l^{(k)} \):

\[ u_i(x) = \sum_{k=1}^{N+L} \sum_{l=1}^{2} \{ \alpha_l^{(k)} f^{(k)}(x) \} \]  \hspace{1cm} (2.20)

where, \( N \) denotes the number of boundary nodes, \( L \) denotes the number of internal nodes.
Then the domain integration in Equation 2.19 becomes:

\[
\int_{\Omega} u_j(x) U^*_i(x^{(A)}, x) d\Omega = \sum_{k=1}^{N+L} \sum_{l=1}^{2} \left\{ \alpha l^{(k)} \left( \int_{\Gamma} f^{(k)}(x) U^*_i(x^{(A)}, x) d\Gamma \right) \right\}
\]  

(2.21)

Now considering the term

\[
\int_{\Omega} f^{(k)}(x) U^*_i(x^{(A)}, x) d\Omega
\]  

(2.22)

in the right hand side of Equation 2.21. This domain integration can be transformed into a boundary integration by finding a displacement field

\[
\hat{u}^{(l)}_j(x, x^{(k)})
\]

with the corresponding stress tensor \( \tau^{(k)}_{jm} \), such that:

\[
\tau^{(l)}_{jm,m}(x, x^{(k)}) = \delta_{jl} f^{(k)}(x)
\]  

(2.23)

where, \( \hat{u}^{(l)}_j(x, x^{(k)}) \) is the displacement at point \( x \), in \( j \) direction, caused by a unit force acting at point \( x^{(k)} \), in direction \( l \), and \( x^{(k)} \) is a point in the domain \( \Omega \).

By substituting Equation 2.23 into Equation 2.22, and integrating by parts, one obtains:

\[
\int_{\Omega} f^{(k)}(x) U^*_i(x^{(A)}, x) d\Omega
\]

\[
= -c_{ij} \hat{u}^{(l)}_j(x^{(A)}, x^{(k)}) + \int_{\Gamma} U^*_i(x^{(A)}, x^{(B)}) \hat{p}^{(l)}_j(x^{(B)}, x^{(k)}) d\Gamma -
\]

\[
- \int_{\Gamma} P^*_i(x^{(A)}, x^{(B)}) \hat{u}^{(l)}_j(x^{(B)}, x^{(k)}) d\Gamma
\]  

(2.24)

where \( \hat{p}^{(l)}_j(x^{(B)}, x^{(k)}) \) is the traction field corresponding to the displacement field \( \hat{u}^{(l)}_j(x^{(B)}, x^{(k)}) \).
By substituting Equation 2.24 into Equation 2.19, the boundary element formulation can be written as:

\[
\begin{align*}
    & c_{ij}(x^{(A)}) u_i(x^{(A)}) + \int_{\Gamma} P_{ij}^{(s)}(x^{(A)}, x^{(B)}) u_j(x^{(B)}) d\Gamma - \\
    & - \int_{\Gamma} U_{ij}^{(s)}(x^{(A)}, x^{(B)}) p_j(x^{(B)}) d\Gamma \\
    = & \omega^2 \rho \sum_{k=1}^{N+L} \sum_{j=1}^{2} \left\{ -c_{ij} \ddot{u}_j^{(l)}(x^{(A)}, x^{(k)}) + \int_{\Gamma} U_{ij}^{(s)}(x^{(A)}, x^{(B)}) \ddot{p}_j^{(l)}(x^{(B)}, x^{(k)}) d\Gamma - \\
    & - \int_{\Gamma} U_{ij}^{(s)}(x^{(A)}, x^{(B)}) \dot{u}_j^{(l)}(x^{(B)}, x^{(k)}) d\Gamma \right\} \alpha_i^{(k)} \quad (2.25)
\end{align*}
\]

The factors \( \alpha_i^{(j)} \) can be found from Equation 2.20.

Now Equation 2.25 can be solved by the regular boundary element approach to give the unknown boundary displacement or traction. Here the non-homogeneous body force term is estimated by boundary integrals. Therefore, the advantages of BEM will not be lost. This method has been called the "dual reciprocity method" and has been extended to solve more general cases in which non-homogeneous terms are involved.

The dual reciprocity method was first proposed to solve the steady state dynamic problem. It has been extended to solve elastic problems in which various kinds of body forces are involved. However, using this method to solve visco-elastic dynamic problems in time domain has not been reported.

2.5.4 Discussion

The finite element method has been used in one-dimensional (1-D) or two-dimensional (2-D) ground response analyses for more than twenty years. Various kinds of constitutive relationships can be introduced into the FEM formulation, and, it can deal with time-dependent problems. Therefore, FEM can be used in
representing the liquefiable soil. The limitation for FEM is the large number of elements required in dynamic analysis. This limitation increases the cost in input and computation.

The boundary element method (BEM) is more efficient than the finite element method in solving linear, homogeneous wave propagation problems. The limitation is that BEM cannot be used to solve the real non-linear problem such as liquefaction of soil. In solving liquefaction problems, a time domain analysis is needed, and the visco-elastic behavior of all the soil layers should be considered. The existing method of solving visco-elastodynamic problems in time domain requires a domain integral in the BEM formulation, which reduces the efficiency of the method.

Some kinds of domain integrals can be transferred to boundary integral by means of the dual reciprocity method. This method has been used in solving steady state elastodynamic problems.

BEM and FEM can be used in combination to solve problems with regions which have different properties, such as the soil-foundation interaction problem (Messafer and Coates, 1989). This hybrid method has the advantages of both BEM and FEM. Therefore, it is possible to use the hybrid method in solving the liquefaction problem.

2.6 SUMMARY

The investigation of earthquake damage shows that the ground motion is affected by liquefaction. This is called the dual effects of liquefaction. Also heavy damage or soil liquefaction occurs more frequently in the areas with uneven thickness of
soil layers, such as the alluvial valley. The reason for this phenomenon is that the seismic shaking is amplified by the valley. At present, however, the theoretical analysis of local site effects on liquefaction has not been reported.

On the other hand, one-dimensional liquefaction problem and the two-dimensional soil-structure interaction problem have been studied by means of the finite element method, and the amplification effect of alluvial valley has been investigated by using the boundary element method. All these methods could be used in the analysis of the dual effects of soil liquefaction and the effects of topography, if the following problems can be solved:

1. the ability of modeling 2-D and 3-D dynamic problems with a reasonable computational cost;

2. the ability of handling time-dependent problems, such as the pore-pressure build-up and liquefaction in sandy soil;

3. the ability of computing the dynamic response of non-linear material properties, such as the visco elastic behavior of soils under cyclic load.
<table>
<thead>
<tr>
<th>Structure</th>
<th>Conditions</th>
<th>Type</th>
<th>Size</th>
<th>Quantitative Predictability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Topography</td>
<td>Sensitive to shape ratio, Largest for ratio 0.2–0.6</td>
<td>Amplification at top of structure, amplification or de-amplification at base, rapid changes in amplitude phase along slopes</td>
<td>Ranges up to a factor of 30 but generally 2–10</td>
<td>Poor: Generally underpredicted Possible reasons are ridge interaction and 3-D effect</td>
</tr>
</tbody>
</table>

*sediment-filled Valleys*

| 1) Shallow and wide, shape ratio <0.25 | Effects most pronounced near edges. Largely vertically propagating shear wave from edges | Broad band amplification across valley because of whole valley modes | 1-D models may underpredict at higher frequency by about two near edges | Good: away from edges 1-D works well, extend amplification near edges |

| 2) Deep and narrow, Shape ratio ≥0.25 | Effects throughout valley width | Broad band amplification across valley because of whole valley modes | 1-D models may underpredict in a wide band-range by 2–4 away from edges Resonant frequency shifted from 1-D | Fair: gives detailed description of vertical and lateral changes in material properties |

| 3) General in shallow sediment thickness | Local changes in duration | Increased duration | Duration of significant motions can be doubled | Fair |

| 4) General | Generation of long period surface waves from body waves at small incidence angles | Increased amplification and duration because of trapped surface waves | Duration and amplification of significant motion may be increased over 1-D predictions | Good at periods exceeding 1 second |

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Figure 2.1: Map of Tsusen River District in Niigata City (Ohsaki, 1966)
Figure 2.2: Soil Profiles of Tsusen River District (Ohsaki, 1966)
Figure 2.3: Liquefaction in Liulin area, Tianjin (Huang et al., 1984)
Figure 2.4: Marina District Zonation (Rosidi and Wigginton, 1991)

Figure 2.5: Soil Profile in Marina District (O’Rourke et al., 1991)
Figure 2.6: Ground Motion in Kushiro-oki (Iai et al., 1995)
Figure 2.7: Ground Motion of an Aftershock (Iai et al., 1995)

Figure 2.8: Cross Section of Kobe Area
Figure 2.9: Tanks on Liquefied Site (EQE, 1995)

Figure 2.10: Tanks on Non-Liquefied Site (EQE, 1995)
Figure 2.11: Foundation Failure of a bridge (EQE, 1995)

Figure 2.12: Ground Failure in Meriken Wharf Chuou-ku, Kobe (ERC, 1995)
Figure 2.13: Ground Acceleration Record in Niigata Earthquake (Aki, 1988)
Figure 2.14: Soil Profile of the Port Island Array Site

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Soil type</th>
<th>Soil profile</th>
<th>SPT N-value</th>
<th>Vs &amp; Vp (km/sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Man-made till (Reclaimed land)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Silty clay</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Layers of gravelly sand and silt</td>
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<tr>
<td>30</td>
<td>Alternate layers of gravelly sand and silt (Diluvium)</td>
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<tr>
<td>40</td>
<td>Silty clay (Diluvium)</td>
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<td>70</td>
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<tr>
<td>80</td>
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</tbody>
</table>
Horizontal Acceleration at 0 m  Vertical Acceleration at 0 m

Horizontal Acceleration at -16 m Vertical Acceleration at -16 m

Horizontal Acceleration at -32 m Vertical Acceleration at -32 m

Horizontal Acceleration at -83 m Vertical Acceleration at -83 m

Figure 2.15: Ground Motion at the Port Island Array Site
Figure 2.16: Map of Kirovaken (Yegian et al., 1994)
Figure 2.17: Soil Profile of Zone 1 in Kirovakan (Yegian et al., 1994)
Figure 2.18: Soil Profile of Zone 2 and 3 (Yegian et al., 1994)
Figure 2.19: Soil Profile of Zone 4 and 5 (Yegian et al., 1994)

Figure 2.20: Map of Osaka Area (Oba, 1995)
Figure 2.21: Acceleration Time History at Point P and Point U (Oba, 1995)
Figure 2.22: Amplification Effect of Topographic Irregularities (Shinozaki, 1995)
Figure 2.23: Reaction of an Alluvial Valley (Bouden et al., 1990)
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Figure 2.25: The Energy Dissipation (Law et al., 1990)
Chapter 3

DEVELOPMENT OF NUMERICAL MODELS

From the discussion in Chapter 2, it is concluded that

1. Soil liquefaction will affect the ground motion;

2. Local geology will affect the ground motion and the liquefaction potential.

In order to estimate the effects of liquefaction on ground motion, a numerical model for soil liquefaction is needed. This model should be able to represent the change of the constitutive relationship of soil during the development of pore-water pressure. Besides the liquefaction model, a formulation to compute the dynamic response of the site is also needed. This formulation should be able to give the non-linear site response in time domain, as discussed in Section 2.5.3.

The existing elastoplastic liquefaction models are suitable for the response analysis, but usually these models require special tests to obtain the model parameters, and the implementation is difficult. The energy approach for estimating soil liquefaction can be introduced into the numerical analysis, with the advantages of
simplicity and efficiency.

In many cases, 1-D analysis gives an acceptable solution for the site response analysis. Numerical methods for 1-D liquefaction analysis have already been developed for the site response analysis, by introducing a liquefaction model into the 1-D finite element analysis.

The dynamic finite element analysis for liquefaction analysis is time consuming because the limitation in the size of elements (Section 2.5.2). By considering the simplicity of the 1-D problem, a more efficient method can be developed by using 1-D BEM analysis.

In order to estimate the effects of local geology, numerical methods for 2-D or 3-D response analysis are required. To be useful in cases of strong seismic shaking, the method should be able to deal with the viscoelastic properties of the soil materials. To study soil liquefaction occurring at a site, a time domain analysis is required because the soil properties change during the seismic shaking.

As mentioned in Section 2.6, in 2-D or 3-D analyses, a finite element method can be used to compute the dynamic response of the liquefiable soils with the help of an appropriate liquefaction model, while the boundary element method is suitable for the analysis of non-liquefiable regions if the solution for viscoelastic problem in time domain is found.

In the following discussion, numerical tools, including a new liquefaction model based on energy approach, and a new boundary element formulation for viscoelastic analysis in time domain, will be developed.
3.1 STATEMENT OF THE PROBLEM

The problem under consideration is a site subjected to seismic shaking. The site will have

1. one or more liquefiable layer(s);

2. one or more non-liquefiable layer(s);

3. bedrock, where the input seismic motion is applied, and

4. an alluvial valley for studying local geological features under the 2-D and/or 3-D conditions.

The physical model of this problem is a layered half space, as shown in Figure 3.1.

There are three types of materials involved in the problem. The first type of material is the liquefiable soil, which is saturated sandy soil with a non-linear stress-strain relationship. The action of seismic loading will cause the pore water pressure build-up in liquefiable material.

In the following discussion, it is assumed that the liquefiable soil elements are under the undrained condition. The permeability of liquefiable soils is about $10^{-4}$ cm/sec. Normally the drainage for this kind of soil should be considered in effective stress analyses. For site response and liquefaction potential analysis, however, only the time period of strong seismic shake, which lasts for 15–30 second, is of interest. The difference in excess pore water pressure between the adjacent soil elements is
not high (Section 4.2.2). Therefore, in this special case, the change in pore pressure caused by drainage is ignored.

The second type of material is the non-liquefiable soil. When a strong earthquake is considered, the shear strain in soil layers will be as high as $10^{-4}$ to $10^{-2}$. Under such a strain level, the damping of the soil is significant. Therefore, the non-liquefiable soil is considered as a viscoelastic material when subjected to the dynamic loading.

The third type of material is the bedrock. It has a much higher shear modulus than soils. When subjected to strong earthquake motion, it behaves elastically.

In the liquefaction analysis, the whole domain can be divided into two regions: the liquefiable region for the liquefiable soil layer, and the non-liquefiable region for the remaining soil layers.

In the liquefiable region, the constitutive relationship of a soil element is described by the liquefaction model, and the model parameters are dependent on the development of pore-pressure, which in turn is related to the energy dissipation in a soil element. Liquefaction is defined as the state when the excess pore-pressure in a soil element reaches the confining pressure.

In the non-liquefiable subregion, the soil is assumed to be a layered half-space with viscoelastic stress-strain relationship, while the bedrock is assumed to be a pure elastic material, for which the damping ratio is zero. The input motion is applied at the top of the bedrock.

At the interface of the two subregions, the compatibility of deformation and the equilibrium of traction are satisfied.
The computation is carried out in time domain, in order to study the development of liquefaction and the change in ground motion during the seismic shaking.

3.2 THE NEW LIQUEFACTION MODEL

The proposed liquefaction model is based on the energy approach. The model is established by assuming that:

1. The stress-strain relationship of the soil is non-linear;

2. The build-up of pore-water pressure is an increasing function of the cumulative dissipated energy, which is defined as the sum of the enclosed area of the hysteresis loop for every loading cycle; and

3. The softening of soil depends on the pore-water pressure build-up.

For the first assumption, the non-linear elastic constitutive relationship is commonly used in the dynamic analysis of site response (Lysmer et al., 1975). Within the strain level of interest ($10^{-4} - 10^{-2}$), this assumption is acceptable (Hu, 1988, Chen, 1990). The second assumption, as discussed in Section 2.4 is based on the test results (Nemat-Nasser and Shokook, 1979, Law et al., 1990, Figueroa and Dahisaria, 1991).

The third assumption states that the only factor controlling the constitutive relationship during the loading procedure is the effective confining pressure, i.e., the stress-strain relationship and the strength of a sandy soil are determined mainly by the confining pressure. For a saturated sandy soil subjected to strong seismic loading, the change in effective confining pressure is significant. The effects of
other factors, such as the volume change or the loading rate, are not significant in this case (Huang, 1983).

3.2.1 Constitutive Relationship

The constitutive relationship used in the liquefaction model should satisfy the following conditions:

1. It can express the change of the stress-strain behavior due to the pore-water pressure build-up.

2. It can yield a reasonable value of the energy dissipation (or damping ratio), especially at the large strain level where large amounts of energy dissipation takes place. The large strain level is reached when the effective stress is considerably reduced by the pore pressure build-up.

Two kinds of constitutive equations will be discussed in the following.

Non-Linear Elastic Equation

For a soil element subjected to cyclic loading, the stress-strain curve can be divided into two parts: a skeleton curve (initial loading curve) and an unloading-reloading curve (Figure 3.2). The initial loading can be in the positive direction (increment of shear stress \( \Delta \tau > 0 \)), or in the negative direction (\( \Delta \tau < 0 \)).

The results of cyclic shear tests have demonstrated that the unloading-reloading curve can be determined by the Masing's rule, which states that the unloading curve is obtained from the negative part of the skeleton curve enlarged by a factor
of two, and the reloading curve is obtained in the same way from the positive part of the skeleton curve (Natasovic and Vucetic, 1993, Ishihara, 1986).

