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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS REÇUE
SWAY EFFECTS

IN

UNBRACED MULTI-STORY STEEL FRAMES

by

M.M. SINGHAL

B.E. (Civil)

A THESIS

Presented to the Faculty of Graduate Studies of Carleton University
in partial fulfillment of the requirements for the degree of

MASTER OF ENGINEERING

Carleton University

Ottawa, Canada

April 1979
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acceptance of the thesis

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submitted by M.M. SINGHAL, B.E. (Civil) in partial fulfillment of the
requirements for the degree of

MASTER OF ENGINEERING

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ABSTRACT

A method for the second order analysis of unbraced multi-story steel buildings has been developed. The method is based on a modification of the stiffness matrices of the columns. It avoids the necessity of assumptions regarding the distribution of moments throughout the length of a column, the location of the point of contraflexure, etc. and does not depend on an iterative procedure. A computer program has also been developed for the application of the above method to the analysis of frames. The program requires only slightly more coding than a comparable first order program and very little additional input. The scope of this method has been extended to account for initial eccentricity in columns resulting from erection tolerances or any other reasons.

True strength of a column in a single story frame has been derived and the relative values of the column moments resulting from primary loads and the sway effects have been obtained. The total moment carrying capacity of the column as well as its primary moment capacity has been plotted and compared with the corresponding capacities obtained from interaction equations. It has been shown that, in general, the moment carrying capacities predicted by interaction equations are conservative when equations applicable to sway permitted cases are used along with a first order analysis, but that the predictions overestimate the strength when equations applicable to the sway prevented case are used along with a second order analysis. A Table for Correction Factors to be used in
design of columns when a second order analysis is used has been prepared for varying values of several important parameters such as the slenderness ratio $h/r$, the ratio of column stiffness to beam stiffness, $G$ and the axial load (non-dimensionalized) $P/P_y$.

A series of frames have been designed by two alternative methods: first by performing a first order analysis and designing the columns as if sway was permitted and second by carrying out a second order analysis and designing the columns as if the sway was prevented. It has been found that no significant economy is achieved on the whole but by using a second order analysis the girders become heavier while the columns become lighter.
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CHAPTER 1

INTRODUCTION

1.1 THE PROBLEM

The mounting pressure on urban land due to industrialization is a big incentive for multi-story construction. One of the materials employed for such construction is steel. Steel multi-story structures are usually of tier type construction consisting of vertical columns and horizontal beams. The framing system, in general, consists of a series of parallel frames joined at floor levels by a floor diaphragm. The vertical loads are carried to the foundation by the columns and stability against lateral loads is provided either by bracing or by continuity of joints.

Bracings have to be avoided at times to provide for unobstructed areas and for other architectural reasons. When such is the case, the structure may consist of rigid jointed frames in one or both directions.

The rigid jointed frame structure is, strictly speaking, a space structure but it is reasonable to consider it as comprised of two orthogonal sets of planar frames, each of which can be analyzed and designed for the loads tributary to it. The analysis is based on the classical approach of elasticity and linearity. Linearity implies not only a linear stress-strain relationship for materials but also small deflections so that equilibrium can be formed on undeformed geometry and the interaction of axial loads with member deformation can be neglected.
While the assumption of linear stress strain relationships is quite valid for steel within the range of working stresses, the non-linearity introduced by the forces acting on the deformed geometry is not always negligible. These non-linearities are generally termed as "secondary effects" and, in some cases, must be taken into consideration for correct and realistic analysis. The following secondary effects may be important in unbraced multi-story frames:

(i) The effect of axial force on the stiffness of the column.

(ii) The effect of vertical floor loads acting through the sway displacements of the stories, the so-called PA effects.

(iii) The deformation of joints.

This work is mainly concerned with the investigation of the secondary effects arising from the sources listed as (i) and (ii) above and are dealt with in detail later.

The secondary effects can be taken into account by using a second order analysis method. However, the additional complexity involved has discouraged the use of such methods for building design. Usually a first order analysis (where equilibrium is based on undeformed geometry of the members) is carried out and the member strengths are purposely underestimated so as to leave a margin for the secondary effects. This approach is permitted by the CSA (Canadian Standards Association) Code on the Design of Steel Structures(44).

To summarize, there are two alternative methods of design depending on the type of analysis performed. One method is to carry out a first
order analysis (exclusive of secondary effects) and then design the member so that only part of its capacity is used while the other approach is to carry out a second order analysis (inclusive of secondary effects) and then to design the member so that its full capacity is used. In spite of additional complexity involved in the second approach, it may be preferable to use it for the following reasons:

(a) The method of predicting the member forces is more rational and it is possible to assess the stability effects correctly.

(b) First order analysis method of design makes only an approximate estimate of the secondary effects and provides spare capacity in the members to take care of such effects. Often, this reserve of capacity is underutilized. In such cases, second order analysis methods may lead to savings in the design.

(c) In general, second order analysis methods predict higher girder moments as compared to first order analysis methods. Design of girders for higher moments usually results in increased ultimate strength of the frame (15).

The above advantages of the second order analysis design method provide enough incentive for adopting this method. To avoid the disadvantages or difficulties involved in using second order computer programs, engineers have always attempted to devise some means by which first order analysis results can be converted into second order analysis results without resorting to a complicated procedure. This work is also an effort in that direction.
1.2 LITERATURE SURVEY

In recent years, extensive analytical and experimental investigations have been carried out on the secondary effects in multi-story frames, frame components and also on the influence of these effects on frame behaviour\(^{(36,37)}\). Tests have also been carried out on large scale frames to examine the instability effects in multi-story frames. The results of an experiment on a single story single bay frame subjected to constant vertical loads and an increasing lateral load \(H\) at the story level carried out at Lehigh University\(^{(2)}\) are shown in Fig. 1.1 in the form of a graph between horizontal load \(H\) and horizontal deflection \(\Delta\). The diagram compares the load deflection relationship obtained from a first order elastic plastic analysis (broken lines) and a similar second order analysis (lower dashed curve) with the results of the actual test (full lines joining solid circles).

The following two significant conclusions can be drawn from the results:

(i) The presence of secondary effects has significantly reduced the horizontal load corresponding to the attainment of ultimate capacity of the frame.

(ii) These effects have produced an increase in the deflections and corresponding moments and shears throughout the frame at each stage of loading.

The results of a similar experiment carried out on a single bay four storied frame with a rigid concrete shear wall subjected to constant
FIGURE 1.1 OBSERVED AND PREDICTED LOAD DEFORMATION RELATIONSHIPS

FIGURE 1.2 LOAD-DEFLECTION RELATIONSHIPS
vertical load and increasing horizontal load H are shown in Fig. 1.2(27). In this case also horizontal load H versus deflection A relationships obtained from a first order (broken line) and a second order analysis (lower dashed line) have been compared with the results of the test (solid line joining solid circles). It is seen that in spite of stiff concrete walls, the PH effect causes reduction in the horizontal load at ultimate strength and increased deflections at all stages of loading. The reduction in ultimate strength is nearly 15% while the reduction in effective lateral stiffness is nearly 7%.

McNamee(30) has presented the results of a test on a three story single bay frame as shown in Fig. 1.3 subjected to vertical loads. The frame was designed and built to be symmetrical. However, because of unavoidable eccentricities introduced during fabrication and erection, the frame undergoes lateral deflection even under purely vertical loads. The second order analysis (lower dashed line) assumes that in addition to the vertical load the frame is also subjected to small lateral loads (1/4 of 1% of P) to simulate the eccentricities. In this case, it was observed that the deflection of the frame increased rapidly at a value of P equal to 24 kips (against-beam mechanism load of 27 kips) and the frame failed at that load. The action of the frame on the whole is similar to that of a frame under combined gravity and lateral loading.

From the above experiments, it is evident that even when stiff shear wall or a bracing system is present, the secondary effects may be significant. Further, such effects may be present even in symmetrical frames.
FIGURE 3.3 LOAD-DEFLECTION RELATIONSHIP, VERTICAL LOADS ONLY
subjected to pure vertical loads. The secondary effects cause a reduction in strength and stiffness of the structure which must be considered.

As mentioned earlier, the secondary effects affect a structure in the following ways:

(i) reduction of stiffness of column due to axial load
(ii) additional moments and shears known as PΔ effects
(iii) deformation of joints.

In the present work, secondary effects due to items (i) and (ii) only are considered, i.e. it is assumed that joints are rigid and do not undergo any deformation.

1.2.1. Reduction of Stiffness of Column Due to Axial Load

When the interaction of axial loads with column deformation is taken into account, the stiffness matrix of a column (Fig. 2.1) is as follows (15):

\[
k = \frac{EI}{L} \begin{bmatrix}
C & S & \frac{C+S}{L} & -\frac{C+S}{L} \\
S & C & \frac{C+S}{L} & -\frac{C+S}{L} \\
\frac{C+S}{L} & \frac{C+S}{L} & 2\frac{(C+S)}{L^2} & -2\frac{(C+S)}{L^2} \\
-\frac{C+S}{L} & -\frac{C+S}{L} & -2\frac{(C+S)}{L^2} & 2\frac{(C+S)}{L^2}
\end{bmatrix}
\]

(1.1)

where

\[
C = \frac{c^2}{c^2 - s^2}
\]

\[
S = \frac{s^2}{c^2 - s^2}
\]
\[ c = \frac{1 - qL}{\tan qL} \frac{L}{(qL)^2} \]

\[ s = \frac{qL}{\sin qL} - 1 \frac{L}{(qL)^2} \]

\[ q = \sqrt{P/EI} \]

\[ P = \text{axial load} \]

\[ L = \text{length of member} \]

\[ E = \text{Young's modulus of elasticity} \]

\[ I = \text{moment of inertia of section} \]

Since the above matrix involves transcendental functions of \( qL \) and hence of \( P \), it is difficult to handle. The problem can be simplified by representing the deflected shape of the column by a cubic polynomial satisfying the differential equation for bending deflection without the axial load and adding the effects of axial loads by calculating a "stability coefficient matrix". The matrix so-obtained, though approximate, has been found to give excellent results\(^{(18)}\). It has the following form:

\[
k = -\frac{p}{L} \begin{bmatrix}
\frac{2}{15} L^2 & -\frac{1}{30} L^2 & \frac{1}{10} L^2 & -\frac{1}{10} L \\
-\frac{1}{30} L^2 & \frac{2}{15} L^2 & \frac{1}{10} L^2 & \frac{1}{10} L \\
\frac{1}{10} L & \frac{L}{10} & \frac{6}{5} & -\frac{6}{5} \\
-\frac{1}{10} L & -\frac{L}{10} & -\frac{6}{5} & \frac{6}{5}
\end{bmatrix}
\]

\[(1.2)\]

The axial force interaction with column deformation can be taken into account in analysis by using either of the two stiffness matrices mentioned.
earlier. However, a problem is that in each case the formulation of stiffness matrix requires a knowledge of the column axial load, and that load is not known till the analysis has been completed.

Previous studies (22) have shown that for frames subjected to large side sway deflection, the deterioration of member stiffness due to axial load has a small effect on the results of the analysis and the major portion of non-linearity is caused by the PA effect alone. It has been shown by Galambos (15) that if the value of q or $\sqrt{PL^2/EI}$ is less than 1.2, the reduction in stiffness due to axial load is less than 5\%. Since, in the majority of beam columns in practical frames, the value of q or $\sqrt{PL^2/EI}$ is found to be less than 1, the effect of reduction in stiffness due to axial load can be safely ignored. However, if the value of Q is 1 or more, the coefficients C and S must be evaluated and the stiffness matrix shown in equation (1.1) or (1.2) must be used.

1.2.2 PA Effect

When a column is acted upon by a lateral force H and a vertical load $\Sigma P$ as shown in Fig. 1.4, the column undergoes a total lateral deflection $\Delta_2$. Of this total deflection, an amount $\Delta_1$ results from the action of horizontal load H and the remaining is caused by the interaction of vertical load and side sway of the column. The moment M at the base of the column is given by:

$$M = Hh + \Sigma P\Delta_2$$ (1.3)
Fig. 1.4. Forces on deflected column.
The term $\Sigma PA_2$ is known as the $PA$ effect and represents the additional moment caused by the vertical load acting on the side sway. In a first order analysis, equilibrium is formed on the undeformed geometry (i.e. $\Delta_2=0$), the second order term $\Sigma PA_2$ becomes zero and base moment becomes

$$M = Hh \tag{1.4}$$

1.2.3 Effect of Inelastic Action

In working stress design, the structure is analyzed on the assumption of elastic behaviour throughout and hence the $PA$ effect included in the analysis is based on elastic deflection only. This may result in an underestimation of the value of $\Delta$ due to the following reasons:

(i) Increased $\Delta$ due to some inelastic deformation in some part of the structure in spite of a consistent factor of safety of 1.67 against the ultimate capacity of critical members.

(ii) Yielding due to axial stress exceeding 0.7 times the yield stress in columns having residual stress 0.3 times the yield stress.

Yielding would mean reduced stiffness for column and an increased value of deflection. This will lead to an increased value of $PA$ effect. However, in the computations, inelastic deflections are ignored on the following grounds:

(i) The column design procedure is quite conservative.

(ii) The test results of large scale specimen have not revealed significant inelastic deflections at loads corresponding to first hinge.
The preliminary results regarding influence of yielding of members indicate that inelastic deflection of girders in structures does not significantly reduce overall stiffness (40).

1.2.4 Secondary Analysis Methods

Several analytical techniques have been proposed by various authors to take into account the secondary forces, particularly those arising from the so-called PA effects. These techniques range from the traditional effective length factor method to second order computer analysis. Some of the more notable methods are summarized as follows.

1.2.4.1 Effective length factor method

The traditional design approach has been to treat the beam column as an isolated member and not an integral member of the frame. The frame action is accounted for indirectly in the column design by the use of an effective length factor. This effective length factor can be read directly from appropriate nomographs (21) constructed for sway permitted and sway prevented cases separately. Use of these nomographs requires a knowledge of the ratio of the sum of column stiffnesses to the sum of beam stiffnesses both at the top and bottom joints of the column.

The nomographs are based on elastic buckling equations of highly idealized and sometimes impractical cases (20). In the derivation of these equations, it is assumed that all columns in the frame reach critical load at the same time and that rotations at the two ends of the beams are equal in magnitude and sense in the sway permitted case while they
are equal but opposite in sense in the sway prevented case.

By considering a frame shown in Fig. 1.5, and plotting the first and second order moments using nomographs, McGregor(28) has demonstrated that there is very little relationship between the actual second order moments and those obtained by using the effective length technique. He has further shown that effective length technique has serious shortcomings in sway frames or partially braced frames where columns have widely varying effective length factors. The agreement is reported to be better if magnification is restricted to the portions of the moments resulting from lateral loads.

1.2.4.2 Approximate method

Since exact analysis of critical and bifurcation load of frame may not be suitable for design office, a number of authors have proposed simplified approximate calculations of the critical loads of tall frames. Based on second order analysis, Goldberg(16) and Stevens(46) have shown that critical load $\Sigma P_{c_i}$ of the $i$th story in a sway frame shown in Fig. 1.6 is equal to:

$$\Sigma P_{c_i} = \frac{k_h h_i}{\gamma} = \frac{H_i}{\gamma} \frac{h_i}{\Lambda_{li}}$$  \hspace{1cm} (1.5)

where

$$k_h = \frac{H_i}{\Lambda_{li}}$$

$H_i = $ horizontal shear in the $i$th story
Fig. 1.3. Comparison of moments in frame by two methods of analysis.

Fig. 1.6. Column subjected to combined loading.
\[ \Delta_{11} = \text{primary sway deflection} \]
\[ \Delta_{21} = \text{lateral deflection inclusive of PA effects} \]
and \[ h_1 = \text{story height}. \]

In Eqn. (1.5), \( \gamma \) is a factor which accounts for the deflected shape of columns and varies from 1.0 for stiff columns and flexible beams to 1.22 for flexible columns and stiff beams. In general, \( \gamma \) approaches 1.0 in the lower story of a tall building.

Eqn. (1.5) shows strong relationship between critical load \( \Sigma P_c \) and horizontal stiffness \( k_h \) and the first order deflection index \( \Delta_1/h \) for a given lateral load \( H \). It can be conveniently used to evaluate the total load in all the columns in a story at instability.

Rosenblueth(41) has also derived a similar expression for the moment:

\[ \Sigma M = Vh(1 + \frac{W}{k - \alpha W}) \]  \hspace{1cm} (1.6)

\[ 1 < \alpha < 1.22 \]  \hspace{1cm} (1.7)

where

\( \Sigma M \) = sum of moments at the base of columns in a story

\( V \) = lateral load at the floor level

\( W \) = total gravity load supported by the story

\( k \) = horizontal stiffness which is equal to

\[ \frac{V}{X} \], \( X \) being the relative horizontal deflection between upper and lower floors of the story under consideration as a result of lateral forces only
\[ h = \text{story height} \]

\[ a = \text{a coefficient which depends on axial load and ratio of the sum of stiffnesses of columns and sum of stiffnesses of beams of the story} \]

O. G. Julien has presented a nomograph that permits rapid calculation of maximum bending moment in an elastic column as a function of its end moments, slenderness ratio and axial load \(^{(21)}\). However, Rosenblueth has observed that the computations do not hold when one or more flexible columns are on the verge of buckling individually. Also, the assumption that axial deformations are negligible may be reasonable for beams but would deserve consideration in the case of columns.

### 1.2.4.3 Moment magnifier method

MacGregor \(^{(28)}\) has derived an expression for determining the second order moment from the first order moment on the assumption that the first and second order deflection shapes are similar. The analysis by this method is approximate and is primarily of use in a preliminary design.

Referring to Fig. 1.4, the bending moment \(M\) at the base of the column is given by:

\[ M = Hh + \Sigma P\Delta_2 \]

\[ (1.8) \]

If the critical load of the column is \(P_{cr}\) and \(Q' = \frac{\Sigma P}{P_{cr}}\) then

\[ \Delta_2 = \left(\frac{1}{1-Q'}\right)\Delta_1 \]

\[ (1.9) \]

and
The term $1/[1-(P/P_e)]$ or the equivalent $1/[1-(f_a/F_e')]$ is the amplification factor and accounts for secondary moments produced by axial load on the deformed member.

The actual boundary conditions at member ends are considered through effective length factor $(K)$ of the member. This factor can be read directly from the monographs (17) for the appropriate cases of sway permitted and sway prevented.

Since second order effects are accounted for in the interaction equations through effective length factor and amplification factor, the structure is analyzed using first order elastic analysis and moments and forces obtained are directly used in the interaction equations for design with appropriate values for effective length factor $K$ and equivalent moment factor $C_m$.

By analyzing a number of cases through interaction equations and comparing the results with true strengths of members, it has been found that interaction equations do not correctly compensate for the $PA$ effect (8,35) and give rise to the following discrepancies:

(i) In most cases, they provide a conservative estimate of the primary bending moment on the member at failure (35).

(ii) The $PA$ moment is not computed in the analysis and the difference between the capacity implied in design and actual total moment at failure is assumed as moment caused by $PA$ effects. From a study of a number of cases (35) with varying slenderness ratios and varying ratios of column stiffness to beam stiffness
1.2.4.4 Iterative PΔ analysis

For tall buildings designed for normal deflection limitations, an acceptable estimate of second order shears, moments and forces in an elastic structure can be obtained by iterative calculations which take into account the sway forces induced by PΔ moments \(^{(14,33)}\).

Adams\(^{(7,8)}\) while describing the method has shown that a first order analysis is carried out initially to obtain the horizontal story deflections. The overturning action of vertical loads acting through the horizontal deflections is then accounted for by applying additional sway forces at story levels. These forces are computed by taking the algebraic sum of additional story shears at each story level. The sway forces are added to the applied lateral loads and a first order analysis of the structure is again carried out, with new horizontal loads. The process is repeated until the displacements at the end of a cycle are the same as those at the beginning. The analysis for the structure shown in Fig. 1.7 proceeds as follows:

First cycle of iteration:

Cumulative vertical loads up to story level \(i = \Sigma P_i\)

Lateral load at level \(i = H_i\)

Additional story shear

\[
V_i' = \frac{\Sigma P_i}{h_i} (\Delta_i + 1 - \Delta_i)
\]

Additional sway force

\[
H_i' = V_i' - V_{i-1}'
\]
\[ V'_i = \frac{\sum P_i}{h_i} (\Delta_{i+1} - \Delta_i) \]

\[ H'_i = V'_i - V'_{i-1} \]

**Figure 1.7** Sway Forces Due to Vertical Loads
Second cycle of iteration:

Vertical load $= \Sigma P_i$

Lateral load $= H_1 + H'$

Deflections $\Delta_{i+1}$ and $\Delta_i$ are again determined and the process is repeated.

