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SETTLEMENT OF A RIGID FOOTING RESTING

ON

A GRANULAR SOIL STRATUM

by

MOSTAPA IBRAHIM SABRY

BSc, ALEXANDRIA UNIVERSITY, 1977

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the
requirements for the degree of

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TO MY MOTHER
ABSTRACT

This thesis examines the load-settlement behaviour of a rigid strip footing resting on the surface of a sand layer underlain by a rigid base. The investigation concentrates on both experimental and theoretical investigations of the problem. Influences of factors such as thickness of the granular stratum, interface conditions, eccentricity of loading, load cycling, etc. are investigated.

The work is also extended to study the influence of a dense soil region on the load-settlement behaviour of the footing. In the case of the "soil composite", factors such as width of the dense region, layer thickness, interface conditions are varied to study the effectiveness of the dense soil region. Where relevant the experimental results are compared with results derived from a finite element analysis which takes into account the non-linear stress-strain response of the granular soil.
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LIST OF SYMBOLS

$B$  
Foundation Width

c  
Cohesion

$D$  
Foundation Depth

$D_r$  
Relative Density

e  
Eccentricity

$E$  
Young's Modulus

$E_{alu}$  
Young's Modulus of the Aluminium Plate

$E_i$  
Initial Young's Modulus

$E_s$  
Young's Modulus of the Sand Layer

$E_l$  
Tangential Young's Modulus

$E_{ur}$  
The Unloading-Reloading Young's Modulus

$H$  
Depth of Compressible Layer From the Foundation Level to the Rigid Base

$I$  
Moment of Inertia

$K$  
Modulus Number

$K_b$  
The Bulk Modulus Number

$K_r$  
Relative Rigidity of Soil and Foundation

$K_{ur}$  
The Unloading-Reloading Modulus Number

$L$  
Length of Foundation

$m$  
The Bulk Modulus Exponent

$M$  
Bending Moment

$n$  
Exponent Determining Rate of Variation of $E_i$ with $\sigma_3$

$N_c$, $N_q$, $N_s$  
Bearing Capacity Factors
$N'_c, N'_q, N'_\gamma$  Modified Bearing Capacity Factors to Take Into Account Effect of the Rigid Base

$q$  Contact Stress

$q_{ult}$  Ultimate Bearing Capacity

$P$  Applied Load

$P_a$  Atmospheric Pressure

$R_f$  Failure Ratio

$S$  The Width of the Dense Sand

$T$  Shearing Force

$T_r$  Shearing Force Developed on Sheet Roughened With Sand

$T_s$  Shearing Force Developed on Sheet of Stainless Steel

$u$  Displacement Component in $x$-Direction

$v$  Displacement Component in $y$-Direction

$V$  Pull-Out Force

$V_r$  Pull-Out Force of Rough Plate

$V_s$  Pull-Out Force of Stainless Steel Plate

$w$  Displacement Component in $z$-Direction

$\gamma$  Unit Weight

$\gamma_l$  The Unit Weight of the Loose Sand

$\gamma_d$  The Unit weight of the Dense Sand

$\delta$  The Settlement of the Footing

$\delta_1, \delta_2$  The Settlements Under the Footing Edges.

$\delta_a$  The Settlement Under the centre of the Footing

$\phi$  Angle of Internal Friction

$\epsilon$  Strain

$\epsilon_1$ or $\epsilon_a$  Axial Strain
\( \nu_{al} \)  
Poisson's Ratio of the Aluminium Plate

\( \nu \)  
Poisson's Ratio

\( \nu_i \)  
Initial Poisson's Ratio

\( \nu_s \)  
Poisson's Ratio of the Sand Layer

\( \nu_t \)  
Tangential Poisson's Ratio

\( \lambda \)  
Angle of Friction

\( \lambda_r \)  
Angle of Friction of Rough Plate

\( \lambda_s \)  
Angle of Friction of Stainless Steel Plate

\( \eta_{qr} \)  
Ratio of Ultimate Bearing Capacity of Foundation Resting on Shallow Layer of Thickness \( H \leq 2B \) to the Bearing Capacity of Similar Foundation Resting on Deep Layer of Thickness \( H \geq 2B \)

\( \sigma_1 \)  
Major Principal Stresses

\( \sigma_3 \)  
Minor Principal Stresses

\( \tau \)  
Shear Stress
Introduction

The analysis of interaction between a structure and the supporting soil is very important to soil mechanics and foundation engineering. Results of such analysis are useful for the design of structural foundations, such as isolated or combined footings and mat or raft foundations.

The foundation is that part of the structure which transmits the load from the superstructure to the supporting soils. The behaviour of the foundation is confined to its ability to adjust itself to soil movements in compression, swelling or even in creep.

The behaviour of the soil is mainly confined to its compressibility or swelling. For granular soils, the allowable pressure which may be applied to a foundation is governed by considerations of settlement rather than of the shear strength of the soil. For this reason, prediction of the settlement of the structures founded on granular materials is of considerable practical importance.

The behaviour of a single footing, near to the surface, resting on a layer of sand has been of great interest for researchers. Most of the cases studied have dealt with a layer of dense sand extending to large width and depth compared to the width of the footing.

In practice, the soil under the footing may contain a composite layer of sand; which consists of a dense soil region contained between two loose regions, as shown in Figure (1.1). This case of composite sand layer may have a considerable influence on the settlement and the bearing capacity of the foundation. Also, the sand layer may have a finite thickness; therefore, the effect of the sand layer thickness on the foundation behaviour should be investigated. The interface conditions between the footing and the sand layer and between the sand layer and the rigid base should be also studied. In addition, another case such as the footing subjected to eccentric loading should be investigated.

In view of the above discussion, the main objective of this thesis was to inves-
FIGURE 11 MODELLING THE PROBLEM OF RIGID STRIP FOOTING SUBJECTED TO LINE LOAD.
(PLANE STRAIN PROBLEM)
tigate the influence of the width of the dense region on the settlement and bearing capacity of a rigid strip foundation. The effect of the rigid base, interface conditions, load eccentricity, sand density and quasi-static load cycling have been investigated.

Literature review of the influence of a rigid base on the settlement and bearing capacity of foundation is discussed in Chapter 1. Chapter 2 describes the experimental program which was developed to study the behaviour of a rigid strip foundation. Only one type of sand with two different densities was used in the experiments. The plane strain apparatus, selection of the rigid model footing, the loading devices and the general properties of Ottawa sand used in the experimental investigations are also described in Chapter 2. The experimental results for rigid footing resting on a layer of dense sand are presented and discussed in Chapter 3. Also the experimental results for rigid footing resting on a composite layer of sand are illustrated and discussed in Chapter 4. Results of finite element analysis and experimental results obtained through this investigation are presented and compared in Chapter 5. The conclusions inferred from the results along with the author's recommendations for future study in this area are reported in Chapter 6.
Chapter 1

LITERATURE REVIEW

This chapter presents a brief review of research which has been carried out to study the effect of a rigid base on the bearing capacity and settlement of a footing. The review is divided into two parts: the first concerns the bearing capacity and the second concerns the settlement.

1.1 Bearing Capacity of a Soil Stratum on a Rigid Base

1.1.1 Introduction

The least pressure that will cause complete failure of the soil in the vicinity of the foundation is defined as the ultimate bearing capacity. In general, methods used for calculating the ultimate bearing capacity form three qualitatively distinct groups of solution.