SKELETON CURVE: The skeleton curve controls the shape of the unloading-reloading curve, which in turn determines the energy dissipation within a load cycle. This curve is directly defined by the constitutive equation.

In order to satisfy the above two requirements for the constitutive equations, and to maintain a certain degree of simplicity, the constitutive equation is chosen as (Huang, 1983):

\[
\tau = \frac{|\gamma|^s}{\frac{1}{G_i} + |\gamma|^s/\tau_{ult}} \tag{3.1}
\]

where: \(\tau\) = shear stress in Pa, \(\gamma\) = shear strain, \(G_i\) = initial tangent shear modulus in kPa, \(\tau_{ult}\) = ultimate shear stress in Pa, and \(0 < s < 1\) is a model parameter.

This constitutive equation is called the modified Duncan-Chang model. The original Duncan-Chang model has been widely used in the computation of deformation and stability of sandy soils for many years, but it gives unacceptably high hysteresis damping when the strain level is high. To describe the damping behavior correctly, an index \(t\) is used in Equation 3.1. The value of \(G_i\), \(\tau_{ult}\), and \(t\) can be determined from triaxial shear tests.

When this equation is used in 2-D and 3-D numerical analyses, the shear stress \(\tau\) should be the maximum shear stress, \(\tau = \tau_{max} = \frac{\sigma_1 - \sigma_3}{2}\), because the parameters in the constitutive equation are obtained from the triaxial test in which the principal stresses \(\sigma_1\) and \(\sigma_3\) are controlled.
Test data (Matasovic and Vucetic, 1993) show that for sand subjected to undrained cyclic shear, the reduction of the effective stress, as a result of the pore-water pressure build-up($\Delta u$), will cause a reduction in the ordinate of the hysteresis loop, and the flattening of the unloading-reloading curve. This reduction and flattening can be expressed through the "degradation" of the skeleton curve (Matasovic and Vucetic, 1993). The degradation is described by a decrease in the model parameters $G_i$ and $\tau_{ult}$. The ordinate of the skeleton curve will be reduced with a decrease in $\tau_{ult}$, while the curve will be flattened with a decrease in $G_i$.

In the following discussion, $G_i$ and $\tau_{ult}$ are used as the model parameters for the initial loading, $G_s$ and $\tau_s$ are used for the subsequent loading cycles. By studying the published data (Matasovic and Vucetic, 1993, Figueroa and Dahisaria, 1991), the following equations are given for the calculation of $G_s$ and $\tau_s$:

$$G_s = G_i \times \sqrt{p_o}$$
$$\tau_s = \tau_{ult} \times p_o$$

(3.2)

where $p_o = \frac{\sigma_e - \Delta u}{\sigma_c} = 1 - \frac{\Delta u}{\sigma_c}$ is the normalized effective stress, $\Delta u$ is the excess pore water pressure, and $\sigma_c'$ is the effective confining pressure.

It can be seen that $G_s$ and $\tau_s$ are increasing functions of $p$. For $p = 1$, $G_s = G_i$ and $\tau_s = \tau_{ult}$; for $p = 0$, which represents liquefaction occurrence, $G_s = 0$, $\tau_s = 0$. It should be noticed that when $p = 0$, the shear modulus and the strength of the soil may not equal to zero because of the existence of residual strength. This residual strength could be defined in computation by assigning a lower limit to the shear modulus and the ultimate shear stress.

By substituting Equation 3.2 into Equation 3.1, the skeleton curve can be
redefined as:

\[ \tau = \frac{|\gamma|^s}{\frac{1}{G_s} + \frac{|\gamma|^s}{\tau_s}} \]  \hspace{1cm} (3.3)

UNLOADING-RELOADING CURVE: By assuming that the hysteresis loop is symmetric with respect to the origin, and applying the Masing's rule to Equation 3.3, the unloading-reloading curve can be expressed as:

For unloading:

\[ (\tau - \tau_a) = \frac{2(\gamma - \gamma_a)}{\frac{1}{G_s} + \frac{(\gamma - \gamma_a)^s}{\tau_s}} \]  \hspace{1cm} (3.4)

For reloading:

\[ (\tau - \tau_a) = \frac{2(\gamma + \gamma_a)}{\frac{1}{G_s} + \frac{(\gamma + \gamma_a)^s}{\tau_s}} \]  \hspace{1cm} (3.5)

where: \( \tau_a \) and \( \gamma_a \) denote the amplitudes of shear stress and shear strain, respectively.

TANGENT SHEAR MODULUS: The tangent shear modulus is obtained by differentiating Equations 3.3, 3.4, and 3.5:

for the skeleton curve:

\[ G_t = \frac{s \gamma^{s-1}}{G_i \left[ \frac{1}{G_i} + \frac{\gamma^s \gamma^2}{\tau_{ult}} \right]} \]  \hspace{1cm} (3.6)
for the unloading curve:

$$G_t = \frac{s(\gamma - \gamma_a)^{s-1}}{G_s \left[ \frac{1}{G_s} + \frac{(\gamma - \gamma_a)^s}{\tau_{ult}} \right]^2}$$  \hspace{1cm} (3.7)$$

for the reloading curve:

$$G_t = \frac{s(\gamma + \gamma_a)^{s-1}}{G_s \left[ \frac{1}{G_s} + \frac{(\gamma + \gamma_a)^s}{\tau_{ult}} \right]^2}$$  \hspace{1cm} (3.8)$$

**Viscoelastic Equation**

In dynamic analysis, the viscoelastic model is widely used to describe the behavior of soil subjected to seismic load. Therefore the linear viscoelastic model has been tested in the new liquefaction model.

The constitutive equation is:

$$\tau_i = G\gamma_i + \mu\dot{\gamma}_i$$  \hspace{1cm} (3.9)$$

where: $\tau =$ shear stress in $Pa$, $\gamma =$ shear strain, $G =$ initial tangent shear modulus in $Pa$, and $\mu$ is the damping coefficient in $Pa\cdot s$. The Cartesian coordinate $i = 1, 2, 3$ is used.

In practice, the damping ratio $\eta = \mu/\mu_{cr}$ is often used to describe the damping behavior of materials, where $\mu_{cr}$ is the critical damping coefficient.

In the liquefaction model, the shear modulus $G$ is assumed to be a function of effective confining pressure, which equals the initial effective confining pressure.
\( \sigma' \) minus the pore-pressure increase \( \Delta u \). The damping ratio \( \eta \) is a function of the shear strain. The shear modulus can be determined by Equation 3.2.

The initial shear modulus \( G_i \) is obtained from the in-situ shear wave velocity test, or from the resonant column test. The value of \( G_i \) for soil ranges from \( 10^7 \, Pa \) to \( 10^8 \, Pa \). The initial viscous damping ratio and subsequent viscous damping ratio, \( \eta_o \) and \( \eta \), are obtained from the resonant column test and the cyclic triaxial test. For sandy soil, \( \eta_o \) ranges from 0.01 to 0.03, \( \eta \) can be as high as 0.4 under high strain level. Further discussion of soil parameters can be found in Chapter 4.

Using a viscoelastic equation in the liquefaction model brings up the following advantages:

1. Unlike other non-linear elastic relationship, the viscoelastic relationship does not require iteration within each time step. The iteration process may cause unstable solution under certain conditions, and it is time consuming.

2. When a viscoelastic relationship is used, the hysteresis loop is given by the viscous damping, the use of Masing's rule and the judgment of loading-unloading-reloading is not necessary.

3. The same constitutive relationship is also used for the non-liquefiable soils. The use of identical constitutive relationships will reduce the error introduced by the change of constitutive equations in the interface of liquefiable and non-liquefiable soil.

There are, however, some problems in using a viscoelastic relationship to represent the dynamic response of soil. First, Equation 3.9 gives an elliptical hysteresis
loop. Under low strain level, the real hysteresis loop is very close to an ellipse. When the soil element approaches the state of liquefaction, because of the high strain level (over $10^{-2}$) and low shear modulus, the soil displays a strong non-linear behavior, and the hysteresis loop is far away from that of an ellipse (Figure 3.3). Therefore the viscoelastic relationship is not able to display the exact hysteresis loop of soils at that stage. Second, the damping of soil is not an ideal viscous damping. It is found that in the frequency range of interest (0.5–5 Hz), the area of the loading–unloading loop, which indicates the damping force, is independent of the frequency of the cyclic strain. In the viscoelastic equation, however, the damping force is a linear function of the frequency (Huang, 1983).

Failure to reproduce the exact shape of the loading–unloading loop may cause an error in the stress–strain curve, and an error in the dissipated energy under high strain level. Failure to show the frequency-independent damping properties results in an underestimate of damping force when the frequency is low.

The error in the stress–strain curve will not significantly alter the reaction of the element, because the average modulus in a loading cycle is under control. However, the error in damping force will affect the cumulative dissipated energy, which in turn determines the shear modulus. For example, the cumulative dissipated energy computed for a uniform cyclic loading with the frequency of 1 Hz will be 30% higher than that with a frequency of 2 Hz. This problem is solved in the procedure of determining the model parameter $\alpha$ and $\beta$. The procedure will be discussed in Section 3.2.4.
3.2.2 Pore Pressure Build-up

The magnitude of the pore pressure is determined by the cumulative dissipated energy, which is defined as the sum of dissipated energy during the loading. The dissipated energy in each stress cycle is defined as the area enclosed by the hysteresis loop (Figure 3.3):

$$\Delta W = \oint_C \tau \, d\gamma$$  \hspace{1cm} (3.10)

where $C$ is the unloading-reloading circle. In the time domain analysis, the integral in Equation 3.10 can be carried out by summing the strain energy in each time step.

Law et al (1990) suggested that the pore-water pressure can be calculated from the cumulative dissipated energy by

$$\frac{u}{\sigma'_{h}} = \alpha W_N^\beta$$  \hspace{1cm} (3.11)

where $\alpha$ and $\beta$ are constants obtained from laboratory tests, $W_N$ is the normalized cumulative dissipated energy given by

$$W_N = \frac{F_1(K_c)F_2(D_\tau)\Sigma(\Delta W)}{\sigma_N}$$  \hspace{1cm} (3.12)

where $\sigma'_{h}$ is the effective horizontal confining pressure, $F_1$ and $F_2$ are functions to account for the consolidation stress ratio $K_c$ and the relative density $D_\tau$, respectively.
3.2.3 Implementation

Calculating the Energy Dissipation

Previous discussion in this section indicates that when a soil element is subjected to cyclic loading, the build-up of pore-water pressure in the element is a function of the cumulative dissipated energy. The energy dissipation in each loading cycle is calculated from the hysteresis loop, which is determined by the constitutive equation of the soil. The parameters in the constitutive equation depend on the effective confining pressure, which in turns are controlled by the pore-water pressure build-up. An error in any link of this chain will affect the subsequent results, and the error may be accumulated in every time step. Therefore the correct calculation and update of the parameters are important.

The dissipated energy is defined by the area enclosed in the unloading-reloading loops. The dissipated energy could be calculated at the end of each complete loading cycle. By calculating the energy dissipation this way, the cumulative dissipated energy will be a step function of time, which "jumps up" at the end of each loading cycle, and remains constant elsewhere within the cycle. According to Equation 3.11, this sharp jump in cumulative dissipated energy will cause a sharp jump in pore-water pressure as well.

The jump in pore-water pressure results in a sudden change in stress-strain curve. This sudden change will cause an artificial disturbance in time domain dynamic analysis. To reduce this disturbance, one can

1. replace a big jump in pore pressure with a series of smaller ones, and,
2. smooth the sharp jump.

In order to reduce the amplitude of the pore pressure jump, the stress-strain loop is examined (Figure 3.4). The energy dissipation in one unloading-reloading loop is defined as the sum of the work done by the cyclic loading. It can be found that the minimum unit for calculating the work done by the cycle loading is one-quater of a complete loading-reloading cycle. For the case of reloading, the shear stress increases from zero to the maximum in a cycle, the energy dissipation during the loading is defined as the area enclosed by the reloading curve AB, the line OB, and the strain axis (area 1 in Figure 3.4); for unloading, the energy dissipation is defined as the area enclosed by the line OB, the unloading curve BC, and the strain axis (area 2). For the negative part of the cycle, the dissipated energy is defined in the same way(area 3 and area 4). The energy dissipation can be calculated four times in a loading-unloading cycle, i.e., at the points of maximum and minimum stress in a cycle, and at the points of zero stress. The dissipated energy in a complete loading-unloading cycle is (area 1 + area 2 + area 3 + area 4), which equals the enclosed area of the stress-strain loop.

Calculating the energy dissipation for every 1/4 cycle gives the smallest jump in the cumulative dissipated energy curve, and therefore in the pore pressure curve. It also gives higher accuracy in the calculation of the parameters for the constitutive equation, because of more frequent update of the parameters (four times in a loading cycle instead of once a cycle). However, it is difficult to apply this model to the case of irregular loading, like seismic loading, where there are spikes or small loading cycles that do not reach the strain axis which gives a zero shear stress (Figure 3.5.)
To solve this problem, the energy dissipation can be calculated in each half loading cycle. The “half cycle” in the computation starts from the point of zero stress, and ends at the next zero stress point. It may include some small cycles that do not pass the strain axis.

In time domain analysis, the whole time history is divided into many small time steps. In each of the time step $\Delta t$, the displacement and force are computed. At the same time, the work done by the dynamic stress in this time step can be found by

$$\Delta w_i = \tau \Delta \gamma$$  \hspace{1cm} (3.13)$$

where, $\Delta w_i$ is the work done by the cyclic loading in $i^{th}$ time step, $\tau$ is the shear stress, $\Delta \gamma$ is the shear strain increment, $\Delta \gamma = \gamma_i - \gamma_{i-1}$.

Equation 3.13 gives a positive work for the reloading curve(increased $|\tau|$), and a negative work for the unloading curve(decreased $|\tau|$). The dissipated energy in a half-cycle is calculated by summing the work $\Delta w_i$ in the half-cycle.

In order to smoothen the sharp jump in the pore-water pressure, the total change in pore pressure at the end of a half cycle can be evenly distributed in $n$ subsequent time steps. $n$ is determined by the length of time step $\Delta t$.

**Initial Loading, Reloading and Unloading**

When using non-linear elastic equations, some criteria are needed to determine the loading state (initial loading, unloading, or reloading) of a soil element.
First, a variable $|\tau|_{max}$ is defined to record the maximum shear stress in the loading history. Then, the stress-strain equation is determined by the following criteria:

1. if $|\tau| \geq |\tau|_{max}$, the stress-strain relationship follows the skeleton curve (Equation 3.3);
2. if $|\tau| < |\tau|_{max}$, and the stress increment $\Delta \tau < 0$, the stress-strain relationship is determined by the unloading curve (Equation 3.4); and
3. if $|\tau| < |\tau|_{max}$, and $\Delta \tau > 0$, the stress-strain relationship is determined by the reloading curve (Equation 3.5).

When the viscoelastic model is used, the judgment of loading/unloading/reloading is not necessary because the shear modulus and the visco coefficient are independent of loading state.

### 3.2.4 Determination of Model Parameters

The parameters in the constitutive equation can be found from undrained cyclic shear test. For non-linear elastic model, the initial shear modulus $G_i$, the ultimate shear stress $\tau_{ult}$, and the parameter $s$ can be obtained by applying a curve fitting scheme to the stress-strain curve of initial loading, while the decrease of the parameters can be defined by the subsequent loading cycles. For a viscoelastic relationship, the initial shear modulus $G_i$ and the initial damping ratio $\eta_i$ can be obtained from the loading-unloading-reloading loop under small strain, and the decreasing function $G_s = f(p')$ and $\eta_s = f(\gamma)$ can be determined from the subsequent loading cycles.
The parameters $\alpha$ and $\beta$ in the pore-pressure build-up equation (Equation 3.11) can be evaluated from the liquefaction test. The test will give a $\tau_d - N$ curve, where $\tau_d$ is defined as the amplitude of the cyclic shear stress, and $N$ is defined as the number of stress cycles needed for pore-water pressure to reach the confining pressure.

From the calculation, another $\tau_d - N$ curve can be obtained. Changing the value of $\alpha$ and $\beta$ in the calculation will change the shape of the calculated $\tau_d - N$ curve. By a trial and error procedure, a pair of parameters $\alpha$ and $\beta$ can be found to give a calculated curve which matches the curve from the test.

When the viscoelastic constitutive equation is adopted, the energy dissipation will be affected by the period of the cyclic loading, as discussed before. In order to obtain the correct pore pressure build-up, the period of the loading used in the calculation of $\alpha$ and $\beta$ should equal the period of the real loading used in the liquefaction analysis. If the loading is an irregular one, such as the earthquake record, the frequency in the calculation of model parameters should be close to the average period of the motion which causes the liquefaction.

In next section, an example of how the parameters $\alpha$ and $\beta$ are obtained will be presented.

3.2.5 Testing the Model

The pore-water pressure build-up and the change of modulus of a soil element can be computed with the model by the following procedure:
1. Calculate the shear stress subjected to prescribed boundary and loading conditions using the shear modulus given by Equation 3.6.