This method is quite simple and convenient to program on a computer. However, convergence may require several cycles for each type of loading considered. More cycles of iteration are needed for convergence in slender structures.

1.2.4.5 Direct $P\Delta$ Analysis

McGregor (28) has derived an equation for obtaining the second order deflections directly without iterations by making use of the fact that in each successive iteration the additional deflection is much smaller. The magnitudes of these successive deflections form a geometric series which can be summed without difficulty as follows. If lateral deflection is proportional to lateral load,

$$\Delta = F_e H$$  \hspace{1cm} (1.16)

where $H = \text{applied lateral load}$

$$\Delta = \text{corresponding first order deflection}$$

$$F_e = \text{story deflection under a unit lateral load.}$$

For the first cycle of iteration,

$$\Delta_{(i=1)} = F_e H_{(i=1)}$$  \hspace{1cm} (1.17)

$$= F_e H (1 + \frac{\Sigma P F}{h})$$  \hspace{1cm} (1.18)
where

\[ \Sigma P = \text{axial load carried by the story} \]

\[ i = \text{number of the cycle of iteration} \]

\[ h = \text{story height} \]

Hence, for the ith cycle of iteration:

\[ \Delta_i = F_e H \left( 1 + \frac{\Sigma PF_e}{h} + \frac{\Sigma PF_e^2}{h^2} + \ldots + \frac{\Sigma PF_e^i}{h^i} \right) \quad (1.19) \]

This is a geometric series that converges because \( \Sigma PF_e/h \) is less than 1.

The second order deflection \( \Delta_s \) is obtained by summing the series and is given by:

\[ \Delta_s = \frac{F_e H}{\Sigma PF_e} \left( \frac{\Delta_i}{1 - \frac{\Sigma PF_e}{h}} \right) \quad (1.20) \]

\[ \Delta_s = \frac{\Delta_i}{1 - \frac{\Sigma PF_e}{Hh}} \quad (1.21) \]

This equation is identical to Eqn. (1.15) derived in the moment magnifier method and also to equations derived by Fe(14), Parme(34) and Goldberg(16).

1.2.4.6 Negative bracing method

Nixon and others(32) have developed a method by which second order deflections and moments can be obtained using a first order computer analysis program without iteration. This is achieved by introducing in each story a fictitious diagonal bracing of negative area. The method is based on the derivation of a second order stiffness matrix based on the
equilibrium of the deflected shape of the column. Such a matrix is as
given below:

\[
\begin{bmatrix}
\frac{4EI}{L} & \frac{2EI}{L^2} & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\
\frac{2EI}{L} & \frac{4EI}{L^2} & -\frac{6EI}{L^2} & \frac{6EI}{L^2} \\
-\frac{6EI}{L^2} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{P}{L} \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{P}{L}
\end{bmatrix}
\]

where

\[
\begin{align*}
E &= \text{Young's modulus of elasticity} \\
I &= \text{moment of inertia of column} \\
L &= \text{length of column} \\
P &= \text{axial load on the column.}
\end{align*}
\]

A first order program will correctly generate all terms in the above
matrix except that it will omit the four \(P/L\) terms. If the structure
contains bracing members shown by the dashed line in Fig. 1.8, stiffness
matrices of the bracing members will be added to the global matrix.

Thus, referring to the top story, the contribution of the brace can
be represented by the following stiffness equation:

\[
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} = \left[ \begin{array}{cc}
\frac{AE}{L_o} & \cos^2 \alpha \\
\cos^2 \alpha & -\cos^2 \alpha
\end{array} \right] \begin{bmatrix}
\Delta_1 \\
\Delta_2
\end{bmatrix}
\]

\[(1.22)\]
Fig. 1.8. Frame with bracing and associated degrees of freedom.
where

\[ L_0 = \text{length of the bracing} \]
\[ \Delta_1 = \text{deflection at top of the first story} \]
\[ \Delta_2 = \text{deflection at bottom of the first story} \]
\[ F_1 = \text{horizontal force corresponding to } \Delta_1 \]
\[ F_2 = \text{horizontal force corresponding to } \Delta_2 \]
\[ \alpha = \text{angle the brace makes with the horizontal} \]
\[ A = \text{cross-sectional area of the bracing} \]

The elements of the above stiffness matrix will get added to the overall stiffness matrix in the same position which the P/L terms occupy. Thus, the secondary effect of the axial loads interacting with sway deflections can be represented by the introduction of a fictitious bracing member in each story. The area of such a fictitious bracing is obtained by equating \((AE/L_0)\cos^2 \alpha\) to \((-P/L)\) so that

\[ A = -\frac{P/L}{E \cos^2 \alpha} \quad (1.23) \]

where

\[ P = \text{sum of axial loads in the columns of story under consideration.} \]

This method is quite useful since a first order analysis program which can handle bracing members can be used directly. The only modification that the structural property of one additional member per story must be specified. However, the method suffers from the following disadvantages:

(1) The vertical component of forces in bracing members will introduce errors in the analysis whenever a vertical column
axial load is calculated. This is examined below in detail.

(ii) In deriving the stiffness matrix of a column, vertical column
extension degrees of freedom have not been considered assuming
axial deformation to be small. When such deformations are
significant and axial degrees of freedom are accounted for,
certain terms in the stiffness matrix of the brace will add to
the global stiffness terms corresponding to the axial degrees
of freedom and will render the analysis inaccurate. The method is
not applicable to frames with sloping roofs.
It is of interest to examine in some detail the effect a fictitious
brace of negative area has on the results of analysis. Consider the fict-
itious bracing forming part of a frame as shown in Fig. 1.9. The global
degrees of freedom are numbered in the figure and include degrees of
freedom corresponding to the axial deformation of columns. Also shown
are the local degrees of freedom of the bracing member.

The stiffness matrix of the bracing member corresponding to these
degrees of freedom is shown by Eqn. (1.24). Also shown (in brackets on the
matrix) are structural degrees of freedom that are in correspondence with
the degrees of freedom of the bracing member.

\[
k = \frac{EA}{L}
\]

\[
\begin{bmatrix}
\cos^2 \alpha & \sin \alpha \cos \alpha & -\cos^2 \alpha & -\sin \alpha \cos \alpha \\
\sin \alpha \cos \alpha & \cos^2 \alpha & -\sin \alpha \cos \alpha & -\sin^2 \alpha \\
-\cos^2 \alpha & -\sin \alpha \cos \alpha & \cos^2 \alpha & \sin \alpha \cos \alpha \\
-\sin \alpha \cos \alpha & -\sin^2 \alpha & \sin \alpha \cos \alpha & \sin^2 \alpha
\end{bmatrix}
\]

(1.24)
(a) Frame and structural degrees of freedom.

(b) Member and its degree of freedom.

Fig. 1.9. Frame and degrees of freedom.
The structural degrees of freedom are 1, 2, 10, 11 corresponding to brace member degrees of freedom 1, 2, 3, 4. The area of bracing member is suitably adjusted so that the four terms \([1,1], (1,10), (10,1), (10,10)\] in the brace matrix modify the global stiffness matrix to correctly account for the second order effect. However, the remaining terms in the brace matrix introduce spurious effects in the global matrix and will cause errors in the results of analysis. Gouwens (49) has also developed a similar method in which the \(P\dot{\alpha}\) terms are incorporated in the stiffness formulation.

1.2.4.7 Second order finite element analysis

A finite element approach has been proposed by Aas Jakobsen (1) to account for second order effects under linear elastic conditions. McNamee and Lu Wu (6) have proposed a similar method for second order effects under elastic plastic behaviour of material. In these methods, the structure stiffness matrix \(k\) is assumed to be the sum of two stiffness matrices \(k_1\) and \(k_2\) where \(k_1\) is the first order matrix and \(k_2\) is a matrix containing the second order terms, obtained through an iteration procedure.

When a unit displacement is applied to the member, the axial load required to maintain equilibrium is unknown and can be obtained only by trial and error. To start the iteration, the axial load can be set to zero. The first order axial force obtained in the first cycle is used in the second cycle and the process is repeated until the axial load computed in a cycle is close to the value computed in the previous cycle.

To take account of inelastic behaviour, McNamee has suggested an incremental load approach method where a small increment of load is applied
to the frame and compatible incremental deformation is computed. A stable deflection position is possible only when incremental sway deformations are positive. This method is also iterative and may not be good as a practical design tool since in comparison to non-iterative methods it may be both expensive and time-consuming.

1.2.4.8 Story stiffness method

Cheong-Siat-Moy (10, 11, 12) has developed a simple and economical method to serve as an alternative to the conventional second order analysis by dividing the frame into a number of sub-assemblages: one for each story. Under certain idealizations, a story can be further subdivided into a series of subassemblages each composed of a column segment and one or two restraining beams depending on the position of the subassemblage in the story.

Story stiffness is then defined as the sum of the stiffnesses of its component subassemblages and is worked out on the consideration that unbraced frame derives its resistance to lateral loads solely from the bending action of its rigidly joined members. Once story stiffness is calculated, relative sway $\Delta$ can be evaluated from the equation

$$\Delta = \frac{\Sigma Q}{S_T}$$  \hspace{1cm} (1.25)

where

$\Sigma Q =$ total lateral shear on the story

$S_T =$ story stiffness.

The story stiffness $S_T$ at a given load factor $\lambda$ is equal to:
\[ S_T = \frac{12E}{h^3} \sum_{i=1}^{m} \left( \frac{I_c}{I_i + \psi l_i} \right) - \lambda \frac{\Sigma P}{h} \]  
(1.26)

where

\[ \Sigma P = \text{total gravity loads on the story at working load level} \]

\[ E = \text{Young's modulus} \]

\[ I_c = \text{column moment of inertia} \]

\[ h = \text{story height} \]

\[ \psi = \frac{I_c}{h}, \quad n \sum_{i=1}^{n} \frac{\alpha_i l_i b_i}{l_i b_i} \]

\[ n = \text{number of beams in a joint} \]

\[ \alpha_i = \text{beam stiffness modifying factor depending on the end conditions} \]

\[ I_b = \text{beam moment of inertia} \]

\[ L_b = \text{beam length} \]

\[ m = \text{number of columns in the story} \]

Since for each column \( Q=SA \), load factor \( \lambda=1 \) and since the point of inflection occurs at mid-height, the column moment \( M_c \) can be evaluated as follows:

\[ M_c = \frac{Q h}{2} + P \frac{\Delta}{2} \]  
(1.27)

\[ = \frac{6EI_c}{h^2(1+2\psi)} \Delta \]  
(1.28)

where stiffness \( S \) of a subassemblage in an intermediate story is as follows:
This method is based on the following assumptions:

(i) Story eccentricities are non-existent.

(ii) When bending occurs due to lateral force, points of inflection are assumed to develop at mid-lengths of members.

(iii) Subassemblage stiffness expression does not depend on the location of the story.

Factors like semi-rigid connections, initial story eccentricities, column axial shortening and column yielding can be accounted for by modification of basic subassemblage stiffness but the fact that inflection points do not always occur at mid-heights of columns makes the method approximate.

1.2.5 Design by Interaction Equations

The columns are usually designed through interaction equations applied to axial load and moments determined from a first order analysis. The interaction equations account for the second order effects in an approximate manner. Two interaction equations are used \((36,44)\) one relating to the failure of member through inelastic instability and the other for failure through attainment of ultimate capacity by the formation of a plastic hinge. The two equations are known as the stability equation and the strength equation respectively and are as follows:

\[
\frac{P - \sigma C_m M}{P_{cr} M (1 - P/P_c)} \leq 1.0
\]  

(1.29)
\[
\frac{P}{P_y} + 0.85 \frac{M}{M_p} \leq 1.0.
\]  
(1.30)

where

\[ P \] = axial load

\[ P_{cr} \] = buckling strength of the member, equal to 1.67 \( F_a A \), \( A \) being the area of cross-section of the member and \( F_a \) the allowable axial stress

\[ P_e \] = elastic buckling strength of the member, equal to \( \frac{286,000 A}{(KL/r_x)^2} \)

\( KL/r_x \) being the effective slenderness ratio in the plane of bending

\[ M_p \] = plastic moment capacity of the member (when no axial load is present)

\[ C_m \] = equivalent moment factor which accounts for a moment distribution other than a uniform moment throughout the length

\[ P_y \] = maximum axial load at yield stress, equal to \( A \sigma_y \), where \( \sigma_y \) is the yield stress

\[ M \] = maximum end moment on the member

When multi-story structures are designed by the allowable stress method\(^{44}\), the interaction equations can be expressed in the following convenient format. A factor of safety of 1.67 is used except in the calculation of Euler's stress when the factor of safety is 1.92.

\[
\frac{f_a}{F_a} + \frac{C_m f_b}{F_b (1 - \frac{f_a}{F_e})} \leq 1.0
\]  
(1.31)
\[
\frac{f_a}{0.6F_y} + \frac{f_b}{F_b} \leq 1.0
\]  

(1.32)

where

\( f_a \) = computed axial stress

\( f_b \) = computed bending stress at member end

\( F_a \) = axial stress permissible in absence of moment

\( F_b \) = bending stress permissible in absence of axial load

\[
F_e = \frac{149,000}{(KL/r_x)^2}
\]

where \( KL \) is the effective slenderness ratio in the plane of bending.

If the primary bending is not uniform over the member length, the strength of the column will be increased. This is taken into account by the use of an equivalent moment factor \( C_m \) given by the following expression:

\[
C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.4
\]  

(1.33)

where

\( M_1 \) = smaller moment at column end

\( M_2 \) = larger moment at column end

The ratio \( M_1/M_2 \) is positive if the member gets deformed in single curvature but negative if the member is in double curvature. Eqn. (1.33) is valid only if the member ends are not allowed to translate. If translation is permitted, \( C_m \) should be taken as 0.85.
The term $1/[1-(P/P_e)]$ or the equivalent $1/[1-(f_a/f_e)]$ is the amplification factor and accounts for secondary moments produced by axial load on the deformed member.

The actual boundary conditions at member ends are considered through effective length factor ($K$) of the member. This factor can be read directly from the nomographs (17) for the appropriate cases of sway permitted and sway prevented.

Since second order effects are accounted for in the interaction equations through effective length factor and amplification factor, the structure is analyzed using first order elastic analysis and moments and forces obtained are directly used in the interaction equations for design with appropriate values for effective length factor $K$ and equivalent moment factor $C_m$.

By analyzing a number of cases through interaction equations and comparing the results with true strengths of members, it has been found that interaction equations do not correctly compensate for the PA effect (8,35) and give rise to the following discrepancies:

(i) In most cases, they provide a conservative estimate of the primary bending moment on the member at failure (35).

(ii) The PA moment is not computed in the analysis and the difference between the capacity implied in design and actual total moment at failure is assumed as moment caused by PA effects. From a study of a number of cases (35) with varying slenderness ratios and varying ratios of column stiffness to beam stiffness
it is found that while the PΔ moment changes considerably, yet in a relative sense, the method of interaction equations assumes PΔ effect to be the same in all cases.

(iii) No recognition is given to the fact that in addition to column moments, PΔ effects also increase the girder moments. Since in most practical cases, the strength of girders controls the strength of frames (15) the net result of the above shortcoming is that the factor of safety gets reduced for those structures where PΔ effects are significant.

1.2.6 Scope of Present Work

The main objectives of this work are as follows:

(i) To develop an analytical method and a computer program to analyze rectangular planar, unbraced frames with rigid joints which would predict second order moments, axial loads, etc. (i.e. inclusive of PΔ effects) without iteration.

(ii) To evolve a procedure to correctly account for PΔ effects resulting from the initial eccentricity (due to imperfection in member or fabrication tolerances) in columns using the above computer program without iteration.

(iii) To compare the strength of column predicted by interaction equations (inclusive of PΔ effects) with its true strength within practical range of column properties to determine the correctness of approximate values predicted by interaction
equations, and also to work out a table of "Correction factors" in case of variation.

(iv) To extend the scope of the above method to include the effects due to column axial shortening on account of non-uniform temperature variation, foundation settlement or due to any other cause.

(v) A comparative study of a few frames designed by interaction equations using the results of first order analysis in two ways, i.e. as sway permitted in the case of analysis exclusive of PA effects and sway prevented in the case of analysis inclusive of PA effects.

As shown in Section 1.1, the building structure is considered to be comprised of two orthogonal sets of planar frames; each of which can be analyzed and designed for the loads tributary to it. This work is concerned with the study of such individual frames. It is assumed throughout that the frame is rigid jointed, that the columns are wide flange sections with their major axis in the plane of the frame and that all girders perpendicular to the plane have simple connections at the columns. This last assumption implies that the columns are not subject to any moment about their minor axes.
CHAPTER 2

THE MODIFIED STIFFNESS METHOD

2.1 INTRODUCTION

As stated in Chapter 1, the secondary effects resulting from the interaction of vertical forces with the sidesway can be of considerable importance in the analysis and design of multi-story buildings. The approximate methods of analysis for the secondary forces are, by their very nature, not precise and may introduce errors of uncertain magnitude in some cases. On the other hand, rigorous second order analysis can be time-consuming and prohibitively expensive. There is therefore a need for a method of analysis which is accurate and yet not too complex.

This Chapter describes a method of analysis which will accurately account for the sway interaction effect (PΔ effect) of vertical forces. The method is based on the standard stiffness method of analysis with the modification that the global stiffness matrix is suitably altered to account for the PΔ effect. It is no more complex than a first order analysis method and does not need any iteration. Since the use of a computer program is inevitable even for a first order analysis, it is assumed that the computations required for the method will be carried out through a computer program. A suitable computer program has therefore been developed and briefly described in this Chapter. Also presented are the advantages and limitations of the method. Finally, examples are given to illustrate the use of the method.
2.2 MEMBER STIFFNESS MATRICES

Multi-story building structures often consist of a series of frames interconnected at floor levels through diaphragms which can usually be considered as infinitely rigid in their own plane. Because of this, and because the axial loads in beams are comparatively small, it is reasonable to assume that the beams are axially rigid. The beam element then has four degrees of freedom as shown in Fig. 2.1, and its stiffness matrix is as below:

\[
\begin{pmatrix}
\frac{4EI}{L} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\
\frac{2EI}{L} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{6EI}{L^2} \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{12EI}{L^3} \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{12EI}{L^3}
\end{pmatrix}
\]

Unlike in the case of beams, the axial loads in the columns are of considerable magnitude and the axial deformations of columns may be quite significant, particularly in tall buildings. Thus, for columns, the axial degrees of freedom must usually be taken into account and a column element has therefore six degrees of freedom as shown in Fig. 2.1. The corresponding stiffness matrix is as follows:
Fig. 2.1. Member degrees of freedom.
\[
\begin{bmatrix}
\frac{4EI}{L} & \frac{2EI}{L} & \frac{6EI}{L^2} & 0 & 0 \\
\frac{2EI}{L} & \frac{4EI}{L} & \frac{6EI}{L^2} & 0 & 0 \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{12EI}{L^3} & 0 \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{12EI}{L^3} & 0 \\
0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \\
0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L}
\end{bmatrix}
\]

In Equations (2.1) and (2.2),

- \(E\) = Young's modulus of elasticity
- \(I\) = moment of inertia of section
- \(L\) = length of the member.

The stiffness matrices given by Eqns. (2.1) and (2.2) are based on the assumption that the effect of the interaction of the axial load with the relative lateral deformations of the two ends is small and therefore equilibrium can be formed on undeformed geometry. This assumption is quite valid for the beam because both the axial loads and the lateral deformations are small. For a column, the situation is quite different. First, the axial load is considerable. Second, under combined loading (vertical as well as lateral), significant sidesway occurs. In fact, even under
vertical loads alone, due to eccentricity, unsymmetry in loading or frame geometry, some sidesway is inevitable. For a rational or a realistic analysis, this effect of the relative movement of ends due to sidesway must be taken into consideration in the formulation of equations of stiffness. This effect is known as the PΔ effect where P represents axial load in the column and Δ the horizontal deflection.

