Computational and Experimental Characterization of a Novel Nonlinear Magnetic Shock Absorber with Applications to Ground Vehicles

by

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Abstract

The aim of this thesis is to propose a novel magnetic nonlinear shock absorber and to study the feasibility of the system in real-world applications. This shock absorber consists of an array of identical repelling magnets with an electromagnetic coil energy harvesting system. The nonlinear dynamics of the shock absorber were developed analytically and are implemented in a simulation environment. A parametric study was conducted to study the effects of the magnet mass, repelling force, inter-lattice equilibrium distance, and coil damping on the response of the system. Additionally, to experimentally validate the nonlinear dynamic modelling of the novel magnetic shock absorber, a magnetic shock absorber prototype was developed. The unknown parameters of the shock absorber, such as the magnetic repelling force relationship, were identified. The static and transient responses of the simulation were compared with those of the manufactured magnetic shock absorber prototype and it was shown that the simulation and the experimental results are in agreement.

The feasibility of a novel nonlinear magnetic shock absorber in road vehicle applications was also investigated. The magnetic shock absorber was implemented in quarter and half car models and their nonlinear dynamics were developed analytically. The developed models were implemented in a simulation environment and the results were validated using a quarter car experimental setup. Furthermore, a parametric study was conducted to understand the effects of varying certain parameters such as the number of magnets, coil damping intensity, etc. on the response of the system. The performance of the novel magnetic shock absorber was evaluated and also compared to a conventional linear one. In response to a continuous rough road surface, the ride quality of the magnetic shock absorber was shown to be similar to the linear one. Further, the road holding capabilities were demonstrated to be superior.
with the magnetic shock absorber. As an additional advantage, the magnetic shock absorber could be utilized to harvest the kinetic mechanical energy of the vertical response. The proposed concept can also be used in developing an active suspension through energization of the coils.
To my family
I would like to express my deepest gratitude and regards to my supervisors, Dr. Fidel Khouli, Dr. Robert Langlois, and Dr. Fred Afagh, for their excellent guidance, patience, and willingness to provide support whenever it was needed during the course of this project. Their wealth of knowledge and experience brought support and guidance that exceeded my expectations.

To my family and friends, I am thankful for their support and guidance. I am forever indebted to my family for their relentless support in every way through my education and through every step of my life.
# Contents

<table>
<thead>
<tr>
<th>Abstract</th>
<th>ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgments</td>
<td>iv</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>v</td>
</tr>
<tr>
<td>List of Tables</td>
<td>viii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>ix</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>xiv</td>
</tr>
<tr>
<td>List of Abbreviations</td>
<td>xxii</td>
</tr>
</tbody>
</table>

## 1 Introduction

1.1 Motivation .......................... 1
1.2 Objectives ........................... 2
1.3 Contributions ...................... 2
1.4 Publications ....................... 3
1.5 Organization ....................... 3

## 2 Literature Review

2.1 Nonlinear Lattices .................. 5
2.2 Energy Harvesting Systems ......... 7
2.3 Electromagnetic Damping .......... 10
2.4 Vehicle Implementation .......... 15
7  Ride Quality and Performance 63

7.1  Quarter Car ................................................................. 63

7.1.1  Single Bump ............................................................... 64
7.1.2  Bumpy Road ............................................................. 66
7.1.3  Ride Quality ............................................................... 71
7.1.4  Transmissibility .......................................................... 72
7.1.5  Road Holding ............................................................. 73
7.1.6  Experimental Validation ............................................. 74

7.2  Quarter Car Parametric Study .......................................... 76

7.2.1  Magnet Count ............................................................. 76
7.2.2  Mass and Repelling Force ............................................ 77
7.2.3  Inter-lattice Equilibrium Distance ................................. 79
7.2.4  Coil Damping ............................................................... 79

7.3  Half Car ................................................................. 82

7.3.1  Single Bump ............................................................... 82
7.3.2  Bumpy Road ............................................................. 88

8  Conclusion 92

8.1  Accomplishments ......................................................... 94
8.2  Future Work ................................................................. 95

Bibliography 96

Appendix A Manufacturing Drawings 108
List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Magnetic shock absorber parameters.</td>
<td>25</td>
</tr>
<tr>
<td>5.1</td>
<td>Magnetic shock absorber parameters obtained experimentally.</td>
<td>46</td>
</tr>
<tr>
<td>7.1</td>
<td>Quarter car and magnetic shock absorber parameters.</td>
<td>65</td>
</tr>
<tr>
<td>7.2</td>
<td>Constant coefficient of the road excitation.</td>
<td>67</td>
</tr>
<tr>
<td>7.3</td>
<td>Ride quality metrics of the novel magnetic shock absorber vs. the linear</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>conventional one in a quarter car model moving on a bumpy surface.</td>
<td></td>
</tr>
<tr>
<td>7.4</td>
<td>Half car and magnetic shock absorber parameters.</td>
<td>82</td>
</tr>
<tr>
<td>7.5</td>
<td>Ride quality metrics of the novel magnetic shock absorber vs. the linear</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>conventional one in a half car model moving on a bumpy surface.</td>
<td></td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Repelling magnetic lattice with identical permanent magnets.</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>A cross-sectional view of a traditional magnetic vibration energy harvester.</td>
<td>8</td>
</tr>
<tr>
<td>2.3</td>
<td>An energy harvester device with multiple levitating magnets.</td>
<td>9</td>
</tr>
<tr>
<td>2.4</td>
<td>A nonlinear energy harvester with disk spring.</td>
<td>10</td>
</tr>
<tr>
<td>2.5</td>
<td>Schematic of an eddy current damper.</td>
<td>11</td>
</tr>
<tr>
<td>2.6</td>
<td>Design of the electromagnetic damper with two layers of magnets.</td>
<td>12</td>
</tr>
<tr>
<td>2.7</td>
<td>Schematic of a linear energy harvester with Halbach array topology.</td>
<td>12</td>
</tr>
<tr>
<td>2.8</td>
<td>Schematic of a linear eddy damper with double layered Halbach array topology.</td>
<td>13</td>
</tr>
<tr>
<td>2.9</td>
<td>Schematic of a regenerative shock absorber utilizing rack and pinion mechanism.</td>
<td>14</td>
</tr>
<tr>
<td>2.10</td>
<td>Hybrid damper utilizing twin tube design.</td>
<td>14</td>
</tr>
<tr>
<td>2.11</td>
<td>Diagram of a linear electromagnetic shock absorber.</td>
<td>16</td>
</tr>
<tr>
<td>3.1</td>
<td>Repelling 1D magnetic chain diagram.</td>
<td>18</td>
</tr>
<tr>
<td>3.2</td>
<td>Interaction of a magnet with a single turn of coil.</td>
<td>20</td>
</tr>
<tr>
<td>3.3</td>
<td>Free body diagrams of 1st, i-th, and n-th magnets.</td>
<td>22</td>
</tr>
<tr>
<td>4.1</td>
<td>System response to a constant force.</td>
<td>25</td>
</tr>
<tr>
<td>4.2</td>
<td>Wave-like behaviour of the system.</td>
<td>26</td>
</tr>
<tr>
<td>4.3</td>
<td>Force vs. displacement for four-magnet setup.</td>
<td>27</td>
</tr>
<tr>
<td>4.4</td>
<td>Force vs. displacement estimation of the n-th magnet in the four-magnet system.</td>
<td>28</td>
</tr>
<tr>
<td>4.5</td>
<td>Displacement of the n-th magnet vs. time for various magnet counts (n) in response to a inputs.</td>
<td>30</td>
</tr>
</tbody>
</table>
4.6 Displacement of the \( n \)th magnet vs. time for various magnet counts \((n)\) in response to step a sinusoidal input. ......................... 31

4.7 Total force experienced by the \( n \)th magnet vs. time for sinusoidal input and various magnet counts \((n)\), excluding certain traces for plot clarity. ........... 31

4.8 Displacement of the \( n \)th magnet vs. time for step input and various magnet masses \((m)\) and repelling force coefficients \((A)\). ......................... 32

4.9 Total force experienced by the \( n \)th magnet vs. time for sinusoidal input and various magnet masses \((m)\) and repelling force coefficients \((A)\). ......................... 33

4.10 Total force experienced by the \( n \)th magnet vs. time for step input and various element dimensions \((D)\). ......................................................... 34

4.11 Displacement of the \( n \)th magnet vs. time for step input and various element dimensions \((D)\). ......................................................... 34

4.12 Displacement of the \( n \)th magnet vs. time for step input and various coil damping magnitude for various electrical resistances of the coil. ......................... 35

5.1 Experimental apparatus of the novel shock absorber. ......................................................... 37

5.2 LVDT sensor calibration, voltage output vs. displacement. ......................................................... 39

5.3 Magnetic repelling force magnitude vs. displacement. ......................................................... 40

5.4 Testing schematic for obtaining friction forces prevailing in the shock absorber prototype. ......................................................... 41

5.5 Displacement of the free oscillating mass. ......................................................... 42

5.6 Experimental and simulation results for force vs. displacement of the prototype shock absorber. ......................................................... 44

5.7 Curve fit for force vs. displacement relation of the prototype shock absorber. ......................................................... 45

5.8 Simulated and experimental displacement of each magnet in response to the weight of a 2.89 kg mass. ......................................................... 47

5.9 Experimentally obtained displacement vs. input frequency of the free oscillating mass. ......................................................... 48

5.10 DFT magnitude spectrum of voltage output of the top coil to a 1.8 Hz sinusoidal displacement excitation. ......................................................... 49
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.11</td>
<td>DFT phase spectrum of voltage output of the top coil to a 1.8 Hz sinusoidal displacement excitation.</td>
</tr>
<tr>
<td>5.12</td>
<td>Voltage output of the top coil to a 1.8 Hz sinusoidal displacement excitation and its DFT reconstructions.</td>
</tr>
<tr>
<td>6.1</td>
<td>Schematic of quarter car with magnetic shock absorber (left), and free body diagram of the unsprung mass and the $i^{th}$ and $n^{th}$ magnets (right).</td>
</tr>
<tr>
<td>6.2</td>
<td>Conventional half car model schematic.</td>
</tr>
<tr>
<td>6.3</td>
<td>Schematic of the half car model with magnetic shock absorbers.</td>
</tr>
<tr>
<td>6.4</td>
<td>Free body diagram of the half car sprung mass.</td>
</tr>
<tr>
<td>6.5</td>
<td>Free body diagram of the rear unsprung mass and the corresponding $i^{th}$ and $n^{th}$ magnets.</td>
</tr>
<tr>
<td>6.6</td>
<td>Free body diagram of the front unsprung mass and the corresponding $i^{th}$ and $n^{th}$ magnets.</td>
</tr>
<tr>
<td>7.1</td>
<td>Quarter car experimental apparatus.</td>
</tr>
<tr>
<td>7.2</td>
<td>Simulation time response of the quarter car with the linear and the novel shock absorber traveling at 5 m/s to a single bump on the road surface.</td>
</tr>
<tr>
<td>7.3</td>
<td>Acceleration of the sprung mass of the quarter car with the linear and the novel shock absorber traveling at 5 m/s in response to a single bump on the road surface obtained from the simulation model.</td>
</tr>
<tr>
<td>7.4</td>
<td>Simulink diagram of road profile generation.</td>
</tr>
<tr>
<td>7.5</td>
<td>Simulation time response of the quarter car with the linear spring (top) and magnetic shock absorber (bottom) to a bumpy road surface.</td>
</tr>
<tr>
<td>7.6</td>
<td>Percentage difference of displacement between the quarter car with the linear spring and the quarter car with the magnetic shock absorber for the bumpy road.</td>
</tr>
<tr>
<td>7.7</td>
<td>Acceleration of the sprung mass of the quarter car model with the linear spring (top) and magnetic shock absorber (bottom) in response to a bumpy road surface obtained from simulation.</td>
</tr>
</tbody>
</table>
7.8 Displacement transmissibility of the quarter car with linear spring and magnetic shock absorber. .............................................. 73
7.9 Road holding of the quarter car with linear spring and magnetic shock absorber. 74
7.10 Simulation vs. experimental transmissibility of the quarter car with linear spring and magnetic shock absorber. ............................. 75
7.11 Displacement transmissibility of the quarter car with various magnet counts. 77
7.12 Road holding of the quarter car with various magnet counts. .................. 77
7.13 Displacement transmissibility of the quarter car with various magnet masses (m) and repelling force coefficients (A). ......................... 78
7.14 Road holding of the quarter car with various magnet masses (m) and repelling force coefficients (A). .................................................. 78
7.15 Displacement transmissibility of the quarter car with various magnetic shock absorber dimensions (D). ........................................ 79
7.16 Road holding of the quarter car with various magnetic shock absorber dimensions (D). ................................................................. 80
7.17 Displacement transmissibility of the quarter car with various damping magnitude. ........................................................................ 80
7.18 Road holding of the quarter car with various coil damping magnitude. .... 81
7.19 Single speed bump as road input to the half car rear and front tires. .... 83
7.20 Speed bump geometry. .................................................................. 83
7.21 Displacement of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump. ..... 84
7.22 Acceleration of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump. ..... 85
7.23 Displacement of the front unsprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump. 85
7.24 Displacement of the rear unsprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump. 86
7.25 Pitch angle, θ, of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump. .... 87
7.26 Pitch angle acceleration, $\dot{\theta}$, of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump. ................................................................. 87

7.27 Random road profile at vehicle front and rear wheels. ........................................... 88

7.28 Displacement of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a bumpy road. .................................................. 89

7.29 Displacement of the front unsprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a bumpy road. .............................................. 90

7.30 Displacement of the rear unsprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a bumpy road. .............................................. 90

7.31 Pitch angle, $\theta$, of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a bumpy road. ......................................................... 91
# Nomenclature