The first group includes the exact mathematical methods of the perfect plasticity theory (slip-line, limit equilibrium and limit analysis methods). The slip-line method attempts first to derive the basic differential equations which then make it possible to obtain the solutions of various problems by the determination of the so-called slip-line network. The limit equilibrium method attempts first to create a simplified mode of failure which then makes it possible to solve various problems by simple static. Various solutions obtained by this method are summarized by
Terzaghi (1943) and Taylor (1948). In contrast to slip-line and limit equilibrium methods, the limit analysis method considers the stress-strain relationship of a soil in an idealized manner. This idealisation, termed normality (or the flow rule), establishes the limit theorems on which limit analysis is based. Within the framework of this assumption, the approach is rigorous and the techniques are competitive with those of limit equilibrium, in some instances being much simpler. The plastic limit theorems of Drucker et al. (1953) may conveniently be employed to obtain upper and lower bounds of the collapse load for stability problems, such as the ultimate bearing capacity of soils.

The second group contains empirical methods. Empirical or semi-empirical formula are used to extrapolate for actual foundation results of loading or complex mechanical tests such as standard and cone penetration tests (Meyerhof, 1956).

The methods in the third group require numerical solutions such as finite element, finite difference and boundary element techniques. The foremost of these methods is the finite element method. The accuracy of the solution from the mathematical standpoint is limited by the capacity of the computer.

In light of the foregoing discussion, the following is review of the effect of a rigid base on the bearing capacity.

**Shield (1955)**

Shield (1955) presented a theoretical solution which can be used to calculate the bearing capacity of a rigid footing on a purely cohesive soil. The cohesive soil is underlain by a rough rigid base. The solution was obtained by using limit analysis techniques and it was based on the following assumptions:

i) The soil is an elastic-perfectly plastic material.
ii) The soil obeys Tresca's yield criterion.

iii) Its associated flow rule will apply.

The solution was obtained for rigid smooth footing with square and strip plan shapes. Shield (1955) found that the rigid base would have no effect on the bearing capacity of the soil stratum if the ratio $\frac{B}{H}$ is less than 3, in the case of strip footing, and $\frac{B}{H}$ is less than 5 in the case of square footing.

**Mandel and Salencon (1969)**

Mandel and Salencon (1969) applied a limit equilibrium technique to examine the bearing capacity of a rigid strip foundation on a clay layer underlain by a rigid base. The footing was assumed axially loaded and friction is fully mobilized at the clay-rigid base interface. The solution was developed for the plane strain problem and for material obeying Coulomb's yield criterion.

For computing the bearing capacity of a strip footing resting on a $c - \phi$ soil, the following equation proposed by Terzaghi (1943) can be used:

$$q_u = cN_c + q_o N_q + 0.5\gamma BN_\gamma$$  \hspace{1cm} (1.1)

where

- $q_u$ = the ultimate bearing capacity
- $c$ = the cohesion of the soil
- $q_o$ = the surcharge
- $\gamma$ = the unit weight of the soil.
- $N_c$, $N_q$ and $N_\gamma$ = the bearing capacity coefficients which are function of the angle of internal friction ($\phi$) of the soil.

Mandel and Salencon (1969) proposed the modification of Terzaghi's equation
(1.1) into the following form to take into account the effect of the rigid base:

\[ q_u = c F_c N_c + q_o F_q N_q + 0.5 \gamma B F_\gamma N_\gamma \] (1.2)

or

\[ q_u = c N'_c + q_o N'_q + 0.5 \gamma B N'_\gamma \] (1.3)

where

\[ N'_c, N'_q \] and \[ N'_\gamma \] are the modified bearing capacity coefficients that depend on \( \frac{H}{B} \), \( \phi \) and the interface conditions (for a layer of soil with \( \phi = 0 \) and \( \frac{H}{B} = 1.20 \), \( N'_c = 5.14 \), \( N'_q = 1.0 \) and \( N'_\gamma = 0 \)).

The correction factors \( F_c, F_q \) and \( F_\gamma \) also depend on the friction conditions on the interface between the soil stratum and the rigid base. For example, the value of \( F_c \) and \( F_q \) (\( \phi = 30 \) and \( \frac{H}{B} = 0.5 \)) are respectively equal to 0.562 and 0.586 in the case of smooth rigid base and equal to 2.5 and 2.42 in the case of rough rigid base.

**Mandel and Salencon (1972)**

Mandel and Salencon (1972) extended their previous study to include the calculation of the bearing capacity for different values of \( \phi \) ranging between \( 8^\circ \) to \( 40^\circ \). They presented the values of the correction factors \( F_c, F_q \) and \( F_\gamma \) mentioned above in both tabular and graphical form as a function of \( \frac{H}{B} \), \( \phi \) and the interface conditions between the footing and the soil, and between the soil and the rigid base. When a perfectly rough contact is assumed the bearing capacity increases steadily with a decrease in the ratio \( \frac{H}{B} \). When a perfectly smooth contact is assumed, the bearing capacity starting from the classical value (e.g. it is equal to 5.14 \( c \) for a purely cohesive soil with \( \phi = 0 \)), decreases as the ratio \( \frac{H}{B} \) decreases. After reaching
a minimum value, the bearing capacity increases with the decrease of the ratio \( \frac{H}{D} \) as shown in Figure (1.2).

**Meyerhof (1974)**

Meyerhof (1974) examined the bearing capacity of a soft layer on a rigid base. Solutions were developed for rectangular footings with rough contact surfaces. The footing was assumed to be located on the surface of a layer of sand resting on a rough rigid base. Meyerhof (1974) proposed the following equation for the ultimate bearing capacity of rectangular foundation:

\[
q_u = \left[ 1 - (1 - S'_q) \frac{B}{L} \right] \gamma DN'_q + \left[ 1 - (1 - S'_\gamma) \frac{B}{L} \right] 0.50 \gamma BN'_\gamma
\]  

(1.4)

where

- \( N'_q \) and \( N'_\gamma \) are the modified bearing capacity coefficients, which depend on \( \phi \) and \( \frac{H}{D} \) and the degree of roughness of the rigid base.
- \( S'_q \) and \( S'_\gamma \) are the modified shape factor.

Meyerhof (1974) gave the values of \( N'_q \), \( N'_\gamma \), \( S'_q \), and \( S'_\gamma \) in a graphical form as a function of \( \phi \) and \( \frac{H}{D} \). Meyerhof (1974) also carried out a number of tests, using rough strip and circular footing models. The models were placed on the surface, and at a shallow depth in sand layers resting on a rough rigid base. For layers of ratios \( \frac{H}{D} > 0.5 \), the bearing capacity values obtained from the experiment were in general higher than those obtained from the theory. For layers of ratios \( \frac{H}{D} < 0.5 \), the bearing capacity values obtained from the experiment were less.

**Chen (1975)**

Chen (1975) presented a solution to problem of the bearing capacity of a strip
FIGURE 12: RELATIONSHIP BETWEEN THE RATIO H/B AND THE BEARING CAPACITY OF A PURELY COHESION SOIL (\(\phi = 0\)) IN SMOOTH CONTACT WITH A RIGID BASE (AFTER MANDEL AND SALENCIN, 1972)

For soils of \(\phi = 0\)

\[ q_u = \gamma' C N' + a \]

\[ N_c = F_c N_c \]

\[ N_c = N_c = 1 \times 21, F_c = 1.0 \]

\( N_c = F_c N_c \)
footing located on the surface of a shallow clay stratum resting on a rough rigid base. The solution was obtained by finite element techniques. The agreement between the finite element solution for a limit load and theoretical limit load was found to be satisfactory.