The above-mentioned secondary effect can be accounted for in the stiffness matrix by considering the equilibrium on the deformed geometry of the column in the direction of horizontal degree of freedom. The modified stiffness matrix of the column is then:

\[
\begin{bmatrix}
\frac{4EI}{L} & \frac{2EI}{L} & \frac{6EI}{L^2} & -\frac{6EI}{L^2} & 0 & 0 \\
\frac{2EI}{L} & \frac{4EI}{L} & \frac{6EI}{L^2} & -\frac{6EI}{L^2} & 0 & 0 \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{P}{L} & \frac{12EI}{L^3} & \frac{P}{L} & 0 & 0 \\
-\frac{6EI}{L^2} & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & +\frac{P}{L} & -\frac{12EI}{L^3} & -\frac{P}{L} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \\
0 & 0 & 0 & 0 & -\frac{EA}{L} & \frac{EA}{L}
\end{bmatrix}
\]

(2.3)

It would be noted that all the terms in the stiffness matrix remain the same as those in a first order analysis except the four terms corresponding to sidesway which are modified by the addition of \( \frac{P}{L} \). Since \( P \), the axial load in the column, is not known till the analysis has been
completed, the direct evaluation of the term \( \frac{12E I}{L^3} - \frac{P_i}{L} \) of stiffness matrix is not yet possible.

2.3 STRUCTURE STIFFNESS MATRIX

The structure stiffness matrix is obtained by the assembly of member stiffness matrices with due consideration to the correspondence of global degrees of freedom to the member degrees of freedom. Consider the sidesway degrees of freedom 4 and 11 of the frame shown in Fig. 2.2. By assembling the column sidesway terms which contribute to the corresponding structure stiffness matrix terms, it can be shown that:

\[
k_{14} = \frac{12E}{L^3} (I_1 + I_2 + I_3) + \frac{12E}{L^2} (I_4 + I_5 + I_6) - \frac{1}{L_1} (P_1 + P_2 + P_3)
\]

\[
- \frac{1}{L_2} (P_4 + P_5 + P_6)
\]

\[
k_{11,11} = \frac{12E I_4}{L^3} + \frac{12E I_5}{L^3} + \frac{12E I_6}{L^3} - \left( \frac{P_4}{L_2} + \frac{P_5}{L_2} + \frac{P_6}{L_2} \right)
\]

\[
= \frac{12E}{L^3} (I_4 + I_5 + I_6) - \frac{1}{L_2} (P_4 + P_5 + P_6)
\]

where

- \( I_i \) = moment of inertia of the \( i \)th column
- \( L_j \) = height of columns in the \( j \)th story
- \( P_i \) = axial load in the \( i \)th column

The terms \( (P_1 + P_2 + P_3) \) and \( (P_4 + P_5 + P_6) \) can be represented as \( (\Sigma P)_i \).
Fig. 2.2. Frame and degrees of freedom.
where

i represents the number of the story and

\[ \Sigma P \] is the sum of axial loads in all columns of a story.

The story load can be readily calculated by simple statics as equal to the total applied gravity loads supported by the story. Thus, in the assembled matrix, only the total gravity load supported by a story is required and not the unknown individual column axial loads. The modifications required in the structure stiffness matrix to account for the secondary effects is thus quite straightforward. However, it should be noted that the procedure detailed above will apply only to frames having columns of equal height in a story.

2.4. LOAD FACTOR FOR THE \( P \Delta \) TERM

For ensuring a uniform factor of safety, the sway forces arising from \( P \Delta \) effects should be computed on the basis of factored axial loads even though the purpose of computation is to assess the magnitude of the \( P \Delta \) effect at working loads. This apparent inconsistency can be explained by considering the equilibrium of a column (Fig. 2.3) in the swayed position. The equilibrium equation is given by:

\[ \Sigma M = Hh + P \Delta \]  \hspace{1cm} (2.6)

where

- \( \Sigma M \) = sum of the bending moments at the top and bottom
- \( H \) = horizontal force carried by the column
- \( h \) = story height
Fig. 2.3. Forces on deflected column.
P = gravity load acting on the column

Δ = sidesway or the horizontal displacement of one end of the column
with respect to the other.

In allowable stress design, the ultimate capacity of a member is divided by a factor known as the factor of safety (α). Both CSA and AISC codes use a safety factor of 1.7. Considering the capacity at ultimate loads, the applied loads will become 1.7H and 1.7P and for consistent factor of safety; the moment capacity should be 1.7 times the moment at working stress. Assuming that the horizontal displacement is directly proportional to the lateral force, the equilibrium equation can be rewritten as:

\[ 1.7 \sum M = (1.7H) \times h + (1.7P) \times (1.7\Delta) \]  \hspace{1cm} (2.7)

To arrive at allowable stress design loads, both sides of Eqn. (2.7) are divided by the factor of safety of 1.7, giving:

\[ \sum M = \frac{Hh + 1.7P\Delta}{1.7} \]  \hspace{1cm} (2.8)

Thus, for a consistent factor of safety, the \( P\Delta \) terms must be derived by using an axial load value of 1.7\( P \), i.e. 1.7 times the gravity load instead of the actual gravity load. The term 1.7\( P \) or in more general terms \( \alpha P \), is known as the factored load. A similar inference has been derived by Adams(7).
2.5 COMPUTER PROGRAM FOR SECOND ORDER ANALYSIS

A first order computer program for the elastic static analysis of rectangular planar frames developed by Humar and available in the Carleton University Library (19) was selected and modified by the author to incorporate the second order effects. The above-mentioned program was originally designed to serve as a complement to a package of seismic analysis programs. The analysis procedure used in the program is based on the following assumptions:

(i) The behaviour of material is linear and elastic.
(ii) All dimensions are referred to the centre line of members, and joint sizes are considered negligible in comparison to lengths of members.
(iii) All members are prismatic and have uniform moment of inertia.
(iv) The effect of axial deformation of beams can be neglected. The beams are thus considered axially rigid.
(v) The shear deformations in beams and columns are negligible.
(vi) The secondary moments induced in columns and beams due to axial loads interacting with deflections are small and can be neglected.

The following are some important attributes of the program:

(i) The gravity loads are applied through the beams and can be uniformly distributed loads throughout the length of beams and/or one or more concentrated loads applied to them.
(ii) The lateral loads can be applied at floor levels.
(iii) In one run, the program can analyze the same frame for a different set of loads or optionally it can analyze different frames each for several different sets of loads.

(iv) The program can handle rectangular frames (with horizontal beams and vertical columns) which are either regular or have one or more setbacks.

The above assumptions and conditions are considered reasonable for a majority of multi-story building frames.

In the modified program, the stiffness matrices for beams and columns are the same as those used in the first order analysis and are shown in Eqns. (2.1) and (2.2). As mentioned earlier, after assembly, the structural stiffness matrix is modified by adding appropriate \( \Sigma P/L \) quantities to the sideway terms, where \( \Sigma P \) represents the gravity load carried by a story and \( L \) is the story height. This modification requires one additional information in the input data, namely the total gravity load carried by each story. This is calculated separately and supplied to the computer along with other information such as the structural properties of the sections, details of loadings, etc.

It may be mentioned here that the program could be modified so as to compute the story loads from the information on loading, but the modification was not done for the following two reasons:

(i) In analysis, the reduced axial load on columns is to be considered as per the National Building Code of Canada\(^{(31)}\).
The working out of reduction coefficients will require a fairly complex logic in the case of setback frames for which the program was originally designed. On the other hand, a manual computation of the gravity loads carried by a story is quite straightforward.

(ii) The factored load ($\alpha P$) can be fed to the computer directly and conveniently where $\alpha$ can have any value (general value being 1.7). Factored loads are required for computation of secondary terms ($\alpha P\delta$) and the factor $\alpha$ may vary because different probability factors are applicable to different combinations of loading.

The program carries out the modification of structure stiffness matrix using the supplied data on factored story loads. The structure displacements are then calculated by the standard stiffness method of analysis. The displacements are now inclusive of the secondary effects. Once the displacements are known, the member forces can be worked out by using the member stiffness equations. Thus, considering the case of a column with degrees of freedom shown in Fig. 2.1 and the stiffness matrix given by Eqn. (2.3), if $\delta_i$ is the displacement along the $i$th degree of freedom and $F_i$ is the force along the same degree of freedom, the relationship between $F$ and $\delta$ can be expressed as follows:
\[
\begin{bmatrix}
\frac{4EI}{L} & \frac{2EI}{L^2} & \frac{6EI}{L^2} & -\frac{6EI}{L^2} & 0 & 0 \\
\frac{2EI}{L} & \frac{4EI}{L^2} & \frac{6EI}{L^2} & -\frac{6EI}{L^2} & 0 & 0 \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{12EI}{L^3} - \frac{P}{L} & -\frac{12EI}{L^3} + \frac{P}{L} & 0 & 0 \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{12EI}{L^3} + \frac{P}{L} & -\frac{12EI}{L^3} - \frac{P}{L} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{EA}{L} & \frac{EA}{L} \\
0 & 0 & 0 & 0 & -\frac{EA}{L} & \frac{EA}{L}
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\delta_4 \\
\delta_5 \\
\delta_6
\end{bmatrix} =
\begin{bmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4 \\
F_5 \\
F_6
\end{bmatrix}
\] (2.9)

From the above equation, it is obvious that the evaluation of moments
(F_1 and F_2) and axial loads (F_5 and F_6) are straightforward. However,
for evaluating the horizontal shears, the term P/L must be known. The
axial load in the column is therefore evaluated next:

\[
P = F_5 = \frac{EA}{L} \cdot \delta_5 - \frac{EA}{L} \cdot \delta_6
\]

\[
= \frac{EA}{L} (\delta_5 - \delta_6)
\] (2.10)

Hence

\[
\frac{P}{L} = \frac{EA}{L^2} (\delta_5 - \delta_6)
\] (2.11)

The horizontal shears can now be worked out by substituting the above
value of P/L in the expression for F_3 and F_4.
2.6 MODIFICATION FOR PINNED BASE

The original program developed by Humar was capable of handling only those frames which had their lowest columns fixed at the base. Since it may sometimes be necessary to analyze rigid jointed frames which have foundations that provide little restraint against rotation, the program was suitably modified so that at the option of the user it would treat the column bases as pinned and would analyze the frame accordingly.

The stiffness matrix of a column with a pinned base is as follows:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{3EI}{L} & \frac{3EI}{L^2} & -\frac{3EI}{L^2} & 0 & 0 \\
0 & \frac{3EI}{L^2} & \frac{3EI}{L^3} & -\frac{3EI}{L^3} & 0 & 0 \\
0 & -\frac{3EI}{L^2} & -\frac{3EI}{L^3} & \frac{3EI}{L^3} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{EA}{L} & -\frac{EA}{L} \\
0 & 0 & 0 & 0 & -\frac{EA}{L} & \frac{EA}{L}
\end{bmatrix}
\] (2.12)

The above expression is used for the stiffness matrices of all the columns which have a pinned base. The assembly of structural stiffness matrix and the subsequent analysis then proceeds as usual.

The modification required in the global stiffness matrix to account for the PA effects is not dependent on the type of constraint at the base of the column. The procedure described earlier for fixed base columns therefore applies without change to this case also.
2.7 EFFECTS OF INITIAL ECCENTRICITY

In practice, columns are seldom perfectly straight. Some eccentricity definitely exists due to imperfection of materials or due to construction or fabrication tolerances. Such tolerances are permissible even under the provisions of Code\(^{(42,44)}\). The maximum out-of-plumb permitted during erection of structures as specified by the codes is 002 times the story height. Since it is the maximum specified limit of out-of-plumb, it would be unduly conservative to take this for granted for all structures. To assess the realistic value of out-of-plumb in a real structure, statistical studies are needed. A few studies carried out by Adams\(^{(4,5)}\) have revealed that a value of \(0.002h\) is unduly conservative except for low rise buildings and a fair estimate of out-of-plumb eccentricity would be as follows:

\[
\Delta_{i+1} - \Delta_i = \frac{0.006h_i}{2.2 \sqrt{n}}
\]

(2.13)

where

- \(\Delta_{i+1}\) and \(\Delta_i\) = movements of top and bottom ends of the columns in the \(i\)th story
- \(h_i\) = height of the \(i\)th story
- \(n\) = total number of columns in the structure.

It is clear from the available studies that initial eccentricity is invariably present but that its magnitude can only be roughly estimated. Any one of the above methods may be used or an alternative rational approach may be followed to determine eccentricity.
According to the National Building Code of Canada, any structure subjected to gravity and lateral loads has to be analyzed for the following combinations of loadings:

(i) dead load
(ii) dead load + live load
(iii) dead load + lateral load
(iv) dead load + live load + lateral load with a probability factor of 0.75

The consideration of initial eccentricity becomes important when only gravity loads are considered and the frame being analyzed is symmetrical with symmetrical loading. Theoretically, there is no side sway in such a frame and therefore it is not possible to consider the stability effects directly. Hence, it is necessary to consider equivalent computed lateral forces on account of initial eccentricity in the analysis while considering stability. Since there is no such problem if lateral loads are present, the effect of initial eccentricity need not be considered in the case of combined gravity and lateral loading (4,5). The equivalent lateral forces mentioned above are worked out by using the method outlined in Section 1.2.4.4.

The author believes that for proper evaluation of the effects of eccentricity, the process should be divided into the following two parts:

(i) deflections (sidesway) caused by equivalent lateral loads due to eccentricity and
(ii) magnification of the above deflections under vertical or gravity loads (PΔ effect).
It was mentioned earlier that for a consistent factor of safety, factored axial loads should be used in the computation of the $P\Delta$ effects. This was because of the fact that both $P$ as well as $\Delta$ resulting from horizontal load $H$ must be multiplied by $\alpha$ at ultimate load. In case of initial eccentricity, the lateral sway is not dependent on horizontal load and therefore does not become $\alpha\Delta$ at ultimate. Hence, the equivalent lateral loads are calculated from gravity loads $P$ and not from factored gravity load $\alpha P$.

Due to the application of equivalent lateral load ($P\Delta/h$), the frame will undergo lateral deflections. These deflections will get magnified under vertical loads. This magnification is automatically accounted for by the second order analysis procedure described earlier. It should be noted that in the second order analysis, factored axial loads are used in modifying the stiffness matrices just as in the case of combined vertical and gravity loads.

To summarize, eccentricity effects are accounted for by applying equivalent lateral loads computed from the expression $P\Delta/h$ for each story and the frame is then analyzed for the combination of these lateral loads and full gravity loads using a second order analysis in which second order terms are worked out from factored loads as usual.

2.8 SWAY DISPLACEMENTS DUE TO OTHER CAUSES

It is apparent that the secondary effects due to $P\Delta$ depend on the magnitude of axial loads and the sway displacement $\Delta$. In the previous
sections, the story sways caused by the lateral forces and initial eccentricities have been considered. In fact, sway displacement may also result from non-uniform temperature change and foundation settlement. Methods are available for estimating these sway deflections. The secondary effects resulting from these additional displacements can be calculated by using exactly the same method as the one used for initial eccentricity. The method consists of evaluating the equivalent lateral loads by the procedure of Section 1.2.4.4 and then carrying out a second order analysis of the frame for these equivalent lateral loads, using the computer program described in the Chapter.

2.9 ADVANTAGES OF THE MODIFIED STIFFNESS METHOD

In the design of a multi-story building, the computations involved are so large that a suitable computer analysis program must invariably be used. The procedure outlined here makes it possible to carry out a second order analysis with a minimal increase in the computer time that will be needed for a first order analysis. The main advantages of this method when compared to several of the other available methods are as detailed below.

(i) The second order frame action is accounted for directly and not in an indirect approximate manner such as, for example, through effective length factor as in the effective length factor method.

(ii) Unlike many other methods of accounting for the secondary effects, such as the moment magnifier method, this method is exact and not approximate.
(iii) It may save considerable computer time and cost in comparison to analysis by the iteration method since iterations are completely avoided.

(iv) It gives correct axial loads and shears in columns unlike in the case of negative bracing method where these forces may be in error due to vertical and horizontal components of the force in fictitious bracing.

(v) Unlike the case of story stiffness method, this method does not depend on assumed locations of the points of contraflexure in columns.

(vi) It can take into consideration eccentricity effects in a more exact and rational manner.

(vii) The input data for the program and the computing time for analysis are only marginally more than those required for a first order program.

However, this method also has the following two limitations:

(i) The method is applicable only to those frames in which all columns in a story are of equal length.

(ii) Like other methods, the reduction of stiffness of column due to axial load is neglected.

2.10 EXAMPLE OF SECOND ORDER FRAME ANALYSIS

A three storied frame with columns hinged at the base is selected for a second order analysis. The frame and the loads acting on it are shown in Fig. 2.4.
Fig. 2.4. Example frame with loading.

Beams  W  18 x 40
Columns  W  10 x 49

E = 29000 ksi
The frame is analyzed by two different methods to obtain member forces and moments inclusive of PA effects. The two methods are as outlined below:

(i) Iterative approach developed by Adams\(^7\). Five iterations are carried out to obtain good convergence.

(ii) Use of computer program based on the modified stiffness matrix method developed by the author. The results obtained are inclusive of PA effects. No iterations are required and only a single run is necessary.

2.10.1 Iterative Approach

In this case, the program developed by the author is used but with second order option omitted, that is, a first order analysis is carried out.

The properties of beams and columns such as the areas, moment of inertia, etc. are obtained from the Handbook of Steel Construction\(^{17}\).

The horizontal and vertical loads are both multiplied by the probability factor of 0.75, since the frame is analyzed for a combination of P and H; the load factor applicable to gravity loads to be used in the PA analysis is worked out as follows:

\[
\alpha = 1.70 \times 0.75 \times 0.75
\]

\[
= 0.956
\]

\[
= 1.0
\]

Hence, the factored story loads are the same as the gravity loads shown on the frame. The method used is described in Section 1.2.4.4.
At each cycle of iteration, equivalent sway forces are computed from the deflection of the previous cycle and the frame is again analyzed for the combination of 0.75P and 0.75M and sway story shears obtained above. The process is repeated till the deflections at the end of a cycle are reasonably close to those at the beginning of the cycle. The results are presented in Table 2.1.

2.10.2 Modified Stiffness Method

In this case, the computer program is used with second order option operative so that the structure stiffness matrix is appropriately modified by the program to include the PA effects. The properties of sections and loadings used remain unchanged. The results of analysis obtained in one run of the program are also shown in Table 2.1.

A comparison of the two sets of results would make it evident that iterations do converge towards the exact values but several iterations may be required in some cases. In fact, the results obtained in one run of the modified computer program are better than the results obtained from five iterations. Besides, there is a considerable saving in computer time as the time taken is 1/5 of the time taken for five iterations.

2.10.3 Eccentricity Effects

As mentioned earlier, these effects are important when a structure is to be designed for gravity loads only. Adams has presented an example of analysis for out-of-plumb effects in his article "Column Design by the PA Method"(7). He calculates equivalent lateral load forces due to out-of-
### Table 2.1. Analysis by iteration process.

<table>
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<tr>
<th>FLOOR LEVEL</th>
<th>APPLIED FORCE</th>
<th>LATERAL DEFLECTION</th>
<th>PA SHEAR IN KIPS</th>
<th>IA FORCE IN KIPS</th>
<th>EFFECTIVE FORCE H'</th>
<th>AS PER MODIFIED STIFFNESS METHOD</th>
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</table>
plumb on the basis of factored axial loads (1.7P), and then carries out four iterations to ensure convergence. The results of four iterations are shown in Table 2.2. For comparison, the results obtained in one run of the second order computer program are also shown in Table 2.2. It is obvious that the results obtained even after four iterations are not as good as those obtained by the present method in one run.

As explained earlier, the author believes that when accounting for the out-of-plumb effects, factored axial loads need not be used for computation of equivalent lateral forces because the initial eccentricity does not get magnified as ultimate load. The author has therefore obtained results of an analysis in which the equivalent horizontal loads were worked out by using unfactored axial loads but the analysis program was run with modified stiffness matrix in which factored axial load was used for $PA$ terms. The deflections so obtained are shown in Column 4 of Table 2.2.

From a comparison of the results, the following conclusions can be drawn:

(i) The iteration process has not fully converged even after 4 iterations and perhaps requires one or two more iterations for results correct to the third place of decimal.