## Latin Characters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Repelling force coefficient</td>
<td>[N/m²]</td>
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<td>$A_w$</td>
<td>Wire cross-sectional area</td>
<td>[m²]</td>
</tr>
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<td>$a_f$</td>
<td>Distance of front suspension from sprung mass CG</td>
<td>[m]</td>
</tr>
<tr>
<td>$a_{ij}$</td>
<td>Distance of magnet $i$ from first turn of coil $j$</td>
<td>[m]</td>
</tr>
<tr>
<td>$a_r$</td>
<td>Distance of rear suspension from sprung mass CG</td>
<td>[m]</td>
</tr>
<tr>
<td>$a_w$</td>
<td>Weighted RMS acceleration</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>$a_w(t)$</td>
<td>Instantaneous weighted acceleration</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>$B_{ij}$</td>
<td>Average magnetic flux density of magnet $i$ on coil $j$</td>
<td>[T]</td>
</tr>
<tr>
<td>$B_r$</td>
<td>Residual magnetism</td>
<td>[T]</td>
</tr>
<tr>
<td>$B_x$</td>
<td>Magnetic induction $x$ component</td>
<td>[T]</td>
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<td>$C_1$</td>
<td>Total damping coefficient of magnet 1</td>
<td>[N · s/m]</td>
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<td>$C_{e_i}$</td>
<td>Total Electromagnetic damping coefficient of magnet $i$</td>
<td>[N · s/m]</td>
</tr>
<tr>
<td>$C_{e_{ij}}$</td>
<td>Electromagnetic damping coefficient of magnet $i$ and coil $j$</td>
<td>[N · s/m]</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Shock absorber damping forces at front suspension</td>
<td>[N]</td>
</tr>
<tr>
<td>$C_{f_1}$</td>
<td>Total damping coefficient of magnet 1 at front suspension</td>
<td>[N · s/m]</td>
</tr>
<tr>
<td>$C_{f_2}$</td>
<td>Total damping coefficient of magnet 2 at front suspension</td>
<td>[N · s/m]</td>
</tr>
<tr>
<td>$C_{f_i}$</td>
<td>Total damping coefficient of magnet $i$ at front suspension</td>
<td>[N · s/m]</td>
</tr>
<tr>
<td>$C_{f_{n-1}}$</td>
<td>Total damping coefficient of magnet $n – 1$ at front suspension</td>
<td>[N · s/m]</td>
</tr>
<tr>
<td>$C_{f_n}$</td>
<td>Total damping coefficient of magnet $n$ at front suspension</td>
<td>[N · s/m]</td>
</tr>
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<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>$C_{fs}$</td>
<td>Front suspension damping coefficient</td>
<td></td>
</tr>
<tr>
<td>$C_i$</td>
<td>Total damping coefficient of magnet $i$</td>
<td></td>
</tr>
<tr>
<td>$C_n$</td>
<td>Total damping coefficient of magnet $n$</td>
<td></td>
</tr>
<tr>
<td>$C_r$</td>
<td>Shock absorber damping forces at rear suspension</td>
<td></td>
</tr>
<tr>
<td>$C_{r1}$</td>
<td>Total damping coefficient of magnet 1 at rear suspension</td>
<td></td>
</tr>
<tr>
<td>$C_{r2}$</td>
<td>Total damping coefficient of magnet 2 at rear suspension</td>
<td></td>
</tr>
<tr>
<td>$C_{ri}$</td>
<td>Total damping coefficient of magnet $i$ at rear suspension</td>
<td></td>
</tr>
<tr>
<td>$C_{rn-1}$</td>
<td>Total damping coefficient of magnet $n-1$ at rear suspension</td>
<td></td>
</tr>
<tr>
<td>$C_{rn}$</td>
<td>Total damping coefficient of magnet $n$ at rear suspension</td>
<td></td>
</tr>
<tr>
<td>$C_{rs}$</td>
<td>Rear suspension spring constant</td>
<td></td>
</tr>
<tr>
<td>$C_s$</td>
<td>Suspension damping coefficient</td>
<td></td>
</tr>
<tr>
<td>$C_v$</td>
<td>Viscous damping coefficient</td>
<td></td>
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<tr>
<td>$C_{vi}$</td>
<td>Viscous damping coefficient of magnet $i$</td>
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<tr>
<td>$c_{eq}$</td>
<td>Equivalent viscous damping coefficient</td>
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</tr>
<tr>
<td>$D$</td>
<td>Total length of the magnetic shock absorber</td>
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</tr>
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<td>$d_0$</td>
<td>Distance between adjacent magnets</td>
<td></td>
</tr>
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<td>$E_{dissipated}$</td>
<td>Dissipated energy</td>
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<tr>
<td>$E_v$</td>
<td>Energy dissipated by a viscous damper</td>
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</tr>
<tr>
<td>$F_{eij}$</td>
<td>Electromagnetic damping force exerted on magnet $i$ by coil $j$</td>
<td></td>
</tr>
<tr>
<td>$F_{f_{mag1}}$</td>
<td>Magnetic forces experienced by magnet 1 at front suspension</td>
<td></td>
</tr>
<tr>
<td>$F_{f_{mag2}}$</td>
<td>Magnetic forces experienced by magnet 2 at front suspension</td>
<td></td>
</tr>
<tr>
<td>$F_{f_{magi}}$</td>
<td>Magnetic forces experienced by magnet $i$ at front suspension</td>
<td></td>
</tr>
<tr>
<td>$F_{f_{mag_{n-1}}}$</td>
<td>Magnetic forces experienced by magnet $n-1$ at front suspension</td>
<td></td>
</tr>
<tr>
<td>$F_{f_{magn}}$</td>
<td>Magnetic forces experienced by magnet $n$ at front suspension</td>
<td></td>
</tr>
<tr>
<td>$F_{mag1}$</td>
<td>Magnetic forces experienced by magnet 1</td>
<td></td>
</tr>
<tr>
<td>$F_{mag2}$</td>
<td>Magnetic forces experienced by magnet 2</td>
<td></td>
</tr>
<tr>
<td>$F_{magi}$</td>
<td>Magnetic forces experienced by magnet $i$</td>
<td></td>
</tr>
<tr>
<td>$F_{mag_{n-1}}$</td>
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<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>-------------</td>
<td>------------------------------------------------------------------</td>
<td>---------</td>
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<tr>
<td>$F_{mag}$</td>
<td>Magnetic forces experienced by magnet $n$</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{magnet}$</td>
<td>Force applied to magnet $i$</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{magnet}$</td>
<td>Force applied to magnet $n$</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{mag1}$</td>
<td>Magnetic forces experienced by magnet 1 at rear suspension</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{mag2}$</td>
<td>Magnetic forces experienced by magnet 2 at rear suspension</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{magi}$</td>
<td>Magnetic forces experienced by magnet $i$ at rear suspension</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{magn-1}$</td>
<td>Magnetic forces experienced by magnet $n-1$ at rear suspension</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{magn}$</td>
<td>Magnetic forces experienced by magnet $n$ at rear suspension</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{repelling}$</td>
<td>Magnetic repelling force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{sprung}$</td>
<td>Force applied to the sprung mass</td>
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</tr>
<tr>
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<td>Force applied to the unsprung mass</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_v$</td>
<td>Viscous damper force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F(x_n)$</td>
<td>External force as a function of displacement</td>
<td>[N]</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Coil fill factor</td>
<td>[−]</td>
</tr>
<tr>
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<td>External force</td>
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<tr>
<td>$K_{f_t}$</td>
<td>Front tire stiffness</td>
<td>[N/m]</td>
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<td>Rear suspension spring constant</td>
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</tr>
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<td>Rear tire stiffness</td>
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</tr>
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<td>[kg]</td>
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</tr>
<tr>
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<td>Mass of the rear unsprung mass</td>
<td>[kg]</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Mass of the sprung mass</td>
<td>[kg]</td>
</tr>
<tr>
<td>$m_U$</td>
<td>Mass of the unsprung mass</td>
<td>[kg]</td>
</tr>
<tr>
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<td>Number of coil turns</td>
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<tr>
<td>$n$</td>
<td>Number of sliding magnets</td>
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<td>Voltage as a function of time</td>
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<td>Displacement of magnet 2</td>
<td>[m]</td>
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<td>[m]</td>
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<td>Road surface undulations at the front tire</td>
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<td>[m]</td>
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<td>Displacement of the sprung mass</td>
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<td>Displacement of the unsprung mass</td>
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<td>Mass velocity</td>
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<td>Velocity of magnet 2</td>
<td>[m/s]</td>
</tr>
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<td>Velocity of $j^{th}$ Coil</td>
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<td>Velocity of magnet 1 at front suspension</td>
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<td>Velocity of magnet 2 at rear suspension</td>
<td>[m/s]</td>
</tr>
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<td>Velocity of the rear unsprung mass</td>
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</tr>
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<td>$\dot{x}_S$</td>
<td>Velocity of the sprung mass</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$\dot{x}_U$</td>
<td>Velocity of the unsprung mass</td>
<td>[m/s]</td>
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<tr>
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<td>Acceleration of magnet 1</td>
<td>[m/s$^2$]</td>
</tr>
<tr>
<td>$\ddot{x}_2$</td>
<td>Acceleration of magnet 2</td>
<td>[m/s$^2$]</td>
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<td>[m/s$^2$]</td>
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<td>[m/s$^2$]</td>
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<td>Acceleration of the front unsprung mass</td>
<td>[m/s$^2$]</td>
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<td>Acceleration of magnet $i$</td>
<td>[m/s$^2$]</td>
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<td>Acceleration of magnet $n - 1$</td>
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<td>Acceleration of magnet $n$</td>
<td>[m/s$^2$]</td>
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<td>$\ddot{x}_{r_1}$</td>
<td>Acceleration of magnet 1 at rear suspension</td>
<td>[m/s$^2$]</td>
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<td>Units</td>
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<td>([m/s^2])</td>
</tr>
<tr>
<td>( \ddot{x}_{ri} )</td>
<td>Acceleration of magnet ( i ) at rear suspension</td>
<td>([m/s^2])</td>
</tr>
<tr>
<td>( \ddot{x}<em>{r</em>{n-1}} )</td>
<td>Acceleration of magnet ( n-1 ) at rear suspension</td>
<td>([m/s^2])</td>
</tr>
<tr>
<td>( \ddot{x}_{rn} )</td>
<td>Acceleration of magnet ( n ) at rear suspension</td>
<td>([m/s^2])</td>
</tr>
<tr>
<td>( \ddot{x}_{RU} )</td>
<td>Acceleration of rear unsprung mass</td>
<td>([m/s^2])</td>
</tr>
<tr>
<td>( \ddot{x}_S )</td>
<td>Acceleration of the sprung mass</td>
<td>([m/s^2])</td>
</tr>
<tr>
<td>( \ddot{x}_U )</td>
<td>Acceleration of the unsprung mass</td>
<td>([m/s^2])</td>
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</table>

**Greek Characters**

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<tr>
<td>( \alpha )</td>
<td>Road profile constant</td>
<td>([\text{rad/m}])</td>
</tr>
</tbody>
</table>
| \( \Delta \) | Change in parameter                                    | \([-\) |}
| \( \delta \) | Logarithmic decrement                                  | \([-\) |}
| \( \zeta \) | Damping ratio                                          | \([-\) |}
| \( \eta(t) \) | Zero-mean Gaussian white noise                        | \([m/s]\) |
| \( \theta \) | Pitch angle of the sprung mass                         | \([\text{Rad}]\) |
| \( \dot{\theta} \) | Angular velocity of the sprung mass                   | \([\text{Rad/s}]\) |
| \( \ddot{\theta} \) | Angular acceleration of the sprung mass                | \([\text{Rad/s}]\) |
| \( \pi \) | Mathematical constant                                  | \([-\) |}
<p>| ( \rho_w ) | Wire resistivity                                       | ([\Omega]) |
| ( \sigma ) | Road roughness variance                                | ([10^{-3} \text{ m}]) |
| ( \tau ) | Integration time                                       | ([s]) |
| ( \Phi ) | Wire diameter                                          | ([\text{m}]) |
| ( \psi ) | Power spectral density                                 | ([\text{m}^2/\text{Hz}]) |
| ( \psi_j ) | Phase of ( j^{\text{th}} ) DFT term                | ([\text{rad}]) |
| ( \psi_\omega ) | Spectral density of white noise                        | ([\text{m}^2/\text{Hz}]) |
| ( \Omega ) | Angular spatial frequency                              | ([\text{rad/m}]) |</p>
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<td>$\xi$</td>
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<td>Frequency of $j^{th}$ DFT term</td>
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<tr>
<td>$\omega_n$</td>
<td>Natural frequency</td>
<td>[rad/s]</td>
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List of Abbreviations

<table>
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<tr>
<th>Abbreviation</th>
<th>Definition</th>
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<td>1D</td>
<td>1-Dimensional</td>
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<tr>
<td>AWG</td>
<td>American wire gauge</td>
</tr>
<tr>
<td>CG</td>
<td>Center of gravity</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier transform</td>
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<tr>
<td>DOF</td>
<td>Degrees of freedom</td>
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<tr>
<td>EOM</td>
<td>Equations of motion</td>
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<td>FBD</td>
<td>Free body diagram</td>
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<tr>
<td>KLT</td>
<td>Kanade-Lucas-Tomasi</td>
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<tr>
<td>LVDT</td>
<td>Linear variable differential transformer</td>
</tr>
<tr>
<td>MR</td>
<td>Magnetorheological</td>
</tr>
<tr>
<td>MTVV</td>
<td>Maximum transient vibration value</td>
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<tr>
<td>NNM</td>
<td>Nonlinear normal mode</td>
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<tr>
<td>PSD</td>
<td>Power spectral density</td>
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<tr>
<td>RMS</td>
<td>Root mean square</td>
</tr>
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<td>ROI</td>
<td>Region of interest</td>
</tr>
<tr>
<td>SSE</td>
<td>Sum of squares error</td>
</tr>
<tr>
<td>VDV</td>
<td>Vibration dose value</td>
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Chapter 1

Introduction

1.1 Motivation

The important problem of vibration isolation has been studied extensively over the years [1]. Commonly, spring-damper combinations are used as isolation devices to reduce the adverse effects of external excitations [2]. Various mechanisms have been developed to alleviate the adverse effects of vibrations across many disciplines. In vehicular applications, suspension systems are developed to attenuate the system’s response [3].

Road vehicle suspension systems provide comfort for the passengers by attenuating the vertical acceleration of the sprung body of the vehicle and enable the tires to maintain solid contact with the ground by limiting the fluctuations of the vertical force that each tire develops with the road [4]. Criteria such as vibration isolation of the sprung mass, tire dynamic load variation, and suspension travel have been used to characterize the suspension performance [5].

In this research project, a novel shock absorber design is proposed that can be utilized in ground vehicle suspension systems. The proposed shock absorber consists of a 1-dimensional (1D) nonlinear lattice of equal strength repelling permanent magnets that are confined between two fixed magnets with a multi-coil setup wrapped around the 1D lattice for energy harvesting. The repelling magnets act as a spring element and the electromagnetic coils act as a damping element due to energy dissipation through induced current in the coils. The electromagnetic coils, in addition to providing damping forces, can be utilized as an
energy harvesting source. The research pertaining to this thesis focuses on characterizing this magnetic shock absorber and investigating its potential in ground vehicle suspension system applications.

1.2 Objectives

The objectives of this thesis are as follows:

1. Investigate the nonlinear dynamics and understand the characteristics of the magnetic shock absorber.
2. Create a simulation model and study the behaviour of the system to various inputs.
3. Verify the simulation results experimentally and analyze any discrepancies.
4. Investigate the coil damping and its effect on the system’s response.
5. Implement the magnetic shock absorber in quarter and half car models and analyze the ride quality and handling performances.

1.3 Contributions

The primary contributions of this thesis are as follows:

1. A novel magnetic shock absorber was proposed and the nonlinear dynamics of the system were developed.
2. The static and transient simulation results were validated experimentally via manufacturing a prototype magnetic shock absorber.
3. The feasibility of the magnetic shock absorber in ground vehicle applications was studied via implementation in quarter and half car models and analysis of ride quality and road holding performance.
1.4 Publications

The following publications were throughout this investigation.


1.5 Organization

This thesis contains 8 chapters. A description of each chapter follows.

**Chapter 1: Introduction** — The introduction provides motivations, objectives, and accomplishments of this research. The publications generated from this thesis are listed. The organization of the thesis is provided.

**Chapter 2: Literature Review** — A review of literature is provided to explore previous work in the field. The review of previous work in the field is necessary to establish the novel direction of the research project.

**Chapter 3: Model Development** — Introduces the design of the novel magnetic shock absorber and explores the forces experienced by the magnets and describes the derivation of the equations of motion (EOM) for the magnetic shock absorber.

**Chapter 4: Simulation and Parametric Study** — Summarizes the simulation results and describes the force versus displacement relationship, which provides insight into
the stiffness of the system. Includes a parametric study that analyzes the effect of different design parameters on the characteristics of the system.

**Chapter 5: Experimental Verification** — Describes the selected experimental setup and the details of the measurement instruments used. The parameters, such as the damping coefficient of the shock absorber are identified experimentally and the testing procedure is explained. The experimental results and the methods used to verify the simulation are also discussed.

**Chapter 6: Vehicle Implementation** — Explores the derivation of equations of motion for the quarter car and half car models with the magnetic shock absorber implemented.

**Chapter 7: Ride quality and Performance** — The ride quality and the handling performance of the quarter car and half car models are evaluated and compared to conventional linear models. The verification of the quarter car simulation is included. A parametric study is conducted on the quarter car.

**Chapter 8: Conclusion** — Conclusions of the research are drawn and the contributions to the field are enumerated. Suggestions for future work are provided as well.
Chapter 2

Literature Review

To demonstrate the novelty of this research project, a literature review is provided exploring previous work in related fields.

2.1 Nonlinear Lattices

Nonlinear lattices are systems of serially-coupled lumped masses interacting with each other through nonlinear forces and they are often represented by nonlinear mass-spring systems. Nonlinear lattices have been studied extensively and various applications have been proposed such as wave control [6 7 8 9], vibration control [10 11], and shock mitigation [12], to name a few. Magnetic lattices can be categorized as a nonlinear lattice due to the nonlinear nature of magnetic forces. A repelling magnetic 1D lattice consists of a number of permanent magnets arranged in a repelling configuration in a chain with the two magnets at each end fixed in place, as demonstrated in Figure 2.1

![Figure 2.1: Repelling magnetic lattice with identical permanent magnets](image)

Figure 2.1: Repelling magnetic lattice with identical permanent magnets [13].
The characteristics of magnetic and repulsive nonlinear lattices have been explored in various studies [14, 15, 16, 17, 18, 19]. Leng et al [14] studied the effects of magnetic field strength on the dynamic response of a granular magnetic chain under strong, intermediate, and weak amplitudes of incident impulses and demonstrated wave propagation modes induced in the magnetic chain by a magnetic field. Avalos et al [15] showed that except for cases where the repulsive potential is very weak, most of the energy due to a velocity perturbation generates a propagating solitary wave in the magnetic chain. They also obtained the propagation velocity of the solitary wave. On the other hand, Mehrem et al [16] studied the propagation of nonlinear waves in a lattice of repelling particles. The particles chosen for that experimental work were permanent magnets arranged in a repelling chain arrangement. They also studied the harmonic waves induced in the lattice and the dilatation or expansion of the magnetic chain caused by the perturbations.

Russell et al [17] studied the propagation of moving breathers in a chain of magnetic pendulums under impulses. Breathers are nonlinear localized excitations with internal oscillations and the paper was successful in visually demonstrating the evolution breathers in the chain. Al Ba’ba’a and Nouh [7] studied the control of the spatial wave profiles of a magnetic chain. They demonstrated that by varying the spacing between the magnets, which affects the stiffness of the system, the wave profile attributes can be controlled. Molerón et al [18] studied a lattice of magnets with long-range interactions and demonstrated that the breather solutions decay algebraically.