2. Calculate the energy dissipation for the first half cycle.

3. Determine the excess pore-pressure in the element by using Equation 3.10 and Equation 3.11, assuming an undrained condition.

4. Calculate the parameters in constitutive equations based on the current pore-pressure. These values are used in the calculation of the shear modulus and the energy dissipation for the next half cycle.

5. Repeat step (1) to (4) for subsequent cycles. The soil element is considered liquefied when \( p = 1 - \frac{u}{\sigma_c} = 0 \), where \( \sigma_c \) is the confining pressure.

By following these steps, the model is checked against published data.

Figure 3.6 shows the computed pore water pressure and the result of a direct cyclic simple shear test published by Matasovic and Vucetic (1993). The soil used is Santa-Monica Beach sand, with the void ratio \( e = 0.56 \). The confining pressure is 196 kPa.

The non-linear elastic constitutive equation is used to represent the skeleton curve and the unloading-reloading curve. The parameters \( G_i = 5.7 \times 10^7 N/m^2 \), \( \tau_{ult} = 340 kPa \), and \( s = 0.5 \) are obtained by curve fitting from the measured shear stress and shear strain in the first loading cycle.

When the parameters in the constitutive equation are obtained, the parameters \( \alpha \) and \( \beta \) can be obtained from the measured pore-pressure, by applying the trial and error procedure described in the following:
1. For stress controlled test, the cyclic loading used in the computation should be defined. In this example, the loading is a sinusoidal function of time, with the same amplitude as used in the test.

2. Assume a pair of values of $\alpha$ and $\beta$, for example, $\alpha = 1.0$, $\beta = 0.5$.

3. Divide the whole time history into small time intervals, and starting the calculation from time $t_0 = 0$.

4. Calculate the shear stress for next time step $t_i = t_{i-1} + \Delta t$, where $\Delta t$ is a small time interval. The stress $\tau_i$ at time $t_i$ is:

$$\tau_i = \sin\left(2\pi \frac{t_i}{T}\right)$$

where $T$ is the period of the cyclic loading.

5. From the constitutive equation (Equations 3.3 to 3.5), calculate the strain at time $t_i$, $\gamma_i$.

6. Calculate the work done by the cyclic stress at a small time interval $\Delta t$ by

$$\Delta w_i = \frac{1}{2}(\tau_{i-1} + \tau_i)(\gamma_i - \gamma_{i-1})$$

7. At the end of the stress cycle (when $t_i = nT$, $n$ is an integer), calculate the cumulative energy dissipation by

$$\Delta W_k = \sum_{j=1}^{i} \Delta w_j$$

where $k$ is the number of cycles. The excess pore pressure at the end of the $k^{th}$ loading cycle can be calculated from $\Delta W_k$, by means of Equation 3.11.
8. Calculate the effective confining pressure by subtracting the excess pore pressure from the initial confining pressure. The parameters in the constitution equation are modified according to the calculated effective confining pressure.

9. Repeat the calculation from step 4 to step 8.

10. Plot the pore pressure – number of cycles \((\Delta u - N)\) curve. Try another pair of \(\alpha\) and \(\beta\) values if necessary, until the calculated \(\Delta u - N\) curve matches with the one obtained from the test.

In this example, \(\alpha = 0.72\) and \(\beta = 0.25\) are obtained from the trial and error procedure.

These parameters are used to predict the energy dissipation and pore-pressure build-up for the subsequent loading cycles. The figure shows that the calculated pore-water pressure agrees well with the test results.

Figure 3.7 shows the computed pore pressure comparing with the test results of a strain controlled torsional shear test (Figueroa and Dahisaria, 1991). The sand used in the test is Ottawa sand, with the relative density equals to 0.7. The confining pressure is 205kPa. The model parameters used for this test, obtained by the same way as for the above mentioned stress-controlled test, are \(G_i = 7 \times 10^7\), \(\tau_{ult} = 0.3\), \(s = 0.5\), \(\alpha = 9.5\), and \(\beta = 0.5\).

Because a large portion of the energy dissipation in a strain-controlled test occurred in the first few cycles, the pore-pressure build-up in the beginning is faster than that in the stress-controlled test. Apart from this, Figure 3.7 illustrated that the model also gives a good estimate of the measured pore-pressure. This result shows that the new model is able to deal with the non-uniform cycle loading.
3.3 THE NEW BOUNDARY ELEMENT FORMULATION

The proposed numerical method for the analysis of visco-elastodynamic problems in time domain is a boundary integral formulation, with the non-homogeneous terms solved by dual reciprocity method. In the development of the formulation, it is assumed that

1. the problem domain described in Section 3.1 is divided into subregions;
2. the material within a subregion is homogeneous and isotropic;
3. the constitutive relationship of the material is viscoelastic, with the damping given by

\[ D = \zeta \ddot{u} \]  \hspace{1cm} (3.14)

where, \( D \) is the damping stress in \( N/m^2 \), \( \zeta \) is the damping factor in \( Pa \cdot s/m^2 \), and \( \ddot{u} \) is the velocity in \( m/s \).

3.3.1 The Governing Equation

The governing differential equation for the general 3-D visco elastodynamic problem is:

\[ \frac{G}{1-2\nu} u_{i,ij} + G u_{j,ii} = \zeta \ddot{u}_j + \rho \dddot{u}_j \hspace{1cm} i, j = 1, 2, 3 \]  \hspace{1cm} (3.15)

where:
\( G, \nu: \) Shear modulus and Poisson’s ratio

\( \zeta: \) Viscous Damping factor,

\( \rho: \) Density of material,

\( u_j: \) Displacement in \( j \) direction,

\( \dot{u}_j = \frac{\partial u_j}{\partial t}: \) Velocity in \( j \) direction, and

\( \ddot{u}_j = \frac{\partial^2 u_j}{\partial t^2}: \) Acceleration in \( j \) direction.

In order to solve the problem mentioned in Section 3.1 by means of BEM, a fundamental solution to the governing differential equation is required. For Equation 3.15, there is no established fundamental solution because the existence of the velocity term (\( \dot{u}_j \)) and the acceleration term (\( \ddot{u}_j \)) complicates the equation.

### 3.3.2 Time Discretization

When a dynamic problem is solved in time domain with the finite element method, the governing equation is solved by applying the “time discretization scheme” (Zienkiewicz and Taylor, 1991).

Following the similar time discretization procedure as used in FEM, Equation 3.15 can be discretized in time domain. Different time discretization schemes are available. For convenience, the simple “linear acceleration” method will be used in this section to show the development of the formulation.

First, the time history is divided into \( N \) time intervals, the length of each interval is \( \Delta t = \frac{T}{N} \), where \( T \) is the total time to be analyzed.
Assuming that in each time interval, from \( t - \Delta t \) to \( t \), the acceleration changes linearly. Then the velocity and acceleration at time \( t \), namely \( \dot{u}_t \) and \( \ddot{u}_t \), can be written in terms of the displacement, velocity, and acceleration in the previous time step, \( t - \Delta t \), and the displacement in time \( t \):

\[
\dot{u}_t = \frac{3}{\Delta t} u_t - \left( \frac{3}{\Delta t} u_{t-\Delta t} + 2 \dot{u}_{t-\Delta t} + \frac{\Delta t}{2} \ddot{u}_{t-\Delta t} \right)
\]

\[
\ddot{u}_t = \frac{6}{\Delta t^2} u_t - \left( \frac{6}{\Delta t^2} u_{t-\Delta t} + \frac{6}{\Delta t} \dot{u}_{t-\Delta t} + 2 \ddot{u}_{t-\Delta t} \right)
\]  (3.16)

Substitute Equation 3.16 into Equation 3.15 yield:

\[
\frac{G}{1 - 2\nu} u_{i,j,j} + G u_{j,ii}
\]

\[
= \zeta \dot{u}_j + \rho \ddot{u}_j
\]

\[
= \left( \frac{3 \zeta}{\Delta t} \right) u_j - \left( \frac{3 \zeta}{\Delta t} u_{j-\Delta t} + \frac{6 \rho}{\Delta t^2} \right) u_{j-\Delta t} - \left( 2 \zeta \right) \dot{u}_{j-\Delta t} - \left( \frac{\zeta \Delta t}{2} + 2 \rho \right) \ddot{u}_{j-\Delta t}
\]  (3.17)

let:

\[
\lambda = \frac{3 \zeta}{\Delta t} + \frac{6 \rho}{\Delta t^2}
\]

\[
\xi = 2 \zeta + \frac{6 \rho}{\Delta t}
\]

\[
\psi = \frac{\zeta \Delta t}{2} + 2 \rho
\]  (3.18)

Equation 3.17 becomes:

\[
\frac{G}{1 - 2\nu} u_{i,j,j} + G u_{j,ii} = \lambda u_j - \left( \lambda u_{j-\Delta t} + \xi \dot{u}_{j-\Delta t} + \psi \ddot{u}_{j-\Delta t} \right)
\]  (3.19)
By applying the time discretization scheme, the functions of time in Equation 3.15 \((\dot{u}, \ddot{u})\) are expressed as functions of location, and a new differential equation (Equation 3.19) is obtained.

Equation 3.19 is not an exact solution to the physical problem but, when the time step \(\Delta t\) tends to zero, Equation 3.19 will converge in the exact solution. In the following discussion, this equation will be used as the governing differential equation of the time-discretized visco elastodynamic problem.

### 3.3.3 Fundamental Solutions

The fundamental solution for Equation 3.19 is not found. However, it is found that the homogeneous part of this equation

\[
\frac{G}{1 - 2\nu} u_{i,i} + G u_{j,ii} = 0
\]

(3.20)

is the governing equation for elastostatic problems. Its fundamental solution has been given for 1-D, 2-D, and 3-D cases (Brebbia and Walker, 1980).

For 1-D analysis, the governing differential equation becomes a Laplace equation:

\[
\frac{d^2 u}{dx^2} = 0
\]

(3.21)

The fundamental solution is:

\[
U^* = r/2, \quad r = |x|
\]

(3.22)
The governing differential equation for 2-D and 3-D elastostatic problems is given in Equation 3.20. The fundamental solution for 2-D problem is:

\[
U_{ij}^* = \frac{(3 - 4\nu) \ln(1/r) \delta_{ij} + r_{,j} r_{,i}}{8\pi G(1 - \nu)}
\]  \hspace{1cm} (3.23)

The fundamental solution for 3-D elastostatic problem is:

\[
U_{ij}^* = \frac{1}{16\pi G(1 - \nu) r} ((3 - 4\nu) \delta_{ij} + r_{,i} r_{,j})
\]  \hspace{1cm} (3.24)

where \( r = |x - x^{(4)}| \).

The procedure for 2-D analysis will be discussed in the following. 1-D and 3-D analyses are similar.

### 3.3.4 Dual Reciprocity Method

When the fundamental solution for the homogeneous part (Equation 3.20) is given, the dual reciprocity method (DRM) can be applied in the attempt of solving Equation 3.19. DRM gives a pure boundary solution to a governing differential equation, as discussed in Section 2.5.3.

First, by following the DRM approach proposed by Partridge et al (1992), the non-homogeneous part of Equation 3.19 can be written in the following form:

\[
\lambda u_j^t - (\lambda u_j^{t-\Delta t} + \xi u_j^{t-\Delta t} + \psi u_j^{t-\Delta t}) = \sum_{k=1}^{N} \sum_{l=1}^{L} \alpha_i^{(k)} f^{(k)}
\]  \hspace{1cm} (3.25)

where \( N \) and \( L \) denote the total number of boundary nodes and the total number of internal points, respectively. Each \( f^{(k)} \) is a function so chosen that a particular
solution \( \hat{u}^{(k)}_j \) for the equation

\[
G \hat{u}^{(k)}_{j,i} + \frac{G}{1 - 2\nu} u^{(k)}_{i,j} = f^{(k)}
\]

(3.26)

can be found.

Substituting Equation 3.25 into the governing differential equation (Equation 3.19), one obtains:

\[
G u_{j,i} + \frac{G}{1 - 2\nu} u_{i,j} = \sum_{k=1}^{N+L} \sum_{l=1}^{2} \alpha^{(k)}_l f^{(k)}
\]

(3.27)

When the governing equation (Equation 3.19) is expressed by Equation 3.27, it can be solved by the boundary element method. Multiplying Equation 3.27 by the fundamental solution (Equation 3.23), and integrating it over the whole domain, one has:

\[
\int_{\Omega} \left[ \frac{G}{1 - 2\nu} u_{i,j} + G u_{j,i} \right] U^*_{ij} \, d\Omega = \sum_{k=1}^{N+L} \sum_{l=1}^{2} \int_{\Omega} \left[ \alpha^{(k)}_l f^{(k)} \right] U^*_{ij} \, d\Omega
\]

(3.28)

Integrating Equation 3.28 by parts gives:

\[
c_{ij}(x^{(A)}) u_j(x^{(A)}) + \int_{\Gamma} P^*_{ij}(x^{(A)}, x^{(B)}) u_j(x^{(B)}) \, d\Gamma - \int_{\Gamma} U^*_{ij}(x^{(A)}, x^{(B)}) p_j(x^{(B)}) \, d\Gamma = \sum_{k=1}^{N-L} \sum_{l=1}^{2} \alpha^{(k)}_l \left\{ c_{ij}(x^{(A)}) \hat{u}^{(l)}_j(x^{(A)}, x^{(k)}) + \int_{\Gamma} P^*_{ij}(x^{(A)}, x^{(B)}) \hat{u}^{(l)}_j(x^{(B)}, x^{(k)}) \, d\Gamma \right\}
\]

(3.29)

where, \( N \) is the total number of boundary nodes, \( L \) is the total number of internal nodes, \( c_{ij}(x^{(A)}) \) is the coefficient related to the location of the source point \( x^{(A)} \),
\( \Gamma \) is the boundary of the interested region, the function \( \tilde{p}^{(l)}_j(x^{(B)}, x^{(k)}) \) is defined as the corresponding traction field of the displacement field \( \tilde{u}^{(l)}_j(x^{(B)}, x^{(k)}) \), which is the displacement at a field point \( x^{(B)} \) in direction \( j \), caused by a unit force acting at a DRM collocation point \( x^{(k)} \), in direction \( l \).

Equation 3.29 contains no domain integral.

### 3.3.5 Solving the DRM Equation

To Solve the DRM equation (Equation 3.29), one needs to determine the factor \( \alpha^{(k)}_i \) and the function \( f^{(k)} \).

In finding the \( \alpha \) factor, it is noticed that in Equation 3.19, all the non-homogeneous terms appeared at the right hand side are functions related to the previous time. These terms for the first time step can be obtained from the "initial condition" in time dependent problems, which gives the displacement, velocity, and the acceleration at time "zero". The solution at time \( t \) can be obtained by starting the computation from the initial state. This result will then be used to determine the non-homogeneous terms for the next time step.

By applying the initial condition, and following the step-by-step time integral, the values of the displacement, the velocity, and the acceleration at previous time step are computed for all the boundary and internal points. These values can be used to establish the \( \alpha \) matrix in Equation 3.27. Let:

\[
b_j = b_j(x) = \lambda \ u_j^{t-\Delta t}(x) + \xi \ \dot{u}_j^{t-\Delta t}(x) + \psi \ \ddot{u}_j^{t-\Delta t}(x) \quad (3.30)
\]
From Equation 3.25,

$$\lambda \{u^i\} - \{b\} = \{\alpha\}[f]$$  \hspace{1cm} (3.31)

Once the approximating function $f = f(x)$ is defined, the vector $\alpha$ can be obtained by

$$\{\alpha\} = [f]^{-1}(\lambda \{u^i\} - \{b\})$$  \hspace{1cm} (3.32)

Now, Equation 3.29 contains no domain integration. By discretizing the boundary $\Gamma$ into $M$ elements, Equation 3.29 can be written as:

$$c_{ij}(x^{(A)})u_j(x^{(A)}) + \sum_{m=1}^{M} \int_{\Gamma_m} P_{ij}^*(x^{(A)}, x^{(B)}) u_j(x^{(B)}) d\Gamma -$$

$$- \sum_{m=1}^{M} \int_{\Gamma_m} G_{ij}^*(x^{(A)}, x^{(B)}) p_j(x^{(B)}) d\Gamma$$

$$= \sum_{k=1}^{N+L} \sum_{l=1}^{2} \left\{ \alpha_{l}^{(k)} \left( c_{ij}(x^{(A)}) \hat{u}_j^{(l)}(x^{(A)}, x^{(k)}) +

- \sum_{m=1}^{M} \int_{\Gamma_m} P_{ij}^*(x^{(A)}, x^{(B)}) \hat{u}_j^{(l)}(x^{(B)}, x^{(k)}) d\Gamma -

- \sum_{m=1}^{M} \int_{\Gamma_m} U_{ij}^*(x^{(A)}, x^{(B)}) \hat{p}_j^{(l)}(x^{(B)}, x^{(k)}) d\Gamma \right) \right\}$$  \hspace{1cm} (3.33)

where, $x^{(A)}$ is the source point, $x^{(B)}$ denotes the boundary point, $x^{(k)}$ denotes a DRM collocation point, $u_j(x^{(B)})$ is the $j^{th}$ displacement component at point $x^{(B)}$, $p_j(x^{(B)})$ is the $j^{th}$ traction component at point $x^{(B)}$.