(ii) A comparison of values in columns 2 and 4 shows that the values obtained by Adams are nearly 66% higher than the values obtained by the author. The values obtained by Adams are
<table>
<thead>
<tr>
<th>FLOOR LEVEL</th>
<th>DEFLECTIONS OBTAINED BY ITERATION METHOD USING FACTORED AXIAL LOAD FOR EVALUATING EQUIVALENT LATERAL FORCES</th>
<th>DEFLECTIONS OBTAINED BY MODIFIED STIFFNESS MATRIX EQUIVALENT LATERAL LOADS AS PER FACTORED AXIAL LOAD</th>
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</tr>
</thead>
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<tr>
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<td>0.30777</td>
<td>0.18239</td>
</tr>
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<td>0.27968</td>
<td>0.16576</td>
</tr>
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<td>0.20715</td>
<td>0.12278</td>
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<tr>
<td>1</td>
<td>0.000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>
obviously conservative due to consideration of factored axial loads in the calculation of equivalent lateral forces.

2.10.4 Influence of PA Effects on the Magnitude of Member Forces

The frame under consideration is analyzed twice, once excluding the PA effects and a second time including the PA effects, to obtain an estimate of the influence PA effects have on the member moments and forces. Fig. 2.5 shows the moment distributions for the leeward column stack and for the girders of level 3. The dashed lines represent the values obtained from a first order analysis (exclusive of PA effects) and the solid curves represent the final moments (inclusive of PA effects). The moments are also indicated in the figures, the final ones being enclosed in parentheses.

Due to PA effects, the bending moments in girders are higher in comparison to those obtained in a first order analysis. As can be seen from Fig. 2.6, the left hand girder of level 2 must be designed for a moment of 4008.7 in.kips instead of a moment of 3958.3 in.kips. A comparison of column moments shows that the bottom column (level 1 to 2) of the leeward stack must be designed for a moment of 1725 in.kips against a primary moment of 1478.9 in.kips.

It should be noted that the higher moment obtained by a second order analysis does not necessarily mean that the column section obtained in a design involving direct compensation for PA effects will be larger than the one obtained in a design in which the PA effects are indirectly accounted for through interaction equations applicable to sway permitted
Fig. 2.5. Leeward column stack.

Fig. 2.6. Right hand girder, Level 3.
cases. In fact, many times, direct compensation will give a more economical design. A more detailed comparison of the two alternative design methods is presented in Chapter 3.
CHAPTER 3

CONSIDERATION OF SWAY FORCES IN DESIGN

3.1 INTRODUCTION

As stated in Section 1.1, two alternative methods can be used for the design of members in an unbraced multi-story frame which depends on its own rigidity for resisting horizontal forces. Both methods are recognized by the Canadian Code for the limit states design of steel structures (45). In one method, the design moments and forces are obtained by a first order analysis and the column is designed to have sufficient spare capacity to resist the secondary effects. In the second method, an approximate second order analysis procedure is used to obtain forces and moments which are inclusive of sway effects and the columns are then designed for these forces using the full available capacity.

The two alternative methods are discussed briefly in this Chapter. It is noted that, when using the second method, there are situations when useful limit of load may be reached before the full capacity of a column member is utilized. After this limit, an increasing portion of this capacity is used up by the sway effects, so that even though the total moment resisted by the column increases, the portion of the capacity available to resist primary or applied moment decreases. This implies that the use of full capacity may lead to unconservatice design. A major portion of this Chapter is devoted to an examination of this aspect.

A single story portal frame is studied and true column strength is cal-
culated. This is compared with the column strengths predicted by the two sets of interaction equations, one applicable to sway prevented cases and the other to sway permitted cases. The strength comparisons are carried out for varying values of different parameters that affect such strengths.

3.2 **TRUE STRENGTH OF MEMBERS**

Since the axial loads carried by beams are small, their moment carrying capacity is not appreciably affected by the presence of such loads. The traditional method for the design of a beam is therefore to calculate the first order moments carried by it and to ensure that these moments are not greater than the moment carrying capacity. In fact, however, the sway effects also cause an increase in the moments carried by the beams. The traditional method therefore leads to beam sections which may be slightly underdesigned.

To obtain the true strength of a column in a frame, consider the equilibrium and compatibility of the frame shown in Fig. 3.1, subjected to gravity load \( P \) and lateral load \( V \) as shown assuming that \( V \ll P \).

For compatibility,

\[
\theta_c + \theta' = \Delta/h \tag{3.1}
\]

and

\[
\theta_c, \theta_b = \theta \text{ (say)} \tag{3.2}
\]

For equilibrium,

\[
M_c = Hh + P\Delta \tag{3.3}
\]

\[
M_b = M_c = M \text{ (say)} \tag{3.4}
\]
(a) Single story frame.

(b) Column geometry.

(c) Column equilibrium.

Fig. 3.1. Analysis of single story frame.
Since the beam is elastic, its end moment is related to the end rotation by the equation:

\[ M_b = \frac{6EI_b}{L} \cdot \theta_b \]  

(3.5)

where

\( I_b \) = moment of inertia of beam
\( E \) = Young's modulus of elasticity
\( L \) = length of beam.

Substituting the value of \( \theta \) obtained from Eqns. (3.2) and (3.5) with Eqn. (3.1):

\[ \frac{ML}{6EI_b} + \theta' = \Delta/h \]  

(3.6)

Equations (3.3), (3.4) and (3.6) give:

\[ M = Hh + Ph \left( \frac{ML}{6EI_b} + \theta' \right) \]  

(3.7)

Dividing both sides of Eqn. (3.7) by the plastic moment capacity \( M_p \)

\[ \frac{M}{M_p} = \frac{Hh}{M_p} + \frac{Ph}{M_p} \left( \frac{ML}{6EI_b} + \frac{Ph}{M_p} \cdot \theta' \right) \]  

(3.8)

Now, let the following symbols be used to indicate column properties:

\( Z \) = plastic section modulus
\( S \) = elastic section modulus
\( \sigma_y \) = yield stress
\( d \) = depth of section
\( r \) = radius of gyration in plane of bending.
Then,

\[ \frac{P_h}{M_p} = \frac{1}{2.24} \cdot \frac{P}{P_y} \cdot \frac{h}{r} \cdot \frac{d}{r} \]

and because for wide flange section \( Z \) is approximately equal to 1.12 times \( S \), the above gives:

\[ \frac{P_h}{M_p} = \frac{1}{2.24} \cdot \frac{P}{P_y} \cdot \frac{h}{r} \cdot \frac{d}{r} \]  \hspace{1cm} (3.9)

If the ratio of column stiffness \( I_c/h \), \( I_c \) being the moment of inertia of the column, to beam stiffness \( I_b/L \) is denoted by \( G \), the following value is obtained for the expression \( PhL/6EI_b \):

\[ \frac{PhL}{6EI_b} = \frac{P}{6E} \cdot G \cdot \frac{h^2}{I_c} \]

\[ = \frac{\sigma_y}{6E} \cdot \frac{P}{P_y} \cdot \left( \frac{h}{r} \right)^2 \cdot G \]  \hspace{1cm} (3.10)

Substitution of the values obtained from Eqns. (3.9) and (3.10) into Eqn. (3.8) gives the following expression for \( M/M_p \):

\[ \frac{M}{M_p} \left[ 1 - \frac{\sigma_y}{6E} \cdot \frac{p}{P_y} \cdot \left( \frac{h}{r} \right)^2 \cdot \frac{h}{r} \right] = \frac{P}{P_y} \cdot \frac{h}{r} \cdot \frac{d}{r} \cdot \frac{1}{2.24} \left[ \frac{H - \theta}{P} \right] \]  \hspace{1cm} (3.11)

For a wide flange section, \( M_{pc} \), the plastic moment capacity in the presence of an axial load is given by:

\[ M_{pc} = 1.18 M_p (1 - P/P_y) \]  \hspace{1cm} (3.12)

Equations (3.11) and (3.12) give:
\[
\frac{M}{M_{pc}} = (1-P/P_y) \cdot 1.18 \cdot \left(1 - \frac{p}{p_y} \cdot \left(\frac{h}{r}\right)^2 \cdot \frac{\sigma_y}{6E}\right)
\]

\[= \frac{1}{2.24} \cdot \frac{p}{p_y} \cdot \frac{h}{r} \cdot \frac{d}{f} \left(\frac{H}{P} + \theta'\right)
\]

(3.13)

Substitution of the following values in Eqn. (3.13) gives Eqn. (3.14):

- \(E = 29000\) ksi
- \(\sigma_y = 56\) ksi
- \(d/f = 2.3\) (fairly constant for wide flange sections).

\[
\frac{M}{M_{pc}} = 1.18 \cdot (1-P/P_y) \cdot \left(1 - .0092069 \cdot \frac{p}{p_y} \cdot \left(\frac{h}{r}\right)^2 \cdot G\right)
\]

\[= \frac{2.3}{2.24} \cdot \frac{p}{p_y} \cdot \frac{h}{r} \left(\frac{H}{P} + \theta'\right)
\]

(3.14)

For specific values of \(P/P_y\), \(h/r\) and \(G\), Eqn. (3.14) can be represented as follows:

\[
\frac{M}{M_{pc}} = k\left(\frac{H}{P} + \theta'\right)
\]

(3.15)

where \(k\) is a function of \(P/P_y\), \(h/r\) and \(G\) and for specified values can be treated as constant.

For a specified value of \(H\), Eqn. (3.15) gives one relationship between the moment \(M\) and rotation \(\theta'\). Another relationship between these two variables is obtained from the known end moment versus end rotation curves of wide flange sections. Such curves have been developed by several
researchers for strong axis bending of wide flange shapes for given values of \( \frac{h}{r} \) and \( \frac{P}{P_y} \) and are available in the literature\(^{33,39}\). A process of numerical integration was used in the calculation of these curves and the presence of residual stresses was taken into account. A typical curve for \( \frac{h}{r} = 40 \) and \( \frac{P}{P_y} = 0.6 \) is shown in Fig. 3.2.

Having two independent relationships between the end moment and end rotation, it is possible to solve for these variables; of course, the value of \( H \) must be specified and a graphical method must be used. The value of \( H \) is not, however, specified; instead, it is required that maximum value of \( H \) and the corresponding \( M \) and \( \theta' \) be obtained. Because the curve represented by Eqn. (3.14) is a straight line inclined to \( \theta' \) axis at an angle whose tangent is \( k \), this is easily achieved. On the \( M-\theta' \) graph for the given column, a tangent having a slope \( k \) is drawn as shown in Fig. 3.2.

The coordinates of the point of tangency represent the maximum value of \( M \) and the corresponding \( \theta' \). Also, the intersection of the tangent with the moment axis is proportional to the maximum horizontal shear resisted by the column. Thus, by setting \( \theta' \) equal to zero in Eqn. (3.15), the maximum value of \( H/P \) is given by:

\[
\frac{H}{P} = \frac{1}{k} \left( \frac{M}{P_{c}} \right)_{\theta' = 0}
\]

(3.16)

where \( \left( \frac{M}{P_{c}} \right)_{\theta' = 0} \) is the intercept of the tangent on the moment axis. The primary moment \( \frac{H}{P} \) is now obtained as follows:
Fig. 3.2. Moment rotation relationship.
\[ \frac{Hh}{M_p} = \frac{H}{P} \cdot \frac{Ph}{M_p} \]  

(3.17)

Using Eqn. (3.9), this reduces to:

\[ \frac{Hh}{M_p} = \frac{1}{2.24} \frac{P}{P_y} \cdot \frac{h}{r} \cdot \frac{d}{\bar{r}} \cdot \frac{H}{P} \]  

(3.18)

Since the total moment capacity of the column as well as its primary moment capacity are now known, it is possible to obtain curves such as \( a \) and \( b \) in Fig. 3.3 by repeating the calculations for several values of \( \frac{P}{P_y} \) for a fixed \( \frac{h}{r} \). The process can then be repeated for other values of \( \frac{h}{r} \) and \( G \), varying one at a time.

An analytical solution is also possible by using the curve fitting technique by which the \( M-\theta' \) curve is transferred into a mathematical expression. Once such an expression is determined, \( \theta' \) and \( M \) can be obtained by the simultaneous solution of two equations, the other of which is Eqn. (3.14) and the maximum value of \( \frac{H}{P} \) sustained by column can be determined by equating \( \frac{\partial (M/M_p)}{\partial (H/P)} \) to zero. The graphical method has been used in this work for the calculation of true strengths.

3.3 CAPACITY OF COLUMN BY INTERACTION EQUATION

Since the normal procedure in the design of a column is to use the interaction equations, it is essential to know the column strengths predicted by them. The interaction equations used are as follows:

\[ \frac{P}{P_{cr}} + \frac{M}{M_{cr}} \cdot \frac{C_m}{(1-P/P_{c2})} = 1 \]  

(3.19)
\[ \frac{P_y}{P} + 0.55 \frac{M}{M_p} = 1 \quad (3.20) \]

and

\[ \frac{M}{M_p} \leq 1.0 \quad (3.21) \]

In the above equations, the value of equivalent moment factor \( C_m \) is given by:

(a) Sway prevented

\[ C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.4 \quad (3.22a) \]

(b) Sway permitted

\[ C_m = 0.85 \quad (3.22b) \]

where

\[ M_1 = \text{smaller end moment} \]

\[ M_2 = \text{larger end moment} \]

For sway prevented cases, with one end of the column pinned, \( C_m \) is always 0.6. The Euler load \( P_E \) is given by:

\[ P_E = \frac{2EA}{(kh/r)^2} \quad (3.23) \]

where \( k \) is the effective length factor determined from the appropriate nomographs for the sway permitted or sway prevented cases. It should be noted that when using these nomographs the ratio of column to beam stiffness (G) is required. The critical axial load that the column can carry
when no moment is present is equal to the tangent modulus load and is obtained from the following expression given by the Code(44):

\[
P_{cr} = 1.67 \frac{A}{r} F_a
\]

\[
\frac{k h}{r} < C_o \quad F_a = 0.60 \sigma_y
\] (3.25a)

\[
C_o < \frac{k h}{r} < C_p \quad F_a = 0.60 \sigma_y - m \left(\frac{k h}{r} - C_o\right)
\] (3.25b)

\[
\frac{k h}{r} > C_p \quad F_a = \frac{149000}{(k h/r)^2}
\] (3.25c)

where

\[
C_o = 30 - \frac{\sigma_y}{5} \leq 20
\]

\[
C_p = \frac{286000}{(\sigma_y - 13)^{0.5}} \geq 78
\]

\[
m = \frac{0.60 \sigma_y - 149000/C_p^2}{C_p - C_o}
\]

Thus, for given column and beam properties, the value of \(\frac{M}{M_p}\) can be obtained for each specified value of \(\frac{P}{\sigma_y}\) both for sway permitted cases and sway prevented cases. Typical curves so obtained are shown as \(c\) and \(d\) in Fig. 3.3. By repeating such calculations, similar curves can be obtained for different values of \(\frac{h}{r}\) and \(G\).
3.4 DESIGNING FOR SWAY EFFECTS

The method of calculating the true strength of the column in a single story frame was described in Section 3.2. By repeating such calculations for different values of axial load $P$, it is possible to obtain a relationship between $P$ and the moment carrying capacity of the column. A part of this capacity is used up by the secondary moment equal to $P \Delta$; the rest is available for resisting the primary moment $M_h$. Typical true strength curves for a frame similar to that shown in Fig. 3.1 are shown in Fig. 3.3 by solid lines. Curve $b$ represents the total moment carried, while curve $a$ represents the primary moment. The difference between curves $a$ and $b$ is the $P \Delta$ moment.

As described in Section 3.3, the total moment carrying capacity of the column can be approximately predicted by the use of strength and stability interaction curves for the sway prevented case. These two curves are also shown in Fig. 3.3. The envelope formed by them represents the total capacity of the column. Fig. 3.3 also shows the capacity predicted by the interaction curves for sway permitted cases. The traditional method of design is to calculate the first order moment in the column and to ensure that this moment is lower than the carrying capacity predicted by the sway interaction equation for sway permitted cases. The reserve of strength represented by the difference between the full strength and the strength predicted by sway permitted interaction equations is then assumed to carry the additional $P \Delta$ moment.
An alternative method of design is to calculate the total moment on the column, that is the sum of the primary moment and the PΔ moment, and then to ensure that this total moment is less than the strength predicted by the interaction equations applicable to the sway prevented case. This method of design, sometimes termed as the direct compensation for sway effects, has been recently accepted by the Canadian Code (45). The method is more rational and has several other advantages which are detailed in the literature (7, 35) and will not be described here.

Reference to Fig. 3.3 will show that, in the design of a column, the use of the traditional method will lead to a conservative design except probably for very low values of the axial load. This is because over most of the length the interaction curves applicable to sway permitted cases predict a lower strength than that given by curve a. On the other hand, it is seen from the same figure that the total moment carrying capacity predicted by the interaction equations applicable to the sway prevented case is higher than the true capacity given by curve b. This is not because the approximate interaction equations are unconservative in the prediction of the strength of a column, but largely because this full capacity cannot be realized when the column is permitted to sway.

The above fact can be illustrated by studying the behaviour of column shown in Fig. 3.4. The end moment, \( M_c \), versus end rotation, \( \theta' \), relationship for the column is known once the column properties are given. Since \( \theta' \) is related to the displacement \( \Delta \), a relationship between \( M_c \) and \( \Delta \) is obtained, and is shown by curve a in Fig. 3.4. As stated earlier,
the moment at the end of the column consists of two components: the primary moment $Hh$ and the secondary moment $P\Delta$. If the full moment carrying capacity of the column is to be utilized, $\Delta$ should reach or exceed a value of $\Delta_u$. However, it is seen from Fig. 3.4, that the maximum primary moment is attained at a value of $\Delta$ smaller than $\Delta_u$. After this, even though the total moment carried by the column increases, a larger portion of it is used up by the $P\Delta$ moment and the primary moment, and hence the horizontal load $H$, carried by the column, in fact decreases. It is clear that the useful limit of moment carrying capacity is lower than the maximum strength of a sway prevented column.

The unconservatism involved in the use of sway prevented interaction curves to estimate the true strength needs investigation for the practical range of values of the different parameters. Such strength curves have been derived as part of this study and are described in Section 3.6.

3.5 EXAMPLE FOR THE CALCULATION OF STRENGTHS

The true strengths and the strengths predicted by the interaction curves for a restrained column such as the one shown in Fig. 3.3 are determined. The following properties are assumed:

- $\sigma_y = 36$ ksi
- $\frac{H}{R} = 40$
- $G = 0$
- $\frac{P}{F_y} = 0.6$
3.5.1 True Strength of Column

By substitution of appropriate values in Fig. 3.14, the following relationship between \( M \) and \( \theta' \) is obtained:

\[
\frac{M}{M_{pc}} = 52.21 \left( \frac{H}{P} + \theta' \right)
\]

Using the moment rotation curves given in Ref. (39) and the graphical method outlined in Section 3.3, the following values are obtained for the ultimate moment, rotation and the horizontal load.

\[
\frac{M_{max}}{M_{pc}} = 0.805
\]

\( \theta'_{max} = 0.0078 \text{ radians} \)

\[
\left( \frac{M}{M_{pc}} \right)_{\theta'=0} = 0.405
\]

Since

\[
\left( \frac{M}{M_{pc}} \right)_{\theta'=0} = 52.21 \left( \frac{H}{P} \right)
\]

then

\[
\frac{H}{P} = 0.007757
\]

Now, \( M_{pc} = 1.18(1-P/P)M_{P} \gamma \)

\[
= 0.472M_{P}
\]

Hence:

\[
\frac{M_{max}}{M_{pc}} = \frac{M_{max}}{M_{pc}} \cdot \frac{M_{pc}}{M_{P}}
\]

\[
= 0.3799
\]
Thus, the non-dimensionalized true strength of the column \( \frac{M}{M_P} \) inclusive of P\Delta effects is .3799.

A part of the above strength is used up by the P\Delta moment. The remainder is equal to the primary moment carried by the column, and is given by Eqn. (3.16b).

\[
\frac{M_{\text{max}}}{M_P} = \frac{Hh}{M_P} = \frac{2.3}{2.24} \frac{P}{p} \frac{h}{x} \frac{M}{P}
\]

Substituting the given values of \( \frac{P}{P_y} \) and \( \frac{h}{x} \) and the value of \( \frac{M}{P} \) determined above

\[
\frac{M_{\text{max}}}{M_P} = 0.1911.
\]

Hence, the non-dimensionalized true strength of column exclusive of P\Delta effects is .1911.