The dynamics of solitary wave propagation in a 1D magnetic lattice was studied by Molerón et al [13] and the possibility of shock mitigation and absorption using such a design was suggested. Despite the vast literature on nonlinear lattices, no previous work has been found that has addressed the feasibility of shock absorption and energy harvesting through utilization of a 1D magnetic lattice. This thesis will investigate the suggestion made by Molerón et al [13] and utilize a nonlinear repelling magnetic lattice as a source of stiffness in the proposed novel shock absorber and explore its feasibility in ground vehicle applications.
2.2 Energy Harvesting Systems

Energy harvesting devices, utilizing magnets and electromagnetic coil setups, have received increasing attention in recent years. The relative motion of the magnets and coils produces electric power, and maximizing this output power has been the driving factor in many articles. Electromagnetic energy harvesters can be designed in the standard or inverse configuration. In the standard configuration, the electromagnetic coils are fixed while the magnets are movable. However, in the inverse configuration, the coils are movable and the permanent magnets are fixed [20]. Linear energy harvesters are only able to extract the maximum power at their resonance frequency [21] and this can limit their performance in many applications. Therefore, to broaden the operating frequency range of the harvesters, researches have intentionally introduced nonlinearities into the system [22, 23, 24].

The most common nonlinear energy harvesters are the magnetic spring based designs [25]. A magnetic spring consists of a single magnet or a stack of magnets sliding between two fixed magnets [26]. The nonlinearity arises from the repelling magnetic forces between the levitating magnets and the fixed magnets. A traditional nonlinear magnetic energy harvester, demonstrated in Figure 2.2 utilizes the magnet spring design and the electromagnetic coils to convert kinetic energy to electrical energy.

Over the years various design concepts for nonlinear energy harvesters have been proposed where, through single [27, 28, 29] or multi magnet arrangements [30, 31, 32], the vibration of a system is converted into electrical energy. Jensen et al [27] studied the full phase space dynamics of an energy harvesting device consisting of two coils and three permanent magnets with one levitating freely between two fixed magnets, where the motion of the levitating magnet is induced by the environmental vibrations. They also experimentally verified the derived expressions of the forces between the magnets and the damping forces between the floating magnet and the coils. Furthermore, they demonstrated that certain driving frequencies and initial conditions allow a large amount of power to be harvested. Munaz et al [28] introduced an energy harvesting system that utilized multi-pole magnet arrangement. They demonstrated that this energy harvester can be used as a human wearable device that can generate up to 4.8 mW of power at a resonance frequency of 6 Hz. In
another study by Imbaquingo et al. [29], a similar design was introduced where the external vibrations were harvested by a levitating magnet confined between two fixed magnets. In that study, an analytical model was developed and used to drive expressions for the characteristic frequency and output power of the energy harvesting system were derived.

Saravia et al. [30] developed a computational approach for modeling a levitation-based energy harvester. This energy harvester consists of two fixed magnets at each end and multiple floating magnets levitating in between that are separated by spacers, as shown in Figure 2.3. They also verified their model experimentally and demonstrated that their model offers excellent prediction of the voltage-time signal and estimation of parameters such as average power and average voltage. Fosial et al. [31] proposed a levitation-based energy harvester system where four separate individual single magnet harvesters were stacked on top of each other. Through optimization of each individual single magnet energy harvester, the operating frequency range of the whole system was extended and it was able to operate over the range of 7 to 10 Hz input frequency. Wang et al. [32] introduced a design of an energy harvesting device with multiple levitating magnets. The system was designed to be a human wearable device and it was experimentally demonstrated that the device is operable.
over a broad range of frequencies. It was also demonstrated that a maximum average output power of 10.66 mW was obtained from the swing motion of a human leg at a speed of 8 km/h with the entire device weighing 218.7 g.

Figure 2.3: An energy harvester device with multiple levitating magnets [30].

Some concepts incorporate springs to aid in the levitation of the magnets [33]. For instance, Aldawood et al [21] used a planar disc spring to improve the power metrics of the harvester, shown in Figure 2.4. Other design approaches have also been incorporated as well, such as vibration-to-rotation conversion mechanism [34]. The electromagnetic energy harvesters have been developed for various applications. Some research projects have proposed applications for magnetic energy harvesters in vehicle suspension systems [35, 36, 37], and some even for rail vehicles [38]. However, no previous research project on the energy harvesting matter was found that utilizes the nonlinear magnetic spring as a source of stiffness. Moreover, the proposed novel magnetic shock absorber utilizes a repelling magnetic chain as a source of stiffness where the individual magnets are free to move, whereas in the designs in literature the levitating magnets are attached to each other and move as a single unit.
2.3 Electromagnetic Damping

When a conductor moves in a magnetic field, eddy currents will be induced in the conductor and a magnetic drag force will be induced opposing the motion of the conductor, which will dissipate the kinetic energy by transforming it into electricity. Electromagnetic dampers utilize this phenomena to produce damping forces. Figure 2.5 demonstrates the schematic of an example of an eddy current damper which utilizes a stack of permanent magnets separated by iron poles and a conductor as an outer tube.

The damping force magnitude in eddy current dampers depends on the magnetic field intensity, conductor or coil properties, and the velocity of the coil with respect to magnet [40]. Therefore, an increase in magnetic field intensity through adjusting the magnet and coil arrangements or amplification of the relative velocity between the magnet and the
coil would result in an increase in damping forces. This has been the focus of the many studies and has resulted in a wealth of papers. Many research projects have looked into increasing the flux density per unit mass. Some have doubled the layers of magnetic stack to increase the flux intensity acting on the coils [41, 42, 43], as shown in Figure 2.6.

Due to weight limitation in practical solutions, increasing the number of magnets is not desirable. Therefore, to further increase magnetic flux density, studies have looked into the optimum configuration for magnetic dampers [43, 44, 45]. For instance, Cheung [46] investigated the effect of spacer thickness variance on the magnitude of induced voltage in the coils. Bissal et al. [47] developed finite-element-method-based models to determine the magnet arrangement, topology, and spacer materials that results in the highest damping forces.

It was found that the Halbach array, demonstrated in Figure 2.7, is the best topology yielding the largest damping force. However, Zhu [48] demonstrated that although a Halbach array results in the highest absolute flux density, the magnetic flux gradient, and therefore the output power, is lower than some layouts studied in the paper. As a result,
Halbach arrays do not offer the best solution for all use cases and there is a compromise between damping force and power output. On the other hand, to further increase the damping forces, some studies have even proposed double layered Halbach arrays [49, 50], as demonstrated in Figure 2.8.
Nevertheless, increasing the magnetic flux intensity while adhering to weight limits of real world applications has proved to be challenging. Therefore, a vast number of researchers have focused on amplifying the relative velocity of magnets and coils. This has resulted in development of various indirect drive systems. In an indirect drive shock absorber, the external vibrations are not directly transferred to the linear motion between the magnets and coils. Instead, through a mechanism, the external linear motion is transferred to the magnets and the input amplitude is magnified. To achieve this, various mechanism such as the ball screw mechanism [51, 52, 53], rack and pinion mechanism [54, 55, 56], hydraulic mechanism [57, 58], etc. have been proposed by many. For instance, Figure 2.9 demonstrates an example of shock absorber that converts the linear motion to a rotational one using a rack and pinion mechanism and the damping is provided by a generator.

In addition to direct and indirect drive systems, various studies have proposed hybrid dampers, combining the magnetic and viscous dampers to increase the damping force magnitude, while benefiting from the controllability of magnetic shock absorbers and their energy harvesting capabilities [59, 60]. Figure 2.10 demonstrates the design of the hybrid damper.
Figure 2.9: Schematic of a regenerative shock absorber utilizing rack and pinion mechanism [55].

proposed by Asadi et al [59]. In this design, the power is solely generated by the permanent magnets and coils, while the damping force is produced by both the electromagnetic and hydraulic components.

Figure 2.10: Hybrid damper utilizing twin tube design [59].

As demonstrated, there has been a great interest in eddy current and regenerative shock absorbers and many have proposed various applications for them. Reference [61] proposed an electromagnetic damper for precision positioning stages. Reference [62] used an electromagnetic damper for a vibration isolation system for a space shuttle payload. Reference [63] introduced electromagnetic dampers for civil applications, and eddy current dampers have even been proposed for vehicle suspension systems [64] [51].
Despite the vast number of articles on the magnetic shock absorbers, to the best of the author's knowledge, no previous work has used a repelling magnetic lattice as a stiffness source and the electromagnetic coils as a source of damping in a single device. The main focus of the study is the stiffness characteristics of this novel magnetic shock absorber and its feasibility in ground vehicle applications, with a brief look into the coil damping and energy harvesting effects as well.

Since the introduction of magnetorheological (MR) fluid by Rabinow [65], MR dampers have gained popularity as they enable a variable damping force to be provided in response to an electrical control signal. MR dampers have been developed for various applications such as automotive suspension [66], airplane landing gear systems [67], medical devices [68], and even seismic applications [69]. The novel shock absorber studied in this research not only could offer the same damping force controllability, but also the capability to generate force and develop active suspension. Additionally, its nonlinear characteristics could be beneficial in certain applications, such as automotive suspension systems.

2.4 Vehicle Implementation

As mentioned previously, electromagnetic and eddy current dampers have been developed for vehicle suspension system by several research groups. Goldner and Zerigian [35] studied the effectiveness of efficiently transforming a vehicle's vibrational energy into electrical power by using optimally designed regenerative electromagnetic shock absorbers. They concluded that a 2500 lb vehicle that utilizes four optimized regenerative magnetic shock absorbers and whose average speed is 45 mph on a typical U.S. highway is likely to convert between 20% and 70% of its vibrational energy to electric power. Zuo et al [36] proposed an electromagnetic damper that utilizes a stack of magnets and coils as a source of damping and energy harvesting which can be retrofitted into a conventional vehicle suspension system, as shown in Figure 2.11. They also demonstrated that the regenerated power will be the largest at a frequency around the resonance frequency of the system and that the voltage and power output increases with increasing excitation amplitude.
Figure 2.11: Diagram of a linear electromagnetic shock absorber \[36\].

In another study by Zuo et al \[37\], a dynamic model for a quarter vehicle with a magnetic energy harvesting system was developed to analyze the energy harvesting and the damping characteristics of a hypothetical vehicle employing this system. They demonstrated that the energy harvester can produce a peak power of 1.08 W with a sinusoidal input having an amplitude of 2mm and frequency of 10 Hz, which they was gauges to power some of the automotive electronics. Kawamoto et al \[51\] developed a model for an electromagnetic damper for ground vehicle applications and validated their modeling approach using a shaker test and implementation in a quarter car model. Their proposed electromagnetic damper converts linear motion to rotational motion and the damping and energy harvesting is provided by a rotary generator. Kim et al \[64\] studied a cylindrical-type electromagnetic actuator with a permanent magnet. They demonstrated that the electromagnetic damper delivered good performance in terms of reducing various vibratory motions in an experimental one degree-of-freedom vehicle model. Babak et al \[41\] proposed a hybrid active damper for vehicle suspension systems that consisted of an active eddy current damper in conjunction with an off-the-shelf passive one. They demonstrated that the addition of the passive damper reduces the damping forces required to be produced from the active eddy current damper which results in cost and weight savings. This provides a synopsis of the broad and active field of electromagnetic dampers with emphasis on applications to ground vehicles; however, these systems differ from the system presented in this investigation. The system proposed and analyzed herein rests on the premise of one dimensional nonlinear lattices. The lattice elements are permanent magnets arranged as to be in a repulsion configuration. As seen
from a shock absorber design perspective, the lattice acts as a nonlinear spring element in addition to a damping element when conducting coils circumscribe it. To the best of the author’s knowledge and the thorough literature review conducted, there is yet to be a research investigation of this system and its applicability as a shock absorber element for ground vehicles.

The dynamics of vehicle suspension systems has been explored extensively over the years. Quarter car [70, 71, 72], half car [73, 74, 75], and full car [76, 77] models have been developed to study the ride quality and handling performance of a vehicle. Semi-active [78, 79, 80], and active [81, 82] suspension systems have been developed to gain further control on the behaviour of the vehicle. Similarly, the performance of the proposed shock absorber design in ground vehicles in terms of ride quality and road holding is investigated using quarter and half car models. As mentioned previously, no previous work has studied this novel shock absorber design nor its application to ground vehicles.
Chapter 3

Model Development

With reference to Figure 3.1, the proposed magnetic shock absorber consists of a number of ring magnets arranged in a 1D lattice configuration with axial magnetic polarization and similar physical properties so that the initial distance between adjacent magnets is the same and assumed to be $d_0$. The magnets at each end are fixed, confining the range of motion of the sliding magnets. With $n$ sliding magnets, the external force is applied to the $n^{th}$ magnet. For each sliding magnet there is a corresponding electromagnetic coil, thus there are $n$ coils as well, which are fixed to the shock absorber body. Each sliding magnet experiences a force due to its interactions with the adjacent sliding or fixed magnets and an additional force due to its interaction with the coils. The Newtonian approach is implemented to obtain the equations of the motion (EOM) of the system.

Figure 3.1: Repelling 1D magnetic chain diagram.
3.1 Magnetic Force

Based on the research conducted by Molerón et al. [13], the repelling force between two adjacent magnets is given by

\[ F_{\text{repelling}} = Ad^p \] (3.1)

where \( A > 0 \) and \( p < 0 \) are empirically-derived constants and \( d \) is the distance between the two magnets. Note that the constants depend on the magnet’s properties, such as grade and size. According to [13], since for typical magnets the repelling force diminishes rapidly as the distance increases, the effects of non-adjacent magnets likely can be ignored in analyzing the dynamics of the magnetic chain.

3.2 Electromagnetic Damping

The repelling magnetic chain alone acts as a nonlinear spring element; however, implementing electromagnetic coils enables the dissipation of energy from the system and provides the ability to harvest the kinetic energy of the magnets into electrical energy. This is due to Faraday’s law of induction, which states that whenever a conductor loop experiences a change in magnetic flux, a voltage will be induced in that loop. The current in the loop in return creates a magnetic field which opposes changes in the initial magnetic field according to Lenz’s law [83].

According to Faraday’s induction law, the electromagnetic force is proportional to the relative speed of the magnet with respect to the coil. Therefore, electromagnetic energy dissipation is velocity dependant and can be modeled similarly to a viscous damper as

\[ F_{e_{ij}} = C_{e_{ij}} \dot{x}_i \] (3.2)

where \( F_{e_{ij}} \) is the electromagnetic damping force exerted on magnet \( i \) by coil \( j \), \( (1 \leq i, j \leq n) \), \( C_{e_{ij}} \) is the damping coefficient due to interaction between magnet \( i \) and coil \( j \), and \( \dot{x}_i \) is the velocity of the \( i^{th} \) magnet with the assumption of fixed coils.

Unlike a simple viscous damper, the damping coefficient is not constant. According to [84] [85] [86], it is dependant on the magnetic field density as given by
\[ C_{eij} = \frac{(NB_{ij} l_c)^2}{R_{\text{load}} + R_{\text{coil}}} \quad (3.3) \]

where \( N \) is the number of coil turns, \( B_{ij} \) is the average magnetic flux density of magnet \( i \) interacting with coil \( j \), \( l_c \) is the coil length, \( R_{\text{coil}} \) is the internal electrical resistance of the coil, and \( R_{\text{load}} \) is the connected resistive load. For a conventionally-wound coil,

\[ R_{\text{coil}} = \rho_w \frac{L_w}{A_w} \quad (3.4) \]

where \( \rho_w \) is the wire resistivity, \( L_w = f_c V_c / A_w \) is the wire length, \( V_c = \pi (r_o^2 - r_i^2) l_c \) is the coil volume, \( A_w \) is the wire cross-sectional area, \( r_o \) is the outer radius of the coil, and \( r_i \) is the inner radius of the coil [87]. Therefore,

\[ R_{\text{coil}} = 16 \rho_w \frac{f_c (r_o^2 - r_i^2) l_c}{\pi \Phi^4} \quad (3.5) \]

where \( \Phi \) is the wire diameter and \( f_c \) is the coil fill factor which is the percentage of conductive material occupying the coil volume. The fill factor of wound coils will vary, but a figure of 50–60\% can be reasonably assumed [87].