**Tournier and Milovic (1977)**

Tournier and Milovic (1977) examined the bearing capacity of foundation on a layer of sand resting on a rough rigid base. They performed a series of tests, using rough rigid strip footing models. The models were of width \( B = 20, 30, 35 \) and 50 cm. Several values of the ratio \( \frac{H}{B} \) (between 0.5 and 6.8) were studied. For all sizes, the bearing capacity increased with a decrease in the thickness of the sand layer. Tournier and Milovic (1977) compared their results with those obtained from the theory of Mandel and Salencon (1972). Tournier and Milovic's results indicate that the influence of the rigid base can still be appreciable until a ratio of \( \frac{H}{B} = 2.0 \). The theoretical results of Mandel and Salencon (1972) reveal that the rigid base would have no influence on the bearing capacity if the ratio \( \frac{H}{B} \) was greater than about 1.2.

**Pfeifle and Das (1979)**

Pfeifle and Das (1979) investigated the problem of the bearing capacity of a rigid rectangular footing on a cohesionless layer underlain by a rough rigid base. Rough rigid steel model foundations of width 5.05 cm. and lengths 5.08, 10.16, 15.24 and 30.48 cm. were used. Their results confirmed those of Tournier and Milovic (1977). Pfeifle and Das (1979) found that the modified shape factors proposed by Meyerhof (1974) agreed closely with the experimental findings.
Rabbaa (1981)

Rabbaa (1981) studied the bearing capacity of strip footing which is located on the surface of a layer of dense sand underlain by a rigid base. The model was of width \( B = 15.2 \) cm. Two experiments were conducted for two specific values of the ratio \( \frac{H}{B} \) ( = 1.0 and 2.5 ). Also two types of interface conditions between the footing and the sand layer and between the sand layer and the rigid base were investigated (smooth and rough interface conditions). In other words, the bottom surface of the footing and the top surface of the rigid base were lined with sheets of polished No.16 gauge of stainless steel in order to simulate a smooth surface. In the case of rough surfaces, these surfaces were lined with sheets roughened by sand.

Results of this investigation have shown that the influence of the ratio of the layer thickness to the foundation width \( \left( \frac{H}{B} \right) \) on the bearing capacity of the foundation depends both on the roughness conditions of the interface between the foundation and the soil and the interface between the soil and the rigid base. For a smooth interface conditions, the bearing capacity decreases as the ratio \( \frac{H}{B} \) decreases. But for rough interface conditions, the bearing capacity increases with the decrease in the ratio \( \frac{H}{B} \). The relationship between the ratio \( \frac{H}{B} \) and the bearing capacity as obtained from the experiments were described by the following equation:

\[
\eta_{qr} = 1.5 \left( \frac{B}{H} \right)^{0.6} ; \quad 0.5 < \frac{H}{B} < 2.0
\]

(1.5)

where

\( \eta_{qr} \) = the ratio of the bearing capacity of the foundation resting on a layer of thickness \( H < 2B \) to the bearing capacity of a similar foundation resting on a relatively deep layer of thickness \( H > 2B \).

\( B \) = width of the strip footing.

\( H \) = layer thickness.
1.2 Settlement of a Soil Stratum Resting on a Rigid Base

1.2.1 Introduction

Engineering structures settle for many reasons: owing to the effect of additional loading of the subsoil by a structure, lowering of the groundwater level, diverse forms of ground surface sinking (mining, sliding, subsidence, underground erosion), etc. (Sower, 1962; Terzaghi and Peck, 1967). In general, methods used for calculating the final settlement form four qualitatively distinct groups of solutions (Figure 1.3).

The first group includes mathematically exact methods of the elasticity theory which satisfy the equilibrium conditions, the constitutive equations and the boundary conditions prescribed. The most refined of these, the theory of elastic, homogeneous, isotropic and linear half space, is mathematically exact but its constitutive equations do not usually correspond to the real foundation soils; hence the results obtained by its means are at considerable variance with reality (Awojobi and Gibson, 1973; Kefin, 1956).

The second group encompasses engineering methods which are most widely used for settlement calculations. This group of methods makes use of the elasticity theory only for stress calculations; stress-strain relations needed for the subsequent phase of the calculation are derived from experiments. This is the reason why these methods, in contrast to the former, direct methods, are called indirect (Giroud, 1972). The methods of this group readily introduce the non-linearity of the stress-strain relation of a real subsoil, and this fact is clearly the main reason why the calculated and the actual settlements are generally in good agreement.

The third group contains empirical methods. Empirical or semiempirical formulas are used to extrapolate for actual foundation results of loading or complex mechanical tests (for example, penetration tests), particularly of cohesionless foun-
FIGURE 3 REVIEW OF METHODS FOR CALCULATING THE FINAL SETTLEMENT

Theory of Elasticity
(Direct Methods)
  Linear
  Nonlinear
  Homogeneous
  Non-homogeneous
  Isotropic
  Anisotropic

Engineering: Indirect Methods
  State Boundary Surface
  Cenometric Compression
  Skempton-Bjerrum Method

Empirical Methods
  Loading Tests
  Dynamic Penetration
  Static Penetration
  Pressuremeter

Numerical Methods
  Finite Element Method
  Method of Finite Differences
  Lumped-Parameter Method
  Boundary Element Method
dation soils.

The methods of the fourth group are highly demanding numerically and therefore require the application of computers. The first and foremost of these methods is the finite element method. They are characterised by the introduction of real non-linear stress-strain relations, and the accuracy of the solution from the mathematical standpoint is limited by the capacity of the computer.

A review of the effect of a rigid base on the bearing capacity is presented and discussed as follows:

Steinbrenner (1934)

Steinbrenner (see Tersaghi, 1943) presented an approximate method of computing the settlement of an elastic isotropic homogeneous layer on a rigid base. Steinbrenner (1934) computed the settlement \( \delta \) of the corners of a uniformly loaded rectangular area on the horizontal surface of a semi-infinite mass as shown in Figure (1.4). Then, he computed the vertical displacement \( \Delta \delta \) of the points located at a depth \( H \) below these corner and assumed that the settlement \( \Delta \delta_H \) of the corners of the loaded area on the surface of an elastic layer with the thickness \( H \) is equal to the difference \( \delta - \Delta \delta \), or

\[
\Delta \delta_H = \Delta \delta \tag{1.6.a}
\]

\[
\Delta \delta_H = \frac{qB}{E} \left[ \left( 1 - \nu^2 \right) F_1 + \left( 1 - \nu - 2\nu^2 \right) F_2 \right] \tag{1.6.b}
\]

\[
\Delta \delta_H = \frac{qB}{E} I_p \tag{1.6.c}
\]

where

\[
I_p = \left[ \left( 1 - \nu^2 \right) F_1 + \left( 1 - \nu - 2\nu^2 \right) F_2 \right]
\]

\( B \) = the width of the loaded area.
Figure 14 Uniformly Distributed Pressure over a Rectangular Area (After Steinbrenner, 1934)
$L$ = the length of the loaded area.

$E$ = elastic modulus.

$\nu$ = poisson's ratio.

\[
F_1 = \frac{1}{\pi} \left[ \log \frac{(1 + \sqrt{1^2 + 1})\sqrt{1^2 + d^2}}{l(1 + \sqrt{1^2 + d^2} + 1)} + \log \frac{(l + \sqrt{1^2 + 1})\sqrt{1 + d^2}}{l + \sqrt{1^2 + d^2} + 1} \right]
\]

\[
F_2 = \frac{d}{2\pi} \tan^{-1} \frac{l}{d\sqrt{1^2 + d^2} + 1}
\]

The values of $F_1$ and $F_2$ are dimensionless numbers and depending on the ratios $(l = \frac{L}{B})$ and $(d = \frac{H}{B})$. Steinbrenner (1934) presented numerical results for $F_1$ and $F_2$ for different values of $\frac{H}{B}$ (0 to 10) and different values of $\frac{L}{B}$ (1, 2, 5, 10, 100) in a graphical form.