3.5.2 Column Strength by Interaction Equation

(a) \( G_T = \bar{G} \)

\[ = 0 \, \text{(given value)} \]

\( G_B = \infty \, \text{(because the lower end of the column is hinged)} \)

From nomographs\(^{(17)}\)

\[ k = 2.0 \]

Hence,
\[
\frac{kh}{r} = 80
\]

For the above value of \(\frac{kh}{r}\) and \(\sigma_y = 36\) ksi, the value of \(F_a\) is obtained from the Handbook of Steel Construction (17) and is 15.3. Thus,

\[
\frac{p}{P_{cr}} = \frac{p}{\frac{\sigma_y}{\sigma_{cr}}} = \frac{36}{25.5}
\]

\[C_m = 0.85\]

Substituting these values in Eqn. (3.19),

\[
0.6 + \frac{36}{25.5} + \frac{0.85}{1 - 0.6(80)^2} \frac{36}{(3.14)^2 \times 29000} \cdot \frac{M}{M_p} = 1
\]

or

\[
\frac{M}{M_p} = 0.0930
\]

Eqn. (3.20) gives:

\[
0.6 + 0.85 \cdot \frac{M}{M_p} = 1
\]

or

\[
\frac{M}{M_p} = 0.4706
\]

Hence the strength of the column predicted by interaction equations for sway permitted cases is lower of the above two values, i.e. 0.0930.
(b) Sway Prevented Case

\[ G_T = G = 0 \text{ (given value)} \]

\[ G_B = \infty \text{ (because the lower end of the column is hinged)} \]

From nomographs \(^{(17)}\)

\[ k = 0.7. \]

Hence,

\[ \frac{kh}{T} = 0.28 \]

From the Handbook of Steel Construction \(^{(17)}\), \(F_a = 20.8\). Thus,

\[ \sigma_{cr} = 20.8 \times 1.67 \]

\[ = 34.67 \text{ ksi} \]

\[ C_m = 0.6 + 0.4 \frac{M_1}{M_2} \]

\[ = 0.6. \]

Substituting these values in Eqn. (3.19) gives:

\[ 0.6 \times \frac{36}{34.67} + \frac{0.6}{1 - \left( \frac{0.6 \times 36 \times (28)^2}{290000 \times (3.14)^2} \right)} \frac{M}{M_p} = 1 \]

or

\[ \frac{M}{M_p} = 0.5911 \]

From Eqn. (3.20),
\[ 0.6 + 0.85 \frac{M}{M_p} = 1 \]

or

\[ \frac{M}{M_p} = 0.4706 \]

Hence the strength of the column predicted by interaction equations for sway prevented cases is lower of the above two values, i.e., 0.4706.

3.5.3 Comparison of Results

From the above simple example, it is obvious that moments in either case (inclusive or exclusive of PA effects) as obtained from interaction equations are different from the true strengths of the beam column. The following points may be specifically noted.

(i) The selection of a member through interaction equations on the basis of first order analysis (exclusive of PA effects) may have considerable capacity of member unutilized \((0.1911 - 0.0930)\frac{M}{M_p}\)

i.e. 

\[ 0.0981 \frac{M}{M_p} \]

which is as much as design capacity in this case.

(ii) The selection of members through interaction equations as per second order analysis (inclusive of PA effects) will result in an overestimation of the member strength by \((0.4706 - 0.3799)\frac{M}{M_p}\)

i.e. 

\[ 0.0907 \frac{M}{M_p} \]

which is 20% of the strength predicted by interaction equations.

(iii) The interaction equations do not correctly compensate for the PA effects. The true value of PA effects in the present
case is \((0.3799 - 0.1111) M_p \) or \(0.1888 M_p \) while the interaction equations leave a reserve capacity of \((0.3799 - 0.0930) M_p \) or \(0.2869 M_p \). Perhaps the effective length factor approach used to account for the frame action is largely responsible for the disparity.

In view of the above results, it is essential to examine the strengths predicted by interaction equations and to compare them with the true strengths to ensure that design is neither unduly conservative nor unsafe and secondly, to assess the magnitude of deviation if any.

3.6 \textbf{STRENGTH CURVES FOR COLUMNS}

As mentioned earlier, two parameters namely \(\frac{h}{r} \) and \(G \) must be varied to obtain a series of curves between axial loads \(\frac{P}{P_y} \) and moments (non-dimensionalized as \(\frac{M}{M_p} \)) carried by the column. Since it will be useful to have such curves for all values of the above parameters which lie in the range of practical columns such practical ranges must be first determined. The design of 7 frames (Chapter 4) subjected to all combinations of lateral and gravity loads indicates the following ranges for these parameters:

\[
\frac{h}{r} = 20-30 \\
G = 0.7-3.0 \\
\frac{P}{P_y} = 0.3-0.7
\]

Curves are derived for three values of \(\frac{h}{r} \) for a pinned column, namely:
20, 30 and 40. These correspond to $\frac{h}{r}$ ratios of 40, 60 and 80 for a column deflecting in double curvature. Four values are assigned to the parameter $G$. These are 0, 1, 2 and 3. For $P/P_y$, all values between the extreme values of 0 and 1 should be considered. Moment rotation curves required for determination of the true strength of columns are, however, available only for $P/P_y$ values ranging from 0.3 and 0.9 (33, 39). Data is therefore obtained for $P/P_y$ values of 0.3 and higher up to a value when the column can carry no moment.

The curves are obtained by the methods described in the earlier sections of this chapter and are shown in Figs. 3.5 to 3.16.

3.7 CONCLUSIONS

An examination of Figs. 3.5 to 3.16 reveals the following important points:

(i) The stability equation invariably governs if the member is designed on the basis of first order analysis.

(ii) In design of members by second order analysis (direct compensation for sway effect), strength equation governs except where $P/P_y$ exceeds the following values:

<table>
<thead>
<tr>
<th>$\frac{h}{r}$ values</th>
<th>Bottom Hinged</th>
<th>Bottom Rigid</th>
<th>$P/P_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>60</td>
<td>80</td>
<td>0.8</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
<td>0.7</td>
</tr>
</tbody>
</table>
Fig. 3.5. Strength curves for columns.
Fig. 3.6. Strength curves for columns.
Fig. 37. Strength curves for columns.

\[ M = h + PA \]

\[ M = hh \]

Stability Equation
Sway Permitted

Stability Equation
Sway Prevented
Fig. 3.8. Strength curves for column.
Fig. 3.9. Strength curves for column.
Fig 3.10. Strength curves for column.
Fig. 3.11. Strength curves for column.
Fig. 3.13. Strength curves for column.
Fig. 3.15. Strength curves for column.
$\frac{h}{r} = 20, \ G = 3$

$M = Hh + PA$

Stability Equation
Sway Prevented

$M = Hh$

Strength Equation

Stability Equation
Sway Permitted

Fig. 3.16. Strength curves for column.
Since it is very rare that a practical column will lie in the above range, for practical purposes, the strength equation will always govern.

(iii) In the case of first order analysis, the ratio of true strength to the strength determined from interaction equations increases i.e. the design becomes more and more conservative when either

(a) the slenderness ratio $\frac{h}{r}$ increases
(b) the relative stiffness coefficient of the column to beam $G$ increases
(c) $\frac{P}{p_y}$ increases or $\frac{M}{M_p}$ decreases.

(iv) In the case of second order analysis, the ratio of strength predicted by interaction equations to the true strength increases, i.e. design becomes more and more unsafe when either

(a) the slenderness ratio $\frac{h}{r}$ increases
(b) the stiffness coefficient $G$ increases
(c) $\frac{P}{p_y}$ increases or $\frac{M}{M_p}$ decreases.

(v) Up to an $\frac{h}{r}$ ratio of 20 which is equivalent to a slenderness ratio of 40 for a column fixed at both ends, the design by either method may be satisfactory as the moments predicted by interaction equations are close to the true moments of column.

3.8 CORRECTION COEFFICIENT

When the $\frac{h}{r}$ ratio exceeds 20 (one end hinged and 40 when both ends are fixed), it becomes essential to account for the variation in true strength and strength predicted by interaction equations. Consideration
of variation in the case of second order analyses is more important because interaction equations may overestimate capacity and lead to unsafe design and a reduction in the factor of safety.

The most simple method of determining the correction coefficient is to consider the appropriate curve available in Figs. 3.5 to 3.16 and then to read the value of true moment and interaction moment (in either case inclusive of PA effects) for appropriate \( \frac{P}{P_Y} \) and work out the ratio of two moments.

The above procedure will often involve repeated interpolation since the values of \( \frac{h}{r} \) or \( G \) will seldom be exactly whole numbers for which curves are readily available.

Since most practical columns have \( \frac{P}{P_Y} \) values in the range of .3-.7, three values of \( \frac{P}{P_Y} \) namely .3, .5 and .7 were selected and values for true moments and interaction moments (both inclusive of PA effects) were read from Figs. 3.5 to 3.16. The value of \( R \) (correction coefficient, equal to true moment/interaction moment) was worked out and plotted against \( \frac{h}{r} \) for four values of \( G \), namely 0, 1, 2 and 3. These curves are available in Figs. 3.17-3.20.

It was assumed that \( R \), i.e. the ratio of moment capacities as obtained from exact analysis and from interaction equations was a quadratic function of the slenderness ratio and was of the form:

\[
R = a + b \left( \frac{h}{r} \right) + c \left( \frac{h}{r} \right)^2
\]

The constants were evaluated and are shown in Table 3.1. From the Table, it is obvious that the values of constants \( a \), \( b \) and \( c \) are not
Fig. 3.17. Reduction coefficient slenderness ratio curve.
Fig. 3.18. Reduction coefficient slenderness ratio curve.
Fig. 3.19. Reduction coefficient slenderness ratio curve.
Fig. 3.20. Reduction coefficient slenderness ratio curve.
Table 3.1. Values of constants for correct coefficient.

<table>
<thead>
<tr>
<th>G</th>
<th>$P/P_y$</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>.3</td>
<td>0.890</td>
<td>.00950</td>
<td></td>
<td>-.08975</td>
</tr>
<tr>
<td>0</td>
<td>.5</td>
<td>0.971</td>
<td>.00355</td>
<td>-.08450</td>
</tr>
<tr>
<td></td>
<td>.7</td>
<td>0.975</td>
<td>.00275</td>
<td>-.07717</td>
</tr>
<tr>
<td>.3</td>
<td>.930</td>
<td>.00650</td>
<td></td>
<td>-.08925</td>
</tr>
<tr>
<td>1</td>
<td>.5</td>
<td>1.015</td>
<td>.00075</td>
<td>-.08330</td>
</tr>
<tr>
<td></td>
<td>.7</td>
<td>0.880</td>
<td>.01050</td>
<td>-.07550</td>
</tr>
<tr>
<td>.3</td>
<td>.965</td>
<td>.00425</td>
<td></td>
<td>-.0883</td>
</tr>
<tr>
<td>2</td>
<td>.5</td>
<td>0.990</td>
<td>.00275</td>
<td>-.0825</td>
</tr>
<tr>
<td></td>
<td>.7</td>
<td>-1.0916</td>
<td>0.1733</td>
<td>-.0036</td>
</tr>
<tr>
<td>.3</td>
<td>.5092</td>
<td>.01271</td>
<td></td>
<td>-.00258</td>
</tr>
<tr>
<td>3</td>
<td>.5</td>
<td>-1.0405</td>
<td>0.1728</td>
<td>-.00360</td>
</tr>
<tr>
<td></td>
<td>.7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
consistent which indicates that the actual relationship is much more complex.

A higher degree curve can be tried but the process in that case would be too cumbersome to be of any practical use and hence would defeat its own purpose.

Hence, a table has been constructed to indicate the ratio of true moment capacity to the moment capacity obtained from interaction equations. The permissible bending stress obtained from the interaction equations must be multiplied by this ratio when testing the suitability of a beam column. These coefficients which can be termed as correction coefficients are given in Table 3.2. From the table, it is further evident that the two consecutive values are so close that the results of linear interpolation will not be very much different from the results obtained through rigorous methods of analysis.

The author believes that with the use of these coefficients the moment capacity obtained will be much better or nearer to the true strength of the beam column than those obtained by using interaction equations alone.

It is further obvious that for a \( \frac{h}{r} \) ratio of 20 (or 40 with both ends fixed) or lower, the coefficient is practically 1.00.
Table 3.2. Correction coefficients.

<table>
<thead>
<tr>
<th>$P/P_y$</th>
<th>$h/r = 40$</th>
<th>$h/r = 30$</th>
<th>$h/r = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0.3</td>
<td>.9528</td>
<td>.9528</td>
<td>.9385</td>
</tr>
<tr>
<td>0.4</td>
<td>.9076</td>
<td>.8975</td>
<td>.8776</td>
</tr>
<tr>
<td>0.5</td>
<td>.8675</td>
<td>.8475</td>
<td>.8225</td>
</tr>
<tr>
<td>0.6</td>
<td>.8072</td>
<td>.7622</td>
<td>.6417</td>
</tr>
<tr>
<td>0.7</td>
<td>.7271</td>
<td>.6619</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>.4062 (3410</td>
<td>.3410</td>
<td>.7824</td>
</tr>
<tr>
<td>0.9</td>
<td></td>
<td></td>
<td>.6923</td>
</tr>
</tbody>
</table>
CHAPTER 4

DESIGN OF FRAMES

4.1 INTRODUCTION

As noted earlier, sway effects result in a small increase in the moments in beams. They have a more pronounced effect on column moments. The magnitude of the increase in a column moment depends on the slenderness ratio \( \left( \frac{h}{r} \right) \) of the column, axial load carried by it (non-dimensionalized as \( \frac{P}{P_y} \)) and the ratio \( G \) of column stiffnesses to the stiffnesses of the restraining beams. The above effects were studied by considering a simple single story frame. A multi-story frame has a far more complex arrangement of beams and columns, and to study the effect of sway forces on such a frame, it would be necessary to design a complete frame (instead of one beam or one column) by the two alternative methods of design for the same loading conditions.

The two methods described earlier in detail are as follows:

(i) Carry out a first order analysis and design the frame as sway permitted using appropriate interaction equations.

(ii) Carry out a second order analysis and design the frame through interaction equations applicable to sway prevented cases.

The second order analysis can be carried out by the modified stiffness matrix method developed by the author.
A series of frames were designed for varying values of gravity and lateral loads using the two methods of design and the resulting designs were then compared with each other. The information obtained from these comparisons and the inferences drawn are presented in this Chapter.

Since sway effects depend on the relative values of the axial loads arising from gravity and the lateral loads due to wind or earthquake, the basic parameters used in this study were: the uniformly distributed gravity loads that is the dead and live loads, the lateral loads due to wind and the number of stories. The last parameter was considered significant because, in a tall building, the lower columns will carry larger accumulated gravity loads and the PA effect may be more pronounced. Both the wind and the gravity loads were varied between the range of maximum and minimum practical values. The procedure used was to vary one of the three parameters mentioned above, keeping the other two constant.

4.2 FRAME GEOMETRY

The 3 bay 10 story frame shown in Fig. 4.1 was selected for analysis under two sets of loadings, one with varying gravity load and the other with varying lateral load. In the frame selected, bays are of 30 ft span each and the height of all the stories is 12' uniformly from top to bottom. The centre to centre distances of two consecutive frames is assumed to be 30 ft.

To study the effect of the number of stories, two other frames, one of 5 stories and the other of 2 stories, were selected. Frame dimensions such as the span, the story height and the centre to centre spacing of frames were kept the same as those in the 10 story frame.
Fig. 4.1. Typical ten storied frame.
4.3 **LOADING**

The magnitudes of various loads used in this study were based on the recommendations of the National Building Code of Canada\(^{(31)}\).

4.3.1 **Live Load**

The minimum specified value of live load is 40 lbs/sq.ft. for residential areas, apartment hotels, homes, etc., while the maximum value is 100 lbs./sq.ft. for office areas, storage areas, theatres, corridors, lobbies, etc. except garages and driveways. The other values specified are 50 lbs/sq.ft. and 75 lbs./sq.ft. The following three values of live load were used in this study:

(i) Maximum value \(100\) lbs/sq.ft.

(ii) Usual value \(60\) lbs/sq.ft.

(iii) Minimum value \(40\) lbs/sq.ft.

4.3.2 **Dead Load**

The minimum value of dead load for the floor systems is of the order of 50 lbs/sq.ft. When live load exceeds 60 lbs/sq.ft., the dead load for the floor system is also likely to be higher. The following values were used for the magnitude of dead load corresponding to each particular value of the live load:

<table>
<thead>
<tr>
<th>Live Load (lbs/sq.ft.)</th>
<th>Dead Load (lbs/sq.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>100</td>
<td>70</td>
</tr>
</tbody>
</table>
4.3.3 Wind Load

As per Clause 4.1.8.1(1) of the National Building Code, the wind load \( P \) in lbs/sq.ft. is given by:

\[
P = q \cdot C_e \cdot C_g \cdot C_p
\]

With an average value of 1.14 for exposure factor \( C_e \), 2.0 for gust factor \( C_g \), and 1.3 for shape factor \( C_p \),

\[P = 2.964 \quad q\]

If the value of reference velocity pressure \( q \) is assumed to vary from 6.7 to 13.5, the maximum and minimum values of \( P \) works out as 40 lbs/sq.ft. and 20.02 lbs/sq.ft. Hence three values adopted for design are as follows:

(i) Maximum value 40 lbs/sq.ft.
(ii) Minimum value 20 lbs/sq.ft.
(iii) Usual value 30 lbs/sq.ft.

4.3.4 Loading Cases Considered

Having decided on the range of loads, the loading cases given in Table 4.1 were selected for the purpose of this study.

4.3.5 Load Combinations Considered

In view of the provisions contained in Section 4.1.3.1 of the National Building Code of Canada, the following load combinations were considered in design:
Table 4.1. Design loads.

<table>
<thead>
<tr>
<th>NO. OF STORIES</th>
<th>LIVE LOAD (i)</th>
<th>DEAD LOAD (ii)</th>
<th>GRAVITY LOAD (i)+(ii)</th>
<th>WIND LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>40</td>
<td>50</td>
<td>90</td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>50</td>
<td>110</td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>70</td>
<td>170</td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>50</td>
<td>110</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>50</td>
<td>110</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>50</td>
<td>110</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>50</td>
<td>110</td>
<td>30</td>
</tr>
</tbody>
</table>

All loads are in lbs/sq.ft.
(i) Dead Load
(ii) Dead Load + Live Load
(iii) Dead Load + Wind Load
(iv) Dead Load + Live Load + Wind Load with a probability factor of 0.75.

4.4 PRELIMINARY DESIGN

For the detailed analysis of a structure, the geometric properties of the sections used for various members are required. It is therefore essential to carry out a preliminary design. The steps involved in such a design are described below.

4.4.1 Design of Beams

For design of beams, the magnitude and distribution of live load and dead load along the length of the beam must be known. The manner in which floor loads are carried to the frame is indeterminate and depends to a large extent on the type of flooring system and framing. For the sake of simplicity, the gravity loads were assumed to be uniformly distributed over the tributary area of the beams shown in Fig. 4.2.

According to Clause 4.1.6.3(S) of the National Building Code of Canada (31), the live load to be used in the design is the specified load multiplied by a live load reduction factor whose value depends on the tributary area of the structural member. The reduction factor is to be worked out by the following formula:

Reduction factor = 0.3 + 10/√A
Fig. 4.2. Tributary area for beams.
where $B$ is the tributary area supported by the member in square feet. Live load reduction used in the design was in conformity with the above provisions of the code.

(a) Moments due to gravity loads

For the preliminary design, the approximate gravity moment coefficients published by ACI and available in Reference (47) were used. These are as follows:

(i) Negative moment at exterior face of the first interior support

$$= \frac{1}{10} WL^2$$

(ii) Negative moment at other faces of interior supports

$$= \frac{1}{11} WL^2$$

(iii) Positive moment in end span

$$= \frac{1}{14} WL^2$$

(iv) Positive moment in interior span

$$= \frac{1}{16} WL^2$$

where $W$ is load per unit length and $L$ is the span of the beam. It is evident from these coefficients that the negative moment at the support governs the design.

(b) Moments due to wind

The wind load is considered to be transmitted to the structure at the floor levels through the panel action of outer walls. The girder moments as well as the column moments were obtained by the portal method of analysis which is based on the assumption that each column in a story
resists a percentage of the total horizontal shear on the story, which is proportional to the width of the aisle the column supports and that the points of contraflexure are located at the mid-lengths of members.

(c) Selection of members

Once the total moments resisted by the beam (worst effect under various combination of loads shown in Section 4.3.5) are known, suitable sections for the beam can be selected from "Beam Selection Tables" for elastic design ASTM A36 Steel given in the Handbook of Steel Construction\(^{17}\). A single section was used for all girders at one floor level.