![Figure 3.2: Interaction of a magnet with a single turn of coil.](image)

On the other hand, with reference to Figure 3.2, the \( x \) component of the magnetic flux density on the axis of symmetry for a ring magnet is given by [88]
\[ B_{ij}(x) = \frac{B_r}{2} \left[ \frac{L + a_{ij}}{\sqrt{R_o^2 + (L + a_{ij})^2}} - \frac{a_{ij}}{\sqrt{R_o^2 + a_{ij}^2}} \right] - \left( \frac{L + a_{ij}}{\sqrt{R_i^2 + (L + a_{ij})^2}} - \frac{a_{ij}}{\sqrt{R_i^2 + a_{ij}^2}} \right) \]  

\[ \text{(3.6)} \]

where \( a_{ij} \) is the distance from the pole face of the magnet \( i \) to the first turn of the coil \( j \), \( 2L \) is the length of the magnet, \( R_i \) is the inner radius of the ring magnet, \( R_o \) is the outer radius of the ring magnet, and \( B_r \) is the residual magnetism which is independent of the magnet’s geometry and grade. For instance, for a N52 neodymium magnet, the residual magnetism ranges from 1.42 to 1.47 Tesla. Note that for relative motion between the magnet and the coil, considering only the \( x \) component of the magnetic induction \( B_x \) suffices as an approximation \[83\]. Therefore, the total magnetic flux density is given by \[89\]

\[ B_{ij} = \frac{1}{l_c} \int_0^{l_c} B_{ij}(x) dx \]  

\[ \text{(3.7)} \]

Note that in Equation \[3.7\], the magnetic flux density calculation does not take into account the interactions of the adjacent magnets. According to Reference \[90\], the existence of other magnets increases the flux density which will result in stronger damping forces. Derivation of the exact analytical electromagnetic damping force for a multi-magnet-coil setup is beyond the scope of this study. However, in the parametric study offered in Chapter \[4\] the effect of coil damping magnitude variation is studied on the response of the system. Finite element methods can also be used to determine the flux density \[28\]. The topology of the magnet and coil is an area that can be further studied to optimize the design of the proposed shock absorber and increase its performance, but such an optimization study is beyond the focus of the current feasibility investigation. The coil damping coefficient also can be obtained through identification of damping force experimentally and analysis of the voltage outputs of the coils \[91\]. However, this simplification allows for simulation efficiency and offers sufficient information to study the feasibility of the shock absorber in real-world applications. Ultimately, the total electromagnetic damping coefficient of the \( i^{\text{th}} \) magnet
can be obtained as

\[
C_{ei} = \sum_{j=1}^{n} C_{eij}
\]  (3.8)

Note that unlike the repelling force case where the effect of non-adjacent magnets was ignored, in this case the effect of all coils is considered since a single magnet throughout its travel due to a perturbation can come within range of any coil where the force due to their interaction is significant. Lastly, the total damping coefficient for each magnet is \( C_i = C_{ei} + C_{vi} \), where \( C_{vi} \) is the viscous friction coefficient of the \( i \)th magnet obtained experimentally. For a more comprehensive friction model, a LuGre model could be used to incorporate stiction force [92], which could play an important role in the back and forth type motion of the magnets.

Figure 3.3: Free body diagrams of 1st, \( i \)th, and \( n \)th magnets.
3.3 Magnetic Shock Absorber Equations of Motion

The forces interacting with each free body can be categorized as the magnetic forces that act as a nonlinear spring forces, the damping forces in the form of friction forces and electromagnetic coil forces, and the inertial forces related to each free body. Based on the free body diagrams provided in Figure 3.3, dynamic force equilibrium equations for the 1st, i\textsuperscript{th}, and n\textsuperscript{th} magnets are, respectively

\[ m\ddot{x}_1 = A(d_0 + x_1 - x_2)^p - A(d_0 - x_1)^p \] (3.9)
\[ m\ddot{x}_i = A(d_0 + x_i - x_{i+1})^p - A(d_0 + x_{i-1} - x_i)^p \] (3.10)
\[ m\ddot{x}_n = A(d_0 + x_n)^p - A(d_0 + x_{n-1} - x_n)^p \] (3.11)

where \( m \) is the mass of each magnet and \( x_i \) is the displacement of magnet \( i \) from the initial equilibrium position. Including the damping forces, the EOM of the system were obtained using Newtonian dynamics as

\[
\begin{bmatrix}
m & 0 & \ldots & \ldots & 0 \\
0 & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & m & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \ldots & \ldots & 0 & m
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_i \\
\ddot{x}_n
\end{bmatrix}
= 
\begin{bmatrix}
A(d_0 + x_1 - x_2)^p - A(d_0 - x_1)^p - C_1\ddot{x}_1 \\
\vdots \\
A(d_0 + x_i - x_{i+1})^p - A(d_0 + x_{i-1} - x_i)^p - C_i\ddot{x}_i \\
\vdots \\
f_n + A(d_0 + x_n)^p - A(d_0 + x_{n-1} - x_n)^p - C_n\ddot{x}_n
\end{bmatrix}
\] (3.12)

where \( f_n \) is the external force applied to the n\textsuperscript{th} magnet.
Chapter 4

Simulation and Parametric Study

To analyze the behaviour of the novel magnetic shock absorber, a nonlinear simulation model was created in Simulink simulation environment utilizing the EOM developed in Chapter 3. This model is used to study the system’s response to various inputs and analyze the effects of different parameters on the response of the system.

4.1 Simulation

Parameters used to simulate the response of the system to an external force are summarized in Table 4.1, some of which were obtained from Reference [13]. The magnets used by Molerón et al [13] to study the dynamics of the repelling magnetic chain were small in size, with a mass of 10.5 grams, and as such are not suitable for shock absorber development for ground vehicle applications. However, these parameters are used for the preliminary simulation development and to study the behaviour of the system and investigate its feasibility in real-world applications. Subsequent to this initial computational investigation, the systems parameters will be updated and scaled to the size of a typical vehicle shock absorber.

Figure 4.1 shows the response of a four-magnet setup (i.e. $n = 4$) to a constant force of magnitude 0.3 N, applied to the fourth magnet at $t = 0.5$ s. The fourth magnet settles to a final displacement of approximately 102 mm due to the externally applied force. The displacement of the magnets decreases as they are further away from the fourth magnet in
Table 4.1: Magnetic shock absorber parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet mass, $m$</td>
<td>$10.5 \times 10^{-3}$</td>
<td>kg</td>
</tr>
<tr>
<td>Repelling force exponent, $p$</td>
<td>-2.73</td>
<td>-</td>
</tr>
<tr>
<td>Repelling force constant, $A$</td>
<td>$6.25 \times 10^{-5}$</td>
<td>N/m$^p$</td>
</tr>
<tr>
<td>Viscous damping coefficient, $C_v$</td>
<td>0.03</td>
<td>N·s/m</td>
</tr>
<tr>
<td>Inter-lattice equilibrium distance, $d_0$</td>
<td>0.3</td>
<td>m</td>
</tr>
<tr>
<td>Magnet inner radius, $R_i$</td>
<td>4.25</td>
<td>mm</td>
</tr>
<tr>
<td>Magnet outer radius, $R_o$</td>
<td>9.55</td>
<td>mm</td>
</tr>
<tr>
<td>Magnet length, $2L$</td>
<td>6.4</td>
<td>mm</td>
</tr>
<tr>
<td>Coil length, $l_c$</td>
<td>15.64</td>
<td>mm</td>
</tr>
<tr>
<td>Number of coil turns, $N$</td>
<td>500</td>
<td>-</td>
</tr>
<tr>
<td>Total resistance, $R_{tot} = R_{load} + R_{coil}$</td>
<td>10</td>
<td>Ω</td>
</tr>
</tbody>
</table>

the chain, with the first magnet having the smallest displacement.

Figure 4.1: System response to a constant force.

4.2 Wave-like Behaviour

As demonstrated in Figure 4.1, various “modes” are detectable in the response of the system.

For nonlinear systems, the modes are called nonlinear normal modes (NNMs). However, the principle of superposition of modes does not apply to nonlinear systems [93] and cannot be
used to study the various oscillations in system’s response. Thus, the complex response of the
magnets cannot be decomposed into modes as in a linear system; however, the compound
motion can be better understood as a wave.

A closer look at the displacement reveals the wave-like behaviour of the repelling chain
and gives better insight into the reason behind the observed complex oscillations. Figure 4.2
demonstrates the initiation of the wave. As the wave starts to propagate, each magnet begins
to move with a slight delay, with magnet number one being the last one to displace. On the
other hand, Figure 4.2 shows that magnet one peaks earlier than the other magnets with
magnet four being the last one to peak. Magnet one then bounces back and causes a second
peak. This second wave travels down the lattice to magnets two and three. However, due
to the existence of damping forces resulting from electromagnetic coils and friction forces,
presented in Equation (3.12), the wave dissipates before reaching magnet 4. This is a sign
of the solitary wave-like behaviour of the system, agreeing with Molerón et al [13].

![Figure 4.2: Wave-like behaviour of the system.](image.png)
4.3 System Stiffness

To comprehend the stiffness characteristics of the system, the force versus displacement curve was generated. To do so, a constant force was applied to the $n^{th}$ magnet and the displacement of each magnet was recorded after the system had reached static equilibrium. This process was repeated for an array of external forces, the result of which is summarized in Figure 4.3. Note that this is a hardening type of system, meaning the higher the displacement, the higher the stiffness. This behaviour can be beneficial in applications where having a soft stiffness is desirable for low displacement amplitudes; however, the ability to limit the displacement amplitude to a certain range is crucial as well, such as vehicle suspension systems where dual rate springs are implemented to combine the relatively low initial stiffness, to absorb minor road undulations and increase grip, and transition to a secondary higher stiffness, to improve vehicle roll control during cornering.

![Figure 4.3: Force vs. displacement for four-magnet setup.](image)

Using an exponential curve fitting algorithm, the force-displacement relation for this particular design was estimated as:

$$F(x_n) = \left(5.34 \times 10^{-3}\right) e^{18x_n} + \left(2.36 \times 10^{-9}\right) e^{62.12x_n}$$  \hspace{1cm} (4.1)
where $F(x_n)$ is the external force, in Newtons, as a function of displacement, in meters. This estimation resulted in a sum of squares due to error (SSE) equal to 0.055. As demonstrated in the zoomed-in view provided in Figure 4.4, the slight deterioration of the fitness is mainly confined to the low external force range. Equation 4.1 can be used to quickly obtain the stiffness of this specific system for a given input. However, for other design choices such as number of magnets, inter-lattice equilibrium distance, etc. a different stiffness equation should be derived, Equation 4.1 highlights the exponential nature of the system’s stiffness.

Figure 4.4: Force vs. displacement estimation of the $n^{th}$ magnet in the four-magnet system.

4.4 Parametric Study

The parametric studies in this subsection are carried out to understand the effect of each parameter on the response of the system.

4.4.1 Magnet Count

The number of magnets that were simulated ranged from 1 to 8, excluding the fixed magnets at each end. Every other aspect of the shock absorber was kept constant, even the overall dimension, implying that as the number of magnets was increased, the inter-lattice
equilibrium distance of the magnets, $d_0$, decreased. This is due to the fact that the overall length of the shock absorber is held constant and the equilibrium distance between adjacent magnets is given by $d_0 = D/(n + 1)$.

The simulation was run for a step input force of magnitude of 0.3 N, as well as a sinusoidal one with the same magnitude and a frequency of 5 rad/s. Figures 4.5 and 4.6 show the displacement of the $n^{\text{th}}$ magnet, the magnet to which the external force is applied, for the step and sinusoidal inputs, respectively. It is observed that a lower magnet count in the lattice results in higher amplitudes of the response and longer settling time.

Figure 4.7 shows spikes in inter-lattice force amplitude for one and two magnet setups. This can be explained by the relatively large equilibrium distance in lattices with low counts of magnets, which allows for the magnets to attain higher velocities and lower inter-magnet distances, and hence higher forces, when perturbed. Depending on the application, higher magnet counts might be desirable, since they do not demonstrate spikes in inter-lattice forces. Also, as shown in Figure 4.5, the response of a single magnet system is similar to a damped harmonic oscillator in the transitory phase for a step input but more complex for a harmonic input.
Figure 4.5: Displacement of the $n^{th}$ magnet vs. time for various magnet counts ($n$) in response to a inputs.
Figure 4.6: Displacement of the $n^{th}$ magnet vs. time for various magnet counts ($n$) in response to step a sinusoidal input.

Figure 4.7: Total force experienced by the $n^{th}$ magnet vs. time for sinusoidal input and various magnet counts ($n$), excluding certain traces for plot clarity.
4.4.2 Mass and Repelling Force

The mass and repelling force coefficient (the constant $A$ in Equation 3.1) of the magnets are coupled to each other, i.e. stronger magnets will be heavier. However, their proportional relation was unknown and magnet manufacturers do not provide any information on the repelling force magnitude between two equal magnets. Therefore, the repelling force of the permanent magnet and its mass is assumed to be linearly proportional in this investigation. Meaning, if a magnet is twice as heavy, it can exert twice the repelling force. Future investigations are proposed to examine this assumptions.

Note that as the coefficient $A$ increases, the amplitude of the displacement and total force decrease, as demonstrated in Figures 4.8 and 4.9. However, the settling times for the stronger magnets are longer, which could be rectified through damper tuning.

![Graph showing displacement vs. time for various magnet masses and repelling force coefficients](image)

Figure 4.8: Displacement of the $n^{th}$ magnet vs. time for step input and various magnet masses ($m$) and repelling force coefficients ($A$).
4.4.3 Inter-lattice Equilibrium Distance

The effect of overall dimension of the shock absorber, $D$, was also studied. Changing the overall dimension affects the inter-lattice equilibrium distance between the magnets, $d_0$. Since in this setup there are four moving magnets, the equilibrium distance can be expressed as $d_0 = D/(n + 1)$, where $n = 4$.

Larger shock absorber length allows the magnets to reach higher velocities before being stopped due to the repulsive force of the adjacent magnets. The higher velocities attained by the perturbed magnets in the lattice are reasoned to cause the spikes in the registered forces in Figure 4.10 since the magnets get closer to each other for the same magnet count and increasing lattice length. Figure 4.11 indicates that larger equilibrium distances result in larger displacements and longer settling times. Also, the ability to adjust the inter-lattice equilibrium distance opens up the possibility to fine tune the shock absorber for various applications, such as meeting the suspension requirements of an off-road versus on-road vehicle.
Figure 4.10: Total force experienced by the $n^{th}$ magnet vs. time for step input and various element dimensions ($D$).

Figure 4.11: Displacement of the $n^{th}$ magnet vs. time for step input and various element dimensions ($D$).
4.4.4 Coil Damping

Coil damping can be adjusted by changing the electric load resistance, which provides the possibility to actively tune the shock absorber. The coil damping, unlike a simple viscous damping, is velocity and displacement dependant; therefore as the magnet is perturbed along the axis of the lattice, it experiences damping forces that range from high to vanishing. As seen in Figure 4.12 as the electrical resistance of the coil decreases, which increases its effective damping according to Equation 3.3, the displacement of the magnet along the axis of the lattice is increasingly damped as expected; however, the oscillations in the response persist. Also, the displacement profile is dependant on the applied force magnitude as it would affect the velocity of the magnets.

![Figure 4.12](image.png)

Figure 4.12: Displacement of the $n^{th}$ magnet vs. time for step input and various coil damping magnitude for various electrical resistances of the coil.
Chapter 5

Experimental Verification

The equations of motion of the magnetic shock absorber developed in Chapter 3 and the simulation model results provided in 4 must be validated experimentally to verify the accuracy the modeling approach.

5.1 Experimental Apparatus

For experimental purposes, a prototype was designed and constructed by the author of the thesis. This prototype consists of a lattice that is assembled using eight identical N45 neodymium ring magnets, six floating and two fixed at each end. The magnets are axially magnetized and have a 2 inch outer diameter, 0.5 inch inner diameter, and 0.25 inch height. To confine the motion of the magnets to the longitudinal (x) axis and to keep the repelling orientation (NS-SN), the magnets are placed around a circular aluminum shaft of 3/8 inch diameter, as shown in Figure 5.1. To reduce the friction levels and avoid damaging the magnets through sliding contact with the aluminum shaft, a Teflon® PFA sleeve with 3/8 inch inner diameter and 1/2 inch outer diameter is pressed onto the shaft. The Teflon® sleeve was machined down in diameter to allow a loose enough fit for the magnets in order to provide low friction but tight enough fit to avoid rotation of the magnets off axis. The manufacturing drawings for the magnetic shock absorber are provided in Appendix A.

The $n^{th}$ magnet is rigidly attached to the shaft and the motion of the shaft is transferred to the $n^{th}$ magnet; however, the rest of the magnets are free to float and slide on the shaft.
The two fixed magnets at each end are held in place by two machined aluminum cups and four threaded rods at each end. The 5/16-18 inch threaded rods allow a fine adjustment of the total length of the shock absorber, $D$, and enable the addition of coil windings around the magnets. The motion of the shaft, and correspondingly the $n^{\text{th}}$ magnet, is measured by a linear variable differential transformer (LVDT) sensor and the displacements of the floating magnets are estimated utilizing a high speed Sony RX100 IV camera and motion tracking software developed in MATLAB®.

![Experimental apparatus of the novel shock absorber.](image)

**Figure 5.1: Experimental apparatus of the novel shock absorber.**

### 5.2 Motion Tracking Process

Measuring the displacement of the floating magnets by LVDT sensors was challenging, as attaching the LVDT probes to the magnets interfered with their motion. Therefore, motion tracking through video recording proved to be a more effective solution.

For vision based object tracking, there are multiple algorithms that are used either individually or in combination with each other, such as: feature based tracking, colour detection, and edge based detection. MATLAB® implements a powerful algorithm, termed the Kanade-Lucas-Tomasi (KLT) algorithm, that tracks points specified by the user throughout the video data. This algorithm is suitable for tracking objects that do not change shape and exhibit visual texture [91].
Throughout the sequence of the video, points that were being tracked can be lost due to lighting variation, out of plane rotation, or articulated motion. Therefore, a high quality video with consistent lightning and stable camera position will result in better accuracy of the motion tracking. As a result, in the conducted experiments, the camera was isolated from the testing environment to minimize vibrations induced by the testing process. Furthermore, to distinguish the individual magnet of interest from the background of the video and the rest of the magnets, in each video recording a single magnet was coloured red so that the tracking algorithm can detect it more easily and robustly. Therefore, each experiment had to be conducted six times in identical conditions to capture the motion of every single magnet.