Steinbrenner's solution is approximate for two reasons: (i) the solution does not take into account the change in both normal and shear stress distributions along the depth (H) due to the rigid base (see Egorov, 1939 and Terzaghi, 1943); and (ii) the solution neglects the effect of friction conditions of the interface between the soil stratum and the rigid base.

**Egorov (1939)**

Harr (1966) presented a solution by Egorov (1939) to compute the distribution of vertical stresses in semi-finite layer bounded by a rigid lower boundary under a uniformly distributed load acting over an infinite strip of width $B$ as shown in Figure (1.5). Also Harr (1966) presented a semiempirical method to determine the vertical displacement of the surface of a layer of finite depth $H$ (underlain by rigid base) at a point under the centre of a uniformly distributed load of intensity $P$ acting over a strip of width $B$ of variable flexibility. The procedure is based upon the investigations of Gorbunov-Pasadov (1946) and Egorov (1958), and requires the
FIGURE 5 DISTRIBUTION OF VERTICAL NORMAL STRESS IN LAYER BOUNDED BY RIGID BASE FOR STRIP LOADING (AFTER EGOROV 1939).
determination of the dimensionless factor \( t \), where \( t \) is equal to:

\[
    t = \frac{5E_i B^3}{4E_f h^3_f}
\] (1.7)

where

\( E_f \) and \( E_i \) = the modulus of elasticity of the soil in the finite layer and the material constituting the footing respectively.

\( h_f \) = the thickness of the footing.

The settlement of the center of the footing is given by the expression:

\[
    W = [K(t) + 2K_r] \frac{PB(1 - \nu^2_i)}{2\pi E_i}
\] (1.8)

where

\( \nu_i \) = poisson’s ratio of the soil.

\( K_r \) is a function of \( t \).

\( K(t) \) is dependent on the value of \( t \) and \( \frac{H}{b} \).

Harr presented the influence factors \( K_r \) and \( K(t) \) in a tubular form.

**Terzaghi (1943)**

Terzaghi (1943) discussed the influence of the rigid base on the normal and shear stresses distribution produced by a strip uniformly loaded area on the surface of semi-finite elastic mass. Terzaghi (1943) explained that the distribution of the normal stresses on a rigid base of semi-finite layer and the distribution of the shear stresses along a vertical section through the edges of the loaded area depend on the friction conditions between the elastic layer and the rigid base as shown at the top of Figure (1.6).

Terzaghi (1943) discussed the influence of rigid base on normal and shear stresses distribution produced by a strip line load on a surface of semi-finite elastic
Figure 1.5 Intensity and distribution of normal stresses on rigid base of elastic layer beneath flexible strip load and of shearing stresses on vertical sections through edges of loaded strip (modified from Terzaghi, 1943)
mass. Also explained that the distribution of normal and shear stresses depend on the friction conditions between the elastic layer and the rigid base as shown in the bottom of Figure (1.6).

Poulos (1967)

Using the theory proposed by Burmister's (1956), Poulos (1967) evaluated a set of influence factors for computing the stresses and the surface displacements of an elastic layer resting on a rough rigid base. The solution was first obtained for a point load on the surface of the layer for different values of poisson's ratio (0, 0.2, 0.4 and 0.5). Then, the solution for the distributed load was evaluated by an integration process. Specific influence factors were given for loading in the form line, strip and sector plan shapes.

Rabbaa (1981)

Rabbaa (1981) investigated the settlement of a rigid strip footing which was located on the surface of a layer of dense sand underlain by a rigid base. A model of strip footing of width \( B = 15.2 \text{ cm} \) was used. Two values of ratio \( \frac{H}{L} \), between the layer thickness and the foundation width, \( \frac{H}{B} = 2.5 \) and \( \frac{H}{B} = 1.0 \), and two cases of interface conditions between the footing and the soil and between the soil and the rigid base were studied (smooth and rough interface conditions). Rabbaa (1981) found that the settlement decreased as the ratio \( \frac{H}{B} \) decreased and the roughness conditions of the interface between the footing and the sand layer and between the sand layer and the rigid base have minor effects on the settlement.

Rabbaa (1981) also found that the shape of load-settlement curve was influenced by the number of loading and unloading cycles and most of the total and
inelastic settlements occur at the first cycle and that the rate of increase in settlement decreased as the number of cycles increased.
1.3 General Comments on Literature Review

From the review of the literature in the area of soil-foundation interaction presented above, it is possible to make the following comments:

1) Experiments carried out for studying the problem of the surface foundation were done by using small scale models resting on a relatively deep soil layer.

2) The effects of the static loading and unloading cycles and the soil density on the settlement are not well researched.

3) The case of a footing subjected to eccentric load has received little attention.

4) The cases of composite layer of finite width have not been examined previously.

1.4 Scope and Objective of the Present Work

The main objectives of this thesis can be summarized as follows:

1) To conduct an experimental investigation which will examine the behaviour of rigid footing located on the surface of a layer of sand and tested under plane strain conditions.

2) To examine the effect of selected key parameters on the behaviour of rigid strip foundations. The footing is located on the surface of a layer of sand underlain by a rigid base. The parameters investigated are the following:
(i) Layer thickness.

(ii) The interface conditions (rough or smooth).

(iii) Density of the sand.

(iv) Loading cycling (loading and unloading).

(v) Load eccentricity.

(vi) Composite layer of sand (loose and dense).

Other factors as the footing depth, shape of the footing, loading sequence, etc. are kept constant throughout the experimental program.

3) To compare the experimental results to nonlinear finite element techniques.
Chapter 2

EXPERIMENTAL PROGRAM

2.1 Modelling of the Plane Strain Problem

This thesis examines the behaviour of a rigid strip footing located on the surface of a layer of sand (dense and loose) or a layer of composite sand which is resting on a rigid base as shown in Figure (1.1). The previous experimental investigation indicates that the study was carried out in small scale tests. For example, the footing used in the study of Pfeifle and Das (1979) was 5.08 cm. wide.

In the present experimental study, a model strip footing of width 15.2 cm is used. The problem modelled in the present investigation is shown in Figure (2.1). The strip foundation is located on the surface of a stratum of sand and is loaded by a line load. Assuming plane strain conditions to exist, a finite section of the strip foundation can be examined. The plane strain conditions are characterized by the displacement field:

\[ u = u(x, y) \quad v = v(x, y) \quad w = 0 \]  \hspace{1cm} (2.1)

where \( u \), \( v \) and \( w \) are the displacement components in \( x \), \( y \) and \( z \) directions respectively. When the deformation exhibits a state of plane strain as defined by the shear stresses \( \tau_{xx} \) and \( \tau_{yx} \) on any plane \( xy \) are zero.

The footing can be subjected to several types of loading configurations and/or displacement conditions consistent with their practical uses. One particular
Figure 2: Boundary Conditions of Plane Strain Model Footing Problem
type of loading configuration assumes that the footing is subjected to concentrated line load located at the centre of the footing. In general, in this situation the strip footing is capable of rotation as illustrated in Figure (2.2). Alternatively, the rigid strip footing can be rigidly connected to the shaft (see Figure (2.3)); in this case the horizontal displacement and the rotation are prevented. In a practical situation the stiffness of the structural connections prevent rotation and the horizontal displacement at the footing base. Consequently, there is uniform settlement of the footing can be expected as can be seen from Figure (2.3).