In an element $m$, the displacement $u_j(x)$ and the traction $p_j(x)$ can be approximated by their nodal values, $\{U_j^{(1)}, U_j^{(2)}, ..., U_j^{(Q)}\}^T$, and $\{P_j^{(1)}, P_j^{(2)}, ..., P_j^{(Q)}\}^T$. where $m_1, m_2, ..., m_Q$ are the node numbers in the element, $Q$ is the number of nodes.
in the element, and \( j = 1, 2 \) for 2-D analysis. The approximation is given by:

\[
\begin{align*}
    u_j(x^{(m)}) &= \sum_{q=1}^{Q} \phi^{(m)}(x) U_j^{(q)} \\
p_j(x^{(m)}) &= \sum_{q=1}^{Q} \phi^{(m)}(x) P_j^{(q)}
\end{align*}
\]

where \( x^{(m)} \) is a point in element \( m \), \( \Phi = \{ \Phi^{(1)}(x), \Phi^{(2)}(x), \ldots, \Phi^{(Q)}(x) \} \) is the shape function. For 2-node linear element, \( q = 2 \), the shape function becomes:

\[
\Phi = \{ \Phi^{(1)}, \Phi^{(2)} \}
\]

with

\[
\begin{align*}
    \Phi^1 &= (1 - \bar{x})/2 \\
    \Phi^2 &= (1 + \bar{x})/2
\end{align*}
\]

where \( \bar{x} \) is the dimensionless local coordinate varying from \(-1\) to \(1\).

Substituting Equation 3.32 and Equation 3.34 into Equation 3.33, one obtains:

\[
\begin{align*}
    c_{ij}(x^{(A)}) u_j(x^{(A)}) + & \sum_{m=1}^{M} \sum_{q=1}^{Q} U_j^{(q)} \int_{\Gamma_m} P_{ij}^{*}(x^{(A)}, x^{(m)}) \Phi^{(q)}(x^{(m)}) d\Gamma \\
    & - \sum_{m=1}^{M} \sum_{q=1}^{Q} P_j^{(q)} \int_{\Gamma_m} U_j^{*} (x^{(A)}, x^{(m)}) \Phi^{(q)}(x^{(m)}) d\Gamma \\
    & = \sum_{k=1}^{N+L} \sum_{l=1}^{2} \left\{ \alpha_{ij}^{(k)} \left[ c_{ij}(x^{(A)}) \hat{u}_j^{(l)}(x^{(A)}, x^{(k)}) + \\
    - \sum_{m=1}^{M} \sum_{q=1}^{Q} \hat{U}_j^{(l)(q)(k)} \int_{\Gamma_m} P_{ij}^{*}(x^{(A)}, x^{(m)}) \Phi^{(q)}(x^{(m)}) d\Gamma \\
    - \sum_{m=1}^{M} \sum_{q=1}^{Q} \hat{P}_j^{(l)(q)(k)} \int_{\Gamma_m} U_j^{*} (x^{(A)}, x^{(m)}) \Phi^{(q)}(x^{(m)}) d\Gamma \right] \right\}
\end{align*}
\]

where, \( \hat{U}_j^{(l)(q)(k)} \) is the nodal value of the displacement \( \hat{u}_j^{(l)}(x^{(m)}, x^{(k)}) \) at node \( q \), \( \hat{P}_j^{(l)(q)(k)} \) is the value of the traction \( \hat{p}_j^{(l)}(x^{(m)}, x^{(k)}) \) at node \( q \). Let:

\[
H_{ij}^{(A)(m)(q)} = c_{ij}(x^{(A)}) \delta_{ij} + \int_{\Gamma_m} P_{ij}^{*}(x^{(A)}, x^{(m)}) \Phi^{(q)}(x^{(m)}) d\Gamma
\]
\[ G^{(A)(m)(q)}_{ij} = \int_{\Gamma} U^*_i(x^{(A)}, x^{(m)}) \Phi^{(q)}(x^{(m)}) d\Gamma \]  

(3.36)

Equation 3.35 becomes:

\[
\sum_{m=1}^{M} \sum_{q=1}^{Q} [H_{ij}^{(A)(m)(q)} U_j^{(q)} - G_{ij}^{(A)(m)(q)} P_j^{(q)}] 
= \sum_{k=1}^{N+L} \sum_{l=1}^{2} \left\{ \alpha_{l}^{(k)} \sum_{m=1}^{M} \sum_{q=1}^{Q} [H_{ij}^{(A)(m)(q)} \dot{P}_{j}^{(l)(k)(q)} - G_{ij}^{(A)(m)(q)} \dot{P}_{j}^{(l)(k)(q)}] \right\} 
\]  

(3.37)

Applying Equation 3.37 to all boundary nodes using a collocation technique, one obtains the matrix form of the DRM equation:

\[
([H] - \lambda[R])\{u^{i}\} - [G]\{q^{i}\} = [R]\{b\} 
\]  

(3.38)

with

\[ R = ([H][\dot{U}] - [G][\dot{P}])\{\alpha\} \]  

(3.39)

By solving Equation 3.38, all the unknowns on the boundary points and the collocation points can be found for every time step, when the boundary conditions and the initial conditions are given.

The functions \( f^{(k)} \) determine the \( [\alpha] \) vector, the \( [\dot{u}] \) matrix, and the \( [\dot{q}] \) matrix. It is found that the accuracy and the efficiency of the computation are affected by the form of \( f^{(k)} \). Therefore, the functions should be carefully selected for different cases. The selection of \( f^{(k)} \) will be discussed later in Chapter 4 and Chapter 5.

### 3.3.6 Summary

A new boundary element formulation is developed by
1. Discretizing the governing equation in time domain, as suggested by Tanaka and Matsumoto (1989);

2. Applying the dual reciprocity method to the discretized governing differential equation to obtain the boundary element solution; and

3. Computing the [$\alpha$] matrix in DRM from the boundary values of the displacement, velocity, and acceleration in the previous computation.

The main feature of this new formulation is that it deals with the viscoelastic properties of material in the time domain, and bring all the domain integral to the boundary. Therefore, dealing with domain integral in boundary integral formulation is not required. It can be used for solving dynamic problems in an infinite domain, such as the layered half space or half plane, where the computation of the domain integration is impossible.
Figure 3.1: The Physical Model

Figure 3.2: Skeleton Curve and Loading-Reloading Curves
Figure 3.3: Stress–strain Curve for Cyclic Shear Test
Figure 3.4: Calculating the Energy Dissipation
Figure 3.5: An Irregular Loading Cycle

Figure 3.6: Calculated and Measured Pore Pressure (Stress Controlled Test)
Figure 3.7: Calculated and Measured Pore Pressure (Strain Controlled Test)
Chapter 4

THE EFFECTS OF LIQUEFACTION ON GROUND MOTION: 1-D ANALYSIS

The dual effects of soil liquefaction have been discussed in Section 2.2. To date, only conceptual explanation of these effects are given, no systematic analysis has been conducted yet.

By combining the liquefaction model developed in Section 3.2 and the BEM formulation developed in Section 3.3, the effects of liquefaction on ground motion can be studied numerically.

In the study it is assumed that the site under consideration is an evenly layered half space with one or more liquefiable soil layers, subjected to seismic shear wave propagating upwards from the bedrock.

The above assumed site can be represented by a 1-D layered soil column subjected to horizontal shear force acting on the base, without vertical and rotational motion, as shown in Figure 4.1.
The analysis is applied to a record taken from Port Island, as shown in Section 2.2. The soil information is obtained from the published test results. The computational results are compared with the record.

4.1 The Numerical Method

4.1.1 Liquefaction Model

As discussed in Section 3.2, a constitutive equation is essential to the liquefaction model. The equation should meet the two requirements mentioned in Section 3.2, that is, be able to represent the change of shear modulus with the build-up in pore water pressure, and yield a reasonable value of damping at high strain level. Also, the parameters in the equation should be easy to obtain.

For the Port Island case to be studied in this chapter, the stress-strain curve of the soil has not been found from publications, so the parameters in the nonlinear elastic equation cannot be obtained for the liquefiable soil layer. On the other hand, all the required parameters for the viscoelastic equation, including the values of shear wave velocity, the liquefaction strength, and the curves of shear strain-damping ratio and shear strain-shear modulus, have been reported (Ishihara et al., 1996, Iwasaki and Tai, 1996). Therefore, the viscoelastic equation will be used as the constitutive equation in the studies presented in this chapter.

The liquefaction strength for the gravel sand in Port Island is obtained from the cyclic triaxial test (Ishihara et al., 1996). In 1-D analysis, a soil element at
the site is under the similar state as that in a cyclic simple shear test. There are two approaches to make the stress state in the computation correspond to the laboratory test. The first approach is to calculate the principal stress in the soil element subjected to seismic shaking. Then use the maximum shear stress \( \tau_{\text{max}} = (\sigma_1 - \sigma_3)/2 \) to calculate the dissipated energy in each cycle. In this approach, calculation of the principal stresses and the maximum shear stress for every element is needed.

The other approach is to modify the liquefaction strength obtained from the cyclic triaxial test, and to use this modified strength to find the model parameters \( \alpha \) and \( \beta \).

Seed and Peacock (1971) studied the relationship between the liquefaction strength obtained from the simple shear test and that obtained from the triaxial test. By means of theoretical analyses, laboratory tests, and case studies, they concluded that the triaxial compression test data should be reduced by a factor of 0.55 to 0.7 to determine the value of \( \tau_{hv}/\sigma'_0 \) causing liquefaction under field conditions.

In converting the triaxial test results to the liquefaction strength under field condition, one may use a factor \( C_r \) defined as:

\[
C_r = \frac{\tau_{hv}/\sigma'_0}{\sigma_d/(2\sigma'_c)} \tag{4.1}
\]

where, \( \tau_{hv} \) is the horizontal shear stress required to cause liquefaction under the field condition, \( \sigma'_0 \) is the vertical effective consolidation pressure in the field, \( \sigma_d \) is the dynamic stress causing liquefaction under triaxial test condition, and \( \sigma'_c \) is the effective consolidation pressure in triaxial test. The value of \( C_r \) depends on the
density of the soil and the duration of the earthquake. For a sand sample with the relative density of 50\%, a $C_r$ value of 0.6 is suggested by Seed and Peacock (1971).

Higher $C_r$ values are suggested by other researchers. Finn et al. (1971) recommended that the shear stress causing liquefaction under the field condition can be found from the triaxial test result by the relationship

$$\frac{\tau_{h0}}{\sigma_c'} = \frac{1 + K_o \Delta \sigma_d}{2}$$  (4.2)

For Port Island case, where $K_o = 0.5$, Finn’s method gives $C_r = 0.75$.

Vaid and Sivathayalan (1996) reported that the commonly adopted value $C_r = 0.6$ (given by Seed and Peacock) is conservative. They found that the commonly used $C_r$ value will be more conservative when the sand is looser. Their tests show that for the Fraser Delta sand, $C_r$ is 0.78 when the relative density is 40\%, $C_r$ ranges from 0.62 to 0.7 when the relative density is 72\%. They have also reported that the $C_r$ value will not change with the confining pressure, when the relative density of sand is low.

In all the reports mentioned above, it has been mentioned that the test conditions for the two types of test are different in terms of the stress distribution within the sample, the rotation of principal stresses, and the boundary conditions. The stress condition in the simple shear test is closer to the 1-D site condition than that in the triaxial test (Finn et al., 1971). Therefore, it is suggested that the liquefaction strength used in 1-D analysis be obtained from cyclic simple shear test.

In the study conducted in this section, $C_r = 0.6$ is used for two reasons:
first, it has been proved by case studies (Seed and Peacock, 1971); second, the computed ground acceleration is slightly lower than the recorded acceleration (see the following discussion of ground motion).

4.1.2 DRM Formulation

As discussed in Section 3.3.5, the function \( f^{(k)}(x) \) in DRM formulation is essential to the efficiency and accuracy of the computation.

Partridge and Brebbia (1992) suggested that the \( f \) function can be one of the following types:

1. Elements of the Pascal triangle;

2. Trigonometric series; or

3. The distance function \( r \) used in the definition of the fundamental solution.

The \( r \) function was adopted first by Nardini and Brebbia (1982). It is believed to be the simplest and most accurate alternative (Partridge and Brebbia, 1992). In the 1-D boundary integral equation, the boundary of the problem domain shrinks to two end nodes. In this case, if no internal nodes are added, the \( r \) function will not give a sufficient accuracy when only two nodes are used in approximating the non-homogeneous terms. Computation shows that to maintain an acceptable accuracy, at least three internal nodes must be added. Because only two nodes are required by the BEM formulation, adding more than three internal nodes will result in at least 150% increase in the total number of equations. This would be considered as a great loss in efficiency. Therefore, in the 1-D visco-elastodynamic
analysis, the \( f \) function should be carefully selected.

To find a good \( f \) function with a high accuracy and a simple form, the physical model used in the 1-D site response analysis will be examined. As mentioned in the beginning of this chapter, the 1-D model can be simplified as a viscoelastic column subjected to pure shear, with one end fixed and the other end free. In this case, the governing differential equation (Equation 3.15) becomes:

\[
G \frac{\partial^2 u}{\partial x^2} = \zeta \ddot{u} + \rho \dddot{u} \tag{4.3}
\]

If one of the particular solutions of the above equation is used as the approximating function in the dual reciprocity boundary element analysis, an exact solution to the 1-D governing differential equation is obtained. Therefore, it is reasonable to believe that when the approximating function is so selected that it is in the similar form as the exact solution, the accuracy in approximating the non-homogeneous terms will be improved.

The theoretical solution for Equation 4.3 has not been given. However, the theoretical solutions for some other 1-D elastodynamic problems can be found. A typical 1-D elastodynamic problem is the longitudinal vibration of a prismatic bar. Its governing differential equation is in a similar form as Equation 4.3

\[
E \frac{d^2 u}{dx^2} = \rho \frac{d^2 u}{dt^2} \tag{4.4}
\]

The theoretical solution of Equation 4.4 is given as (Weaver et al., 1990):

\[
u = A \sum_{i=1,3,5,...}^{\infty} \frac{(-1)^{(i-1)/2}}{i^2} \sin \frac{i\pi x}{2l} \cos \frac{i\pi ct}{2l} \tag{4.5}
\]
where, \( c = \sqrt{\frac{E}{\rho}} \) is the wave velocity, \( A \) is the amplitude of the vibration.

Comparing Equation 4.4 with Equation 4.3, it is found that the two equations are in the similar form except that Equation 4.3 has an extra term \( \zeta \ddot{u} \). Normally the effect of this damping term is much smaller than that of the displacement term \( C \frac{\partial^2 u}{\partial x^2} \) and the acceleration term \( \rho \ddot{u} \). Therefore Equation 4.5 can be considered as an approximate solution of Equation 4.3.

Therefore, the approximating function \( f^{(k)} \) can be selected as a trigonometric series similar to Equation 4.5:

\[
f^{(k)} = \sum_{i=1,3,5,...}^{\infty} \frac{(-1)^{i-1/2}}{i^2} \sin \frac{i\pi r^{(k)}}{2l} \cos \frac{i\pi c t}{2l}, \quad r^{(k)} = |x^{(k)} - x| \quad (4.6)
\]

Computation shows that by taking only the first two terms in Equation 4.6 will give an acceptable accuracy.

Applying the DRM approach (Section 3.3.4) to the governing equation (Equation 4.3), one obtains:

\[
\frac{\partial^2 u(x,t)}{\partial x^2} = \sum_{k=1}^{N+L} \alpha^{(k)} f^{(k)} \quad (4.7)
\]

where, \( N \) is the number of boundary nodes, \( L \) is the number of internal nodes used in DRM procedure. In 1-D analysis, the boundary of an element is the two ends of the element, \( N = 2 \).

Substitute Equation 4.6 into Equation 4.7, the governing differential equation now becomes:

\[
\frac{\partial^2 u(x,t)}{\partial x^2} = \sum_{k=1}^{N+L} \left\{ \alpha^{(k)} \left( \sin \frac{\pi r^{(k)}}{2l} \cos \frac{\pi c t}{2l} - \frac{1}{9} \sin \frac{3\pi r^{(k)}}{2l} \cos \frac{3\pi c t}{2l} \right) \right\} \quad (4.8)
\]
The fundamental solution to the homogeneous part of Equation 4.8 is:

\[ U^* = \frac{1}{2}[1 - r], \quad r = |x - x^{(A)}| \quad (4.9) \]

where \( x^{(A)} \) is the source point. The corresponding traction field is obtained by differentiating Equation 4.9:

\[ P^* = -\frac{1}{2}\text{sign}(x - x^{(A)}) \quad (4.10) \]

Multiplying both side of Equation 4.7 by the fundamental solution \( U^* \), and integrate over the domain \( \{0, L\} \) by parts twice, one obtains

\[ u(x^{(A)}) + \left[ U^*(x, x^{(A)}) \frac{\partial u(x)}{\partial x} - P^*(x, x^{(A)})u(x) \right]_0^L \]

\[ = \left[ U^*(x, x^{(A)})\hat{p}(x, x^{(A)}) + P^*(x, x^{(A)})\hat{u}(x, x^{(A)}) \right]_0^L \quad (4.11) \]

where, the function \( \hat{u} \) is a particular solution of Equation 4.8:

\[ \hat{u} = -\frac{4l^2}{\pi^2} \sin \frac{\pi r^{(k)}}{2l} \cos \frac{\pi ct}{2l} + \frac{4l^2}{81\pi^2} \sin \frac{3\pi r^{(k)}}{2l} \cos \frac{3\pi ct}{2l} \quad (4.12) \]

with the corresponding traction:

\[ \hat{p} = \frac{\partial \hat{u}}{\partial x} = \frac{2l}{\pi} \cos \frac{\pi r^{(k)}}{2l} \cos \frac{\pi ct}{2l} - \frac{2l}{27\pi} \cos \frac{3\pi r^{(k)}}{2l} \cos \frac{3\pi ct}{2l} \quad (4.13) \]

where \( r^{(k)} = |x^{(k)} - x| \), \( x^{(k)} \) indicates the DRM collocation point.