The video that is to be analyzed is loaded in MATLAB® using the VideoReader function. This function reads the video data and obtains information about the video file, such as frame rate, resolution, etc. Frame rate is an important parameter in the motion tracking process, as it allows the conversion of frame sequence to time. Thus, the video has to be recorded at a constant frame rate.

Then, a region of interest (ROI) must be selected by the user. The drawrectangle function in MATLAB® is utilized to select the ROI. This allows the user to select a rectangle from the first frame of the video. In this case, the red coloured magnet is selected as the ROI. After determining the ROI, the corner points inside the ROI are detected using the detectHarrisFeatures function which uses the Harris-Stephens algorithm [95, 96].

The algorithm obtains the position of the detected points frame by frame throughout the video data. The displacement of the magnet is then estimated by averaging the displacement of all of the corner points. However, the obtained displacement estimate is expressed in image coordinates of the video, i.e. in pixels. To convert pixels to desired units, millimeters in this case, the conversion rate needs to be determined. An object with known length was placed in the background of the video recording scene, at the same distance from the camera as the shock absorber. By counting the number of pixels per millimeter at the target distance, the scaling factor is determined. Moreover, to minimize error in motion estimations, the camera longitudinal and vertical axes were carefully set parallel to the $x$ and $y$ axes of the shock absorber.
5.3 Sensor Calibration

The LVDT sensors output a voltage reading that needs to be converted into the desired displacement data. The voltage output is linearly proportional to the displacement value. Due to the lack of data sheets available from the manufacturer, the proportional relation between voltage and displacement needed to be established. To obtain this linear relation, the LVDT sensors were displaced precisely using a digital caliper and the voltage output was recorded by a data acquisition system connected to the LVDT sensors. Figure 5.2 demonstrates the displacement versus the voltage output for one of the LVDT sensors. Using a linear fit, the conversion rate is established between the voltage and the displacement for each sensor. This process was repeated multiple times to assure accuracy.

![Figure 5.2: LVDT sensor calibration, voltage output vs. displacement.](image)

5.4 Parameter Identification

For numerical analysis purposes, the parameters such as repelling force characteristics and inherent friction forces, need to be identified.

5.4.1 Repelling Force

The repelling force was determined by placing two magnets on top of each other, allowing one to float freely. Various weights were stacked on top of the floating magnet and the
distance between the two magnets was recorded using the LVDT sensor. According to the curve fit in Figure 5.3, the coefficients introduced in Equation (3.1) were obtained to be $p = -2.151$ and $A = 6.7838 \times 10^{-3} \text{ N/m}^p$.

![Figure 5.3: Magnetic repelling force magnitude vs. displacement.](image)

5.4.2 Viscous Damping

Despite the addition of the Teflon® PFA sleeve, the friction forces were still prominent in the shock absorber. To model the friction forces as a viscous damper, the damping coefficient needed to be obtained. To this end, a mass of 8.8 kg was attached to the end of the shock absorber, as demonstrated in Figure 5.4. This 8.8 kg mass was chosen as it is the same mass that will be used in the quarter car apparatus testing that is introduced in Chapter 7.

The mass was displaced a total of 50 mm and released. The displacement of the mass was recorded by an LVDT sensor, as demonstrated in Figure 5.5. Utilizing the displacement data gathered, the logarithmic decrement was calculated using

$$\delta = \frac{1}{N-1} \ln \frac{X_1}{X_N}$$

(5.1)

where $X_1$ and $X_N$ are the amplitudes of the first and the $N^{th}$ peak of the oscillation respectively. The damping ratio is then found from the logarithmic decrement using
\[ \zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} = 0.1269 \]  \hspace{1cm} (5.2)

For a single degree of freedom system, the viscous damping is then obtained as

\[ C_v = 2(M + m)\zeta\omega_n = 38.30 \text{ N} \cdot \text{s/m} \]  \hspace{1cm} (5.3)

where \( M \) is the mass of the attached weight to the shock absorber, \( m \) is the mass of the \( n^{th} \) magnet, and \( \omega_n \) is the natural frequency of the free oscillating mass. Observe that this is the friction force applied to the \( n^{th} \) magnet, since the external mass is directly mounted to the shaft and the shaft is rigidly attached to the \( n^{th} \) magnet. Therefore, \( C_{vn} = C_v = 38.3 \text{ N} \cdot \text{s/m} \). This damping is due to the friction of all magnets sliding on the shaft. Therefore, the damping for each of the sliding magnets can be assumed to be \( C_{vi} = C_v/(n - 1) \), (for \( i = 1 \) to \( n - 1 \)), since they act as dampers in series. It is to be noted that this is a simplification and due to manufacturing imperfections, the friction levels for each magnet would be slightly different; however, it offers good enough accuracy for simulation purposes.
5.5 Simulation Verification

5.5.1 Stiffness

To comprehend the stiffness characteristics of the system, the force versus displacement curve is to be generated. Experimentally, various weights having mass $M$ were placed on the shock absorber and the static equilibrium position of the $n^{\text{th}}$ magnet on the shaft was recorded. For this experiment, six moving magnets were chosen and the total length of the shock absorber was set to be $D = 14.7$ cm. This was achieved by adjusting the position of the aluminum mounting plates that hold the two fixed magnets at each end, as shown in Figure 5.1. Note that, varying the number of magnets and the total length of the shock absorber affects the stiffness and the behaviour of the system, as concluded in Section 4.4. These values were chosen such that the magnetic shock absorber would have similar stiffness to the linear spring available for comparison purposes.

Figure 5.6 summarizes the simulation and the experiment results. The simulation results followed the experimental data closely for low force values; however, the results started to deviate as the force level increased. As pointed out in the derivation of the EOM in Section 3.3, the forces of nonadjacent magnets were ignored. Inclusion of the attractive forces of the second adjacent magnets resulted in a more accurate simulation, as demonstrated in
Figure 5.6. Note that the attraction forces of the magnets were taken to be similar to the repelling forces, given by (3.1). Moreover, using numerical approaches such as the finite element method, the influence of the adjacent magnets on the magnetic field of each magnet can be analyzed and determined. However, these effects were ignored in this simulation and the model offers a good approximation of the experimental results, as demonstrated in Figure 5.6.

Since the repelling or attraction force between magnets quickly diminishes with increasing distance, ignoring the nonadjacent forces is a good approximation for low displacement. However, as the magnets get closer and closer, the interactions of the nonadjacent magnets cannot be ignored and their impact on the response of the system becomes significant. Therefore, for a more accurate simulation, these forces should be implemented in the EOM. However, inclusion of repelling force of the third adjacent magnets does not affect the results of this system significantly. The modified EOM, which includes the forces of second adjacent magnets, and is utilized in the simulations is given by

\[
\begin{bmatrix}
m & 0 & 0 & \ldots & \ldots & \ldots & 0 \\
0 & m & 0 & \ldots & \ldots & \ldots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & m & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & 0 & 0 \\
0 & 0 & \ldots & \ldots & 0 & m & 0 \\
0 & 0 & \ldots & \ldots & 0 & 0 & m + M
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\vdots \\
\ddot{x}_i \\
\vdots \\
\ddot{x}_{n-1} \\
\ddot{x}_n
\end{bmatrix}
= 
\begin{bmatrix}
F_{mag1} - C_1\dot{x}_1 \\
F_{mag2} - C_2\dot{x}_2 \\
\vdots \\
F_{mag_i} - C_i\dot{x}_i \\
\vdots \\
F_{mag_{n-1}} - C_{n-1}\dot{x}_{n-1} \\
f_n + F_{mag_n} - C_n\dot{x}_n
\end{bmatrix}, \quad (5.4)
\]
where,

\[ F_{mag_1} = A \left[ - (d_0 - x_1)^p + (d_0 + x_1 - x_2)^p - (2d_0 + x_1 - x_3)^p \right], \]

\[ F_{mag_2} = A \left[ (2d_0 - x_2)^p - (d_0 - x_2 + x_1)^p + (d_0 + x_2 - x_3)^p - (2d_0 + x_2 - x_4)^p \right], \]

\[ F_{mag_i} = A \left[ (2d_0 - x_i + x_{i-2})^p - (d_0 - x_i + x_{i-1})^p + (d_0 + x_i - x_{i+1})^p \right. \]
\[ \left. - (2d_0 + x_i - x_{i+2})^p \right], \]

\[ F_{mag_{n-1}} = A \left[ (2d_0 - x_{n-1} + x_{n-3})^p - (d_0 - x_{n-1} + x_{n-2})^p \right. \]
\[ + (d_0 + x_{n-1} - x_n)^p - (2d_0 + x_{n-1})^p \right], \]

\[ F_{mag_n} = A \left[ (2d_0 - x_n + x_{n-2})^p - (d_0 - x_n + x_{n-1})^p + (d_0 + x_n)^p \right]. \]

Figure 5.6: Experimental and simulation results for force vs. displacement of the prototype shock absorber.

Note that this is a hardening type of system, meaning the higher the displacement, the higher the stiffness. Using an exponential curve fitting algorithm, the force-displacement relation was estimated as

\[ F(x_n) = 8.123e^{38.518x_n} + 4.185e^{-561.701x_n} \]  \hspace{1cm} (5.5)

where the displacement, \( x_n \), is in meters and the external force, \( F(x_n) \), is in Newtons.
Figure 5.7 shows the simulation vs the curve fit. Equation 5.5 can be used to quickly obtain the stiffness of the system for a given input and shows its hardening nature. It is also provided to highlight the exponential-like nature of the system’s stiffness.

\[ F(x_n) = 8.123e^{38.518x_n} - 4.18474e^{-561.701x_n} \]

Figure 5.7: Curve fit for force vs. displacement relation of the prototype shock absorber.

5.5.2 Transient Time Response

In this section, the transient time simulation is to be validated. To do so, various masses were placed onto the shock absorber and the motion of the magnets resulting from the weight of the external mass was captured utilizing a high speed camera. Custom-developed motion tracking software was used to estimate position of individual magnets. In the simulation environment, the same mass was added to the shock absorber under the same conditions. The parameters used are summarized in Table 5.1. It is important to emphasize that in the simulation, replacing the mass with a simple constant external force with the same magnitude as the weight of the mass used in the experiment would be incorrect, as this would ignore the effect of inertial forces associated with the mass.

The results of the motion tracking and the simulation are summarized in Figure 5.8. In this specific case, the chosen weight had a mass of 2.89 kg. The experimental data show that the magnets reach equilibrium faster than what the simulation estimates by a small amount. This is attributed to the existence of stiction friction forces. A more suitable
Table 5.1: Magnetic shock absorber parameters obtained experimentally.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet mass, ( m )</td>
<td>10.5 ( \times 91^{-3} )</td>
<td>kg</td>
</tr>
<tr>
<td>Repelling force exponent, ( p )</td>
<td>-2.151</td>
<td>–</td>
</tr>
<tr>
<td>Repelling force constant, ( A )</td>
<td>6.784 ( \times 10^{-3} )</td>
<td>N/m(^p)</td>
</tr>
<tr>
<td>Viscous damping coefficient, ( C_v )</td>
<td>38.30</td>
<td>N \cdot s/m</td>
</tr>
<tr>
<td>Shock absorber length, ( D )</td>
<td>12.82</td>
<td>mm</td>
</tr>
<tr>
<td>Total electrical resistance, ( R_{tot} )</td>
<td>0.4</td>
<td>Ω</td>
</tr>
</tbody>
</table>

A friction model that incorporates stiction forces such as the LuGre model can be utilized to further increase the accuracy of the simulation [92]. Also, the overshoot of the fifth and sixth magnets is underestimated by the simulation while it is overestimated for first and second magnets. This is again suspected to be due to varying friction forces between each magnet and the Teflon\textsuperscript{®} sleeve. Each magnet is slightly different geometrically due to generally loose manufacturing tolerances in magnets. As a result, its frictional interaction with the Teflon\textsuperscript{®} sleeve is different. Moreover, the friction forces were modeled as a simple viscous damper that do not take into account nonlinearity of friction damping. Therefore, since magnets five and six experience higher velocities, there is the possibility that the modeled friction forces differed from the friction forces experienced by the magnets in the experiment. On the other hand, the motion tracking has its own shortcomings. As the magnets travel, they pass in and out of the focal point of the camera and this introduces inherent inaccuracies. However, overall the simulation and the experimental results are in good agreement.

### 5.5.3 Coil Damping

It was proposed by Reference [98] that the addition of electromagnetic coils will introduce damping forces and enable energy harvesting capabilities. In the experimental setup, three separate coil windings with approximately 150 turns, which were made of 24 AWG (American Wire Gauge) enameled copper wire, were added to the shock absorber. The coils were added to the system as a proof of concept but the effects of their parameter values such as number of turns, winding method, wire type, location on the shock absorber, etc, were not selected based on a rigorous investigation. As a result, there exists significant opportunity for improving performance by optimizing these parameters. A mass of 8.8 kg was attached...
to the shaft and it was subjected to a sinusoidal displacement input at multiple frequencies. The sinusoidal displacement input was provided to the system by a cam mechanism that was driven by an electric motor. The test was run with and without the coils and the voltage output of the coils and the displacement of the mass were recorded. Figure 5.9 shows the displacement of the mass over a range of input frequencies obtained from the aforementioned experiment. The amplitude of displacement for the system with coils is smaller, particularly near the system’s resonance frequency, thereby demonstrating the effectiveness of the coils as a damper.

The damping forces exerted by the coils can be equated to a viscous damping by determining the energy loss. The induced currents in the coils will dissipate at a rate of \( V_{RMS}^2/R_{tot} \). Therefore, a Discrete Fourier Transform (DFT) was conducted on the voltage output of the coils which was in response to a sinusoidal displacement input to the shock absorber. Utilizing the DFT, the sinusoidal function of the voltage output was reconstructed. Figures 5.10 and 5.11 indicate the magnitude and the phase of the frequency components of the voltage output respectively, for one of the coils. Figure 5.12 demonstrates the experimental voltage output and the DFT reconstruction for one of the coils in response to a sinusoidal displacement input to the shock absorber. Through the DFT reconstruction,
Figure 5.9: Experimentally obtained displacement vs. input frequency of the free oscillating mass.

The voltage can be represented by a sum of multiple alternating voltages as

\[ V(t) = \sum_{j=1}^{q} V_j \cos(\omega_j t + \psi_j) \]  

(5.6)

where \( q \) is the number of terms used to reconstruct the DFT, \( V_j \) is the amplitude of the \( j^{th} \) term, and \( \omega_j \) and \( \psi_j \) are the frequency and phase of the \( j^{th} \) term, respectively. Utilizing Equation (5.6), the energy dissipated in one cycle can be determined to be

\[ E_{dissipated} = \Delta t P_{avg} = \Delta t \sum_{j=1}^{q} \frac{V_{RMS,j}^2}{R_{tot}} \]  

(5.7)

where \( P_{avg} \) is the average electric power generated by the induced current in the coils, \( V_{RMS,j} = V_j/\sqrt{2} \) is the root mean square voltage of the \( j^{th} \) alternating voltage, \( R_{tot} \) is the total electrical resistance, \( \Delta t = 2\pi/\omega \) is the duration of one cycle of system oscillation, and \( \omega \) is the input frequency. On the other hand, the energy dissipated by a viscous damper during a single cycle in a harmonic oscillator can be obtained as
\[ E_v = \int_0^{\Delta t} F_v \, dx = \int_0^{2\pi/\omega} F_v \dot{x} \, dt \]  \hspace{1cm} (5.8)

where \( F_v = -c_{eq} \dot{x} \) is the force exerted by the viscous damper and \( \dot{x} = \omega X_0 \cos(\omega t) \) is the velocity of the mass. Denoting the amplitude of the oscillation as \( X_0 \), and the equivalent viscous damping coefficient as \( c_{eq} \), one gets,

\[ E_v = -c_{eq} \int_0^{2\pi/\omega} \dot{x}^2 \, dt = -c_{eq} \pi \omega X_0^2 \]  \hspace{1cm} (5.9)

Therefore, equating the energy dissipated by the coils, \( E_{\text{dissipated}} \), to the energy dissipated by a damper, the equivalent damping coefficient can be obtained by

\[ c_{eq} = \frac{\sum_{j=1}^{q} V_j^2}{R_{\text{tot}} \omega^2 X_0^2} \]  \hspace{1cm} (5.10)

For simulation efficiency, the coil damping can be simplified to a viscous damper as given in Equation 5.10, which depends on the voltage and displacement amplitude, input frequency, and the electric resistance.