The following sections present a description of the plane strain apparatus and the properties of the soil used in the experimental investigation. The details of the experimental program will also be discussed in relation to a flow chart.

### 2.2 The Plane Strain Apparatus

#### 2.2.1 Test Tank

All the tests were performed in a rigid steel tank with inner dimensions of 152 cm. × 38 cm. × 40 cm. (see Figure (2.4) and Plate 1). The basic design of the test tank was developed and was constructed in the Civil Engineering Laboratory at Carleton University in Ottawa. The base and the ends of the tank were made of C381 × 60 channels, the side walls were made of 1.25 cm thick steel sheets. Each wall was fastened with lengths of C76 × 6 channels. Additional rigidity was incorporated by two lengths of 15.2 cm. × 10.2 cm × 0.95 cm. rectangular hollow steel sections. In each test, these removable sections were placed where the maximum lateral pressures were to be expected. The lateral displacement of the side of walls of the tank was prevented by constraining each rectangular hollow steel section against the main steel frame by means of a screw jack as displayed in
FIGURE 2.2 TILT OF THE FOOTING ACCORDING TO THE CONNECTION BETWEEN THE FOOTING AND THE LOAD SHAFT
FIGURE 2.3 UNIFORM SETTLEMENT OF THE FOOTING ACCORDING TO THE CONNECTION BETWEEN THE FOOTING AND THE LOAD SHAFT
FIG 24.a THE PLANE STRAIN APPARATUS

(LONGITUDINAL SECTION)
FIGURE 24 b THE PLANE STRAIN APPARATUS (CROSS SECTION)
Figure (2.4). The vertical displacement of the main steel frame was measured by means of dial gauges. When the maximum load was applied, measurements observed for the lateral and the vertical displacements of the side walls and the main steel frame indicated that this method had been fairly successful in minimising these displacements.

2.2.2 Study of the Effect of the Side Friction

Friction on the interior faces of the side walls of the test tank was minimized by using sheets of polished No.16 gauge stainless steel. The effectiveness of the stainless steel in minimizing the friction was established by two different techniques by Rabbaa (1981). The first technique involved the determination of the coefficient of friction of the stainless steel, the second technique involved the determination of the effect of the friction on the test results. A brief description of the tests carried out for this purpose by Rabbaa (1981) is given in the subsequent sections.

(a) Determination of Coefficient of Friction

To determine the coefficient of the friction of the stainless steel, two types of tests were carried out: these were, namely, the shear box tests and pull-out tests.

(i) Shear Box Tests

Series of tests were carried out, where a block of stainless steel was sheared against the sand. The stainless steel block had the same inside dimensions as the upper half of the shear box. The bottom surface of the block was polished No.16 gauge. The lower half of the shear box was filled with sand by raining, using the same technique described in section 2.3.2. The surface of the sand filling the lower
half of the shear box was carefully levelled. The two halves of the shear box were fitted together as shown in Figure (2.5.a). The stainless steel block, working as a cap, was loaded and sheared against the sand. Several tests were performed using different values of the normal stress \( \sigma_n \). For each normal stress the corresponding shear strength \( \tau \) was determined, the relationship between \( \tau \) and \( \sigma \) is shown in Figure (2.5.b). The maximum angle of friction of the stainless steel \( \lambda_s \) was found to be about 8°. However, there are some limitations in the determination of the angle of friction of the stainless steel by using this method; these limitations are:

1) due to the scale effect, the half sample of sand in the lower half of the shear box may have a relative density which is different from that obtained from raining the sand in the test tank.

2) the leveling process causes disturbance of the top surface of the sand.

3) there is a possibility of locking of grains of sand between the stainless steel block and the shear box.

(ii) Pull Out Test

The limitations in the determination of the angle of the stainless steel by shear box can be avoided by determining the ratio between the angle of friction of a plate of stainless steel \( \lambda_s \) and the angle of friction of another plate roughened with sand \( \lambda_r \). This ratio is used to determine the effect of the angle of friction of the stainless steel on the test results obtained for the test tank with interior wall lined with sheets of stainless steel, as will be explained in the next section. The two plates are of dimensions 38 cm. × 36 cm. × 0.1 cm. Each plate is vertically placed at the centre of the tank and then the sand was rained until the tank is completely filled. Pull out loads are applied to each plate and the vertical displacements were
**Figure 25a** Shear box with half sample of sand

**Figure 25b** Relationship between shear and normal stresses of the stainless steel surface

(After Rabbaa (1981))
recorded. Following this procedure, the pull out load-displacement curve of the two cases are shown in Figure (2.6). The maximum vertical pull out forces in both cases were determined. In the case of stainless steel plate, the relationship between the maximum pull out force \( V_s \) and the maximum shearing resistance forces \( T_s \) is as follows:

\[
V_s = 2T_s = 2P \tan \lambda_s
\]  

(2.2)

where:

\( P = \) the total force of the horizontal earth pressure

\( \lambda_s = \) the angle of friction of the stainless steel plate

In the case of rough plate, the relationship between the maximum pull out force \( V_r \) and the maximum shearing resistance force \( T_r \) take the following form:

\[
V_r = 2T_r = 2P \tan \lambda_r
\]

(2.3)

where:

\( P = \) the total force of the horizontal earth pressure

\( \lambda_r = \) the angle of friction of the plate roughened with sand

From equations (2.2) and (2.3), we obtained

\[
\frac{V_s}{V_r} = \frac{T_s}{T_r} = \frac{\tan \lambda_s}{\tan \lambda_r}
\]

(2.4)

The values of \( V_s \) and \( V_r \) can be obtained from the experimental results in Figure (2.6) as follows:

\[
\frac{V_s}{V_r} = \frac{0.17 \gamma BH^2}{0.72 \gamma BH^2} = 0.133
\]

(2.5)
Figure 2.5 Relationship between normalized pull-out force and normalized displacement for rough and smooth plates. (After Rabbaa [1981])

- $\gamma = 174.4 \text{ kN/m}^3$
- $\theta = 41^\circ$
- $H = 38 \text{ cm}$
- $B = 36 \text{ cm}$
\[
\frac{\tan \lambda}{\tan \nu} \quad (2.6)
\]

(b) **Influence of the Side Friction on the Test Results**

Series of tests were carried out by Rabbah (1981) in the same test tank to investigate the effect of the friction of the interior walls of the test tank on the ad settlement relationship and the ultimate load. Three different series of tests were performed. The first was carried out with both interior walls lined with sheets of stainless steel. The second was carried out with the two walls lined with sheets roughened by sand. The third was carried out with one wall lined with a polished stainless steel sheet and the other lined with a rough sheet.

The results of these tests are shown in Figure 2.7. From these results and those of the pull out test mentioned before, it was found that the effect of the angle of friction of the stainless steel was to increase the ultimate load by 4% see Appendix A1. It could also be noted that the side friction had a significant influence on the settlement of the footing if the load level is beyond the working load limit.

### 2.2.3 Modelling of the Footing

The footing used in all experiments performed in this thesis was a rectangular aluminum plate of dimensions 37.8 cm. \( \times \) 15.2 cm. \( \times \) 5.1 cm. The bottom surface of the plate was lined with a sheet of polished No 16 gauge of stainless steel in order that the plate simulated a footing with a smooth surface. In the case of rough surface of the footing, the bottom surface of the plate was lined with a roughened sheet.