Equation 4.11 can be solved by routine DRM procedure. It can be found that there is no integration involved in Equation 4.11, therefore no spatial discretization is needed to solve the equation.
A simple example is computed by means of the 1-D DRM formulation. The example is the longitudinal vibration of a column, with one end fixed and the other end free. The column is subjected to an initial displacement at the free end. The time step used in the analysis is 0.01 second. A theoretical solution of this problem has been given by Eringen and Suhubi (1975). Figure 4.2 shows that the computed free-end displacement agrees well with the theoretical solution.

4.2 Case Study: Port Island, Kobe

Port Island is located south to Kobe city, about 17 km from the epicenter of the 1995 Hyogoken-Nanbu (Kobe) earthquake. As mentioned in Section 2.1.5, extensive liquefaction in the reclaimed land on Port Island was found during the 1995 Kobe earthquake. Liquefaction resulted in ground damages such as ground settlement, lateral displacement, and sand boils.

An acceleration recording system was installed in the borehole array station in Port Island. The acceleration time histories were recorded at four different depths during the earthquake. The location of the recording station is shown in Figure 4.3.

By studying the recorded and computed ground motion, the effects of liquefaction on seismic damage can be revealed.

4.2.1 Site Condition and Soil Properties

Port Island is a reclaimed island with an area of 436 hectare, constructed from 1966 to 1980. The island was built by placing a 16.2m to 22.6m thick sandy gravel fill on the old sea bed, which is a 7.6 m to 8 m thick layer of silty clay with an
SPT value less than 10. The old sea bed is underlain by alternative layers of dense gravelly sand and silt. These deposits extend to the depth of 79 m, where stiff silty clay is found. Bedrock has not been found up to the depth of 85m. A cross section of the Port Island is shown in Figure 4.4.

Figure 4.4 indicates that the thickness of soil layers in the Port Island is fairly even. Therefore, 1-D analysis can be adopted in the liquefaction and site response analysis of the site.

The soil properties have been reported by Ishihara et al. (1996), Iwasaki and Tai(1996), Inagake et al.(1996), and Sugito et al.(1996).

The sandy gravel fill contains less than 1% cohesive particles (particle size smaller than 0.005mm), 5% of fines (particle size between 0.005mm and 0.074mm), 40% sand (grain size 0.074mm to 2mm), and 55% gravel (grain size greater than 2mm). The relative density of the gravelly sand is believed to be in the range of 47% to 50%. From the grain size distribution curve (Figure 4.5), it can be found that for the gravelly sand, the mean size ($d_{50}$) is 2mm, the uniformity coefficient ($d_{60}/d_{10}$) is 12.7. The low relative density and low cohesive particle content make the soil liquefiable, in spite of its good grading and large mean grain size.

In the viscoelastic constitutive equation, the shear modulus and the damping ratio are affected by the effective confining pressure and the peak shear strain in a loading cycle. As will be discussed in this section, the shear modulus is mainly determined by the effective confining pressure, and the damping ratio is mainly determined by the peak shear strain. To simplify the analysis, it is assumed that the reduction of shear modulus depends on the effective confining pressure only, and the damping ratio is a function of shear strain amplitude only.
Table 4.1: The reduction in shear modulus

<table>
<thead>
<tr>
<th>soil layer</th>
<th>max. acc.</th>
<th>max. strain</th>
<th>pwp</th>
<th>shear modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>main shock</td>
<td>saturated</td>
<td>341 gal</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>unsaturated</td>
<td>341 gal</td>
<td>high</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>after shock</td>
<td>satiated</td>
<td>10 gal</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>unsaturated</td>
<td>10 gal</td>
<td>low</td>
<td>low</td>
<td>high</td>
</tr>
</tbody>
</table>

It should be noted that the shear modulus used in viscoelastic constitutive equation is an average model in a loading-unloading loop, which represents the inclination of the loop. This modulus is different from the tangent modulus. The tangent modulus will change with the shear strain (Figure 4.6).

The two assumptions about the shear modulus and damping ratio are supported by an analysis of the differences in shear wave velocity and damping ratio during the main shock and the after shock in 1995 Kobe earthquake, reported by Sato et al. (1996).

This analysis shows that for the main shock, the reduction from the initial value in shear modulus is 80% for the saturated reclaimed soil (from the depth of 0m to 20m) and 50% for the unsaturated reclaimed soil. During the after shock 2 minutes later, with the peak ground acceleration of 10 gal, the reduction is 71.5% for the saturated soil, and 23.5% for the unsaturated soil. (Figure 4.7).

The above analysis can be summarized in the following table:

Table 4.2.1 can be explained as follows: during the main shock, the pore-pressure build-up in the saturated soil is high, therefore a large amount of reduction in shear modulus is found. The pore-pressure build-up in the unsaturated soil is much lower than that in the saturated soil, and a large portion of the excess pore-pressure in the unsaturated soil may come from the pore pressure dissipation in
the underlying saturated soil. During the after shock that occurred two minutes after the main shock, the strain level must be much lower than that during the main shock with a peak ground acceleration of 341 gal. The shear modulus of the saturated soil, however, is still low because the excess pore pressure generated in the main shock has not dissipated during the short time. In the unsaturated soil, the pore-pressure dissipation is much faster than that in the saturated soil, therefore the reduction of shear modulus is low.

From the above discussion, it can be concluded that the reduction in shear modulus is mainly caused by the build up of pore-water pressure.

Figure 4.8 shows the damping ratio during the main shock and during the after shock obtained from analysis (Sato et al., 1996). The initial damping ratio in the figure is from laboratory test. The increase in damping ratio during the main shock is 25 times, for both the saturated soil and the unsaturated soil. During the after shock, the increase is 5 times in the unsaturated soil, and 6 times in the saturated soil.

It is believed that the increase in damping ratio is mainly determined by the shear strain because of the following reasons.

1. The increase in damping ratio in the saturated soil is very close to that in the unsaturated soil, while the pore pressure in the latter is much lower.

2. The increase in damping ratio during the after shock is much lower than that in the main shock. In the after shock, the shear strain is much lower than that in the main shock, even though a large portion of the excess pore-pressure in the saturated soil has not dissipated.
The initial shear modulus of each layer is calculated from the shear wave velocities given by Sugito et al. (1996). The compressional modulus is calculated from the pressure wave velocities. The reduction in shear modulus is calculated by Equation 3.2:

\[
\frac{G_s}{G_i} = \sqrt{p_o}
\]  

(4.14)

where \( G_i \) is the initial shear modulus before the acting of seismic shock, \( p_o \) is the normalized confining pressure.

A curve describing the reduction of shear modulus \( G \) is also presented in Figure 4.9. However, when saturated soil subjected to cyclic shear, the shear strain is not the only factor controlling the shear modulus. When the pore-pressure build-up is significant, the reduction of shear modulus is mainly due to the decrease of the effective confining pressure, as discussed above. In stress controlled undrained test, the strain \( v_s \), shear modulus curve may also indicate the increase in shear strain due to the reduction of shear modulus. Therefore, when the decrease in effective confining pressure is not given, the shear modulus - shear strain curve given in Figure 4.9 cannot be used to define the non-linear elastic constitutive relationship.

The initial damping ratio for each soil layer can be found in Figure 4.8. The deduction in damping ratio of the reclaimed soil is given by Iwasaki and Tai (1996) in Figure 4.9. The figure shows that the initial damping ratio is about 2%, while the maximum damping ratio at the strain level of 2% is about 45%. The relationship
between the damping ratio and the shear strain can be found from the figure as:

\[
\frac{\eta_s}{\eta_i} = 1 + 2000 \log(1 + \gamma)
\]  

(4.15)

where \(\eta_i\) is the initial damping ratio.

A curve describing the reduction of shear modulus \(G\) is also presented in Figure 4.9. However, when saturated soil subjected to cyclic shear, the shear strain is not the only factor controls the shear modulus. When the pore-pressure build-up is significant, the decrease of the effective confining pressure will affect the shear modulus, as discussed above. In stress controlled undrained test, the strain \(\nu_s\), shear modulus curve may also indicate the increase in shear strain due to the reduction of shear modulus. Therefore, if the decrease in effective confining pressure is not given, the shear modulus vs. shear strain curve cannot be used to represent the real stress-strain relationship of the soil.

A series of cyclic triaxial tests, as reported by Ishihara et al.(1996), were carried out prior to the earthquake. Soil samples used in the testing were recovered in blocks and frozen in the field before being brought to the laboratory. The block samples were carefully cut into specimens of 7.5 cm in diameter and 15 cm in height. The location of sampling is shown in Figure 4.3. Another set of cyclic triaxial tests was carried out on undisturbed samples recovered by “triple-tube sampling”. The liquefaction strength of the fill obtained from these two sets of tests are summarized in Figure 4.10 (Ishihara et al., 1996).

With the above information, the pore-water pressure build-up factors \(\alpha\) and \(\beta\) in Equation 3.11 can be obtained by a trial and error procedure. Pairs of \(\alpha\) and \(\beta\) values are assigned to give a relationship between the liquefaction strength \(\tau_f\)
and the number of stress cycles to liquefaction, \( N_f \). This is carried out using a computer program, until a pair of \( \alpha \) and \( \beta \) values are found to give a \( \tau_f \) vs. \( N_f \) curve matches the test results shown in Figure 4.10. For the Port Island case, \( \alpha = 100 \) and \( \beta = 0.5 \) are obtained. These values are used in the computation conducted in this section.

The frequency of cyclic loading used in the calculation of \( \alpha \) and \( \beta \) is selected by examining the recorded horizontal ground motion shown in Figure 4.13. In the record, the first cycle of strong motion starts at 4.2 second, ends at 5.2 second. The period is about 1 second. According to the computation, a great portion of the excess pore pressure is generated in this cycle. Therefore the period of 1 second is used to calculate liquefaction parameters.

One of the advantages of the new liquefaction model is that the simple viscoelastic constitutive relationship can be used in the liquefaction analysis, and the time consuming iteration in computing the non-linear material properties is not necessary. The model can be further improved by introducing a viscoelastic constitutive model with better simulation to the energy dissipation properties of the soil.

4.2.2 Soil Liquefaction

A C++ program is developed by applying the above mentioned BEM formulation and liquefaction model. With this program, the ground motion at the Port Island site is computed. In the computation, it is assumed that the input seismic motion acts at the depth of 83 m, where the deepest recording point is located.
The time interval used in the computation is 0.005 second. The 83 m thick soil mass is divided into 33 elements, in which 14 elements are used to represent the 14 m thick liquefiable soil (the sandy gravel, under the ground water table). The BEM mash is shown in Figure 4.11.

Figure 4.17 is the pore-water pressure build-up at the depth of 12 m. It can be seen that the pore-water pressure build-up becomes significant at the 4th second in the record, after the arrival of the first strong shaking. The soil element fully liquefies (the effective confining pressure equals zero) at the 7th second.

Computation shows that the liquefaction starts from the bottom of the sandy layer, because the transmission of seismic energy is from the bottom to the top. The energy dissipation at the bottom causes a quicker pore-water pressure build-up, and therefore reduces the energy transferred to the upper soil mass.

On the other hand, the upper soil elements are subjected to lower confining pressure, and therefore have a lower liquefaction strength.

In general, the pore-water pressure build-up in each liquefiable soil element occurs at more or less the same time. The top elements liquefy at 0.3 second later than the bottom elements. The whole sandy layer liquefies at the 7th second. Figure 4.18 shows the development of liquefaction in the liquefiable layer.

Figure 4.17 indicates that the pore-water pressure time history is determined by the ground acceleration time history. A pulse of strong shake will result in a jump in pore-water pressure, and the value of the jump depends on the amplitude of the pulse, and the pore-pressure at that moment. A pulse in a certain amplitude will cause a higher increase in pore-pressure when the pore-pressure is low, or a
lower increase when the pore-pressure is close to the confining pressure.

From the above discussion, it can be concluded that using a uniform cyclic loading as the equivalent seismic loading in the calculation of pore-pressure (Seed's method) will not give the correct pore-pressure time history. Therefore, the "equivalent cyclic loading" method cannot be used in the analysis of site response at a liquefied site.

4.2.3 Ground Motion

The computed acceleration time histories are shown in Figure 4.12 and Figure 4.14. The recorded time histories are presented again in Figure 4.13 and Figure 4.15 for reference. It is found that the computed motions agree reasonably well with the recorded motion.

The acceleration response spectra of the computed motions and recorded motions are shown in Figure 4.16. It is found that the two spectra are similar in shape. The computed maximum spectral acceleration is about 20% lower than the recorded one. This is possibly due to the error in the soil parameters used in the computation.

Both the computed results and the field records show that the frequency components of the vertical motion have not been changed by the propagation through soil layers. For the horizontal motion, the high frequency component at the ground surface is filtered out. This phenomenon can be explained by further study of the time histories.

The acceleration time histories of the horizontal ground motion can be divided
into three stages: The first stage is from the beginning of the record to the 4th second, when the pore-water pressure is low. In this stage, the ground motion corresponds to the case where soil does not liquefy. The second stage is from the 4th second to the 7th second, in which the pore-water pressure in the liquefiable soil layer increases quickly, causing the shear modulus to decrease. From the 7th second to the end of the record, the sandy layer is fully liquefied, and this significantly affects the ground motion.

Figure 4.19 is a good example to demonstrate the effect of soil liquefaction to the ground motion. It shows the horizontal velocity response spectra of the computed ground motion at the ground surface before and after the sandy layer liquefies. The velocity response spectra for the case with no liquefaction are also plotted.

Figure 4.19a shows the responses from the beginning of the record to the 6th second, which is in stage 1 and stage 2. In this time period, the pore pressure build-up in the liquefiable layer is low (at 5.8 second, the excess pore-water pressure is less than 50% of the confining pressure). The spectra for the cases with and without liquefaction show no difference.

Figure 4.19b is the response from the 6th second to the 30th second, which covers the end of stage 2 (in most of the liquefiable elements, pore-pressure build-up is over 90% of the total confining pressure at 6.4 second) and stage 3. It is found that for the shorter period motion, the “liquefied” response is significantly weaker than that of the “non-liquefied” response. For the motion with the period longer than 2.5 second, liquefaction does not change its amplitude.

This result explains the reduction of damage to structures at sites where soil
liquefaction has been observed. From the period of 0.2 to 2.5 second, which is the range of the resonant periods for most structures, the seismic energy transmitted to the structures is lower in the liquefied sites. That is, the liquefaction will cause damage to the ground and shallow foundations, but on the other hand, it will reduce the seismic energy transmitted to the structures.

In the case discussed in Section 2.1, the reduction in damage is found mostly in structures with short resonant period, such as the multi-story residential buildings in Niigata. The reason is that the damage-reducing effect of liquefaction is significant when the resonant period of the structure is shorter than 2 second.

For a structure with long resonant period, the occurrence of liquefaction will not reduce the seismic damage to the structure.

The record of ground motion also demonstrates the “dual effect” of liquefaction. Figure 4.20 is the velocity response spectra for the motion at ground surface and the motion at −16m (liquefiable layer is at −5 to −19 meter at the site). It is found that at ground surface, where the shear wave has passed through the 19 m thick liquefied sandy layer, the short period motion is significantly weaker than that at the depth of 16 m.

Based on the computational results and the analysis of the ground record, it is suggested that in earthquake zone with high liquefaction potential, structures with short resonant period should be considered, to reduce the structural damage. Pile foundations are recomended in liquefiable sites, to prevent the foundation failure.

Under the 1-D condition, the vertical motion is caused by the propagation of the pressure wave, which depends on the compressional modulus of the soil. Because
the soil liquefaction does not affect the compressional modulus, the frequency components of the vertical motion are not significantly altered.

4.3 SUMMARY

The time-domain boundary element formulation for viscoelastic materials, combined with a liquefaction model, can be used in the analysis of soil liquefaction and seismic response.

When BEM is used in 1-D analysis, the boundary of the problem reduces to two nodes at the ends of the 1-D domain, therefore no spatial discretization is required. This gives a higher accuracy than the FEM analysis in which spatial discretization is needed. In the time domain dynamic analysis mentioned above, approximation function is chosen as the similar form to the theoretical solution to the 1-D elastodynamic problem, the accuracy of the approximation is high.

The liquefaction model based on an energy approach can be used in the 1-D BEM analysis. One of the advantage of using this model is that a simple constitutive relationship can be adopted in the computation, and the time-consuming non-linear iteration is not necessary. However, the simple viscoelastic model cannot reflect the rate-independent hysteresis damping of soil. To improve the model, a constitutive equation which is able to representing the energy dissipation characteristics of soil is needed.