4.4.3 Design of Columns

For design of columns, it is essential to determine the axial loads and end moments resulting from the applied loads. The most severe combination of axial loads and moments must be selected from the values obtained for all combinations of loads given in Section 4.3.5.

(a) Axial loads

For a column, the axial load is determined by the magnitude of dead and live loads acting over the tributary areas supported by the column. As in the case of beams, the live load used in design is the specified live load multiplied by the live load reduction factor.

(b) Moments

The moments induced in the column arise from two sources:

(i) due to gravity loads

(ii) due to wind loads.
In a symmetrical frame with symmetrical loading moments due to gravity load exist only in the exterior columns and are equal to or half of the restraining beam moment depending on whether that moment is shared by one or two columns. As mentioned earlier, the wind load moments were calculated by the portal method.

(c) Selection of member

The choice of a suitable column can be made by using the Table "Allowable Axial Loads (P) in the Columns" for ASTM A36 Steel given in Reference (17). In these tables, the allowable axial load has been tabulated against the ratio of effective length in feet to the least radius of gyration. The effective length factor \( k_x \) or \( k_y \) can be taken from Table 4-12 of Reference (17). A simple procedure is used to account for the presence of moment. This procedure is to convert the applied moment (larger of the two end moments) to an equivalent axial load, by multiplying the former by a bending factor \( B \), the appropriate values of which are listed in Table 4-13 of Reference (17).

A nominal size is first selected (W14, W12 or W10, etc.) and the bending factor is read from Table 4-13 (17). The total equivalent design axial load is now determined from the following equation:

\[
P' = P + M_B \]

where

- \( P' \) = equivalent total axial load
- \( P \) = applied axial load
\[ M_2 = \text{larger applied end moment} \]
\[ B = \text{appropriate bending factor} \]

4.5 **DETAILED ANALYSIS**

Once the beam and column sections are determined through a preliminary design, it is possible to carry out a detailed analysis using the program developed by the author. The input to the program consists of the properties of the sections for beams and columns and also the loads applied to them. Since the combination of dead load and full live load throughout the structure does not usually constitute the worst loading condition for the members, various patterns of live load distribution are to be considered. Precisely, to obtain maximum moments in all members, it shall be necessary to analyze the building for a large variety of patterns of loading and hence some approximations are made which are as follows:

(i) The live load may be considered to be applied only to the floor or roof under consideration and the far ends of the columns may be assumed as fixed.

(ii) Consideration may be limited to combinations of dead loads on all spans with full live load on two adjacent spans (for negative support moments) and with full alternate spans (for positive moments).

(iii) Columns may be designed to resist the axial loads on all floors plus the maximum bending moment due to loads on a single adjacent span of the floor under consideration.
The above assumptions are justified on the following considerations:

(i) Spans far removed from one under consideration make small contribution to the moment in the latter.

(ii) The traditional loading is very unlikely and has a small probability of occurrence.

(iii) In columns, maximum axial loads are produced when all floors above are loaded. The pattern for maximum moment is quite different. However, the difference will get minimized in view of first two assumptions as the number of stories above the column become large.

Hence, four patterns of live load as shown in Figs. 4.3 to 4.6 need to be considered and the load cases considered for analysis are as follows:

(i) Dead load only
(ii) Four cases of live load
(iii) Wind load

The forces under any particular combination of loading can be obtained by combining the results of individual load cases.

The eccentricity effects in the case of gravity loads alone as described in Section 2.7 are also considered. The equivalent lateral loads that simulate the effect of initial eccentricity are calculated assuming that the initial eccentricity is .002 times the height. This is in conformity with the erection tolerance provided in the Code.
Fig. 4.3. Live load for maximum negative moment in beam.

Fig. 4.4. Live load for maximum positive moment in beam and end moment in columns.
Fig. 4.5. Live load for maximum positive moment in beam and end moment in columns.

Fig. 4.6. Live load for maximum axial load.
4.6 DESIGN USING FIRST ORDER ANALYSIS

Once the basic data such as geometric properties of sections and
details of applied loadings are known, a first order analysis is carried
out by running the computer program with the first order option. From
the results of analysis for individual loads like dead loads, the four
cases of live load and wind load, forces are derived for the prescribed
loading combinations by superposition and the most critical combination
is determined for each member.

4.6.1 Design of Beams

Girders are designed for the maximum moment by making use of the
table on Page 5-100 to 5-106 of Reference (17). Only one girder size is
used for one floor for ease of construction.

4.6.2 Design of Columns

A column can be designed if the axial load and two end moments are
known under the worst combination of loading. The moments can be cal-
culated readily for the worst combination of loading by algebraic addition
of moments obtained for the individual case. The axial load calculated
in this manner will not, however, be correct because live load reduction
coefficients applicable to columns are different from those applicable
to beams.

The live load reduction coefficients used in the program are appro-
priate for beams but require modification for each column because of the
difference in area supported. Appropriate modification factor in this
case were worked out and corrected or modified axial loads were used in the design in conformity with the code.

In some cases, it is not readily possible to determine the worst combination of axial load and end moment and under these circumstances, the column section must be checked for 2 or even larger number of cases.

The columns can now be designed by the interaction equations, treating the former as sway permitted because secondary effects have not been accounted for in the analysis.

The interaction equations (mentioned in Section 1.2.4.9) are as follows:

\[
\frac{f_a}{.6F_y} + \frac{f_b}{F_b} \leq 1.0
\]

\[
\frac{f_a}{F_a} + \frac{C_m f_b}{(1 - \frac{f_a}{F_a})F_b} \leq 1.0
\]

At this stage, the effective length factor \( k \) can be read from nomographs knowing the values of \( G_U \) and \( G_L \) where \( U \) and \( L \) are the subscripts which refer to the joints at the two ends of the column and \( G \) is the ratio of the column stiffness to the beam stiffness meeting at a joint.

Being a sway permitted case, the value of \( C_m \) is 0.85.

Once the value of effective length factor \( k \) and moment factor \( C_m \) is known, the interaction equations mentioned above can be used to design the column section.
4.7 DESIGN USING SECOND ORDER ANALYSIS

When the second order analysis of the frame is carried out using modified stiffness matrix, the effect of movement of one end of the column with respect to the other is included in it and need not be considered again. Hence, in this case, the frame can be designed as if it is prevented from swaying.

A second order analysis is carried out by using the appropriate option when running the program. One additional information of factored story loads and story heights is required to use this option. The factored story loads can be determined taking into consideration appropriate probability factors for the load combination considered as mentioned in Section 2.5. For dead loads, live load and wind loads acting simultaneously, the probability factor is 0.75 and the value of factor of safety works out as follows:

\[ \alpha = 1.70 \times 0.75 \times 0.75 \]
\[ = 0.95625 \]
\[ \approx 1.00 \]

Just as in the case of a first order analysis, all load combinations prescribed in the code are used to determine the worst load case for each member. This would be the maximum moment in the case of a beam and the most critical combination of axial load and end moment for the column.

For design, the interaction equations method (Section 1.2.4.9) is used. The interaction equations are as follows:
\[
\frac{f_a}{0.6F_y} + \frac{f_b}{F_b} \leq 1.0
\]

\[
\frac{f_a}{F_a} + \frac{C_m f_b}{F_b (1 - \frac{f_a}{F_y})} \leq 1.0
\]

These are the same equations as used in the sway permitted method and the only difference lies in determining the effective length factor \(k\) and the value of \(C_m\). The value of \(k\) in turn affects \(F_a\) which is dependent on the effective slenderness ratio \(kL/r\).

The value of \(k\) is determined now by using the nomographs \(^{(17)}\) for the sway prevented case. The value of \(C_m\) is given by:

\[
C_m = 0.6 + 0.4 \frac{M_1}{M_2} \geq 0.4
\]

where

- \(M_1\) = smaller end moment
- \(M_2\) = larger end moment

It may be noted here that \(\frac{M_1}{M_2}\) is positive in the case of single curvature bending while it is negative in the case of double curvature bending.

Having known the value of effective length factor \(k\) and the moment factor \(C_m\), the column can be designed in the usual manner using the interaction equations.
4.8 RESULTS

The seven frames with different live loads, wind loads and number of stories detailed in Table 4.1 were analyzed and designed by the above two methods. The beam and column sections as designed are shown in Figs. 4.7 to 4.13. The member sizes based on first order analysis and design through interaction equations for sway permitted conditions are shown in ordinary type while sizes based on second order analysis and design for sway prevented conditions is shown in script type. The following important observations can be made:

(i) The moments in beams or girders obtained from a second order analysis are higher than those obtained in a first order analysis. This results in heavier beam sections when direct compensation for sway effects is used.

(ii) Second order analysis gives increased moments and axial loads in columns but these increased moments and axial loads do not necessarily mean heavier sections. In fact, usually smaller column sections are obtained in the direct compensation method, the value of \( C_m \) becomes 0.4 against 0.85 and the value of \( F_a \) goes up due to reduction in the value of \( k \).

(iii) A comparison of the weight of frames obtained by the two alternative methods is presented in Table 4.2. From the results, it is apparent that practically no economy has been achieved, on the whole by the use of direct compensation method since the reduction in weight achieved by smaller column sections is largely compensated for by the heavier section of beams. However, because in most cases the ultimate strength of a frame depends on the strength of beams\(^{(15)}\), the capacity of the frame increases.
LOADING

Dead Load = 50
Live Load = 40
Wind Load = 30

Fig. 4.7. Frame design by two alternative methods.
LOADING

Dead Load = 50 psf
Live Load = 60 psf
Wind Load = 30 psf

---

Fig. 4.8. Frame design by two alternative methods.
LOADING

Dead Load = 70 psf
Live Load = 100 psf
Wind Load = 50 psf

Fig. 4.9. Frame design by two alternative methods.
LOADING

Dead Load = 50 psf
Live Load = 60 psf
Wind Load = 20 psf

---

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</tbody>
</table>

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Fig. 4.10. Frame design by two alternative methods.
LOADING

Dead Load = 50 psf  
Live Load = 60 psf  
Wind Load = 40 psf

Fig. 4.11. Frame design by two alternative methods.
LOADING

Dead Load = 50 psf
Live Load = 60 psf
Wind Load = 30 psf

Fig. 4.12. Frame design by two alternative methods.

Fig. 4.13. Frame design by two alternative methods.
(iv) The second order analysis design method is more rational because in this case the $PA$ effects are exactly accounted for and are included in the forces (axial load or end moment). The alternative method of ignoring the $PA$ forces in the analysis and leaving a reserve strength in the member to counteract such forces is approximate and not quite rational. In fact, for beams, the section adopted by the first order analysis may be on the unsafe side or result in a reduced factor of safety.

Normally, some overall economy can be expected when designing through a second order analysis rather than through a first order analysis. However, as noted earlier, such economy is not apparent in the example frames presented. This is because of the following reasons:

(i) In most cases, while the stability equation governs the design in sway permitted cases, strength equation governs the design in sway prevented cases. Hence, the economy which should have resulted due to reduction in value of effective length factor, $k$ and corresponding $F_a$ and $C_m$ is lost to a large extent.

(ii) Further, in many cases, the potential economy cannot be realized because discrete sizes must be used for members. In design, it was observed that a member selected by the sway permitted method proved somewhat conservative when the design was repeated using the sway prevented method but the use of the next lower section made it unsafe.
(iii) A closer examination of Table 4.2 reveals that for most of the columns the parameters $\frac{h}{r}$, $P/P_y$, and $G$ lie in the following range:

- $\frac{h}{r} - 22.01-28.01$
- $P/P_y - .30-.754$
- $G - .77-3.21$

From the curves drawn under Section 3.6, it is evident that for columns with hinged base having an $h/r$ ratio up to 20 which is equivalent to an $h/r$ ratio of 40 for columns fixed at both ends, the method adopted for design is immaterial since the strength predicted by either method is fairly close to the true strength of the column. Hence, it would be reasonable to expect that for the frames presented there would not be much difference in the sections of columns designed by the two alternative methods.
Table 4.2. Comparative weights of frames and range of column properties.

<table>
<thead>
<tr>
<th>NO. OF STORIES</th>
<th>LOADING</th>
<th>h/y</th>
<th>P/F</th>
<th>G</th>
<th>WEIGHT OF FRAME (DESIGNED AS SWAY PERMITTED)</th>
<th>WEIGHT OF FRAME (DESIGNED AS SWAY PREVENTED)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>50</td>
<td>60</td>
<td>20</td>
<td></td>
<td>22.85-27.90 .405-.754 .679-3.14 86.95 85.52</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>60</td>
<td>30</td>
<td></td>
<td>22.53-27.90 .324-.558 .77-3.21 93.25 93.25</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>60</td>
<td>40</td>
<td></td>
<td>22.32-27.90 .30-.46  .77-3.24 102.10 102.06</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td></td>
<td>22.53-28.01 .30-.506 .64-3.21 91.60 92.09</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>70</td>
<td>100</td>
<td>30</td>
<td></td>
<td>22.01-27.27 .365-.625 .65-3.12 115.84 113.82</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>60</td>
<td>30</td>
<td></td>
<td>26.76-27.90 .321-.514 .77-2.21 31.55 31.38</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>60</td>
<td>30</td>
<td></td>
<td>27.90        .380         .77-1.14 12.60 12.60</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 5

SUMMARY AND CONCLUSION

5.1 GENERAL REMARKS

The objectives of this work were to investigate the effects of the interaction of gravity forces with the sway deflections in an unbraced multi-story frame, to develop an analytical technique for the analysis of such frames, and to study the applicability of the empirical interaction equations to design. As a part of this investigation, a method of analysis which avoids the complexity of a second order analysis, yet gives results that correctly account for the PΔ effect, was developed. The method, named as the modified stiffness method, is only marginally more complex than the comparable first order analysis method yet does not depend on additional simplifying assumptions or an iterative procedure.

The true strengths of a column both inclusive and exclusive of PΔ effects (total and primary moment carrying capacity) were derived and compared with the corresponding moment carrying capacities predicted by the interaction equations by studying a single story frame. Curves indicating the moment carrying capacities of columns were obtained for all practical values of the important parameters that affect the column strength such as the slenderness ratio, $\frac{h}{r}$, the ratio of column stiffness to beam stiffness $\frac{P}{G}$, and the axial load $\frac{P}{F}$ (non-dimensionalized).

Further, a series of frames were designed for varying gravity and lateral loads by two alternative methods. In one method, a first order
analysis was used along with the interaction equations applicable to the sway permitted. In the other method, a second order analysis was used along with the interaction equations applicable to the sway prevented case. The resulting designs were compared for possible differences in overall weight of the structure.

The inferences derived are applicable mainly to unbraced multi-story steel frames designed on the basis of working stress design methods, though some of the inferences may have wider application.

5.2 DEVELOPMENT OF THE METHOD OF ANALYSIS

A series of experiments conducted by several researchers have proved that under the action of loads, frames definitely sway and non-linearity is introduced primarily by the gravity forces acting on the deformed geometry. This interaction of gravity forces give rise to secondary forces which are known as PÂ effects. For a rational analysis, they must be taken into consideration.

To avoid the complexity of second order analysis, many engineers have suggested approximate methods based on simplifying assumptions. These methods are designed to convert the results of a first order analysis to those of a second order analysis. Since the assumptions made in these methods may not always be justified or valid and may introduce errors of unknown magnitudes, alternative methods have been suggested which rely on an iteration process. Although such methods are not based on any simplifying assumptions and are quite simple in principle, they may be time-consuming and expensive to use since five to six cycles of iterations, that is, five to six first order analyses may be needed for convergence.
The method developed as a part of this work and named as modified stiffness method solves most of these problems. In this method, the stiffness matrix of a column must be modified to account for the interaction of vertical loads with sway deflections.

For individual columns, such modification will require knowledge of the axial load carried by the column. Since these loads are not known in advance, it would appear as if an iterative procedure must be used. However, when the global stiffness matrix is assembled, the terms which account for the secondary effects contain only the total story loads and not the axial loads in individual columns. These story loads can be readily determined. It is to be noted that for a consistent factor of safety, the story loads should be factored loads even though the analysis is being performed for working loads level. A computer program has also been developed to apply this method to the analysis of multi-story frames. The program is based on a first order frame analysis programme developed earlier at Carleton University by Humar (19). The modified program can carry out both a first order analysis and a second order analysis. All joints in the frame are treated as rigid except that the base columns may be fixed or hinged at the foundations.

5.3 ECCRITICITY EFFECTS

In practice, columns are seldom perfectly straight and some eccentricity definitely exists due to imperfection of materials or due to construction and fabrication tolerances. The consideration of eccentricity effects becomes important when only gravity loads are present and the frame analyzed is symmetrical with symmetrical loading.
The eccentricity effects can be accurately accounted for by calculating a set of equivalent lateral forces and analyzing the frame for the combined effect of gravity forces and these lateral forces. For the evaluation of eccentricity effects, the analysis should account for the following:

(i) deflections caused by the equivalent lateral loads due to eccentricity,
(ii) magnification of the above deflections under vertical or gravity loads (PA effect).

Since the initial eccentricity is independent of the forces acting on the structure, factored loads are not used for part (i). For (ii), that is, for modification of stiffness matrix, factored load should be used as usual.

By using a method similar to that used for the eccentricity effect, it is possible to take into consideration the sway displacements due to non-uniform temperature change and foundation settlement.

5.4 CONSIDERATION OF SWAY EFFECTS IN DESIGN

As per the CSA Code, two methods can be used for the design of steel frames. In one method, the design forces and moments are obtained by first order analysis, and the columns are designed to have sufficient spare capacity to resist the secondary effects. In the other method, a second order analysis is carried out to obtain the forces and moments in the members inclusive of sway effects and the members are then designed so that full capacity is used. The design in either case is carried out by interaction equations which are approximate in nature. To examine
the validity of these methods, the true strengths of columns have been compared with the strengths predicted by the corresponding interaction equations. The comparisons have been made for varying values of important parameters such as the slenderness ratio \( \frac{h}{t} \), the ratio of column stiffness to beam stiffness \( G \), and the axial load (non-dimensionalized) \( \frac{P}{f_y} \), by studying a single story frame. The following important inferences have been drawn.

(i) The stability equation invariably governs if the member is designed by using the results of a first order analysis.

(ii) Within practical range of column properties, the strength equation invariably governs when the results of a second order analysis are used in design. This makes the selection of members much quicker, simpler and less time-consuming since the necessity of determining the effective length factor is avoided altogether.

(iii) In case of first order analysis, the strength predicted by interaction equations is always conservative and the design becomes more and more conservative as:

(a) the slenderness ratio \( \frac{h}{t} \) increases

(b) the ratio of the column stiffness to the beam stiffness \( G \) increases

(c) \( \frac{P}{f_y} \) increases or \( \frac{M}{f_p} \) decreases.

(iv) In case of second order analysis, the ratio of strength predicted by interaction equations to the true strength increases, i.e. design becomes more and more unsafe as:
(a) the slenderness ratio $\frac{h}{r}$ increases
(b) the stiffness ratio $C$ increases
(c) $\frac{P}{P_y}$ increases or $\frac{M}{M_p}$ decreases.

(v) Up to an $\frac{h}{r}$ ratio of 20 which is equivalent to a slenderness ratio of 40 for a column fixed on both ends, design by either method may be satisfactory as the moments predicted by interaction equations are close to the true moments of column. However, in case of second order analysis, when the $\frac{h}{r}$ ratio exceeds 20 (40 when both ends are fixed) it becomes essential to account for the difference between the true strength and the strength predicted by interaction equations. This can be done easily with the help of the Table of Correction Coefficients presented in this thesis.

A study of the relationship between the end moment and lateral deflection of a column carrying a given axial load $\frac{P}{P_y}$ (non-dimensionalized) reveals that when secondary sway effects are present there are situations when the useful limit of lateral load is reached before full capacity of the column is utilized. After this limit, an increasing portion of this capacity is used up by the sway effects, so that even though the total moment resisted by the column increases the portion of capacity available to resist primary or applied moment decreases. This explains to a large extent why the interaction equations applicable to the sway prevented case over-estimates the strength of the column.
5.5 DESIGN OF FRAMES

With a view to study the implication of the use of direct compensation for sway effects in design, a series of frames have been designed for varying values of gravity and lateral loads by the two alternative methods under identical conditions and the results compared. Because sway effects depend on the relative values of axial loads arising from gravity and the lateral loads due to wind or earthquake, the basic parameters used in study are the uniformly distributed gravity loads, i.e. the dead load and live load, the lateral load due to wind and the number of stories. The following important inferences have been drawn:

(i) The girder sections obtained when a second order analysis method is used in design are heavier than those obtained when a first order analysis method is used.