The rigidity of the plate was checked by estimating the deflection of the plate.
FIGURE 2: EFFECT OF ROUGHNESS OF THE SIDE WALLS OF THE TEST TANK ON LOAD-DISPLACEMENT CURVES (AFTER RABBAA 1981)
during application of the maximum pressure, causing failure of sand. Calculations were made to estimate the relative rigidity of the plate as defined by Borowicka (1939). These calculations are given in Appendix (B). Also, dial gauges were placed at different locations on the surface of the plate during the test in order to check the flexural deflection of the plate (see Plate 2). Results of both experimental measurements and theoretical calculations indicate that the plate can be considered rigid.

2.2.4 Measurement of Displacements

The vertical displacements of the test plate were measured by means of four dial gauges with an accuracy of 0.001 cm., located at each corner of the plate. The readings of the gauges indicate that the plate was settling uniformly in the case of central loading conditions. The settlement of the plate was taken as the average of the four readings. On the other hand, in the case of eccentric load conditions, the average of each pair of dial gauges were considered to represent the settlement on each side of the plate.

2.2.5 Loading Devices

As illustrated in Figure (2.4), the loading rig consists of a steel frame, a hydraulic jack and a loading cell. The steel frame, which was designed and fabricated at the Civil Engineering laboratories. Its outside dimensions are approximately 155 cm. × 110 cm. The frame had two columns consisting of two W105 × 19 I-sections which were connected with two horizontal beams; one on the top and the other in the middle. The beam at the top was a MC 152 × 24 channel and the one in the middle was a C 229 × 22 channel. The two columns of the frame were bolted to MC 152 × 24 channels which were welded together to form the base of the frame.
Despite the fact that the frame was able to stand firmly under its own weight and the weight of the test tank. To provided extra support, the frame was anchored to the floor by means of two inclined struts. Each strut consisted of a C 76 × 6 channel that was welded to a steel plate anchored firmly to the floor as shown in Figure (2.4). In general the frame was designed to safely transmit the maximum vertical reaction of the hydraulic jack.

The load was applied by means of a manually operated pump and a hydraulic jack. The upper end of the hydraulic jack was attached to the MC 152 × 24 channel at the top of the frame, and the lower end was connected to a steel shaft of 7.8 cm. diameter. Throughout the experiments, the shaft was kept vertically aligned by two precision ball bushings enclosed in two pillow blocks. These pillow blocks were mounted on a C 229 × 22 channel positioned over the plate and the tank by means of a steel frame as shown in Figure (2.4).

The steel shaft was attached rigidly to a 45 kN capacity load cell by a threaded connection. This connection was designed to minimise both tilting of the test plate and horizontal deflection. The purpose of the load cell was to measure the load increment applied directly to the rigid test plate by the hydraulic jack. The system was capable of measuring a minimum applied load of 0.01 kN. The design of the hydraulic load application system was not capable of unloading in an incremental fashion. (It was not possible to reverse the flow of the hydraulic fluid without completely releasing the incipient pressure and the corresponding load). Nevertheless, it was possible to maintain a high initial load of about 25 kN without appreciable loss of load.

The entire loading apparatus was designed to ensure that the load could be applied vertically with or without eccentricity with the axis of the footing so that no horizontal displacement of the footing would occur.
2.3 Materials Used in the Experimental Program

2.3.1 General Properties of Ottawa Sand

The material used in this study was flint grade sand supplied by the Ottawa Silica Company, Ottawa, Illinois. The flint grade sand (Ottawa sand) was a medium white quartz sand with rounded to subrounded particles. The sand was poorly graded as indicated by a uniformity coefficient of 1.4, and the grain size distribution shown in Figure (2.8). By using the method outlined by Bowles (1978), it was found that the minimum and the maximum densities were 15.0 \( kN/m^3 \) and 17.8 \( kN/m^3 \) respectively.

2.3.2 Method of Placement of Sand in Test Tank

It is known that both settlement and bearing capacity are highly dependent on the density of the soil. Since investigation of the effect of some other variables was of concern in this study, it was necessary to keep the density of the sand constant throughout the entire tests. This was done by using two specific techniques for placement of the sand in the test tank; the first technique gives a loose sand and the second gives a dense sand.

In the first technique which involved pouring the sand into the test tank from a constant height, it was possible to control the rate of deposition. In order to determine the density of the deposited sand, containers, 6.2 cm. in diameter and 4.4 cm. in height were placed at different locations and levels inside the test tank during the pouring process of the sand. The densities obtained were almost the same, with a value of 1640 \( \pm \) 15 kg/m\(^3\). This gave a relative density \((D_r)\) of 44 \( \pm \)3%.

The second technique that was employed, has been used by several researchers:
FIGURE 28 GRAIN SIZE DISTRIBUTION OF OTTAWA SAND
Atefi (1975), Kempthorne (1978), Moore (1979) and Rabbaa (1981). This technique entailed raining the sand into the test tank at a pre-determined rate of deposition, using a sand hopper which traversed the tank at an almost a constant velocity (see Plate 3). Although it was possible to control the rate of deposition, it was not possible to move the manually operated hopper at constant a velocity. The rate of the deposition was controlled by allowing the sand to rain through a mesh of openings 0.16 cm. in diameter located at the base of the hopper. In order to determine the density of the deposited sand, containers, 6.2 cm in diameter and 4.4 cm in height were placed at different locations inside the test tank, during the raining process of the sand. It was found that the densities obtained were almost the same, with a value of 1775 ±10 kg/m³. In this case the relative density \(D_r\) was of 88 ±3%.

In this section, the technique used to construct the composite soil layer will be briefly outlined. Two stainless steel plates (of dimensions 38 cm. × 38 cm. × 0.1 cm.) were placed vertically at the distances located \(x = \pm \frac{5}{2}\) from the centre of the test tank (see Plate 4). The sand was then rained in the central region until it was completely filled which gave a layer of dense sand as mentioned above. In the other region, the sand was poured until it completely filled and this gives a layer of a loose sand as also mentioned above (see Figure (1.1)). After the tank was filled, the two plates were carefully removed.

To achieve a level surface of a sand deposit, a straight-edged metal ruler was used. During the levelling process, the metal ruler was supported against the straight edges of the test tank; this ensured a perfectly levelled surface. Great care was taken during the process of placing the rectangular aluminium plate on the surface of the sand layer.
2.3.3. Shear Strength and Deformability Characteristics of Ottawa Sand

A series of triaxial compression tests were carried out by Rabbaa (1981) in order to determine the angle of internal friction $\phi$ and the stress-strain properties of Ottawa sand (dense state). The samples to be used in the triaxial compression test were prepared by following procedures outlined by Bishop and Henkel (1962) and Bowles (1978). Seven tests were performed using different values of confining pressure ($\sigma_3$). All the tests were performed on specimens having almost the same relative density of $D_r = 90\%$ which was slightly different from the value obtained in section 2.3.2. ($D_r = 88\%$). The samples were made by allowing the sand to rain into a mould using the same method as that outlined in section 2.3.2. (the case of dense sand).

The resulting stress-strain curves are shown in Figure 2.9. The angle of internal friction $\phi$ was determined from the $p$ vs $q$ diagram illustrated in Figure 2.10. The value of $\phi$, as obtained by using the line of best fit, was approximately 41°.

Another series of triaxial compression tests were carried out to determine the angle of internal friction of Ottawa sand in its loose state. The samples to be used in the triaxial compression test were prepared by following the procedure outlined in section 2.3.2 (the case of the loose sand). The resulting Mohr circles at failure are shown in Figure 2.11. The angle of internal friction $\phi$ was determined from the $p$ vs $q$ diagram illustrated in Figure 2.12. The value of $\phi$, as obtained by using the line of best fit, was approximately 30°.