Figure 5.10: DFT magnitude spectrum of voltage output of the top coil to a 1.8 Hz sinusoidal displacement excitation.
Figure 5.11: DFT phase spectrum of voltage output of the top coil to a 1.8 Hz sinusoidal displacement excitation.

Figure 5.12: Voltage output of the top coil to a 1.8 Hz sinusoidal displacement excitation and its DFT reconstructions.
Chapter 6

Vehicle Implementation

To study the feasibility of the novel magnetic shock absorber in ground vehicle applications, the shock absorber is implemented in quarter and half car simulation models. Using the relationships and properties developed in Chapters [3] and [5] the EOM of both systems are obtained in this Chapter.

6.1 Quarter Car Model

In a conventional quarter car model the sprung mass is connected to the unsprung mass via a linear spring in parallel with a viscous damper. The equations of motion for the linear quarter car model are

\[
\begin{align*}
    m_S \ddot{x}_S + C_S (\dot{x}_S - \dot{x}_U) + K_S (x_S - x_U) &= 0 \\
    m_U \ddot{x}_U + C_S (\dot{x}_U - \dot{x}_S) + K_S (x_U - x_S) + K_t (x_U - x_R) &= 0
\end{align*}
\]

(6.1) (6.2)

where \( m_S \) is the mass of the sprung mass, \( m_U \) is the mass of the unsprung mass, \( C_S \) is the viscous damping coefficient, \( K_S \) the linear spring constant, \( K_t \) is the tire stiffness, \( x_S \) is the displacement of the sprung mass, \( x_R \) is the elevation of the road surface irregularities and is the input to the system, and \( x_U \) is the displacement of the unsprung mass [5].

In this investigation, the nonlinear quarter car model is developed such that the linear spring is replaced by the magnetic shock absorber. The viscous damper is not removed
from the system to account for any friction forces persisting in the experimental apparatus. This is due to the fact that in the experimental apparatus, the motion of the sprung mass is confined in the vertical direction through two bearings that slide on two rigid shafts and these bearings introduce friction forces which are modeled as a viscous damper. Utilizing the established relationships for the repelling force and coil damping, the equations of motion of the quarter car are then developed.

![Schematic of quarter car with magnetic shock absorber (left), and free body diagram of the unsprung mass and the $i^{th}$ and $n^{th}$ magnets (right).](image)

Based on the free body diagrams (FBDs) provided in Figure 6.1, the sum of the forces applied to the $i^{th}$ magnet is

$$\sum F_{magnet_i} = A(2d_0 - x_i + x_{i-2})^p - A(d_0 - x_0 + x_{i+1})^p + A(d_0 + x_i - x_{i+1})^p$$

$$- A(2d_0 + x_i - x_{i+2})^p - C_v(\dot{x}_i - \dot{x}_S) - \sum_{j=1}^{n} C_{e_{ij}}(\ddot{x}_i - \ddot{x}_{coil_j})$$

(6.3)

where $C_v$ is the viscous damping coefficient accounting for sliding friction forces of the magnet. The length of arrows provided in the FBD diagram in Figure 6.1 are not proportional to the magnitude of the forces they represent and are arranged for clarity and legibility.
Note that the coils are rigidly connected to the external surface of the shock absorber body which is connected to the sprung mass, and as a result, $x_{\text{coil}_j} = x_S$. On the other hand, according to Newton’s third law of motion, the coils are subjected to the same force they exert on the magnets, acting in the opposite direction. Hence, coil $j$ experiences a sum of forces of $\sum_{i=1}^{n} C_{e_{ij}} (\dot{x}_{\text{coil}_j} - \dot{x_i})$.

The total forces applied to the $n$th magnet and the sprung mass can be obtained from inspection of Figure 6.1. Same with the electromagnetic coils, each fixed magnet is rigidly connected to the shock body. Correspondingly, the fixed magnet’s displacement is represented by $x_S$ in Figure 6.1. The resulting equations are

$$\sum_{i=1}^{n} F_{\text{magnet}_i} = A(2d_0 - x_n + x_{n-2})^p - A(d_0 - x_n + x_{n-1})^p + A(d_0 + x_n - x_S)^p - C_v(\dot{x}_n - \dot{x}_S) - \sum_{j=1}^{n} C_{e_{ij}} (\dot{x}_n - \dot{x}_{\text{coil}_j}) \tag{6.4}$$

$$\sum F_{\text{unsprung}} = K_t(x_U - x_R) - C_s(\dot{x}_U - \dot{x}_S) \tag{6.5}$$

The $n$th magnet is rigidly connected to the unsprung mass; therefore, $x_n = x_U$. As a result, Equations 6.4 and 6.5 can be merged into one. On the other hand, the shock absorber forces that were applied to the unsprung mass will be applied in the opposite direction to the sprung mass. In [100], it was shown that coil damping can be modeled as viscous damping for any specific input frequency and amplitude. Thus, the coil damping and friction forces can be combined into a single force with a new coefficient $C_i$ for the $i$th magnet. As a result, the system’s EOM can be summarized as
\[
\begin{pmatrix}
  m & 0 & 0 & \ldots & \ldots & \ldots & 0 \\
  0 & m & 0 & \ldots & \ldots & \ldots & 0 \\
  0 & 0 & m & \ldots & \ldots & \ldots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  \vdots & \vdots & \vdots & \ddots & m & 0 & 0 \\
  0 & 0 & \ldots & \ldots & 0 & m + m_U & 0 \\
  0 & 0 & \ldots & \ldots & 0 & 0 & m_S 
\end{pmatrix}
\begin{pmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \vdots \\
  \dot{x}_i \\
  \vdots \\
  \dot{x}_{n-1} \\
  \dot{x}_U \\
  \ddot{x}_S 
\end{pmatrix}
= \begin{pmatrix}
  F_{mag_1} - C_1 (\dot{x}_1 - \dot{x}_S) \\
  F_{mag_2} - C_2 (\dot{x}_2 - \dot{x}_S) \\
  \vdots \\
  F_{mag_i} - C_i (\dot{x}_i - \dot{x}_S) \\
  \vdots \\
  F_{mag_{n-1}} - C_{n-1} (\dot{x}_{n-1} - \dot{x}_S) \\
  F_{mag_n} + K_1 (x_U - x_R) - C_S (\dot{x}_U - \dot{x}_S) - C_n (\dot{x}_n - \dot{x}_S) \\
  -F_{mag_n} - C_S (\dot{x}_S - \dot{x}_U) - \sum_{i,j=1}^{n} [C_{e_{i,j}} + C_v] (\ddot{x}_S - \ddot{x}_i)
\end{pmatrix}
\]

(6.6)

where,

\[
F_{mag_1} = A \left[ -(d_0 + x_S - x_1)^p + (d_0 + x_1 - x_2)^p - (2d_0 + x_1 - x_3)^p \right],
\]

\[
F_{mag_2} = A \left[ (2d_0 + x_S - x_2)^p - (d_0 - x_2 + x_1)^p + (d_0 + x_2 - x_3)^p - (2d_0 + x_2 - x_4)^p \right],
\]

\[
F_{mag_i} = A \left[ (2d_0 - x_i + x_{i-2})^p - (d_0 - x_i + x_{i-1})^p + (d_0 + x_i - x_{i+1})^p \\
- (2d_0 + x_i - x_{i+2})^p \right],
\]

\[
F_{mag_{n-1}} = A \left[ (2d_0 - x_{n-1} + x_{n-3})^p - (d_0 - x_{n-1} + x_{n-2})^p \\
+ (d_0 + x_{n-1} - x_n)^p - (2d_0 + x_{n-1} - x_S)^p \right],
\]

\[
F_{mag_n} = A \left[ (2d_0 - x_n + x_{n-2})^p - (d_0 - x_n + x_{n-1})^p + (d_0 + x_n - x_S)^p \right].
\]
6.2 Half Car Model

The quarter car model, which represents one-quarter of the car body and suspension system connected to one tire, is a simple and useful tool to study the characteristics of the vertical motion of the sprung and unsprung masses. However, to analyze both the bouncing and pitching motions of a vehicle, a half car model must be utilized.

A conventional linear half car model has 4 DOF, the vertical motion of front and rear unsprung masses, the vertical motion of the sprung mass, and the pitch motion of the sprung mass. Based on Figure 6.2, the EOM of the conventional half car model can be obtained as

\[
m_S \ddot{x}_S = -K_{rs}(x_S - x_{ru} - a_r \sin \theta) - C_{rs}(\dot{x}_S - \dot{x}_{ru} - a_r \dot{\theta} \cos \theta) - K_{fs}(x_S - x_{fu} + a_f \sin \theta) - C_{fs}(\dot{x}_S - \dot{x}_{fu} + a_f \dot{\theta} \cos \theta) \tag{6.7}
\]

\[
I_{S} \ddot{\theta} = a_r K_{rs}(x_S - x_{ru} - a_r \sin \theta) + a_r C_{rs}(\dot{x}_S - \dot{x}_{ru} - a_r \dot{\theta} \cos \theta) - a_f K_{fs}(x_S - x_{fu} + a_f \sin \theta) - a_f C_{fs}(\dot{x}_S - \dot{x}_{fu} + a_f \dot{\theta} \cos \theta) \tag{6.8}
\]

\[
m_{ru} \ddot{x}_{ru} = K_{rs}(x_S - x_{ru} - a_r \sin \theta) + C_{rs}(\dot{x}_S - \dot{x}_{ru} - a_r \dot{\theta} \cos \theta) - K_{ri}(x_{ru} - x_{rR}) \tag{6.9}
\]

\[
m_{fu} \ddot{x}_{fu} = K_{fs}(x_S - x_{fu} + a_f \sin \theta) + C_{fs}(\dot{x}_S - \dot{x}_{fu} + a_f \dot{\theta} \cos \theta) - K_{fi}(x_{fu} - x_{fR}) \tag{6.10}
\]

where \(m_S\) is the mass of the sprung mass representing the half of the car body, \(I_S\) is the mass moment inertia of the sprung mass about its center of gravity (CG), \(m_{ru}\) is the mass of the rear unsprung mass, \(m_{fu}\) is the mass of the front unsprung mass, \(C_{rs}\) is the viscous damping coefficient of the rear suspension, \(C_{fs}\) is the viscous damping coefficient of the front suspension, \(K_{rs}\) is the linear spring constant of the rear suspension, \(K_{ri}\) is the linear spring constant of the front suspension, \(K_{ri}\) is the rear tire stiffness, \(K_{fi}\) is the front tire stiffness, \(x_S\) is the displacement of the sprung mass, \(\theta\) is the pitch angle of the sprung mass, \(x_{ru}\) is the displacement of rear unsprung mass, \(x_{fu}\) is the displacement of front unsprung mass, \(x_{rR}\) is the elevation of the road surface undulations for the rear tire, \(x_{fR}\) is the elevation of the...
road surface undulations for the front tire, $a_R$ is the distance of the rear suspension from the sprung mass CG, and $a_f$ is the distance of the rear suspension from the sprung mass CG [101]. Note that the pitch angle introduces nonlinearity to the system. Conventionally, the small angle assumption is made for $\theta$ in order to linearize the EOM of the half car model. However, such simplifications are not necessary in this study, since the magnetic forces are nonlinear as well. So the EOM for both the conventional model and the model with magnetic shock absorbers will be nonlinear.

In the half car model with two magnetic shock absorber, the degrees of freedom of the system increases to $2n + 2$. Two magnetic shock absorbers connect the front and rear unsprung masses to the sprung mass replacing the linear springs. The method used for obtaining the EOM of the half car model is similar to the process followed for the quarter car model in Section 6.1. However, now the pitch motion has to be considered as well. Employing the annotation used for the parameters, Figure 6.3 can be referenced for the schematic of the half car model.

Herein, the subscripts $r$ and $f$ denote the parameters related to the rear and front unsprung masses, respectively, as denoted in Figure 6.3. The FBDs of the sprung mass and the rear and front unsprung masses are provided in Figures 6.4, 6.5 and 6.6 respectively.
Figure 6.3: Schematic of the half car model with magnetic shock absorbers.

Figure 6.4: Free body diagram of the half car sprung mass.
Figure 6.5: Free body diagram of the rear unsprung mass and the corresponding $i^{th}$ and $n^{th}$ magnets.
Figure 6.6: Free body diagram of the front unsprung mass and the corresponding $i^{th}$ and $n^{th}$ magnets.
With reference to the provided FBDs the half car model is developed. The EOM is split into three separate matrices, one for each unsprung mass and the corresponding magnetic shock absorber and one for the sprung mass as summarized below by Equations (6.11) through (6.13).

\[
\begin{bmatrix}
m & 0 & 0 & \ldots & \ldots & 0 \\
0 & m & 0 & \ldots & \ldots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & m \\
0 & 0 & \ldots & \ldots & 0 & m + m_{f_U}
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_{f_1} \\
\ddot{x}_{f_2} \\
\vdots \\
\ddot{x}_{f_i} \\
\ddot{x}_{f_{n-1}} \\
\ddot{x}_{f_U}
\end{bmatrix}
= 
\begin{bmatrix}
F_{f_{mag1}} - C_{f_1}(\dot{x}_{f_1} - \dot{x}_S - a_f \dot{\theta} \cos \theta) \\
F_{f_{mag2}} - C_{f_2}(\dot{x}_{f_2} - \dot{x}_S - a_f \dot{\theta} \cos \theta) \\
\vdots \\
F_{f_{mag_i}} - C_{f_i}(\dot{x}_{f_i} - \dot{x}_S - a_f \dot{\theta} \cos \theta) \\
\vdots \\
F_{f_{mag_{n-1}}} - C_{f_{n-1}}(\dot{x}_{f_{n-1}} - \dot{x}_S - a_f \dot{\theta} \cos \theta) \\
F_{f_{magn}} + K_{f_i}(x_{f_U} - x_{f_R}) - C_{f_S}(\dot{x}_{f_U} - \dot{x}_S - a_f \dot{\theta} \cos \theta) - C_{f_{n}}(\dot{x}_{f_{n}} - \dot{x}_S - a_f \dot{\theta} \cos \theta)
\end{bmatrix}
\]
\[
\begin{bmatrix}
m & 0 & 0 & \ldots & 0 \\
0 & m & 0 & \ldots & 0 \\
0 & 0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots \ & 0 \\
0 & 0 & \ldots & 0 & m + m_{ru}
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_{r_1} \\
\ddot{x}_{r_2} \\
\vdots \\
\ddot{x}_{r_i} \\
\vdots \\
\ddot{x}_{r_{n-1}} \\
\ddot{x}_{ru}
\end{bmatrix}
= \\
\begin{bmatrix}
F_{r_{mag}} - C_{r_1} (\dot{x}_{r_1} - \dot{x}_S + a_r \dot{\theta} \cos \theta) \\
F_{r_{mag_2}} - C_{r_2} (\dot{x}_{r_2} - \dot{x}_S + a_r \dot{\theta} \cos \theta) \\
\vdots \\
F_{r_{mag_i}} - C_{r_i} (\dot{x}_{r_i} - \dot{x}_S + a_r \dot{\theta} \cos \theta) \\
\vdots \\
F_{r_{mag_{n-1}}} - C_{r_{n-1}} (\dot{x}_{r_{n-1}} - \dot{x}_S + a_r \dot{\theta} \cos \theta) \\
F_{r_{mag_n}} + K_{r_1} (x_{ru} - x_{r_u}) - C_{r_2} (\dot{x}_{ru} - \dot{x}_S + a_r \dot{\theta} \cos \theta) - C_{r_n} (\dot{x}_{ru} - \dot{x}_S + a_r \dot{\theta} \cos \theta)
\end{bmatrix},
\]

\[
\begin{bmatrix}
m_S & 0 \\
0 & I_S
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_S \\
\dot{\theta}_S
\end{bmatrix}
= \\
\begin{bmatrix}
-F_{f_{mag}} - F_{r_{mag}} - C_{f_S} (\dot{x}_S - \dot{x}_{fu} + a_f \dot{\theta} \cos \theta) - C_{r_S} (\dot{x}_S - \dot{x}_{ru} - a_r \dot{\theta} \cos \theta) - C_f - C_r \\
a_r \left[ F_{r_{mag}} + C_{r_S} (\dot{x}_S - \dot{x}_{ru} - a_r \dot{\theta} \cos \theta) + C_r \right] - a_f \left[ F_{f_{mag}} + C_{f_S} (\dot{x}_S - \dot{x}_{fu} + a_f \dot{\theta} \cos \theta) + C_f \right]
\end{bmatrix},
\]
where,

\[ F_{mag_1} = A \left[ -(d_0 + x_1 + s) + (d_0 + x_1 - x_2)^p - (2d_0 + x_1 - x_3)^p \right], \]

\[ F_{mag_2} = A \left[ (2d_0 + x_1 + s) - (d_0 - x_2 + x_1)^p \right. \]
\[ \left. + (d_0 + x_3 - x_3)^p - (2d_0 + x_2 - x_4)^p \right], \]

\[ F_{mag_3} = A \left[ (2d_0 - x_1 + x_1 - x_2)^p - (d_0 - x_1 + x_3)^p \right. \]
\[ \left. + (d_0 + x_4 - x_4)^p - (2d_0 + x_2 - x_5)^p \right], \]

\[ F_{mag_{n-1}} = A \left[ (2d_0 - x_{n-1} + x_{n-3})^p - (d_0 - x_{n-1} + x_{n-2})^p \right. \]
\[ \left. + (d_0 + x_{n-1} - x_{n-2})^p - (2d_0 + x_2 - x_5)^p \right], \]

\[ F_{mag_n} = A \left[ (2d_0 - x_{n-1} + x_{n-2})^p - (d_0 - x_{n-1} + x_{n-2})^p \right. \]
\[ \left. + (d_0 + x_{n-1} - x_{n-2})^p - (2d_0 + x_2 - x_5)^p \right], \]

\[ F_{mag_{n-1}} = A \left[ (2d_0 - x_{n-1} + x_{n-3})^p - (d_0 - x_{n-1} + x_{n-2})^p \right. \]
\[ \left. + (d_0 + x_{n-1} - x_{n-2})^p - (2d_0 + x_2 - x_5)^p \right], \]

\[ F_{mag_n} = A \left[ (2d_0 - x_{n-1} + x_{n-2})^p - (d_0 - x_{n-1} + x_{n-2})^p \right. \]
\[ \left. + (d_0 + x_{n-1} - x_{n-2})^p - (2d_0 + x_2 - x_5)^p \right], \]

\[ C_r = \sum_{i,j=1}^{n} [C_{r_{ij}} + C_{v}] (\dot{x}_i - a_r \dot{\theta} \cos \theta - \dot{x}_i), \]

\[ C_f = \sum_{i,j=1}^{n} [C_{f_{ij}} + C_{v}] (\dot{x}_i + a_f \dot{\theta} \cos \theta - \dot{x}_i). \]
Chapter 7

Ride Quality and Performance

Utilizing the EOM developed in Chapter 6 simulations of the quarter and half car models were created. The nonlinear simulations were created in the Simulink environment since it facilitates easy nonlinear modeling.