(ii) The second order analysis method gives lighter column sections.

(iii) Although a comparison of the weights of the two alternative designs shows that no significant economy is achieved mainly because member sections are available only in discrete sizes, the second order analysis method has the following advantages:

(a) The strength of the frame goes up since, in general, frame strength depends on the strength of beam or girder.\(^{(15)}\)

(b) The second order analysis method is more rational since P\&A effects are accounted for exactly.

(iv) If the slenderness ratio of columns is less than 20 (40 when both ends are fixed), both methods give almost equally good results.
5.6 RECOMMENDED FUTURE STUDIES

During the course of this work, certain factors which may influence the sway effects significantly have become identified. There is scope for additional study of the extent and nature of such influences. Some of the potential areas of study are described below.

1. In evaluating the sway effects at working load levels, it has been assumed that the lateral deflections increase proportionally with the load until ultimate load has been reached. In fact, however, before the ultimate load is reached, there may be considerable inelastic deformations in individual members of the frame. As a result, the sway deflections are likely to be more than those determined on the basis of proportionality. Additional sway deflections may arise from other sources such as slip of bolts and joint deformations. Working stress sway deflection values that are consistent with the total response of the frame under factored loads inclusive of the effects described above should be determined and used in a PΔ analysis. It has been argued that the increase in PΔ effect due to the above cause is compensated by stiffening effects such as those due to non-structural components, finite joint sizes, etc. but the topic is worth additional study.

2. As mentioned in 1 above, certain factors add to the stiffness of the structure but are not usually accounted for in analysis. The most difficult to analyze correctly is the effect of exterior cladding and interior partitions which may interact with the
structural frame and contribute to the stiffness of the structure. This topic needs additional investigation.

3. It is recognized that the average live loads on entire floors are generally a fraction of the live loads that are used to design individual columns. The live load reduction factors that may be applied to the total story P forces used in a PΔ analysis need further study.

4. Since PΔ effects may be more significant in taller frames, it is suggested that some frames taller than 10 stories may be analyzed for a comparative study.
REFERENCES


APPENDIX A

PROGRAM USERS MANUAL
COMPUTER PROGRAM FOR THE
STATIC ANALYSIS OF ELASTIC MULTI-STORY
RECTANGULAR FRAMES
WITH OR WITHOUT
SWAY EFFECTS

INSTRUCTIONS FOR USE

BY
M.M. SINGHAL

September 1978

Department of Civil Engineering
Faculty of Engineering
Carleton University
Ottawa, Canada
PREFACE

The program described in this User's Manual has been developed for carrying out a first order or a second order analysis of an orthogonal planar frame with rigid joints. The base columns may either be hinged or fixed at the foundation level. In writing this program, use has been made of a program developed earlier by Humar for a first order static analysis of orthogonal frames. This earlier program was developed to serve as a complement to a package of seismic analysis programs available through the Seismic Programming Library of Carleton University. The input data format was therefore designed to be compatible with that used for the seismic analysis program. Essentially, the same input format has been maintained in this program.
DISCLAIMER

While every effort has been made to ensure the accuracy of the information presented in this manual, and the reliability of the program described therein, use of this material can be made only with the understanding that neither the author nor Carleton University will assume any liability of any kind arising from such use.
CONTROL CARDS

The following control cards are used to execute the program on the
XDS Sigma 9 Computer as of April, 1976. With the passage of time, some
changes may occur in the control cards. If the control cards do not
cause successful executions of the program, it is suggested that the SPL
Librarian at the Computing Centre be consulted.

Execution of SPL 7304 Using Remote Slow Speed Terminal and Disk Input
and Output Files

:JOB XXXXXXXX, XXXXXX, 7, 3, SEND TO ENGINEERING

:LIMIT, (TIME, 2) , (LO, 200) , (CORE, 40)

:ASSIGN F:1 , (FILE, SE730401) , (IN) , (SAVE)

:ASSIGN F:2 , (FILE, SU730401) , (OUT) , (SAVE)

:RUN (LMN, LNS)

:DATA

:EOD

The above control command file is submitted to BATCH through the
terminal. The input data is on File SE730401, and the output will be
produced on File SU730401.

Execution of SPL 7304 Using RBT in Computer Room of C.J. Mackenzie
Building and Cards for Input Data

:JOB XXXXXXXX, XXXXXXXXX, 7

:LIMIT (TIME, 2) , (LO, 200) ; (CORE, 40)

:ASSIGN F:1 , (DEVICE, CR)

:ASSIGN F:2 , (DEVICE, LP)
RUN (LMN, LNS)

DATA

---
data cards

EOD
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2. Assumptions and Conventions Used in Assigning the Degrees-of-Freedom
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1. INTRODUCTION

This computer program has been developed for a first or a second order elastic static analysis of orthogonal planar frames with rigid joints. The base columns may either be hinged or fixed at the foundation level. It is assumed that gravity loads are applied through beams and can be uniform loads distributed throughout the length of beam or/and one or more concentrated loads applied to them. The lateral loads are applied at floor levels. The program has been designed so that during the same run a number of different structures can be analyzed, each for several different sets of loads.

Input data for the program comprises of beam and column properties, and the loading data. In addition, for a second order analysis, the total gravity load carried by each story as well as the height must be specified. The output comprises of the beam end moments, column end moments, column axial loads, and lateral displacements at each floor level.

Since the program was originally developed by Humar to serve as a complement to a package of seismic analysis programs the data input for the two sets of programs is designed to be compatible. Some of this data input will be redundant for the present program. If, therefore, only a static analysis of the frame is desired, the redundant data need not be specified and the relevant card columns may be left blank. This has been described in greater detail in the specification of input data.
2. **ASSUMPTIONS AND CONVENTIONS USED IN ASSIGNING THE DEGREES-OF-FREEDOM**

The stiffness or displacement method is used in the analysis and it is therefore implicitly assumed that the material is linear and elastic. Following further assumptions are made.

1. **The effect of axial deformation of beams can be neglected.**
   The beams are thus considered axially rigid and the corresponding degrees-of-freedom are omitted. It is further assumed that shear deformations are negligible. For columns, the flexural and axial deformations are accounted for but shear deformations are considered negligible.

2. **It is assumed that all members are prismatic and have a uniform moment of inertia throughout their lengths.**

3. **All dimensions are referred to the center line of members and the joint sizes are considered negligible in comparison to the lengths of members.**

4. **The secondary moments introduced in columns and beams due to axial loads interacting with member deformations are small and can be neglected. Similarly, in a first order analysis, the frame displacements are assumed to be small so that the equations of equilibrium can be formulated on the undeformed geometry of the structure. However, in the second order analysis, the interaction of gravity loads with the sway displacement is taken into account.**
Assumption 1 implies that only one sidesway degree-of-freedom is associated with each story as indicated in Fig. 1. Each column element will have six degrees-of-freedom and each beam element four degrees-of-freedom. The convention used in numbering the element degrees-of-freedom is shown in Fig. 2.

The element stiffness matrices for beams and columns are shown below. In the following,

\[ E = \text{modulus of elasticity} \]
\[ I = \text{moment of inertia of the section} \]
\[ L = \text{length} \]

**Stiffness matrix for beam element:**

\[
\begin{bmatrix}
\frac{4EI}{L} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{6EI}{L^2} \\
\frac{2EI}{L} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{6EI}{L^2} \\
\frac{-6EI}{L^2} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & \frac{-12EI}{L^3} \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{12EI}{L^3}
\end{bmatrix}
\]

**Stiffness matrix for column element:**

\[
\begin{bmatrix}
\frac{4EI}{L} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{-6EI}{L^2} & 0 & 0 \\
\frac{2EI}{L} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{-6EI}{L^2} & 0 & 0 \\
\frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{-12EI}{L^3} & 0 & 0 \\
\frac{-6EI}{L^2} & \frac{-6EI}{L^2} & \frac{-12EI}{L^3} & \frac{12EI}{L^3} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{EA/L}{-EA/L} \\
0 & 0 & 0 & 0 & \frac{-EA/L}{EA/L}
\end{bmatrix}
\]
Fig. 1. Structure coordinates with columns axially deformable.
(a) Beam.

(b) Column.

Fig. 2. Member degrees of freedom.
In accordance with assumption 2, the effect of axial load interaction has been neglected in deriving the member stiffness matrices. Having determined the individual element stiffness matrices, the system stiffness matrix can be assembled using the correspondence of element degrees-of-freedom to structure coordinates. When a second order analysis is to be carried out, the assembled structure stiffness matrix is modified by the algebraic addition of P/L terms to the degrees-of-freedom corresponding to the sidesway of the frame.

Before preparing the data, the structural degrees-of-freedom have to be numbered. A specific scheme of numbering has been adopted to take advantage of the banding of structural stiffness matrix. As in Fig. 1, starting from the lowest story, the story rotational degrees-of-freedom are numbered first, then the degree-of-freedom corresponding to the sidesway deflection of the story is numbered, the degrees-of-freedom relating to the column extensions coming last. The positive sense of degrees-of-freedom is also shown in Fig. 1.

The above scheme ensures minimum bandwidth for the structural stiffness matrix, the value of bandwidth being equal to the maximum difference between a rotational degree-of-freedom in one story and the degree-of-freedom corresponding to sidesway deflection of the story above or below plus one. Thus, in Fig. 1, the maximum difference between the above said degrees-of-freedom is 14 - 1 = 13 and the bandwidth is 13 + 1 = 14.

The beams and columns have also to be assigned numbers by which they are recognized in input data and in the static analysis output. There is no limitation on the scheme of numbering and any sequence may be
followed as long as the beams or columns are assigned consecutive numbers starting from 1. It will be found convenient to say start from the lowest story and proceed from left to right.

Having numbered the degrees-of-freedom of the beams and columns, it is straightforward to work out the structural coordinate numbers that correspond to element degrees-of-freedom. Thus, in Fig. 1, for column 5

\[
\begin{align*}
NCC (5,1) &= 1 \\
NCC (5,2) &= 10 \\
NCC (5,3) &= 5 \\
NCC (5,4) &= 14 \\
NCC (5,5) &= 6 \\
NCC (5,6) &= 15
\end{align*}
\]

where NCC is an array containing structural coordinate numbers corresponding to column degrees-of-freedom. For columns like 1, 2, 3 and 4, it will be observed that there are no structure coordinates corresponding to element degree-of-freedom numbers 1, 3 and 5. In such cases if the column is fixed at the base the corresponding structure coordinate number is simply assigned a value of zero. Thus

\[
\begin{align*}
NCC (1,1) &= 0 \\
NCC (1,3) &= 0 \\
NCC (1,5) &= 0
\end{align*}
\]

In case column is pinned at the base, the degree of freedom assigned for rotation is 9999. Thus,

\[
NCC (1,1) = 9999.
\]
3. **INTERNAL STORAGE OF STIFFNESS MATRIX**

   The structural stiffness matrix for the frame will be symmetrical and banded as shown in Fig. 3. All elements outside the band will be zero and if the elements inside one half band including the diagonal are stored in the computer all relevant information will be available. Only the lower half band and diagonal are therefore stored in a special matrix S of size NBAND by NDF so that elements on diagonal occupy the first row of S matrix.

   \[
   \begin{array}{cccccccc}
   A & B & C & D & E & I & M & Q & U & X & Z \\
   B & E & F & G & H & & & & & & \\
   C & F & I & J & K & L & A & E & I & M & Q & U & X & Y \\
   D & G & J & M & N & O & P & B & F & J & N & R & T & Y \\
   H & K & N & Q & R & S & T & C & G & K & O & S & W & - \\
   L & O & R & U & V & W & D & H & L & P & T & - & - \\
   \end{array}
   \]

   Stiffness Matrix

4. **SIGN CONVENTIONS AND UNITS**

   4.1 **Sign Conventions**

   Joint rotations, displacements and moments are treated as positive in the positive directions of structural degrees-of-freedom shown in Fig. 1. The axial forces in columns are considered positive when compressive.

   Uniformly distributed or concentrated vertical loads acting on beams are considered positive when acting downwards.
4.2 Units

Any set of consistent units may be used; the program does not carry out any transformation. A typical set of units is shown below:

Length \( L \) \( \text{in} \)

Moment of Inertia \( I \) \( \text{in}^4 \)

Modulus of Elasticity \( E \) \( \text{kips/in}^2 \)

U.D.L. UDL \( \text{kips/in} \)

Conc. Load & Story Load \( \text{kips} \)

Area \( A \) \( \text{in}^2 \)

5. Program Specifications

The program has been written in Standard Fortran IV and has been tested on Zerox Sigma 9 computer. With minor modifications, it will run on any machine that accepts Fortran IV language. The limiting dimensions in the present version of the program are as follows:

- No. of degrees-of-freedom 260
- Bandwidth 20
- No. of stories 20
- No. of beams 100
- No. of columns 120

A 20 story 5 frame can be analyzed within these dimensions.

The program when run in double precision needs the following storage in the core.
Storage requirements in decimal words:

1. Main program 15759

Subroutines

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BEAM</td>
</tr>
<tr>
<td>2</td>
<td>COLUMN</td>
</tr>
<tr>
<td>3</td>
<td>LOAD</td>
</tr>
<tr>
<td>4</td>
<td>FORE</td>
</tr>
<tr>
<td>5</td>
<td>BACK</td>
</tr>
<tr>
<td>6</td>
<td>ELST</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Grand Total</td>
</tr>
</tbody>
</table>
## INPUT DATA

<table>
<thead>
<tr>
<th>CARD GROUP</th>
<th>NO. OF CARDS</th>
<th>FORTRAN VARIABLE</th>
<th>FORMAT</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>NAME = array containing case title</td>
<td></td>
<td>For identification of the problem</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>NB = no. of beams</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NC = no. of columns</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS = no. of stories</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NBAND = maximum bandwidth of stiffness matrix</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NDF = no. of degrees-of-freedom</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>E = modulus of elasticity</td>
<td>F12</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>K = variable to specify whether a first or a second order analysis is required</td>
<td>I5</td>
<td>K=0 for first order analysis, K=1 for second order analysis</td>
</tr>
<tr>
<td>D</td>
<td>NB = no. of columns</td>
<td>I = beam no.</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>BL(I) = length</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR(I) = moment of inertia</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>BPM(I) = plastic moment of section</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>SB(I) = section modulus</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NBC(I,J), J=1 to 4</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Structure coordinates corresponding to beam degrees of freedom</td>
<td>415</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>NC = no. of columns</td>
<td>I = column no.</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CL(I) = length</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CR(I) = moment of inertia</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CPM(I) = plastic moment of section</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A(I) = cross-sectional area</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>SC(I) = section modulus</td>
<td>F10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NCC(I,J), J=1 to 6</td>
<td>6I4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Structure coordinates corresponding to column degrees of freedom</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### INPUT DATA

<table>
<thead>
<tr>
<th>CARD GROUP</th>
<th>NO. OF CARDS</th>
<th>FORTRAN VARIABLE</th>
<th>FORMAT</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>NS = no. of stories</td>
<td>I = story no. N = structure coordinate corresponding to story sidesway</td>
<td>IS IS</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>SL = story load XL = story height</td>
<td>F10 F10</td>
<td>Omit the card if K=0</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>I = beam no. UDL = uniformly distributed NCONC = no. of concentrated loads on the beam</td>
<td>IS F12 IS</td>
<td>Leave blank if no UDL Leave blank if no concentrated loads</td>
</tr>
<tr>
<td>I</td>
<td>At the rate of one card for three conc. loads or less</td>
<td>D(J), P(J), J = 1 to NCONC</td>
<td>6F10</td>
<td>Omit this card if NCONC = 0 Present dimensioning handles up to 10 concentrated loads on a beam</td>
</tr>
</tbody>
</table>

**Note** - Repeat cards H and I for each loaded beam

<table>
<thead>
<tr>
<th>J</th>
<th>1</th>
<th>Blank</th>
<th>To signal end of vertical load data</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>No. of stories to which lateral loads are applied</td>
<td>I = story no. W = lateral load applied at the story level</td>
<td>IS F12</td>
</tr>
<tr>
<td>L</td>
<td>1</td>
<td>Blank</td>
<td>To signal end of lateral load data</td>
</tr>
<tr>
<td>M</td>
<td>1</td>
<td>NEXT = flag to test for new load case or new structure</td>
<td>IS</td>
</tr>
</tbody>
</table>

**NOTE**

If NEXT = 1, repeat card groups G to M.
If NEXT ≠ 2, repeat card groups A to M
7. EXAMPLE

The frame shown in Fig. 4 is to be analyzed for the two load cases indicated there. Second order effects are to be included.

The input data is shown on Page 172 and the result of the analysis is on Pages 173 through 175.
(a) Vertical loads.

(b) Lateral loads.

Fig. 3. Frame with degrees of freedom and loading.
### Analysis of Multistory Frame

**FOR STATIC LOADS**

**PROGRAMMED BY**

**J L HUIRAN**

**CARLETON UNIVERSITY**

SEPT 1978

---

**CASE TITLE**

**EXAMPLE FRAME**

<table>
<thead>
<tr>
<th>NB</th>
<th>NC</th>
<th>NS</th>
<th>NBRAND</th>
<th>NDF</th>
<th>Fb</th>
<th>.30000E 05</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>12</td>
<td>.F</td>
<td></td>
</tr>
</tbody>
</table>

---

**SECOND ORDER ANALYSIS**

<table>
<thead>
<tr>
<th>BEAM NO.</th>
<th>LENGTH</th>
<th>INERTIA</th>
<th>PLASTIC NOW.</th>
<th>MODULUS</th>
<th>STRUCTURE COORDINATES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.24800E 03</td>
<td>.80000E 03</td>
<td>.00000E 00</td>
<td>.00000E 00</td>
<td>1 2 3 5 6</td>
</tr>
<tr>
<td>2</td>
<td>.25800E 03</td>
<td>.80000E 03</td>
<td>.00000E 00</td>
<td>.00000E 00</td>
<td>2 3 6 7</td>
</tr>
<tr>
<td>3</td>
<td>.26300E 03</td>
<td>.70000E 03</td>
<td>.00000E 00</td>
<td>.00000E 00</td>
<td>8 9 11 12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>COL NO.</th>
<th>LENGTH</th>
<th>INERTIA</th>
<th>PLASTIC NO.</th>
<th>AREA</th>
<th>MODULUS</th>
<th>STRUCTURE COORDINATES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.14400E 03</td>
<td>.60000E 03</td>
<td>.00000E 00</td>
<td>.15000E 02</td>
<td>0 1 0 4 0 5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>.14400E 03</td>
<td>.60000E 03</td>
<td>.00000E 00</td>
<td>.15000E 02</td>
<td>2 3 4 6 6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>.14400E 03</td>
<td>.60000E 03</td>
<td>.00000E 00</td>
<td>.15000E 02</td>
<td>0 3 4 0 6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.14400E 03</td>
<td>.60000E 03</td>
<td>.00000E 00</td>
<td>.15000E 02</td>
<td>2 8 4 10 6 11</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>.14400E 03</td>
<td>.60000E 03</td>
<td>.00000E 00</td>
<td>.15000E 02</td>
<td>3 9 4 10 7 12</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STORY NO.</th>
<th>STORY LOAD</th>
<th>ZIGGY HEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.15140000</td>
<td>100.00</td>
</tr>
<tr>
<td>2</td>
<td>-43.800000</td>
<td>100.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BEAM NO.</th>
<th>UDL</th>
<th>CONC LOADS</th>
<th>LOAD NO.</th>
<th>DIST FROM LEFT</th>
<th>LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.14400E 03</td>
<td>1</td>
<td>.14400E 03</td>
<td>.25000E 02</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>.14400E 03</td>
<td>0</td>
<td>.14400E 03</td>
<td>.25000E 02</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>.14400E 03</td>
<td>1</td>
<td>.14400E 03</td>
<td>.25000E 02</td>
<td></td>
</tr>
</tbody>
</table>
### Beam No.