Researchers have shown that the angle of internal friction, as determined from an axially symmetrical triaxial compression test, is several degrees less than that determined from a plane strain test (Bishop, 1961; Cornforth, 1964; Lee, 1970 and others). According to Hansen (1970), Bishop (1961) and Lee (1970), the following
FIGURE 2.9 STRESS-STRAIN RELATIONSHIP OF OTTAWA SAND AT DIFFERENT CONFINING PRESSURES.

(AFTER RABBAH, 1981)
\[ \sin \phi = \tan \psi = 0.66 \]
\[ \gamma = 17.3 \text{ kN/m}^3 \]
\[ P = \frac{\gamma}{\gamma} (\sigma_1 - \sigma_3) \]
\[ q = \frac{1}{2} (\sigma_1 - \sigma_3) \]

**FIGURE 2.10** \( p-q \) FAILURE ENVELOPE OF OTTAWA SAND

(AFTER RABBAA 1981)
FIGURE 2.11  THE MOHR CIRCLES OF OTTAWA SAND FROM DRAINED TRIAXIAL COMPRESSION TESTS

Dr. = 44%
γ = 16 kN/m²
\[ \begin{align*}
\gamma & = 16 \text{ kN/m}^3 \\
p & = \frac{1}{2} (\sigma_1 - \sigma_3) \\
\sigma & = \frac{1}{2} (\sigma_1 + \sigma_3)
\end{align*} \]

**Figure 2.12** P-q Failure Envelope of Ottawa Sand
formula can be used to determine the plane strain equivalent of the angle of internal friction determined from triaxial tests, i.e.

\[ \phi_{\text{plane strain}} = 1.1 \phi_{\text{triaxial}} \]  \hspace{1cm} (2.7)

\[ \phi_{\text{plane strain}} = 1.1 \times 41 = 45^\circ \]

\[ \phi_{\text{plane strain}} = 1.1 \times 30 = 33^\circ \]

Rabbaa (1981) performed a series of tests which included the plane strain assumption to get the angle of internal friction of Ottawa sand. The basic test consisted of the passive action of a retaining wall with a smooth vertical back. The retaining wall supported a layer of Ottawa sand with horizontal surface. Two series of experiments were carried out using two heights of Ottawa sand \( H \) \( (H = 20 \text{ and } 38 \text{ cm.}) \). For each height, the test was repeated three times. By using Rankine's equation (1857), Rabbaa (1981) found that the angle of internal friction under plane strain condition was approximately equal to 45° (equal the same value obtained using Hansen's equation (2.7)).
2.4 Experimental Investigation

It is generally agreed that the soil-structure interaction is a very complicated engineering problem. A comprehensive experimental or theoretical examination of the variables that govern the behaviour of these soil-structure interaction would be a formidable task.

The flow chart given in Figure (2.13) presents an outline of the important components of the testing program carried out in this research. As can be seen in this figure, five primary variables and six secondary variables were investigated.

The primary variables that influence the foundation behaviour and were considered in this investigation, are the following:

1) Layer thickness ($H$).
2) Interface conditions.
3) Sand density.
4) Composite layer of sand.
5) Central and eccentric line load.

The secondary variables can be summarised as follows:

1) Ratio between the layer thickness to the footing width ($\frac{H}{B} = 2.5, \frac{H}{B} = 1$).
2) Interface conditions between the footing and the sand layer and between the sand layer and the rigid base (rough, smooth).
3) Density of the sand layer ($\gamma = 17.4 \text{ kN/m}^3, \gamma = 16.1 \text{ kN/m}^3$).
4) Central and eccentric line load ($e = 0, e = 1.5 \text{ cm}$).
5) Ratio between the width of the dense region to the width of the footing ($\frac{W}{B} = 0, 1.2, 2, 3, 4, 5, 6, 7 \text{ and } \infty$).
6) Loading and unloading cycles.

Also the schedule of experimental program is given in Table 2.1.
Figure 2.13 Flowchart of Research Plan
<table>
<thead>
<tr>
<th>Layer Type</th>
<th>Layer Thickness</th>
<th>Interface Conditions</th>
<th>Load Eccentricity</th>
<th>Width of the Dense Region</th>
<th>Cyclic Loading and Unloading Cycles</th>
<th>No. of Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense Sand Layer</td>
<td>H = 15.2 cm</td>
<td>Rough</td>
<td>e = 15 cm</td>
<td>15 x 22</td>
<td>70</td>
<td>12</td>
</tr>
<tr>
<td>Loose Sand Layer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
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<tr>
<td>Composite Sand Layer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>78</td>
</tr>
</tbody>
</table>

Total No. of Tests = 44 x 3 = 132

* No of Repeated Tests

Tests Completed

Tests Completed
Chapter 3

INFLUENCE OF RIGID BASE AND LOADING
AND UNLOADING CYCLING

3.1 Influence of Rigid Base

The review of the literature, mentioned in chapter 1, concerning the settlement and the bearing capacity of foundation located on the surface of a layer of soil resting on a rigid base, indicates that behaviour depends on many factors. The ratio of the layer thickness (H) to the foundation width (B), load eccentricity and the roughness or the smoothness of the rigid base are among the important factors. Other factors such as the foundation shape and shear strength parameters of the soil, also have significant influence on the behaviour.

The main purpose of this section is to present the results of the experimental investigations of the effect of the rigid base on the settlement of a rigid strip foundation. Three main variables have been investigated; the ratio of the layer thickness to the foundation width (\( \frac{H}{B} \)), central line load, eccentric line load with eccentricity equal to 10% of the width of the footing (\( e = 1.5 \text{ cm.} \)) and the interface conditions between the footing and the dense sand layer and between the dense sand layer and the rigid base.

Eight different problems were investigated, a summary of the investigated cases are shown in Table 3.1.
3.1.1 Influence of Rigid Base on the Settlement

The load-settlement curves of the first four problems mentioned in Table 3.1 are shown in Figure (3.1). As can be seen from the Figure (3.1), at a given load, the settlement decreases as the thickness of the sand layer (expressed by the ratio \( \frac{H}{D} \)) decreases. The settlement also decreases when the interface conditions are changed from smooth to rough. But the effect of the interface conditions in the case of \( \frac{H}{D} = 1 \) is greater than in the case of \( \frac{H}{D} = 2.5 \), also this means that the interface conditions have only a slight effect on the settlement in the case of \( \frac{H}{D} = 2.5 \).

3.1.2 Effect of Rigid Base and Load Eccentricity on the Settlement

Figures (3.2.a) and (3.2.b) show the eight problems mentioned in Table 3.1. In the case of deep layer (\( \frac{H}{D} = 2.5 \)), the load eccentricity has a slight effect on the settlement under the center of the footing in both cases of interface conditions (rough or smooth) as shown in Figure (3.2.a). The same effect of the load eccentricity was observed for the case of shallow layer (\( \frac{H}{D} = 1 \)) with rough interface conditions. However, in the case of shallow layer (\( \frac{H}{D} = 1 \)) with smooth interface conditions, the load eccentricity has a significant effect on the settlement under the center of the footing as shown in Figure (3.2.b).

The effect of the eccentricity and the rigid base on the load-settlement curves are shown in Figure (3.3). At a given load, the settlement under the centre of the footing decreases as the thickness of the dense sand layer decreases. The settlement under the centre of the footing decreases when the interface conditions are changed from smooth to rough. In the case of deep layer (\( \frac{H}{D} = 2.5 \)), the interface conditions have only a slight effect on the settlement (under the centre of the footing). Figure (3.3) illustrates that, in the case of shallow layer (\( \frac{H}{D} = 1 \)), the interface conditions
have a significant effect on the settlement. The failure occurs in the case of smooth interface conditions before the applied load reaches the same value as in the case of rough interface conditions as shown in Figure (3.3).