7.1 Quarter Car

The quarter car Simulink model was used to study the response of the system in various scenarios. The quarter car setup used for testing the model consisted of a sprung mass connected to the unsprung mass via a linear spring and a viscous damper, demonstrated in Figure 7.1.

The motion of the masses is confined to the vertical axis through bearings that can slide on two rigid vertical shafts. The linear spring can be replaced by the magnetic shock absorber and the viscous damper can be disconnected to study the undamped response. A second spring, called the tire spring, connects the unsprung mass to a cam mechanism representing the road surface. The cam is driven by a motor for which the speed can be controlled. The parameters of the quarter car setup and the shock absorber are obtained from Reference [100] and are summarized in Table 7.1.
7.1.1 Single Bump

To reach a basic understanding of the system’s behaviour and analyze its response to a simple excitation, the quarter car was subjected to a single half-sinusoidal bump with an amplitude of 0.15 m. The response of the quarter car with the magnetic shock absorber is compared to the one with conventional linear spring. The stiffness of the magnetic shock absorber was set to be the same as the linear spring for this particular input amplitude, which was achieved by the adjustment of the overall length of the shock absorber, $D$. Figures 7.2 and 7.3 demonstrate the displacement for the sprung and unsprung masses and the acceleration of the sprung mass, respectively. Despite the nonlinear nature of the shock absorber, it has similar behaviour to the linear model in this case.
Table 7.1: Quarter car and magnetic shock absorber parameters.

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter Car</td>
<td>Sprung mass, $m_S$</td>
<td>8.3</td>
<td>kg</td>
</tr>
<tr>
<td></td>
<td>Unsprung mass, $m_U$</td>
<td>6.2</td>
<td>kg</td>
</tr>
<tr>
<td></td>
<td>Tire stiffness, $K_t$</td>
<td>27074.87</td>
<td>N/m</td>
</tr>
<tr>
<td></td>
<td>Suspension spring stiffness, $K_S$</td>
<td>2718.88</td>
<td>N/m</td>
</tr>
<tr>
<td></td>
<td>Suspension damping coefficient, $C_S$</td>
<td>25.94</td>
<td>N·s/m</td>
</tr>
<tr>
<td>Shock Absorber</td>
<td>Repelling force exponent, $p$</td>
<td>-2.151</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Repelling force constant, $A$</td>
<td>$6.784 \times 10^{-3}$</td>
<td>N/m²</td>
</tr>
<tr>
<td></td>
<td>Magnet mass, $m$</td>
<td>$10.5 \times 91^{-3}$</td>
<td>kg</td>
</tr>
<tr>
<td></td>
<td>Viscous damping coefficient, $C_v$</td>
<td>38.30</td>
<td>N·s/m</td>
</tr>
<tr>
<td></td>
<td>Shock absorber length, $D$</td>
<td>12.82</td>
<td>mm</td>
</tr>
</tbody>
</table>

Figure 7.2: Simulation time response of the quarter car with the linear and the novel shock absorber traveling at $5 \text{ [m/s]}$ to a single bump on the road surface.
Figure 7.3: Acceleration of the sprung mass of the quarter car with the linear and the novel shock absorber traveling at 5 [m/s] in response to a single bump on the road surface obtained from the simulation model.

### 7.1.2 Bumpy Road

The random bumpy road profile was generated utilizing the white noise method introduced by Lenkutis et al. in [102]. The profile of the random road excitation can be expressed by its power spectral density (PSD) as follows

\[
\psi(\omega) = \frac{2\alpha v \sigma^2}{\pi} \frac{1}{(\alpha v)^2 + \omega^2} \quad \text{or} \quad \psi(\Omega) = \frac{2\alpha \sigma^2}{\pi} \frac{1}{\alpha^2 + \Omega^2}
\]

(7.1)

where \(v\) is the vehicle longitudinal velocity [m/s], \(\omega\) is an angular frequency in the time domain [rad/s], and \(\Omega\) is the angular spatial frequency [rad/m]. The constant \(\alpha = 0.127\) [rad/m] is independent of the road profile shape and it is termed as a low frequency cut off in units of spatial angular frequency, and \(\sigma^2\) is road roughness variance, which is presented in Table 7.2 according to [103]. The power spectral density of road excitation can be rewritten as
\[ \psi(\omega) = \frac{2\alpha v \sigma^2}{\pi} \frac{1}{(\alpha v - j\omega)(\alpha v + j\omega)} = H(\omega) \psi_\omega H^T(\omega) \] (7.2)

where \( H(\omega) = \frac{1}{\alpha v + j\omega} \) is the frequency response function of the shaping filter and \( \psi_\omega = 2\alpha v \sigma^2 \) is the spectral density of a white noise process. Assuming that the vehicle is moving at a constant velocity, the profile of road excitation in the time domain can be defined as

\[ \frac{d}{dt} x_R(t) = -\alpha v x_R(t) + \eta(t) \] (7.3)

where \( \eta(t) \) is a zero-mean Gaussian white noise having a velocity dimension [m/s] with a PSD as \( 2\alpha v \sigma^2 \). This equation can be implemented in the Simulink environment as demonstrated in Figure 7.4.

Table 7.2: Constant coefficient of the road excitation.

<table>
<thead>
<tr>
<th>Road Type</th>
<th>Very Good</th>
<th>Good</th>
<th>Average</th>
<th>Bad</th>
<th>Very Bad</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma ) [10^{-3} m]</td>
<td>2</td>
<td>4</td>
<td>16</td>
<td>18</td>
<td>32</td>
</tr>
</tbody>
</table>

Figure 7.4: Simulink diagram of road profile generation.

Figure 7.5 displays the road profile and the displacement of the sprung and unsprung masses. Note that since the system is nonlinear, its stiffness will vary based on the input amplitude. Therefore, even though the stiffness of the magnetic shock absorber and the linear spring were chosen to be equal for the single bump case, their response will differ if the input amplitude changes and this variation in response is observable in the bumpy road simulation. Nevertheless, the responses are similar for the linear and nonlinear models, as demonstrated in Figure 7.6, where the difference between the displacement of the linear quarter car and the one with the magnetic shock absorber is less than 2%. Figure 7.7
shows the acceleration of the sprung mass. The absolute maximum acceleration value for the linear model is slightly higher. In spite of that, both systems demonstrate very similar performance.

Figure 7.5: Simulation time response of the quarter car with the linear spring (top) and magnetic shock absorber (bottom) to a bumpy road surface.
Figure 7.6: Percentage difference of displacement between the quarter car with the linear spring and the quarter car with the magnetic shock absorber for the bumpy road.
Figure 7.7: Acceleration of the sprung mass of the quarter car model with the linear spring (top) and magnetic shock absorber (bottom) in response to a bumpy road surface obtained from simulation.
7.1.3 Ride Quality

The ISO 2631 international standard, titled Mechanical Vibration and Shock Evaluation of Human Exposure to Whole-Body Vibration, is a standard primarily defining methods for quantifying vibration in relation to human health, comfort, and the incidence of motion sickness, even though it does not provide limits for vibration exposure [104]. The first method is the weighted root-mean-square acceleration (RMS), being the simplest one. The weighted RMS acceleration, $a_w$, can be calculated using

$$a_w = \left\{ \frac{1}{T} \int_0^T [a_w(t)]^2 \, dt \right\}^{1/2} \quad (7.4)$$

where $a_w(t)$ is the weighted acceleration as a function of time and $T$ is the duration of the measurement. The second evaluation method, called the running RMS, takes into account occasional shocks and transient vibration by using a short integration time constant. The vibration magnitude is defined as a maximum transient vibration value (MTVV), given as the maximum in time of $a_w(t_0)$, defined by

$$a_w(t_0) = \left\{ \frac{1}{\tau} \int_{t_0-\tau}^{t_0} [a_w(t)]^2 \, dt \right\}^{1/2} \quad (7.5)$$

where $a_W(t)$ is the instantaneous weighted acceleration, $\tau$ is the integration time for running averaging, $t$ is the time (integration variable), and $t_0$ is the time of observation (instantaneous time). Utilizing Equation 7.5, the MTVV can be calculated as

$$MTVV = \max [a_w(t_0)] \quad (7.6)$$

The third method, called fourth power vibration dose, is more sensitive to the peaks in the acceleration levels than the basic evaluation method since it uses RMS acceleration raised to the fourth power and demonstrates the severity of the vibration exposure [104]. The fourth power vibration dose value (VDV) in metres per second to the power of 1.75 [m/s^{1.75}], is defined as
\[ VDV = \left\{ \int_0^T [a_w(t)]^4 \, dt \right\}^{1/4} \] (7.7)

Methods described in the ISO 2631 standard can be utilized to evaluate the acceleration data obtained from the bumpy road simulation. The results summarized in Table 7.3 demonstrate that both systems perform similarly and the differences are insignificant, with the \( a_w \) and \( MTVV \) values being 2.06% above the linear model’s response for the quarter car as compared with the magnetic shock absorber.

Table 7.3: Ride quality metrics of the novel magnetic shock absorber vs. the linear conventional one in a quarter car model moving on a bumpy surface.

<table>
<thead>
<tr>
<th>Method</th>
<th>Magnetic</th>
<th>Linear</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted root mean square acceleration, ( a_w )</td>
<td>1.682</td>
<td>1.648</td>
<td>m/s²</td>
</tr>
<tr>
<td>Maximum transient vibration, ( MTVV )</td>
<td>1.682</td>
<td>1.648</td>
<td>m/s²</td>
</tr>
<tr>
<td>Fourth power vibration dose, ( VDV )</td>
<td>4.62</td>
<td>4.62</td>
<td>m/s(^{1.75})</td>
</tr>
</tbody>
</table>

7.1.4 Transmissibility

Displacement transmissibility is another evaluation method of a system’s vibration isolation performance. The transmissibility which is the ratio of output amplitude, \( x_S \), to the input amplitude, \( x_R \), is generally studied over a range of input frequencies [5]. For passenger cars this frequency usually ranges from 0.25 to 25 Hz [4]. Since the the linear model is a 2-DOF system, two resonance modes are observed, commonly known as the bounce and wheel hop frequencies, as shown in Figure 7.8. However, the quarter car model with the magnetic shock absorber experiences multiple peaks and demonstrates complex behaviour. This arises from the nonlinearity of the repelling magnetic chain and system’s higher number of degrees of freedom given that each free magnet in the lattice is a degree of freedom. The multiplicity of peaks in the high-frequency region is an undesirable behaviour, which can contribute to passengers’ discomfort when traveling over bumpy roads at high speeds. However, it should be noted that the design of the magnetic shock absorber in this investigation is not optimized in anyway and future investigations are anticipated to better tailor and optimize the response to supersede the performance of conventional shock absorbers for all frequency
range.

![Figure 7.8: Displacement transmissibility of the quarter car with linear spring and magnetic shock absorber.](image)

7.1.5 Road Holding

The normal force acting between the tire and the road varies due to the vibrations caused by road undulations. Since the longitudinal and lateral forces generated by a tire depend directly on the normal tire load, minimizing fluctuations in the normal tire loads is key to improving road holding performance [105]. The normal force between the tire and the road during vibration can be represented by the dynamic tire deflection [5]. As a result, improving road holding performance is equivalent to minimizing the relative displacement between the unsprung mass and the road which represents the tire deflection. According to Figure 7.9, the linear model demonstrates two undesirable peaks at the system’s natural frequencies. However, the quarter car with the magnetic shock absorber generates multiple smaller peaks. For instance, the linear model’s dynamic tire deflection ratio has a second peak with a magnitude of approximately 14 but the quarter car with the magnetic shock absorber has a maximum value of 3 around the same frequency range. This behaviour can be beneficial as the magnitude of the tire deflection is lower, showing better road holding capability. Moreover, high peaks and valleys in the dynamic tire deflection demonstrates that the vehicle could experience unpredictable behaviour since the magnitude of the longitudinal
and lateral forces the tire can generate fluctuates.

![Graph showing dynamic tire deflection vs. frequency]

Figure 7.9: Road holding of the quarter car with linear spring and magnetic shock absorber.

7.1.6 Experimental Validation

Even though the EOM of the shock absorber were verified in Reference [100], the prototype shock absorber without the coils was implemented in the quarter car apparatus to validate the simulation results. The inherent friction forces of the system were identified and modeled as viscous dampers. To recreate the transmissibility data, a frequency sweep was conducted. The process was repeated for the linear model as well. Figure 7.10 demonstrates that the simulation follows the experimental results closely. Due to limitations in the testing apparatus, reaching higher input frequencies was not possible. Nevertheless, the data for the quarter car with the magnetic shock absorber shows that the transmissibility ratio peaks at around 3 Hz, followed by a decrease, to then start to increase again at around 5 Hz, as the simulation predicted. An improved input mechanism would enable the analysis of the system’s response to higher frequency inputs and further investigation of the complexities of the system’s behaviour.
Figure 7.10: Simulation vs. experimental transmissibility of the quarter car with linear spring and magnetic shock absorber.
7.2 Quarter Car Parametric Study

The purpose of this parametric study is to analyze the effects of various shock absorber parameters on ride quality and handling performance of the quarter car. For each parameter the displacement transmissibility and road holding is obtained in the simulation environment and the results are compared to the conventional linear model.

7.2.1 Magnet Count

The number of magnets that were simulated ranged from 4 to 8, excluding the fixed magnets at each end. Every other aspect of the shock absorber was kept constant, even the overall dimension, implying that as the number of magnets was increased, the inter-lattice equilibrium distance of the magnets, $d_0$, decreased. To obtain the displacement transmissibility, each system was exited by a sinusoidal input at a range of frequencies. Figure 7.11 shows the displacement transmissibility of the sprung masses for quarter cars with magnetic shock absorbers containing 4, 6, and 8 magnets, as well as for the linear model.

As mentioned in Section 4.4, increasing the number of magnets results in a stiffer shock absorber. Figure 7.11 demonstrates that the higher the magnet count, the higher the resonance frequencies, which is expected from a stiffer system. Moreover, the magnitude of the transmissibility ratio increases as the number of magnets increases. On the other hand, in the high-frequency region, the system with 4 magnets behaves similarly to a linear system; however, as the number of magnets increase, the response of the system becomes more and more complex.

Figure 7.12 depicts the road holding for the quarter car with various number of magnets and compares these against the linear model’s results. Similar to the transmissibility results, increasing the magnet count increases the dynamic tire deflection magnitude. Despite this, the magnitude of the dynamic tire deflection in nonlinear systems never exceeds the linear one’s, resulting in better road holding performance. Moreover, the frequency at which the peaks of dynamic tire deflection occur increases with the increase in the number of magnets.
7.2.2 Mass and Repelling Force

As discussed in Section 4.4, the mass and repelling force coefficient (the constant $A$ in Equation 3.1) of the magnets are coupled to each other, i.e., stronger magnets will be heavier. However, their proportionality relation was unknown, so it is assumed they are linearly proportional. In this section, two additional quarter car models were studied, one system with magnets with half the original mass and repelling force magnitude (denoted by $0.5 \times (m, A)$) and another system with magnets with twice the original mass and repelling force magnitude (denoted by $2 \times (m, A)$).
Figures 7.13 and 7.14 demonstrate the transmissibility and road holding results, respectively. Increasing the mass and repelling force magnitude, results in a higher first resonance frequency and magnitude. However, unlike the magnet count case, lower mass and repelling force magnitude increases the complexity of the system’s behaviour in the high-frequency region. As a matter of fact, a new resonance frequency appears in the transmissibility diagram for the system with half the original mass and repelling force magnitude.