When the soil layer liquefies, the high frequency components of shear wave are filtered out because of the reduced shear modulus in the liquefied layer. This reduction in high frequency motion results in less damage to structures with short
resonant periods. On the other hand, the liquefaction of subsoil will cause ground failure and the damage to the foundation, as shown in Section 2.1. These dual effects of liquefaction should be carefully considered in designing foundations in earthquake zone.

The vertical component of ground motion is not affected by the soil liquefaction, because its propagation depends on the compressional modulus, which is not reduced when the soil element liquefies.
<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Soil Type</th>
<th>Soil Profile</th>
<th>SPT N-value</th>
<th>Vs m/sec</th>
<th>Vp m/sec</th>
<th>Gs * Pa</th>
<th>Damping ratio *</th>
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<td>10</td>
<td>W.T.</td>
<td></td>
<td>&lt; 10</td>
<td>170</td>
<td>330</td>
<td>4x10^7</td>
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<td>7 - 20</td>
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<td>780</td>
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<td>0.02 to 0.4</td>
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<tr>
<td>20</td>
<td>Silty clay</td>
<td></td>
<td>&lt; 10</td>
<td>180</td>
<td>1480</td>
<td>5.3x10^7</td>
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</tr>
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<td>30</td>
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<td>Alternate layers of gravelly sand and silt (Diluvium)</td>
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<td>305</td>
<td>1530</td>
<td>18x10^7</td>
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<td>Silty clay (Diluvium)</td>
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<td>12</td>
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<td>1610</td>
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</tr>
</tbody>
</table>

* Parameters used in computing

** Shear modulus and damping ratio change with pore-pressure build-up

Vs: Shear wave velocity  Vp: Pressure wave velocity

Figure 4.1: Soil Profile and Soil Properties in Port Island
Figure 4.2: Computed Displacement vs. Theoretical Solution
Figure 4.3: Location of Ground Motion Recording and Block Sampling in Port Island (Ishihara et al., 1996)
Figure 4.4: A Cross Section in Port Island (Ishihara et al., 1996)
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Figure 4.17: Pore-Pressure Build-up at the Depth of -12 m
Element No. 1 is at the bottom of the liquefiable layer (-19 m)

Element No. 14 is at the top of the liquefiable layer (-5 m)

Figure 4.18: Development of Liquefaction
Figure 4.19: Computed Velocity Response Spectrum
Figure 4.20: Velocity Response Spectrum for Recorded Motion
Figure 4.21: Acceleration Response Spectrum for Recorded Motion
Chapter 5

THE EFFECTS OF LOCAL GEOLOGY ON GROUND MOTION AND LIQUEFACTION: 2-D ANALYSIS

As discussed in Section 2.3.1, the amplification effects of alluvial valleys have been observed during major earthquakes. Different patterns of amplification are reported: in some cases heavy damage is found in the middle of the alluvial valley, while in some other cases, the heavy damage is found at the edges of the valley.

The results of numerical analyses on the amplification effect of alluvial valley have been discussed in Section 2.3.2. It is found that the amplification effect highly depends on the shape of the valley. Different results have been reported by different researchers.

The effect of alluvial valley on the liquefaction potential has been discussed in Section 2.1. Higher liquefaction potential is observed at the sites with alluvial
valleys. No numerical analysis on this effect has been reported.

In order to study the combined effects of alluvial valley on the ground motion and the liquefaction potential, numerical analysis is required. As discussed in Chapter 2, the numerical method should be able to model the dynamic response of a viscoelastic medium in time domain, with a reasonable computation cost.

A new BEM approach has been developed in Chapter 3 for the ground response analysis. In this chapter, the application of the new approach on solving 2-D problems will be discussed, and examples will be presented to test the method and to reveal the effects of alluvial valleys. The effects of alluvial valleys on soil liquefaction will also be studied.

5.1 NUMERICAL METHOD

The new boundary element formulation developed in Chapter 3, combined with an appropriate approximating function, can be used in the analysis of 2-D visco-elastodynamic problems in time domain. Different from the 1-D analysis, the combination of the subregions with different material properties must be carefully considered.

The new liquefaction model has been applied to 1-D response analysis in previous chapters. When the maximum dynamic shear stress in a soil element is given, this model can be used to calculate the pore-water pressure build-up in the 2-D analysis, as discussed in Section 3.2.

In summary, the 2-D site response and liquefaction problems can be studied by means of the new BEM formulation that incorporates the liquefaction model.
To carry out the 2-D analysis, the following work is required:

1. Incorporation of the liquefaction model into the 2-D BEM analysis;

2. Combination of the liquefiable region and the non-liquefiable region; and

3. Definition of the approximating function in Equation 3.27, and provision of solution to the equation.

5.1.1 The Liquefaction Analysis

As discussed in Section 2.5, FEM is useful in the computation of the liquefiable soil layers. In the analysis of the stability of the thick liquefiable soil layer, FEM might be the only choice because the material will become strongly non-homogeneous due to the asynchronous development and dissipation of pore pressure among the elements.

When the thickness and the lateral extent of a liquefiable layer are small, the build-up in pore pressure will be fairly homogeneous during the main shock, as shown in Section 4.2. In this case, boundary element method can be used to represent the response of the liquefiable layer.

In applying the liquefaction model to BEM, the first step is to divide the liquefiable layer into several rectangular or triangular cells, and assume that the pore-water pressure is equal everywhere within a cell.

The second step is to solve the governing differential equation for the cell by means of the BEM formulation developed in Section 3.3 in the first time step. When the traction at each boundary element is obtained, the average traction in
the global $x_1$ and $x_2$ directions can be found.

Assuming the cell has two straight sides, one parallel to the $x_1$ axis, and the other parallel to the $x_2$ axis, the average of the traction along these two sides can be found as:

$$P_{11} = \frac{1}{N_1} \sum_{k=1}^{N_1} p_1^{(k)}$$

$$P_{12} = \frac{1}{N_1} \sum_{k=1}^{N_1} p_2^{(k)}$$

$$P_{21} = \frac{1}{N_2} \sum_{l=1}^{N_2} p_1^{(l)}$$

$$T_{22} = \frac{1}{N_2} \sum_{l=1}^{N_2} l_2^{(l)}$$

(5.1)

where $k, l$ denote elements in the sides parallel to the $x_1$ and $x_2$ axes, respectively; $P_{ij}$ is the average traction in the $x_j$ direction along the edge parallel to the $x_i$ direction, $p_i^{(k)}$ is the traction in the $i$ direction in element $k$, and $N_i$ is the number of elements at the side parallel to the $x_i$ axis.

The “average principal stress” $\bar{\sigma}_1$ and $\bar{\sigma}_3$ in the cell are defined by:

$$\bar{\sigma}_1 = \frac{P_{11} + P_{22}}{2} + \sqrt{\left(\frac{P_{11} - P_{22}}{2}\right)^2 + \left(\frac{P_{12} + P_{21}}{2}\right)^2}$$

$$\bar{\sigma}_3 = \frac{P_{11} + P_{22}}{2} - \sqrt{\left(\frac{P_{11} - P_{22}}{2}\right)^2 + \left(\frac{P_{12} + P_{21}}{2}\right)^2}$$

(5.2)
Then the average maximum shear stress \( \bar{\tau}_{\text{max}} \) in the cell can be obtained from

\[
\bar{\tau}_{\text{max}} = \bar{\sigma}_1 - \bar{\sigma}_3
\] (5.3)

The third step is to calculate the pore-pressure build-up in the cell from the maximum shear stress, and the corresponding maximum shear strain, by means of the liquefaction model given in Section 3.2. Then, the computation for the next time step can be carried out, with the updated shear modulus and damping ratio.

The main advantage of using BEM in liquefaction analysis is that number of elements needed is significantly less than that in FEM. In FEM, the size of elements is controlled by the cutoff frequency, which is decided by the shear wave velocity and the dimension of the element. Normally, the cutoff frequency of a finite element system must be 5 to 10 times higher than the highest frequency required in the analysis.

For liquefaction analysis, the shear wave velocity in a liquefiable element will be extremely low when the pore-water pressure is high. Therefore, the size of the element must be very small. The limitation in element size will significantly increase the cost of the computation. For example, when the shear wave velocity in a liquefied sandy layer is 20 m/s, and the highest frequency required in the analysis is 10 Hz, the size of the element must be smaller than \( 0.2 \times (20\text{m})/(10\text{HZ}) = 0.4\text{m} \), in order to maintain acceptable accuracy. With this limitation of element size, 1000 element is needed to represent a liquefiable layer in a 2-D case, with the thickness of 8 m and the extent of 20 m.

In BEM analysis, because the spatial descritization is taken in the boundary only, the number of elements needed in solving the same problem can be greatly
reduced.

As discussed before, the use of BEM in liquefaction is restricted to the cases where the pore-pressure build-up in the liquefiable layer is homogeneous. If this condition is not satisfied, for example, for a thick and widely extended soil deposit combined with a complicated local geological condition, more cells will be needed to ensure the homogeneity in a cell. Thus, the cost of computation will be greatly increased.

Therefore, in the analysis of general 2-D liquefaction problems, the hybrid method with the liquefiable soil represented by FEM is preferred, even though the use of FEM analysis for liquefaction analysis will significantly increase the cost of computation. Pure BEM analysis can be adopted only in some simple cases.

5.1.2 Solving the Boundary Integral Equation

In the 2-D analysis, the governing equation for the general visco-elastodynamic analysis is given by Equation 3.27:

\[ G u_{j,ii} + \frac{G}{1-2\nu} u_{i,ij} = \sum_{k=1}^{N+L} \sum_{l=1}^{2} a_{k}^{(l)} f^{(k)} \]  \hspace{1cm} (5.4)

with the coordinate indicator \( i, j, l = 1, 2. \)

The fundamental solution for the homogeneous part is given as:

\[ U_{ij}^{*} = \frac{-1}{8\pi G (1 - \nu)} \left\{ (3 - 4\nu) \ln(r_{ij}) - r_{ij}r_{ii} \right\} \] \hspace{1cm} (5.5)
with the corresponding traction field:

$$P_{ij}^* = \frac{-1}{4\pi(1 - \nu)r} \left\{ ((1 - 2\nu)\delta_{ij} + 2r_{ii} r_{jj}) \frac{\partial r}{\partial n} - (1 - 2\nu)(r_{ni} n_j - r_{nj} n_i) \right\} \quad i, j = 1, 2$$

(5.6)

where, \( r = \sqrt{(x_1 - x_1^{(A)})^2 + (x_2 - x_2^{(A)})^2} \), \( x^{(A)} \) is the source point.

In order to build a boundary integral equation from Equation 5.4, the usual approach in developing BEM solution is followed. First, multiplying the fundamental solution (Equation 5.5) to both sides of Equation 5.4, and integrating by parts, one obtains:

$$c_{ij}(x^{(A)})u_j(x^{(A)}) + \int_{\Gamma} P_{ij}^*(x^{(A)}, x^{(B)})u_j(x^{(B)})d\Gamma -$$

$$- \int_{\Gamma} U_{ij}^*(x^{(A)}, x^{(B)})p_j(x^{(B)})d\Gamma =$$

$$\sum_{k=1}^{N+L} \sum_{l=1}^{2} \alpha_{ij}^{(k)} \left\{ c_{ij}(x^{(A)})\tilde{u}_j^{(l)}(x^{(A)}, x^{(k)}) + \right.$$  

$$+ \int_{\Gamma} P_{ij}^*(x^{(A)}, x^{(B)})\tilde{u}_j^{(l)}(x^{(B)}, x^{(k)})d\Gamma -$$

$$- \int_{\Gamma} U_{ij}^*(x^{(A)}, x^{(B)})\tilde{p}_j^{(l)}(x^{(B)}, x^{(k)})d\Gamma \right\}$$

(5.7)

where \( x^{(A)} \) denotes the source boundary point in BEM analysis, \( N \) is the total number of boundary nodes, \( L \) is the total number of internal nodes, \( c_{ij}(x^{(A)}) \) is the coefficient related to the location of the source point \( x^{(A)} \). \( \Gamma \) is the boundary of the interested region, the function \( \tilde{u}_j^{(l)}(x^{(B)}, x^{(k)}) \) and \( \tilde{p}_j^{(l)}(x^{(B)}, x^{(k)}) \) are related to the approximating function \( f^{(k)} \), as defined in Equation 3.26, and, when the \( f \) function is defined, \( \alpha \) can be obtained from the displacement, velocity, and acceleration at previous time step, as shown in Equation 3.32.

To solve Equation 5.7, the approximating function \( f \) needs to be defined. In 1-D analysis, because the available nodes used to define the non-homogeneous unknown function are limited, the \( f \) function is chosen as a trigonometric series,
which is in the similar form as the theoretical solution of a 1-D elastic vibration problem, to provide good accuracy and efficiency (Section 4.1.2). In 2-D analysis, the accuracy of approximating the non-homogeneous terms is generally higher than that in 1-D analysis, because the number of the boundary nodes and internal nodes for defining the approximating function \( f \) is much larger. Therefore, the concerns in choosing the \( f \) function are solution stability and implementation simplicity.

As suggested by Nardini and Brebbia (1985), the distance function used to define the fundamental solution is adopted as the approximating function in Equation 5.7. This function is the simplest in form, and it has been used in many cases of DRM analysis (Partridge et al., 1992).

The \( f \) function is given as:

\[
f^{(k)} = C + r^{(k)}
\]

(5.8)

where, \( C \) is a constant. In the analysis conducted in this chapter, \( C \) is chosen as a unit length, \( C = 1 \). \( r^{(k)} \) is the distance from a field point \( x \) to a DRM collocation point, \( x^{(k)} \), i.e. \( r^{(k)} = \sqrt{(x_1 - x_1^{(k)})^2 + (x_2 - x_2^{(k)})^2} \).

When the approximating function is defined, the function \( \hat{u} \) and \( \hat{q} \) in Equation 5.7 can be obtained.

The function \( \hat{u} \) in Equation 5.7 is the particular solution to the differential equation

\[
G\hat{u}_{i,j}^{(k)(l)} + \frac{G}{1 - 2\nu} \hat{u}_{j,i}^{(k)(l)} = \delta_{j,l} f^{(k)} \quad (i, j, l = 1, 2)
\]

(5.9)
Solving Equation 5.9, one obtains:
\[ u_j^{(l)} = \frac{1 - 2\nu}{(5 - 4\nu)G} r_{ij} r_{il} r^2 + \]
\[ + \frac{30(1 - \nu)}{30(1 - \nu)} \left( \frac{10\nu}{3} \delta_{jl} - r_{ij} r_{il} \right) r^3 \]  
(5.10)

The traction field corresponding to Equation 5.10 is:
\[ \hat{p}_j^{(l)} = \frac{2(1 - 2\nu)}{5 - 4\nu} \left\{ \frac{1 + \nu}{1 - 2\nu} r_{ij} n_{il} + \frac{1}{2} r_{il} n_{j} + \frac{1}{2} \delta_{jl} \frac{\partial r}{\partial n} \right\} r + \]
\[ + \frac{1}{15(1 - \nu)} \left\{ (4 - 5\nu) r_{il} n_{j} - (1 - 5\nu) r_{ij} n_{l} + \right. \]
\[ \left. + [(4 - 5\nu) \delta_{jl} - r_{ij} r_{il}] \frac{\partial r}{\partial n} \right\} r^2 \]  
(5.11)

Discretizing the unknowns over the boundary \( \Gamma \), and following the procedure shown in Section 3.3.5, Equation 5.7 can be expressed in matrix form:

\[ ([H] - \lambda[R])\{u^t\} - [G]\{p^t\} = [R]\{b\} \]  
(5.12)

where, \( H \), \( R \), \( u^t \), \( p^t \), and \( b \) have been defined in Section 3.3.4.

Applying the boundary conditions (\( N \) values of \( u^t_i \) and \( p^t_i \) are known over the entire boundary \( \Gamma \)), and rearrange Equation 5.12 by moving all the unknowns to the left-hand side of the equation, and all the known terms to the right-hand side, one obtains the reduced form of the DRM equation:

\[ A\{y\} = \{v\} \]  
(5.13)

where \( A \) is a \( 2(N + L) \times 2(N + L) \) matrix, \( y \) contains \( 2(N + L) \) unknown model values of \( u^t_i \) and \( q^t_i \), and \( v \) is determined by the boundary condition and the nodal values of displacement, velocity, and acceleration at the previous time step.

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5.1.3 Combination of Regions

In the physical model shown in Figure 3.1, it can be seen that the whole domain is divided into regions. Because of the differences in material properties, each of the regions must be represented by one BEM equation, which represents the material of the region. The global BEM equation for the entire domain is constructed by combining the equations for all the regions.

When two regions $A$ and $B$ are combined, both the equilibrium equation

$$p_i^{(A)} = -p_i^{(B)}$$  \hspace{1cm} (5.14)

and the compatibility equation

$$u_i^{(A)} = u_i^{(B)}$$  \hspace{1cm} (5.15)

at the interface $\Gamma_I$ should be satisfied. At a smooth interface, the BEM equation for the combined region is given by considering the above two equations (Brebbia and Walker, 1980):

$$\begin{bmatrix} H^{(1)} & H^{(1)} & -G^{(1)} \\ 0 & H^{(2)} & G^{(2)} \\ \end{bmatrix} \begin{bmatrix} U^{(1)} \\ U_I \\ P_I \\ \end{bmatrix} = \begin{bmatrix} G^{(1)} \\ 0 \\ G^{(2)} \\ \end{bmatrix} \begin{bmatrix} P^{(1)} \\ P^{(2)} \\ \end{bmatrix}$$  \hspace{1cm} (5.16)

where,

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$
and the subscript \( I \) denotes the interface \( \Gamma_I \).