<table>
<thead>
<tr>
<th>Moment 1</th>
<th>Moment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-848.1E 03</td>
</tr>
<tr>
<td>2</td>
<td>-360.6E 03</td>
</tr>
</tbody>
</table>

### Column No.

<table>
<thead>
<tr>
<th>Moment 1</th>
<th>Moment 2</th>
<th>Axial Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.403E 03</td>
<td>-8931E 03</td>
</tr>
<tr>
<td>2</td>
<td>-3035E 03</td>
<td>5178E 03</td>
</tr>
<tr>
<td>3</td>
<td>-3045E 03</td>
<td>2012E 03</td>
</tr>
<tr>
<td>4</td>
<td>-3239E 03</td>
<td>9435E 03</td>
</tr>
<tr>
<td>5</td>
<td>-3039E 03</td>
<td>8755E 03</td>
</tr>
</tbody>
</table>

### Story Displacement

<table>
<thead>
<tr>
<th>Story</th>
<th>Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.10017E 00</td>
</tr>
<tr>
<td>2</td>
<td>.5194E-01</td>
</tr>
</tbody>
</table>

### New Load Case

### Story No. | Story Load | Story Height |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>131.4E000</td>
<td>144.00</td>
</tr>
<tr>
<td>2</td>
<td>48.8E000</td>
<td>144.00</td>
</tr>
</tbody>
</table>

### Beam No. | UDL | Grsc Loads | Load No. | Dist from Left | Load |
|------------|-----|------------|----------|----------------|------|

### Story Coord. | Load |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

### Beam No.

<table>
<thead>
<tr>
<th>Moment 1</th>
<th>Moment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23865E 03</td>
</tr>
<tr>
<td>2</td>
<td>12769E 03</td>
</tr>
<tr>
<td>3</td>
<td>13359E 03</td>
</tr>
</tbody>
</table>

### Column No.

<table>
<thead>
<tr>
<th>Moment 1</th>
<th>Moment 2</th>
<th>Axial Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.403E 03</td>
<td>-8931E 03</td>
</tr>
</tbody>
</table>
APPENDIX B.

LISTING OF THE COMPUTER PROGRAM
**THE PROGRAM HAS BEEN DESIGNED FOR THE STATIC ANALYSIS OF RECTANGULAR, **
**ISOTROPIC PLATES. ALL JOINTS ARE ASSUMED TO BE RIGID.**
**BENDING MOMENTS ARE TAKEN INTO ACCOUNT BUT BEAMS ARE ASSUMED**
**ANISOTROPIC RIGID**

**PROMITTED BY: J L SHAR & M M SINGHAL, CARLETON UNIVERSITY, OTTAWA, S**
**SEP. 1975**

**THE PROGRAM HAS BEEN WRITTEN IN FORTRAN 4.**

**DIMENSION DL(100), BR(100), BPM(100), NBC(100,4), BM(100,2), SB(120)**
**DIMENSION CL(120), CR(120), CP(120), NCC(120,2), CH(120,2), SC(120)**
**DIMENSION X(50), Y(50)**
**DIMENSION A(120), P(120)**
**DIMENSION S(20,260), X(260), B(260), PM(100,2), NSC(20), T(20,260)**
**DIMENSION NAME(20)**
**EQUIVALENCE (SM, KM)**
**EQUIVALENCE (SM, CP), (SB, SC)**

**IIR=1**
**IJK=2**
**WRITE(IWW,100)**
**WRITE(IWW,101)**
**WRITE(IWW,102)**
**WRITE(IWW,101)**
**WRITE(IWW,103)**
**WRITE(IWW,101)**
**WRITE(IWW,118)**
**WRITE(IWW,104)**
**WRITE(IWW,119)**
**WRITE(IWW,101)**
**WRITE(IWW,105)**
**WRITE(IWW,101)**
**WRITE(IWW,100)**

**READ AND PRINT CASE TITLE.**

1 READ(IIR,106) NAME
WRITE(IWW,107) NAME

**READ AND PRINT FRAME SPECIFICATIONS**

READ(IIR,108) NB, NC, NS, NBAND, NDF, E
WRITE(IWW,117) NB, NC, S, NBAND, NDF, E
DO 2 I=1, NBAND
DO 2 J=1, NDF
O (I,J) = 0
READ(IIR,121) X
IF (A.EQ.0) GO TO 25
WRITE(IWW,131)
GO TO 15
25 WRITE(IWW,135)

**READ AND PRINT BEAM AND COLUMN PROPERTIES.**
**FILL IN THE STIFFNESS MATRIX**

15 CALL BEAM(BL, BR, BPM, NBC, S, NB, E, SB)
CALL COLUMN(CL,CR,CPM,A,NCC,S,NC,E,SC)

C READ STRUCTURE COORDINATE CORRESPONDING TO SIDE SWAY
C DEGREE-OF-FREEDOM OF EACH STORY.

DO 35 I=1,NS
READ(IWR,105)I,N
NSC(I)=N
35 CONTINUE
IF(K.EQ.0)GO TO 36
50 DO 16 I=1,NBAND
DO 15 J=1,NDF
T(I,J)=S(I,J)
15 CONTINUE
MODIFY STRUCTURE STIFFNESS MATRIX FOR SECOND ORDER ANALYSIS

WRITE(IWW,130)
DO 12 I=1,NS
READ(IWR,120)SL(I),XL(I)
12 WRITE(IWW,125)I,SL(I),XL(I)
N=NSC(I)
S(I,:)=S(I,N2)-SL(I)/XL(I)
IF(NS.EQ.2)GO TO 35
DO 30 I=2,NS
N=NSC(I-1)
N2=NSC(I)
N3=N2-N1+1
S(I,N1)=S(I,N1)-SL(I)/XL(I)
S(I,N2)=S(I,N2)-SL(I)/XL(I)
S(N3,N1)=S(N3,N1)+SL(I)/XL(I)
30 CONTINUE

C DECOMPORSE MATRIX S USING CHOLESKY METHOD

36 CALL FORE(S,NDF,NBAND)
45 DO 3 I=1,NDF
B(I)=0.
3 X(I)=0.

C READ AND PRINT VERTICAL LOADS
5 CALL LOAD(NSC,FM,NB,BL,B)

C READ AND PRINT HORIZONTAL LOADS
WRITE(IWW,114)
6 READ(IWR,109)I,W
IF(I.EQ.0)GO TO 7
N=NSC(I)
WRITE(IWW,115)I,N,W
B(N)=B(N)+W
GO TO 6

C SOLVE BY CHOLESKY METHOD

7 CALL BACK(S,B,X,NDF,NBAND)

C CALCULATE AND PRINT ELEMENT FORCES

CALL ELST(NBC,X, BR, BL, NB, E, BM, FM, NCC, CR, CL, CM, P, NC, A)

C PRINT STORY DISPLACEMENT

WRITE(IWW,110)
DO 9 I=1,NS
L=NSC(I)
9 WRITE(IWW,111)I,X(L)
TEST FOR NEXT CASE
IF NEXT=0 END OF JOB
OF NEXT=1 NEW LOAD CASE
IF NEXT=2 NEW STRUCTURE

READ (IRR, 108): NEXT
IF (NEXT .EQ. 0) GO TO 10
IF (NEXT .EQ. 1) GO TO 12
WRITE (IWW, -12)
GO TO 1
12 WRITE (IWW, 113)
IF (X .EQ. 0) GO TO 45
DO 4 I = 1, N.BAND
DO 4 J = 1, NDF
S(I, J) = T(I, J)
4 CONTINUE
GO TO 50
10 STOP

***********************************************************************

100 FORMAT(36X, 36H*************************************)
101 FORMAT(3X, 1H*, 34X, 1H*)
102 FORMAT(36X, 1H*, 3X, 28H ANALYSIS OF MULTISTORY FRAME, 3X, 1H*)
103 FORMAT(36X, 1H*, 5X, 16H FOR STATIC LOADS, 9X, 1H*)
104 FORMAT(36X, 1H*, 3X, 13H PROGRAMMED BY)
105 FORMAT(36X, 1H*, 1X, 32H CARLETON UNIVERSITY SEPT 1978, 1X, 1H*)
106 FORMAT(20A4)
107 FORMAT(13H CASE TITLE --- , 20A4, /*)
108 FORMAT(51S, F12.5)
109 FORMAT(1S, F12.5)
110 FORMAT(1S, 36H, 5HISTORY, 6X, 12H DISPLACEMENT, /*)
111 FORMAT(36X, 4X, 7X, 12.5)
112 FORMAT(13H NEW STRUCTURE, /*)
113 FORMAT(13H NEW LOAD CASE, /*)
114 FORMAT(20X, 5HISTORY, 5X, 6HCOORD., 7X, 4HLOAD, /*)
115 FORMAT(20X, 14.6X, 14.6X, 14.6X, 12.5)
116 FORMAT(10H, 3X, 3HNB=, I5, 3X, 3HNC=, I5, 3X, 3HNS=, I5, 3X, 6HNBAND=, I5,
13X, 4H NDF=, I5, 3X, 2HE=, E13.5, /*)
120 FORMAT(2F10.5)
121 FORMAT(I5)
122 FORMAT(20X, 12, 13X, F10.5, 12X, F6.2)
130 FORMAT(17X, 9HISTORY NO., 8X, 10HISTORY LOAD, 8X, 12HISTORY HEIGHT, /*)
131 FORMAT(38X, 1H SECOND ORDER ANALYSIS, /*)
135 FORMAT(25X, 20H FIRST ORDER ANALYSIS, /*)
138 FORMAT(36X, 1H*, 20X, 9HJ L HUMAR, 5X, 1H*)
119 FORMAT(36X, 1H*, 20X, 11H M SINGHAL, 3X, 1H*)
END
SUBROUTINE BEAM(BL, BR, BPM, NBC, S, NB, E, SB)

C READS AND PRINTS BEAM DATA AND ADDS BEAM STIFFNESS TO S MATRIX

DIMENSION BL(100), BR(100), BPM(100), NBC(100, 4), S(20, 260), SB(100)
IRR=1
IWW=2
WRITE (IWW, 102)
DO 3 L = 1, NB
1 READ (IRR, 100) I, BL(I), BR(I), BPM(I), SB(I), (NBC(I, J), J = 1, 4)
3 N1=NBC(I, 1)
N2=NBC(I, 2)
N3=NBC(I, 3)
N4=NBC(I, 4)
S(I, N1)=S(I, N1)+4.*E*BR(I)/BL(I)
S(I, N2)=S(I, N2)+4.*E*BR(I)/BL(I)
S(1,N3) = S(1,N3) + 12 * E * BR(I) / BL(I)^2
S(1,N4) = S(1,N4) + 12 * E * BR(I) / BL(I)^2
J = N2 - N1 + 1
S(J,N1) = S(J,N1) + 2 * E * BR(I) / BL(I)
J = N3 - N1 + 1
S(J,N1) = S(J,N1) - 6 * E * BR(I) / 3L(I)^2
J = N4 - N1 + 1
S(J,N1) = S(J,N1) + 6 * E * BR(I) / BL(I)^2
J = N4 - N3 + 1
S(J,N1) = S(J,N1) - 2 * E * BR(I) / BL(I)^2
WRITE(IW,100) I, BL(I), BR(I), BP(I), SB(I), (NBC(I,J), J=1,4)
3 CONTINUE
2 RETURN
100 FORMAT(5,F10.5,145)
101 FORMAT(17X,15,2X,4E14.5,145)
102 FORMAT(/, '17X, 8HBEAM NO., 5X, 6LENGTH, 8X, 7HINERTIA, 4X,
       412PLASTIC HOM,, 4X, 7HMODULUS, 1X, 21HSTRUCTURE COORDINATES, //)
END
SUBROUTINE COLUMN(CL, CR, CPM, A, NCC, S, NC, E, SC)
C READS AND PRINTS COLUMN DATA AND ADDS COLUMN STIFFNESS TO S MATRIX.

DIMENSION CL(120), CR(120), CPM(120), A(120), NCC(120, 6), S(20, 260),
       SC(120)
IRR=1
IWV=2
WRITE(IW,102)
DO 4 L=1, NC
1 READ(IRR, 100) I, CL(I), CR(I), CPM(I), A(I), SC(I), (NCC(I,J), J=1,6)
   N1=NCC(I,1)
   N2=NCC(I,2)
   N3=NCC(I,3)
   N4=NCC(I,4)
   N5=NCC(I,5)
   N6=NCC(I,6)
IF(N1.EQ.9999) GO TO 5
IF(N1.EQ.0) GO TO 2
S(1,N1) = S(1,N1) + 4 * E * CR(I) / CL(I)
S(1,N3) = S(1,N3) + 12 * E * CR(I) / CL(I)^2
S(1,N5) = S(1,N5) + E * A(I) / CL(I)
J = N2 - N1 + 1
S(J,N1) = S(J,N1) + 2 * E * CR(I) / CL(I)
J = N3 - N1 + 1
S(J,N1) = S(J,N1) + 6 * E * CR(I) / CL(I)^2
J = N4 - N1 + 1
S(J,N1) = S(J,N1) - 6 * E * CR(I) / CL(I)^2
J = N2 - N3 + 1
S(J,N3) = S(J,N3) + 6 * E * CR(I) / CL(I)^2
J = N4 - N3 + 1
S(J,N3) = S(J,N3) - 12 * E * CR(I) / CL(I)^2
J = N6 - N5 + 1
S(J,N5) = S(J,N5) - E * A(I) / CL(I)
2 S(1,N2) = S(1,N2) + 4 * E * CR(I) / CL(I)
S(1,N4) = S(1,N4) + 12 * E * CR(I) / CL(I)^2
S(1,N6) = S(1,N6) + E * A(I) / CL(I)
J = N4 - N2 + 1
S(J,N2) = S(J,N2) - 6 * E * CR(I) / CL(I)^2
GO TO 6

STIFFNESS MATRIX FOR A HINGED COLUMN
5 S(1,N2) = S(1,N2) + 2 * E * CR(I) / CL(I)
S(1,N4)=S(1,N4)+3*E*CR(I)/CL(I)**3
S(1,N6)=S(1,N6)+E*A(I)/CL(I)
J=N4-N2+1
S(J,N2)=S(J,N2)-3*E*CR(I)/CL(I)**3
6 WRITE (IW,011)I,CL(I),CR(I),CM(I),A(I),SC(I),(HCC(I,J),J=1,6).
1 CONTINUE
3 RETURN
101 FORMAT (I5,F10.5,6I4)
102 FORMAT (3X,I5,3X,F13.5,5I3)
103 FORMAT ($(3X,7HCOL NO.,5X,6HLENGTH,7X,7HINEPTIA,4X,
1.2HPLASTIC MOM.,4X,4HAREA,8X,7HMODULUS,5X,2HSTRUCTURE COORDINATES
2,/) END
SUBROUTINE LOAD(NBC,FM,NB,AL,BL,B)

DIMENSION NBC(100,4),FM(100,2),BL(100,B(260))
DIMENSION D(10),P(10)
IRR=1
LM=2
DO 1 K=1,NB
FM(K,1)=0.
1 FM(K,2)=0.
WRITE(IW,04)
2 READ(IRR,100)I,UDL,NCONC
IF(I.EQ.0)GO TO 5
WRITE(IW,101)I,UDL,NCONC
N1=NBC(I,1)
N2=NBC(I,2)
N3=NBC(I,3)
N4=NBC(I,4)
IF(UDL.EQ.0.)GO TO 25
FM(I,1)=FM(I,1)-UDL*BL(I)**2/12.
FM(I,2)=FM(I,2)+UDL*BL(I)**2/12.
B(N3)=B(N3)-UDL*BL(I)/2.
B(N4)=B(N4)-UDL*BL(I)/2.
25 IF(NCONC.EQ.0.)GO TO 4
READ(IRR,102)(D(J),P(J),J=1,NCONC)
DO 3 J=1,NCONC
WRITE(IW,103)J,D(J),P(J)
FM=P(J)*D(J)**2/(BL(I)-D(J))**2/BL(I)**2
FM1=P(J)**2*(EL(I)-D(J))/BL(I)**2
FM(I,1)=FM(I,1)-FM1
FM(I,2)=FM(I,2)+FM2
R1=(P(J)**2*(FM1+FM2))/BL(I)
B(N3)=B(N3)-(P(J)-R1)
B(N4)=B(N4)-R1
3 B(N1)=B(N1)-FM(I,1)
B(N2)=B(N2)-FM(I,2)
GO TO 2
5 RETURN
100 FORMAT (I5,F12.5,15)
101 FORMAT (22X,I5,3X,E13.5,3X,I5)
102 FORMAT (6F10.5)
103 FORMAT (5X,I5,5X,E13.5,2X,E13.5)
104 FORMAT ($(3X,8HLOAD NO.,7X,7HCOL NO.,7X,7HINEPTIA,4X,
1.2HPLASTIC MOM.,4X,4HAREA,8X,7HMODULUS,5X,2HSTRUCTURE COORDINATES
2,/) END
SUBROUTINE FORE(A,N,NBAND)

DIMENSION A(20,260)
A(1,1)=SQRAT(A(1,1))
DO 1 I=2,NBAND
C SOLVES EQUATIONS USING CHOLESKY METHOD AND DECOMPOSED S MATRIX

DIMENSION A(20,260),B(260),X(260)
B(I)=B(I)/A(I,I)
DO 7 I=2,N
SUM=0.
I=I-1
INTL=I-NBAND+1
IF(INTL.LT.I)INTL=1
DO 6 J=INTL,I
II=I-J+1
6 SUM=SUM+A(II,J)*B(J)
7 B(I)=(B(I)-SUM)/A(1,1)
I=N
X(I)=B(I)/A(1,1)
8 I=I-1
II=I+1
SUM=0.
LAST=I+NAND-1
IF(LAST.GT.N)LAST=N
DO 9 J=II,LAST
JJ=J-I+1
9 SUM=SUM+A(JJ,I)*X(J)
X(I)=(B(I)-SUM)/A(1,1)
IF(I.EQ.1)GO TO 10
GO TO 8
10 RETURN
END
SUBROUTINE ELST(NBC,X,BR,BL,NB,E,BM,FM,NCC,CR,CL,CM,P,NC,A)
C CALCULATES AND PRINTS MEMBER FORCES

DIMENSION NBC(100,4),X(261),BR(100),BL(100),BM(100,2),NCC(120,6),
FM(100,2),CR(120),CL(120),CM(120,2),P(120),A(120)
IWW=2
WRITE(IWW,102)
DO 1 I=1,NB
N1=NBC(I,1)
N2=NBC(I,2)
N3=NBC(I,3)
N4=NBC(I,4)
BM(I,1)=4.*E*BR(I)/BL(I)*X(N1)+2.*E*BR(I)/BL(I)*X(N2)-
16. *E*BR(I)/BL(I)**2*X(N3)+6.*E*BR(I)/(2*(N4))-FM(I,1)
17. M(I,2)=2.*E*BR(I)/BL(I)*X(N1)+4.*E*BR(I)/(2*(N2))-6.*E*BR(I)/
    BL(I)**2*X(N3)+6.*E*BRI(BL(I)**2*X(N4)+FM(I,2)
18. WRITE(IW,100,1,DM(I,1),DM(I,2)
19. IF(N..EQ.5999) GO TO 5
20. IF(N.EQ.0) GO TO 15
21. C(I,1)=4.*E*CR(I)/CL(I)**2*X(N1)+2.*E*CR(I)/CL(I)**2*X(N2)
22. L=5.*E*CR(I)/CL(I)**2*X(N3)-6.*E*CR(I)/CL(I)**2*X(N4)
23. CL(I,1)=2.*E*CR(I)/CL(I)**2*X(N1)+4.*E*CR(I)/CL(I)**2*X(N2)
24. CM(I,1)=6.*E*CR(I)/CL(I)**2*X(N3)-6.*E*CR(I)/CL(I)**2*X(N4)
25. P(I)=E*A(I)/CL(I)**2*X(N5)-X(N6)
26. GO TO 2
27. CM(I,2)=0.0
28. CM(I,1)=3.*E*CR(I)/CL(I)**2*X(N2)-3.*E*CR(I)/CL(I)**2*X(N4)
29. P(I)=E*A(I)/CL(I)**2*X(N6)
30. WRITE(IW,10)I,CM(I,1),CM(I,2),P(I)
31. 100 FORMAT(28X,I5,9X,E13.5,5X,E13.5)
32. 101 FORMAT(28X,I5,9X,E13.5,5X,E13.5,5X,E13.5)
33. 102 FORMAT(/,,28X,8HBEAM NO.,10X,8HMOMENT 1,10X,8HMOMENT 2,/,)
34. 103 FORMAT(/,,28X,8HCOL. NO.,10X,8HMOMENT 1,10X,8HMOMENT 2
35. 1,10X,10HAXIAL LOAD,/,)
36. RETURN
37. END