Figure 7.13: Displacement transmissibility of the quarter car with various magnet masses ($m$) and repelling force coefficients ($A$).

Figure 7.14: Road holding of the quarter car with various magnet masses ($m$) and repelling force coefficients ($A$).
7.2.3 Inter-lattice Equilibrium Distance

The effect of the overall dimension of the shock absorber, $D$, was also studied. Changing the overall dimension affects the inter-lattice equilibrium distance between the magnets, $d_0$. It was concluded in Section 4.4 that smaller $D$ results in a stiffer shock absorber.

Figures 7.15 and 7.16 demonstrate the displacement transmissibility and road holding for systems with various $D$ values, respectively. Note that as the overall dimension of the shock absorber decreases, the amplitude of the resonance increases. Moreover, the resonance frequencies shift to higher values as the $D$ decreases. Further, the system with the largest $D$ does not demonstrate any complex behaviour at higher frequencies and it exhibits similar behaviour to the linear system in the high-frequency region. On the other hand, Figure 7.16 shows the stiffness of the system with the largest $D$ is so low that the first resonance frequency almost disappears.

![Figure 7.15: Displacement transmissibility of the quarter car with various magnetic shock absorber dimensions ($D$).](image)

7.2.4 Coil Damping

Finally, the effect of coil damping on system’s transmissibility and road holding performance was studied as well. As seen in Figures 7.17 and 7.18, increasing the damping, decreases the resonance magnitude as expected. However, unlike in a conventional quarter car system, increasing the damping also shifts the resonance frequency, although by a small amount.
Figure 7.16: Road holding of the quarter car with various magnetic shock absorber dimensions ($D$).

Note that the system with almost zero damping exhibits extreme fluctuations both in transmissibility and road holding values. Also, the 0.1 and 5 scaling factors were chosen for the damping case to highlight the difference between the responses of the systems.

Figure 7.17: Displacement transmissibility of the quarter car with various damping magnitude.
Figure 7.18: Road holding of the quarter car with various coil damping magnitude.
7.3 Half Car

The Simulink model created for the quarter car was modified to create the half car model utilizing the EOM developed in Section 6.2. A conventional half car model was also developed for comparison purposes. The parameters used for the simulation are summarized in Table 7.4 with the parameters of the magnetic shock absorber being the same ones used in the quarter car simulation.

Table 7.4: Half car and magnetic shock absorber parameters.

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sprung mass, ( m_S )</td>
<td>16.4 kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mass moment of inertia of sprung mass, ( I_S )</td>
<td>1.49 kg \cdot m^2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Front suspension distance from CG, ( a_f )</td>
<td>0.6 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rear suspension distance from CG, ( a_r )</td>
<td>0.4 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Front unsprung mass, ( m_{fu} )</td>
<td>6.2 kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rear unsprung mass, ( m_{ru} )</td>
<td>6.2 kg</td>
<td></td>
</tr>
<tr>
<td>Half Car</td>
<td>Front tire stiffness, ( K_{f_t} )</td>
<td>27074.87 N/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rear tire stiffness, ( K_{r_t} )</td>
<td>27074.87 N/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Front suspension spring stiffness, ( K_{f_S} )</td>
<td>2718.88 N/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rear suspension spring stiffness, ( K_{r_S} )</td>
<td>2718.88 N/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Front suspension damping coefficient, ( C_{f_S} )</td>
<td>25.94 N \cdot s/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rear suspension damping coefficient, ( C_{r_S} )</td>
<td>25.94 N \cdot s/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Longitudinal velocity, ( v )</td>
<td>15 km/h</td>
<td></td>
</tr>
<tr>
<td>Shock Absorber</td>
<td>Repelling force exponent, ( p )</td>
<td>-2.151</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Repelling force constant, ( A )</td>
<td>( 6.784 \times 10^{-3} ) N/m(^p)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Magnet mass, ( m )</td>
<td>( 10.5 \times 91^{-3} ) kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Viscous damping coefficient, ( C_v )</td>
<td>0 N \cdot s/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shock absorber length, ( D )</td>
<td>14.7 mm</td>
<td></td>
</tr>
</tbody>
</table>

7.3.1 Single Bump

To compare the behaviour of the half car model with magnetic shock absorbers to the conventional half car model, both systems were subjected to a single bump as a road input. Figure 7.19 demonstrates the sinusoidal bump with an amplitude of 7.15 millimeters in the road surface for both the front and rear tires for a vehicle traveling at 15 km/h. The sinusoidal shape of the bump is not easily detectable in Figure 7.19 due to the scaling of axes and vehicle velocity. Therefore, Figure 7.20 is provided to demonstrate the speed bump geometry as a function of road distance rather than time.
Figure 7.19: Single speed bump as road input to the half car rear and front tires.

Figure 7.20: Speed bump geometry.
Figures 7.21 and 7.22 demonstrate the displacement and acceleration experienced by the sprung mass due to the bump in the road. Note that, as the front tire travels over the bump, the displacement of the sprung mass for both systems is reasonably similar, but the model with the magnetic shock absorbers demonstrate a compound acceleration response for the sprung mass. Also, as soon as the rear wheel hits the bump, the amplitude of the displacement of sprung masses increases for both systems; however, the displacement and acceleration peak amplitudes for the conventional half car model are slightly lower. But overall, the performance of both systems is similar.

On the other hand, Figures 7.23 and 7.24 compare the displacement of the front and rear unsprung masses of a half car with linear springs to the one with magnetic shock absorbers. Note that, unlike the sprung mass, the amplitude of displacement of unsprung masses are slightly higher for the conventional model. Moreover, the rear unsprung mass experiences a slight disturbance before hitting the bump in the road surface. This is due to the fact that front tire has already traveled over the bump and has caused a pitching motion in the sprung mass which in return has disturbed the rear unsprung mass.
Figure 7.22: Acceleration of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump.

Figure 7.23: Displacement of the front unsprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump.
Figure 7.24: Displacement of the rear unsprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump.

Figures 7.25 and 7.26 indicate the pitch angle and pitch acceleration of the sprung mass of the both half car models in response to the speed bump. The pitch angle amplitude is shown to be similar for both systems. Although, the pitch angle acceleration values are slightly higher for the half car model with the magnetic shock absorber. Nevertheless, the differences between the two systems are negligible.
Figure 7.25: Pitch angle, $\theta$, of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump.

Figure 7.26: Pitch angle acceleration, $\ddot{\theta}$, of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a single speed bump.
7.3.2 Bumpy Road

The bumpy road was generated using the same methodology introduced in Section 7.1.2. Figure 7.27 depicts the road profile at vehicle’s front and rear wheels for a vehicle traveling at 2 m/s. The rear wheel experiences the same road with a time delay which depends on the distance between the front and rear wheels and the velocity of the vehicle and can be calculated as

\[
\text{Time Delay} = \frac{a_r + a_f}{\text{Vehicle Velocity}}
\]

This time delay can be implemented in the Simulink model using Transport Delay block which delays the input by a specified amount of time. The input to this block should be a continuous signal. Note that the rear tire starts its motions on a completely flat road, with zero elevation change before traveling on the bumpy road.

![Figure 7.27: Random road profile at vehicle front and rear wheels.](image)

Figures 7.28, 7.29, and 7.30 compare the displacement of the sprung mass and the front and rear unsprung masses of a conventional half car with linear springs to the one with magnetic shock absorbers. The displacement of both rear and front unsprung masses of the conventional model exhibit larger displacement amplitudes. The same behaviour can
be noticed for the sprung mass as well, with the conventional model demonstrating higher peaks.

Figures 7.31 depicts the pitch angle of the sprung mass of both half car models in response to the bumpy road. The performance of both models are similar to each other.

![Diagram](image)

Figure 7.28: Displacement of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a bumpy road.

Methods described in the ISO 2631 standard are utilized to evaluate the acceleration data obtained from the bumpy road simulation. The results summarized in Table 7.5 demonstrate that both systems perform similarly; however, the half car model with the magnetic shock absorber reached higher VDV values which is the result of higher peaks in sprung mass acceleration. Also, the overall performance of the both half car models is worse than the quarter car models since the motion of the both wheels as well as the pitch motion is contributing to the acceleration of sprung masses.

Table 7.5: Ride quality metrics of the novel magnetic shock absorber vs. the linear conventional one in a half car model moving on a bumpy surface.

<table>
<thead>
<tr>
<th>Method</th>
<th>Magnetic</th>
<th>Linear</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted root mean square acceleration, $a_w$</td>
<td>2.461</td>
<td>2.457</td>
<td>m/s²</td>
</tr>
<tr>
<td>Maximum transient vibration, $MTVV$</td>
<td>2.461</td>
<td>2.457</td>
<td>m/s²</td>
</tr>
<tr>
<td>Fourth power vibration dose, $VDV$</td>
<td>7.37</td>
<td>6.822</td>
<td>m/s^{1.75}</td>
</tr>
</tbody>
</table>
Figure 7.29: Displacement of the front unsprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a bumpy road.

Figure 7.30: Displacement of the rear unsprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a bumpy road.
Figure 7.31: Pitch angle, $\theta$, of the sprung mass of the half car model with linear springs and the magnetic shock absorbers in response to a bumpy road.
Chapter 8

Conclusion

In this study, a novel shock absorber element with energy harvesting capabilities was proposed. The nonlinear dynamics of the element were studied and the Simulink simulation software was utilized to obtain the transient response of the system for various force inputs. The stiffness characteristics of the device were obtained revealing a hardening behaviour. Lastly, a parametric study was conducted to analyze the effects of the number of magnets, magnet mass, repelling force amplitude, inter-lattice equilibrium distance, and coil damping on the response of the system. The adjustability of the stiffness of the element via equilibrium distance, magnet grades, and size demonstrated its tunability.

Then, the derived dynamics of the novel shock absorber element were validated experimentally. Utilizing a manufactured magnetic shock absorber, the parameters such as repelling force and friction forces needed for the simulation were identified. The obtained parameters were then used to simulate various experiments. Firstly, the stiffness of the unit element was obtained and the developed EOM were modified to further increase the accuracy of the simulation. Then, the simulated transient time response of the system was compared to experimental results which were obtained by tracking the motion of the individual magnets using computer vision. The dynamic simulation results followed the experimental results closely, validating the correctness of the EOM and the assumptions. It was also shown that the electromagnetic coils act as an energy dissipation source, producing an alternating current which can be converted to direct current by a bridge rectifier to be easily stored in a battery system. It was demonstrated that the damping due to the coils
can be modeled as a viscous damper for simulation efficiency.

The feasibility of using a novel magnetic shock absorber in road vehicle applications was also investigated. The novel shock absorber that has been proposed was implemented in quarter and half car models. Firstly, the dynamics of the both quarter and half cars were studied and the equations of motion were developed. Utilizing the nonlinear EOM, a simulation model was created in Simulink. Various metrics such as ISO2631 standard, displacement transmissibility, and road holding were used to study the performance of the system in ground vehicle applications and to compare its characteristics to conventional linear quarter car and half car models.

Based on the ISO2631 standard, it was shown that the novel shock absorber performs similar to the conventional one in isolating the vibrations for passengers. The displacement transmissibility analysis indicates that due to the nonlinear nature of the system and its higher number of degrees of freedom, the quarter car with the magnetic shock absorber delivers undesirable performance in the high-frequency region but has similar performance to the conventional one in the low-frequency region. On the other hand, the system with the novel shock absorber demonstrates superior road holding performance with lower variations in tire loads.

Moreover, an experimental quarter car apparatus was used to validate the simulation response. The transmissibility plots were recreated experimentally, demonstrating the accuracy of the modeling approach. A parametric study was conducted on the quarter car to analyze the dependency of ride quality and road holding of the quarter car on parameters such number of magnets, inter-lattice equilibrium distance, and so on.

It was shown that, this magnetic shock absorber can be implemented in road vehicle suspension systems, replacing conventional viscous dampers which behave in a time-variant way due to dependence on viscous fluid temperature. It was also shown that the stiffness of the system is exponential in nature which in automotive suspension systems can be designed to combine the relatively low initial stiffness, to absorb minor road undulations and increase grip, and transition to a secondary higher stiffness, to improve vehicle roll control during cornering. Moreover, the exponential stiffness of the system offers the possibility for it to act as a bump stop without the usual discontinuity in the force-displacement characteristic.
In addition to its satisfactory dynamic performance, the novel shock absorber has a number of other potential advantages. Its stiffness is easily adjustable by changing the inter-lattice equilibrium distance, magnet grade, and size. It offers controllability over damping forces by varying the electric resistive load of the coils. The non-contact nature of the stiffness and damping mechanism may lead to improved wear performance. Additionally, it is capable of converting the system’s kinetic energy to electric power via electromagnetic induction. Furthermore, it has potential for developing active suspension systems through energization of the electromagnetic coils.

8.1 Accomplishments

The primary accomplishments of this thesis are:

1. The dynamics of magnetic repelling forces and damping forces due to electromagnetic coils were studied and the nonlinear equations of motion of the system were developed.

2. A parametric study was conducted on the novel magnetic shock absorber to study the effects of parameters such as the number of magnets, coil damping, etc. on the response of the shock absorber and to fully characterize the system’s behaviour.

3. The static and transient simulation results were validated experimentally via manufacturing a prototype magnetic shock absorber.

4. The damping due to the electromagnetic coils were estimated through calculating the energy dissipated by the induced current in the coils.

5. The magnetic shock absorber was implemented in a quarter and a half car model and the corresponding equations of the motion were developed and simulation models were created.

6. Displacement transmissibility and ISO2631 standard evaluation methods were used to compare the ride quality of the quarter car with the magnetic shock absorber to a conventional linear model.
7. Dynamic tire deflections were calculated to analyze the handling performance and road holding of the quarter car.

8. Effects of parameters such as the number of magnets, amplitude of the repelling force, and damping on ride quality and road holding was studied through a parametric study of the quarter car.

9. The half car model was used to further study the ride quality of the magnetic shock absorber and to additionally analyze the pitching motion of the vehicle.

8.2 Future Work

A few notable directions that can be considered for future work related to this research project include the following.

- Optimize the design of the shock absorber for vehicle suspension system.

- Further study the coil damping effects and methods to maximize the energy harvesting capabilities.

- Investigate the controllability of the coil damping via adjusting the electrical resistance.

- Explore developing active suspension systems via energizing of coil dampers.

- Extend the testing apparatus to a half or full car model.
Bibliography


Appendix A

Manufacturing Drawings
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<th>PART NUMBER</th>
<th>DESCRIPTION</th>
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<td>1</td>
</tr>
<tr>
<td>2</td>
<td>94435A518</td>
<td>Aluminum Threaded Rod</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>90670A030</td>
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</tr>
<tr>
<td>4</td>
<td>Ring_Magnet_0D2_ID_0.5_z0.25</td>
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<td>Upper_Cap_V2</td>
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<tr>
<td>7</td>
<td>92620A141</td>
<td>Hex Head Screw</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>Coupler</td>
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<td>1</td>
</tr>
<tr>
<td>9</td>
<td>Adapter</td>
<td>1</td>
<td>1</td>
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</tbody>
</table>
1) This Teflon tube is available on McMaster-Carr with part # 51805K11 or Grainger.ca with item # USS2USA-HT1608. Prices and specifications in McMaster-Carr are preferable.

2) Dimensions are in inches, unless otherwise stated.
Note:
1) This threaded rod is available on McMaster-Carr with part # 94435A518 or Grainger.ca with item # GUS10P784

2) Dimensions are in inches, unless otherwise stated.
Note:
1) Please chamfer the ends of the threaded holes.
2) Dimensions are in inches, unless otherwise stated.
4x Ø 8.43 THRU ALL
Clearance hole for 5/16 screw
Ø 0.332 inches

2x Ø 8-32 UNC THREAD
4x 8-32 UNC THREAD

Ø 65.0

Close fit for 2 inch magnets

B

SECTION B-B

Broken-out section to demonstrate the threads' location

Note:
1) Please chamfer the ends of the threaded holes.
2) Dimensions are in Millimeters, unless otherwise stated.
Note:
1) Please chamfer the the ends the threaded holes.
2) Dimensions are in Millimeters, unless otherwise stated.
1. Please chamfer the threaded holes.
2. Dimensions are in Millimeters, unless otherwise stated.

Note:

Adapter

Material: 6061 Alloy

Dimensions:
- 4X Ø 5.11 mm x 19.00 mm
- Ø 6.38 mm x 100°, Near Side
- UNC 1/4 - 20 Thread

Tolerances:
- Fractional
- Angular: Mach
- Bend
- Three Place Decimal
- Two Place Decimal

Scale: 1:1

Weight: 171.91 g

Sheet 1 of 1