Corners at the interface need special treatment because the geometric discontinuity. Figure 5.1 shows two interface elements \( J \) and \( J - 1 \). In general, the traction in element \( J - 1 \) will not be equal to the traction in element \( J \). Assuming that element \( J \) and element \( J - 1 \) join at node \( j \), one can write the BEM equation at \( j \) as:

\[
\begin{bmatrix}
  g^{j-1}_{11} & g^{j-1}_{12} & 0 & 0 & h^{j-1}_{11} & h^{j-1}_{12} \\
  g^{j-1}_{21} & g^{j-1}_{22} & 0 & 0 & h^{j-1}_{21} & h^{j-1}_{22} \\
  0 & 0 & g^{j}_{11} & g^{j}_{12} & h^{j}_{11} & h^{j}_{12} \\
  0 & 0 & g^{j}_{21} & g^{j}_{22} & h^{j}_{21} & h^{j}_{22}
\end{bmatrix}
\begin{bmatrix}
  p^{j-1}_1 \\
  p^{j-1}_2 \\
  p^j_1 \\
  p^j_2 \\
  u_1 \\
  u_2
\end{bmatrix}
= \begin{bmatrix}
  b^{j-1}_1 \\
  b^{j-1}_2 \\
  b^j_1 \\
  b^j_2
\end{bmatrix}
\]  

(5.17)

where \( b_i \) is a known factor obtained from the DRM operation.

If node \( j \) is on a smooth interface, \( p^{j-1}_k \) and \( p^j_k \) \((k = 1, 2)\) are equal, then there are four equations with four unknowns in Equation 5.17. If \( j \) is at a corner, the discontinuity in geometry leads to difference in traction in element \( J \) and element \( J - 1 \). In this case, there would be six unknowns at the node with only four independent equations. In order to solve the problem, two more equations are needed.

The extra equations can be found by means of finite element analysis. When the 2-node linear element is used in the BEM analysis, a triangular finite element can be defined at the corner of the interface, as shown in Figure 5.2. In the triangular
element, the strain components are given by:

\[ \epsilon_{11} = b_1 u_1^{(1)} + b_2 u_1^{(2)} + b_3 u_1^{(3)} \]
\[ \epsilon_{22} = c_1 u_2^{(1)} + c_2 u_2^{(2)} + c_3 u_2^{(3)} \]
\[ \epsilon_{12} = \epsilon_{21} = c_1 u_1^{(1)} + b_1 u_2^{(1)} + c_2 u_1^{(2)} + b_2 u_2^{(2)} + c_3 u_1^{(3)} + b_3 u_2^{(3)} \]  \hspace{1cm} (5.18)

where \( u_i^{(j)} \) is the displacement in \( i \) direction at node \( j \); \( b_j \) and \( c_j \) are given by

\[ b_j = x_2^{(k)} - x_2^{(l)}, \quad c_j = -x_1^{(k)} + x_1^{(l)}, \quad (i, j, k) \]

The stress components in the element are given by

\[ \sigma_{ij} = 2G\epsilon_{ij} + \frac{2G\nu}{1-2\nu}\epsilon_{kk}\delta_{ij} \]  \hspace{1cm} (5.19)

The traction in a boundary element is defined as:

\[ p_i^k = \sigma_{ij} n_j^k \]  \hspace{1cm} (5.20)

where \( k \) denotes the \( k^{th} \) boundary element. In Figure 5.2, the vector \( n_j^k \) is given by:

\[ n_1^{(1)} = 0 \]
\[ n_2^{(1)} = 1 \]
\[ n_1^{(2)} = \sin \theta \]
\[ n_2^{(2)} = \cos \theta \]
therefore, the traction can be expressed as

\[
p_1^{(1)} = \sigma_{12} \\
p_2^{(1)} = \sigma_{22} \\
p_1^{(2)} = \cos \theta + p_1^{(1)} \cos \theta \\
p_2^{(2)} = p_1^{(1)} \sin \theta + p_2^{(1)} \cos \theta
\]  

(5.21)

Substituting Equation 5.18 and Equation 5.19 into Equation 5.21, one obtains:

\[
p_1^{(2)} = 2G\{ (b_1 u_1^{(1)} + b_2 u_2^{(1)} + b_3 u_3^{(1)}) + \frac{1}{1 - 2\nu} (b_1 u_1^{(1)} + c_1 u_2^{(1)} + + b_2 u_2^{(2)} + c_2 u_1^{(2)} + b_3 u_3^{(2)} + c_3 u_2^{(3)})\} \sin \theta + p_1 \cos \theta \\
p_2^{(2)} = p_1^{(1)} \sin \theta + p_2^{(1)} \cos \theta
\]  

(5.22)

Equation 5.22 provides the two extra equations required at the corner node.

By using a triangular finite element, the problem of region combination has been converted into a well-posed one. It should be noticed that the assumption of constant stress within the triangular element will produce an error in computing the traction at the corner, therefore, the size of the elements connected to the corner node should not be too large.

Now the 2-D liquefaction and ground response problem can be solved by means of the boundary element method. In the following section, examples will be presented, and the effects of alluvial valley will be discussed.

## 5.2 PARAMETRIC STUDY

In Section 2.3, it is mentioned that the maximum amplification in ground motion may be observed in different locations for different alluvial valleys. It is also
discussed that the amplification may be controlled by the shape of the valley. It is believed that the “shape ratio” of an alluvial valley might be a controlling factor for the amplification effect (Table 2.1). The shape ratio is the ratio of the depth to the width of the valley.

In order to evaluate the effects of alluvial valley on ground motion and liquefaction, artificial valleys will be analyzed by means of the numerical method developed in Section 3.3 and Section 5.1.2. The effects of the geometry of the valley, and the effects of material properties, mainly the shear modulus, will be evaluated.

5.2.1 Geometry and Material

The geometry of the example valleys is shown in Figure 5.3. Eight valleys in different depth and width are used in the analysis. The depths of the valleys, $D$, and the widths of the valleys, $W$, are listed in Table 5.1.

As shown in Figure 5.3, the example sites contain two layers, one soft soil layer(fill), and one stiff layer(base).

Because of the geometrical symmetry of the problem, only half of the valley is used in the computation. The model used in the computing is shown in Figure 5.4. The boundary element mesh for a deep valley (Valley No. 7 in Table 5.1) is shown in Figure 5.5. Meshes for other valleys are similar.

Different kinds of materials are used in the analysis. The shear wave velocity in the soft layer varies from 158 m/s to 316 m/s. The shear wave velocity in the stiff layer varies from 500 m/s to 929 m/s. The material properties for the soft layer and the stiff layer are listed in Table 5.2.
It is assumed that there exists a rigid bedrock at the depth of 54 m. The horizontal input motion is applied at the top of the rigid bedrock.

5.2.2 Input Motion

A real earthquake record and an artificial wavelet are used in the analysis of the artificial-valleys.

Real earthquake records are used in many cases. When a real record is used as the input motion, it is difficult to study the response over an interested frequency range because the frequency content in the record is complex.

Artificial wavelet can be adopted in the study of the wave propagation characteristics of a site. By using a special designed wavelet, the effects of the geometry or material parameters on certain frequency components can be revealed.

The artificial input motion used in the study of site response analysis should be created to meet the following requirements:

1. it contains only the frequency of interest;
2. the frequency components are uniform in amplitude;

In studying the reaction of the alluvial valley, an artificial wavelet called δ–pulse (Li, 1988) can be used as the input motion.
The δ–pulse is expressed as:

\[
F(t) = \begin{cases} 
16 \left( \frac{t}{T_o} \right) & 0 \leq t \leq 0.25T_o \\
1 - 48 \left( \frac{t}{T_o} - 0.5 \right)^2 \left( \frac{t}{T_o} \right) & 0.25T_o \leq t \leq 0.5T_o \\
1 + 48 \left( \frac{t}{T_o} - 0.5 \right)^2 \left( \frac{t}{T_o} - 1 \right) & 0.5T_o \leq t \leq 0.75T_o \\
-16 \left( \frac{t}{T_o} - 1 \right)^3 & 0.75T_o \leq t \leq T_o \\
0 & t < 0 \text{ or } t > T_o 
\end{cases}
\] (5.23)

This function represents a pulse in the time axis.

The frequency content in a δ–pulse is controlled by the factor $T_o$. The wavelet contains all the frequency components lower than $f_{\text{main}} = \frac{1}{T_o}$, with an equal amplitude.

The δ pulse used in this section has a $T_o = 0.1$. It contains no high frequency components. By using this wavelet as the input, the ground motion in the frequency range of interest (0.5 to 10 Hz) can be shown clearly.

The time history and the frequency contents of the δ–pulse are shown in Figure 5.6.

### 5.2.3 Ground Motion in Valleys

The ground responses in artificial valleys have been computed by means of the boundary element method developed in Section 3.3 and Section 5.1.

The 2–D BEM program is tested by comparing the computational result obtained from the 1-D analysis and the result obtained by the 2–D program for the same example. The example is a simple site with two evenly layered soil layers.
The top layer has a shear modulus of $1.425 \times 10^8$ Pa, the base layer has a shear modulus of $1.425 \times 10^9$ Pa.

Figure 5.7 shows the acceleration time histories obtained from 1-D and 2-D computation, respectively. It can be found that the result from 2-D computation agrees well with that from 1-D computation, which has been compared with a theoretical solution (Section 4.1.2).

It is found that when the time interval $\Delta t$ is shorter than 0.05 second, the computation will be unstable.

The effects of the shape factor and the material properties will be studied by using the $\delta$ pulse as the input motion. The real record recorded at Port Island site will be used for the further studies of the valley, and for the liquefaction analysis.

In the following discussion, an amplification factor is defined to describe the amplification effect of the valley. The amplification factor of spectral acceleration is defined as the peak spectral acceleration of the valley divided by the peak spectral acceleration of a reference site with the depth of soft layer equals to the depth of the valley. The amplification factor of spectral velocity is similarly defined.

**The Effects of Shape Ratio**

The ground motions in eight alluvial valleys are computed (Table 5.3 and Table 5.4). Valley No.1, 2, 3, and 4 are shallow valleys with the depth of 12 m. The shape ratios are 0.2, 0.15, 0.10, and 0.075, respectively. Valley No.5, 6, 7, and 8 are deep valleys. The depth of valley No.5 is 36 m. The depths of valley No.6, 7, and 8 are 24 m. The shape ratios are 0.45, 0.4, 0.3, and 0.2, respectively. The
material for the soft layer is material No.T–3, with a shear modulus of $1.425 \times 10^8$ Pa, the material for the stiff layer is material No.B–3, with a shear modulus of $1.425 \times 10^9$ Pa. Values of material parameters for both of the materials can be found from Table 5.2.

The acceleration time histories at three points on the ground surface are plotted for each valley. The locations of the points are: Point 1, at the centerline of the valley, Point 2, 8 m from the edge, Point 3, at the edge of the valley.

The ground acceleration time histories and response spectra of four sites with evenly layered soils are plotted as references. Reference site R–1 represents the site far from the valleys, where the thickness of the soft layer is 4 m (top soil). Reference site R–2 has the thickness of the soft layer of 16 m, which equals the thickness of the top soil plus the depth of valleys No.1 to No.4. At reference site R–3, the thickness of soft layer equals 28 m, which is the thickness of the top soil plus the depth of valleys No.6 to No.8. Reference site R–4 has the thickness of soft layers of 40 m, corresponding to site No.5. The computed ground acceleration time histories of the reference sites are plotted in Figure 5.8. The response spectra are shown in Figure 5.9 and Figure 5.10.

Figure 5.11 and Figure 5.12 are the computed ground acceleration time histories. The figures illustrate that for valley No.1, which has the lowest shape ratio, the peak ground acceleration at the edge is slightly higher than that at the middle of the valley. When the shape ratio is greater than 0.1, the peak accelerations at the middle of the valleys are higher than that at the edges.

Figure 5.13 to Figure 5.16 are the acceleration response spectra and the velocity response spectra of the computed ground acceleration histories. The response
spectra for the reference sites R–2 and R–3 are also plotted in the figures.

From Figure 5.13 to Figure 5.16, it is found that:

1. For both the spectral acceleration and the spectral velocity, the maximum amplification is observed at the middle of the valley.

2. At the middle of a valley, the amplification factors of both the spectral acceleration and the spectral velocity are significantly high in the period ranges from 0.2 second to 0.5 second.

3. Higher shape ratio results in a higher amplification factor throughout the width of the valley. When the shape ratio is smaller than 0.2, the spectral velocity at the edges of a valley will not be amplified.

Table 5.3 and Table 5.4 show the amplification factors for shallow valleys and deep valleys, respectively. These two tables indicate that in general, greater shape ratio gives a higher amplification to the motion.

The two tables indicate that the deep valley ($D = 24$ m) gives greater amplification factors than the shallow valley, at the same shape ratio $R = 0.2$ (Valley No.1 and valley No.8). It is also found that for valley No.5, which has a depth of 36 m and a shape ratio of 0.45, the amplification for both the spectral acceleration and the spectral velocity is lower than that for valley No.6, which has a depth of 24 m and a shape ratio of 0.4. Therefore, it is believed that the amplification factor is affected by the depth of the valley as well.
The Effects of Shear Modulus

Valleys with different material properties are studied in order to find the effects of material properties. From test computing, it is found that the effect of Poisson’s ratio is not significant (Figure 5.17). Test computing also indicates that changing the damping ratio from 0.01 to 0.03 only affects the amplitude of the ground acceleration (Figure 5.18). It does not change the amplification effect of the valley.

Table 5.5 and Table 5.6 are the amplification factors of the spectral acceleration and the spectral velocity, obtained from different shear moduli. From the two tables, it is found that:

1. The amplification factor of both the spectral acceleration and the spectral velocity increases with the decrease of the shear modulus of the fill material (the soft layer).

2. The amplification factor increases with the decrease of the shear modulus of the base material (stiff layer).

These two phenomena are observed when the shear modulus of the fill material ranges from \(4.75 \times 10^7\) Pa to \(19 \times 10^7\) Pa, and the shear modulus of the base material ranges from \(4.75 \times 10^8\) Pa to \(19 \times 10^8\) Pa.

Figure 5.19 and Figure 5.20 show the acceleration response spectra and the velocity response spectra for the shallow valleys, with the shear modulus for the stiff layers \(G_{s2}\) equals \(14.25 \times 10^8\), and the shear modulus for the soft layers \(G_{s1}\) equals \(4.75 \times 10^7\) Pa, \(9.5 \times 10^7\) Pa, \(14.25 \times 10^7\) Pa, and \(19 \times 10^7\) Pa, respectively.

From the response spectra, it is found that with the decrease of shear modulus of
the fill material, the maximum value of the amplification factor of spectral velocity moves to the short period side. This is because that in the reference site with evenly layered soils, site R-2, the maximum spectral velocity appears at a longer period when the shear modulus of the top layer is lower. As the shear modulus of the soft layer becomes higher, the peak of the velocity response spectrum for the site R-2 moves to the short period side. This change in the peak of the velocity response spectrum is not significant in a valley. Figure 5.20 indicates that when the shear modulus $G_{s1}$ is $4.75 \times 10^7$ Pa, the maximum spectral velocity for the valley appears at the period of 0.4 second, and that for the reference site R-2 occurs at the period of 0.7 second. When the shear modulus $G_{s1}$ is $19 \times 10^7$ Pa, the maximum spectral velocities for both the valley and the site R-2 appear at the period of 0.3 second.

The Responses to a Real Earthquake Record

A recorded acceleration time history is applied to valley No.7 (depth = 24 m, width = 80 m), in order to study the response of alluvial valleys to the real earthquake motion. The time history used in the analysis is the horizontal motion recorded at the Port Island.

The amplification effect of the valley is observed again. Figure 5.21 shows that at the middle of the valley, the peak spectral acceleration is amplified by 189%, and the peak spectral velocity is amplified by 128%.

Table 5.9 and Table 5.10 illustrate the amplification in peak spectral acceleration and the peak spectral velocity, respectively. The tables show that at certain combination of the shear moduli $G_{s1}$ and $G_{s2}$, the peak spectral velocity and the peak spectral acceleration for the valley are lower than those for the reference site.
From these two tables, it is difficult to find out how the shear modulus affects the amplification of valleys.

For further study of the amplification effects of alluvial valleys, acceleration time histories and response spectra for valleys with different fill materials are plotted in Figures 5.22 and Figure 5.23. It is found that, as discussed above, the amplification effects of the valley are different for the seismic motion with different frequency. This phenomenon explains the irregularity shown in Table 5.9 and Table 5.10: The motion within a certain frequency range will be amplified by the valley. This frequency range depends on the material properties of the soil. Under certain combination of material properties, the peak spectral acceleration or velocity may not be amplified as it may not fall into the “amplified” frequency range.

From the above discussions, it can be found that when the $\delta$ wavelet is used as the input motion, the amplification effects of a site can be easily revealed, because of its frequency components are in the same amplitude within the designed range ($T \leq T_o$).

### 5.2.4 Liquefaction

The analysis of liquefaction is carried out on a simple example. The BEM model is shown in Figure 5.24. The geometry of the valley is similar to the deep valley discussed in the previous section. The whole valley is filled by liquefiable soil, with the liquefaction strength as same as that for the fill material in Port Island.

The liquefiable soil layer is divided into 3 zones, as shown in Figure 5.24. The
input motion is taken from the Port Island record at the depth of 32m. The time interval used in the computation is 0.02 second.

Figure 5.25 shows the build-up of pore-water pressure. At the beginning of the time history, the pore-pressure build-up in the edge zone is slower than that in the other two zones. After the 5th second, the pore pressure in the edge zone increase quickly and become higher than that in the other two zones. The liquefaction starts from the edge zone.

Figure 5.25 also shows that the build-up in pore water pressure in the valley (zone 1, 2, and 3) is faster than that in a free field. This result explains the observations in major earthquakes, as mentioned in Section 2.1. In river sites, the liquefaction potential is high because of two reasons: First, the high water table and the loose sandy deposit are often found in the site. These site conditions result in a low liquefaction strength. Second, in the alluvial valley formed by the old river course, the generation of excess pore water pressure is faster than that in a site with evenly layered soils.

5.2.5 Discussion

The seismic motion is amplified throughout the width of an alluvial valley. The maximum spectral acceleration and the maximum spectral velocity occur at the center of the valley. This explains the heavy damage found in Zone 2 in Kirovaken. The cross section shown in Figure 2.18 indicates that the heavily damaged zone is located in the middle of an alluvial valley.

The shape ratio will affect the amplification effect of a valley. It is found from
the analysis that a higher shape ratio results in a higher amplification in both the spectral acceleration and the spectral velocity.

It is also found that the shape ratio is not the only parameter to control the effect of geometry on the amplification of a valley. The amplification also depends on the depth of the valley.

The differences between the patterns of the amplification for shallow valleys and deep valleys, as suggested in Table 2.1, have not been observed.

Shear modulus is another important factor to determine the amplification effects of an alluvial valley. It is found that the decrease in the shear moduli of both the fill material and the base material will result in an increase in the spectral acceleration and the spectral velocity.

In an alluvial valley, the amplification to the motion mainly occurs at the short period side in the response spectrum. The maximum amplification will move further to the short period side when the shear modulus of the fill material is low.

When saturated sandy soil is subjected to seismic loading, the pore pressure build-up in the soil will lead to a liquefaction. In an alluvial valley, the pore pressure build-up is faster than that in an evenly layered site. Therefore, for a site with liquefiable soil filled in an alluvial valley, the liquefaction potential will be high. This phenomenon has been observed in major earthquakes.

Based on the computational results, it is believed that in earthquake zone, a site with alluvial valley is not suitable for construction. The problems caused by valleys include the amplified ground motion, the higher liquefaction potential, and the significant change in the magnitude and frequency components of the ground
motion within a small area.

5.3 SUMMARY

The boundary element formulation developed in Chapter 3 can be used in 2-D response analysis. At the corner nodes on the interface of two regions, where the discontinuity in geometry is presented, the discontinuous traction can be calculated by extra equations. The concept of stress analysis in FEM is used in deriving these two equations.

The liquefaction model developed in Chapter 3 can be adopted in the boundary element analysis. The BEM analysis for soil liquefaction is limited to simple cases only. For a detailed analysis of pore-pressure build-up, FEM should be used to represent the liquefiable soil.

The amplification effects of alluvial valleys have been studied by computing the response of artificial valleys in different shapes. It is found that the increase in shape ratio will cause an increase in the amplification factor throughout the width of the valley. The amplification factor is also affected by the depth of the valley.

The effects of material properties on the amplification effect of valleys have also been studied. The analysis shows that the decrease in the shear modulus of both the base and the fill material will result in an increase in the amplification factor.

The liquefaction potential in an alluvial valley is higher than that in an evenly layered site. The liquefaction starts from the edges of the valley.
### Table 5.1: Geometries of artificial valleys

<table>
<thead>
<tr>
<th>Valley No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth $D$ (m)</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>36</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>Width $W$ (m)</td>
<td>60</td>
<td>80</td>
<td>120</td>
<td>160</td>
<td>80</td>
<td>60</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>Shape Ratio $R$</td>
<td>0.2</td>
<td>0.15</td>
<td>0.1</td>
<td>0.075</td>
<td>0.45</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### Table 5.2: Material Properties

<table>
<thead>
<tr>
<th>Material No.</th>
<th>Shear Wave Velocity, $V_s$ (m/s)</th>
<th>Shear Modulus $G_s$ (Pa)</th>
<th>Poisson's Ratio, $\nu$</th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Damping ratio $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-1</td>
<td>158</td>
<td>$4.75 \times 10^7$</td>
<td>0.3</td>
<td>1900</td>
<td>0.02</td>
</tr>
<tr>
<td>T-2</td>
<td>224</td>
<td>$9.5 \times 10^7$</td>
<td>0.3</td>
<td>1900</td>
<td>0.02</td>
</tr>
<tr>
<td>T-3</td>
<td>274</td>
<td>$14.25 \times 10^7$</td>
<td>0.3</td>
<td>1900</td>
<td>0.02</td>
</tr>
<tr>
<td>T-4</td>
<td>316</td>
<td>$19 \times 10^7$</td>
<td>0.3</td>
<td>1900</td>
<td>0.02</td>
</tr>
<tr>
<td>B-1</td>
<td>500</td>
<td>$4.75 \times 10^8$</td>
<td>0.3</td>
<td>1900</td>
<td>0.01</td>
</tr>
<tr>
<td>B-2</td>
<td>707</td>
<td>$9.5 \times 10^8$</td>
<td>0.3</td>
<td>1900</td>
<td>0.01</td>
</tr>
<tr>
<td>B-3</td>
<td>804</td>
<td>$14.25 \times 10^8$</td>
<td>0.3</td>
<td>2200</td>
<td>0</td>
</tr>
<tr>
<td>B-4</td>
<td>929</td>
<td>$19.0 \times 10^8$</td>
<td>0.3</td>
<td>2200</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 5.3: Effects of Shape: Amplification factors in Shallow Valleys

<table>
<thead>
<tr>
<th>Valley No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth/Width</td>
<td>12 m/60 m</td>
<td>12 m/80 m</td>
<td>12 m/120 m</td>
<td>12 m/160 m</td>
</tr>
<tr>
<td>Shape Ratio</td>
<td>0.2</td>
<td>0.15</td>
<td>0.1</td>
<td>0.075</td>
</tr>
<tr>
<td>Amplification in Spec. Acc.</td>
<td>1.77</td>
<td>1.63</td>
<td>1.39</td>
<td>1.26</td>
</tr>
<tr>
<td>Amplification in Spec. Vel.</td>
<td>1.22</td>
<td>1.24</td>
<td>1.23</td>
<td>1.20</td>
</tr>
</tbody>
</table>

### Table 5.4: Effects of Shape: Amplification factors in Deep Valleys

<table>
<thead>
<tr>
<th>Valley No.</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth/Width</td>
<td>36 m/80 m</td>
<td>24 m/60 m</td>
<td>24 m/80 m</td>
<td>24 m/120 m</td>
</tr>
<tr>
<td>Shape Ratio</td>
<td>0.45</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Amplification in Spec. Acc.</td>
<td>1.57</td>
<td>1.75</td>
<td>1.68</td>
<td>1.54</td>
</tr>
<tr>
<td>Amplification in Spec. Vel.</td>
<td>1.25</td>
<td>1.31</td>
<td>1.29</td>
<td>1.25</td>
</tr>
</tbody>
</table>
Table 5.5: Amplification Factors of Spectral Acceleration: Shallow Valley

<table>
<thead>
<tr>
<th>$G_s_2$</th>
<th>$G_s_1 = 4.75 \times 10^7$</th>
<th>$G_s_1 = 9.5 \times 10^7$</th>
<th>$G_s_1 = 14.25 \times 10^7$</th>
<th>$G_s_1 = 19 \times 10^7$</th>
<th>$G_s_1 = 47.5 \times 10^7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.75 \times 10^8$</td>
<td>1.68</td>
<td></td>
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<td>1.52</td>
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<td>1.68</td>
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<tr>
<td>$14.25 \times 10^8$</td>
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<td>1.66</td>
<td>1.63</td>
<td>1.70</td>
<td>1.39</td>
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Table 5.6: Amplification Factors of Spectral Velocity: Shallow Valley

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<th>$G_s_2$</th>
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<th>$G_s_1 = 19 \times 10^7$</th>
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<td>1.36</td>
<td>1.33</td>
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<tr>
<td>$14.25 \times 10^8$</td>
<td>1.33</td>
<td>1.29</td>
<td>1.24</td>
<td>1.22</td>
<td>1.20</td>
</tr>
<tr>
<td>$19 \times 10^8$</td>
<td></td>
<td></td>
<td>1.24</td>
<td>1.20</td>
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Table 5.7: Amplification Factors of Spectral Acceleration: Deep Valley

<table>
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<th>$G_s_1 = 14.25 \times 10^7$ Pa</th>
<th>$G_s_1 = 19 \times 10^7$ Pa</th>
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<tr>
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<td>1.92</td>
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<tr>
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<td>1.92</td>
<td>1.74</td>
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<td>$14.25 \times 10^8$ Pa</td>
<td>1.96</td>
<td>1.87</td>
<td>1.68</td>
<td>1.67</td>
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Table 5.8: Amplification Factors of Spectral Velocity: Deep Valley

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<th>$G_{s2}$ = 9.5 × 10^7 Pa</th>
<th>$G_{s2}$ = 14.25 × 10^7 Pa</th>
<th>$G_{s2}$ = 19 × 10^7 Pa</th>
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</thead>
<tbody>
<tr>
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<td>1.42</td>
<td>1.37</td>
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<td>1.30</td>
<td>1.29</td>
<td>1.27</td>
</tr>
<tr>
<td>$G_{s2}$ = 19 × 10^8 Pa</td>
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<td>1.29</td>
<td>1.26</td>
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Table 5.9: Amplification Factors of Spectral Acceleration: Real Record

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<th>$G_{s2}$ = 4.75 × 10^8 Pa</th>
<th>$G_{s2}$ = 9.5 × 10^8 Pa</th>
<th>$G_{s2}$ = 14.25 × 10^8 Pa</th>
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<td>$G_{s2}$ = 19 × 10^8 Pa</td>
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Table 5.10: Amplification factors of Spectral Velocity: Real Record

<table>
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<th>$G_{s1}$ = 4.75 × 10^7 Pa</th>
<th>$G_{s2}$ = 4.75 × 10^8 Pa</th>
<th>$G_{s2}$ = 9.5 × 10^8 Pa</th>
<th>$G_{s2}$ = 14.25 × 10^8 Pa</th>
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<td>1.01</td>
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<tr>
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<td>1.28</td>
<td>1.0</td>
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<tr>
<td>$G_{s2}$ = 19 × 10^8 Pa</td>
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<td></td>
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<td>1.05</td>
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</table>
Figure 5.1: The Interface of Two Regions
Figure 5.2: The Corner Node on the Interface
Figure 5.3: The Geometry of the Artificial Valley
Figure 5.4: The Physical Model Used in Computation
Figure 5.5: The BEM Mesh for a Deep Valley (Valley No. 7)
Figure 5.6: Delta Pulse
Evenly layered site with two layers
Thickness of soft layer (top) = 16 m
Thickness of stiff layer (base) = 38 m
Gs_1 = 14.25e7
Gs_2 = 14.25e8
Input motion: Delta wavelet

Figure 5.7: The Acceleration Time Histories for 1-D and 2-D analysis
Figure 5.8: Acceleration Time Histories at Reference Sites
Figure 5.9: Acceleration Response Spectra for Reference Sites
Figure 5.10: Velocity Response Spectra for Reference Sites
Figure 5.11: Acceleration Time Histories for Shallow Valleys (D = 12 m)
Figure 5.12: Acceleration Time Histories for Deep Valleys (D = 24 m and D = 36 m)
Figure 5.13: Acceleration Response Spectra for Shallow Valleys
Figure 5.14: Acceleration Response Spectra for Deep Valleys
Figure 5.15: Velocity Response Spectra for Shallow Valleys
Figure 5.16: Velocity Response Spectra for Deep Valleys
a) Point 1 (at the center of the valley)

b) Point 2 (24 m from the centerline)

c) Point 3 (at the edge of the valley)

Figure 5.17: The Effect of Poisson’s Ratio
Figure 5.18: The Effect of Damping Ratio
Figure 5.19: Acceleration Response Spectra for Shallow Valley with Different Material Properties
Figure 5.20: Velocity Response Spectra for Shallow Valley with Different Material Properties
Figure 5.21: Acceleration Time Histories for Deep Valley Subjected to Recorded Seismic Motion
Figure 5.22: Acceleration Response Spectra for Deep Valley Subjected to Recorded Seismic Motion
Figure 5.23: Velocity Response Spectra for Deep Valley Subjected to Recorded Seismic Motion

$G_{S1} = 14.24 e^7$ Pa, $G_{S2} = 14.2 e^8$ Pa
Figure 5.24: The Physical Model Used in Liquefaction Analysis
Figure 5.25: Excess Pore-water Pressure Build-up
Chapter 6

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

6.1 CONCLUSIONS

The effects of local geological condition on seismic ground motion and liquefaction potential, and the effects of soil liquefaction on seismic ground motion, have been studied in this thesis, to improve design of structures in earthquake zones.

The effects of local geology on seismic ground motion and liquefaction potential have been observed for a long time. However, detailed studies of the effects have not yet been reported. The study of these effects requires a 2-D time domain numerical method, with the ability of estimating the pore pressure build-up in liquefiable soil. The existing FEM method requires huge computational resource. The existing BEM methods are not suitable for the analysis of site response and soil liquefaction problems.

In this thesis, numerical tools have been developed to conduct the 1-D and 2-D
site response and liquefaction analyses. A new liquefaction model is established to represent the pore-pressure build-up and the subsequent softening process in a soil element. This model is based on the energy approach, which defines the excess pore-pressure as a function of the accumulative energy dissipation. This model is simple in form. The model parameters are easy to obtain. Different kinds of constitutive relationships of soil can be adopted in this model.

A new boundary integral equation has been developed for the analysis of the dynamic response of the soil. The visco-elastic constitutive relationship of the soil material is considered. The analysis is conducted in time domain, in order to estimate the pore-pressure build-up in liquefiable soil layers. The BEM formulation is developed by applying the time discretization to the governing equation, and adopting the DRM method to solve the non-homogeneous differential equation. An approximation function based on the theoretical solution has been derived for 1-D analysis.

Both case studies and numerical analyses conducted in this thesis show that the liquefaction of subsoil layers will change the characteristics of seismic ground motion. During an earthquake, the high frequency component in the horizontal ground motion is filtered out by the liquefied soil layers. The change in the frequency content of the ground motion reduces its power of damage. On the other hand, soil liquefaction will cause ground failure and loss of bearing capacity. This effect of liquefaction is called "the dual effects of liquefaction". In this thesis, the dual effects of liquefaction have been studied by analyzing the Port Island case. Both the computational results and the ground motion record show significant reduction in the spectral acceleration and spectral velocity for the motion with the period of 0.2 second to 2.5 second. The vertical motion is not affected by soil
liquefaction.

The amplification effects of alluvial valleys have been studied. It is found that the ground motion is amplified in valleys with different shape and material properties. The amplification can be described by an amplification factor defined in Chapter 5. The maximum value of the amplification factor is always observed at the middle of an alluvial valley.

The amplification effects of a valley are determined by the shape ratio and the depth of the valley, and the shear moduli of the fill and base materials. It is found that for a valley with certain depth, the increase in shape ratio results in an increase of amplification factor. It is also found that the amplification factor increases with the decrease of shear moduli of both the fill and the base materials.

For the valleys examined in the parametric studies, the amplification factor of the spectral acceleration ranges from 1.2 to 2.2, the amplification factor of the spectral velocity ranges from 1.2 to 1.5.

The 2-D liquefaction shows that higher liquefaction potential is expected in an alluvial valley. This result explains the fact that liquefaction potential is generally high in the sites with old river courses or other kinds of underground unevenness.

6.2 SUGGESTIONS FOR FUTURE WORK

Considering the work presented in this thesis, the following work is suggested for future study on the effects of local geology and liquefaction on ground motion:
1. The study of constitutive relation of liquefiable soils. The liquefaction model based on energy approach can be improved by adopting a constitutive model with better description of the rate-independent hysteresis damping of soil.

2. The development of a BEM/FEM hybrid program. FEM analysis is required in the detailed study of pore-pressure build procedure, and the liquefaction analysis for real cases with complicated geometry.

3. The development of a new BEM system for 2-D liquefaction analysis. A pure BEM liquefaction analysis program can be developed by improving the existing 2-D program. The improvement includes selecting an appropriate element to build the liquefiable cells, and adopting an optimized procedure in building and renewing the parametric matrix. The pure BEM program will be able to use sufficient number of cells to represent the liquefiable soil, with less computing cost than the BEM/FEM hybrid program.
Bibliography


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