3.1.3 Influence of Rigid Base on the Bearing Capacity

The main purpose of this section is to present the results of the experimental investigations of the effect of the rigid base on the ultimate bearing capacity of a rigid strip foundation subjected to central line load. Two main variables were investigated, the ratio of the layer thickness to the footing width \( \frac{H}{B} \), and the interface conditions between the strip footing and the dense sand layer and between the dense sand layer and the rigid base.

Four different problems were investigated as shown in Figure (3.4). In the four cases, the load was applied until failure occurred. The load-settlement curves for the four different problems are shown in Figure (3.4). All load-settlement curves show a definite peak, or failure point which indicates the ultimate load. It may be noted that the ratio of the layer thickness to the footing width \( \frac{H}{B} \) has a significant influence on the ultimate bearing capacity. It is also important to note that the roughness or smoothness of the interface conditions between the footing and the dense sand layer and between the dense sand layer and the rigid base have an influence on the bearing capacity of the footing.

As can be observed from Figure (3.4), for smooth interface conditions, the ultimate load decreases as the ratio \( \frac{H}{B} \) decreases. A decrease of the ratio \( \frac{H}{B} \) from 2.5 to 1.0 resulted in a decrease in the ultimate load in the order of 30%. However, in the case of rough interface conditions, the ultimate load increases as the ratio \( \frac{H}{B} \) decreases. A decrease of the ratio \( \frac{H}{B} \) from 2.5 to 1.0 resulted in an increase in the ultimate load in the order of 65%. In the case of a shallow layer \( \frac{H}{B} = \)
Figure 3.24: Effect of load eccentricity and interface conditions on load-settlement relationship for a rigid strip footing resting on a layer of sand.
FIGURE 3.2b EFFECT OF LOAD ECCENTRICITY AND INTERFACE CONDITIONS ON LOAD-SETTLEMENT RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.
FIGURE 3.3 EFFECT OF LOAD ECCENTRICITY, INTERFACE CONDITIONS AND LAYER THICKNESS ON LOAD-SETTLEMENT RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.
have a significant effect on the settlement. The failure occurs in the case of smooth interface conditions before the applied load reaches the same value as in the case of rough interface conditions as shown in Figure (3.3).

3.1.3 Influence of Rigid Base on the Bearing Capacity

The main purpose of this section is to present the results of the experimental investigations of the effect of the rigid base on the ultimate bearing capacity of a rigid strip foundation subjected to central line load. Two main variables were investigated, the ratio of the layer thickness to the footing width \((\frac{h}{B})\), and the interface conditions between the strip footing and the dense sand layer and between the dense sand layer and the rigid base.

Four different problems were investigated as shown in Figure (3.4). In the four cases, the load was applied until failure occurred. The load-settlement curves for the four different problems are shown in Figure (3.4). All load-settlement curves show a definite peak, or failure point which indicates the ultimate load. It may be noted that the ratio of the layer thickness to the footing width \((\frac{h}{B})\) has a significant influence on the ultimate bearing capacity. It is also important to note that the roughness or smoothness of the interface conditions between the footing and the dense sand layer and between the dense sand layer and the rigid base have a influence on the bearing capacity of the footing.

As can be observed from Figure (3.4), for smooth interface conditions, the ultimate load decreases as the ratio \((\frac{h}{B})\) decreases. A decrease of the ratio \((\frac{h}{B})\) from 2.5 to 1.0 resulted in a decrease in the ultimate load in the order of 30%. However, in the case of rough interface conditions, the ultimate load increases as the ratio \((\frac{h}{B})\) decreases. A decrease of the ratio \((\frac{h}{B})\) from 2.5 to 1.0 resulted in an increase in the ultimate load in the order of 65%. In the case of a shallow layer \((\frac{h}{B} =\)
FIGURE 3 - EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON LOAD-SETLEMENT RELATIONSHIP
changing the interface conditions from smooth to rough resulted in an increase of the ultimate load in the order of about 185%. In the case of a deep layer \( \frac{h}{b} = 2.5 \), changing the interface conditions from smooth to rough resulted in a increase in the ultimate load in the order of about 10%.

These results confirm the theoretical findings by Mandel and Saleencon (1972) and the experimental findings by Rabbaa (1981).
tre of the footing) relationship in the re-loading cycle exhibited a shape of a curve until the tenth cycle in the case of smooth interface conditions. For rough interface conditions the load–displacement (under the centre of the footing) relationship of the re-loading cycle exhibited a shape of a curve until the fourth cycle. Starting from the fifth cycle the relationship exhibited a straight line shape. For relatively shallow layer ($\frac{h}{D} = 1$) with rough interface conditions, the load–displacement relationship in the re-loading cycle exhibited a straight line shape starting from the first cycle as shown in Figure (3.5.f).

In all of the tests performed, changing the interface conditions from smooth to rough caused the total and inelastic displacement to be smaller and the hysteresis loop to be narrower and the slopes of the load–displacement curves in the re-loading cycle to be higher. Also, most of the inelastic deformation occurred in the first cycle of loading and unloading, and increased with additional cycles at a decreasing rate. The width of the hysteresis loop was also relatively large in the first cycle and decreases with subsequent cycles.

The relationship between the total and inelastic displacements and the number of cycles for the seven problem above are shown in Figure (3.6.a) to Figure (3.6.g). Regardless of the layer thickness or the interface conditions or load eccentricity, the change in the inelastic displacement with number of cycles is almost the same as that of the total displacement, which implies that the elastic displacement is constant with the number of cycles.

Figure (3.7.a) and Figure (3.7.b) show the percentage increase in the total displacement at the first cycle with the increase in the cycle number. As can be seen from these figures, the displacements continued increasing with increase of cycles at a decreasing rate with an indication that this increase will not stop until a few cycles, after the tenth cycles already performed. From Figure (3.7.a), it can be
3.2.1 Influence of Load Cycling on the Total and Inelastic Displacements

The loading and unloading settlement curves for seven different problems are shown in Figure (3.5.a) to Figure (3.5.g). The first and the second problems concern a rigid strip footing resting on a layer of dense sand \( \frac{H}{D} = 2.5 \) and \( \frac{H}{D} = 1 \) and the footing is subjected to a line load acting at the centre line of the footing. The interfaces between the footing and the sand layer and between the sand layer and the rigid base were smooth. The third and the fourth groups of experiments are similar to the first and the second respectively, with rough interface conditions. The fifth and the sixth groups of experiments concern a rigid strip footing resting on a layer of a dense sand of thickness ratios \( \frac{H}{D} = 2.5 \) and \( \frac{H}{D} = 1 \). The footing is subjected to an eccentric line load and the eccentricity was 10% of the width of the footing \( B \). The interface conditions between the footing and the sand layer and between the sand layer and the rigid base were rough. The seventh problem is similar to the fifth problem but with smooth interface conditions.

As can be seen from Figure (3.5.a) to Figure (3.5.d), the layer thickness and the interface conditions have an influence on the load–displacement relationship in the reloading cycle. For the deep layer \( \frac{H}{D} = 2.5 \), the load–displacement relationship in the reloading cycle was in shape of a curve until the fifth cycle. Starting from the sixth cycle the relationship exhibited a straight line shape for both rough and smooth interface conditions. For a relatively shallow layer \( \frac{H}{D} = 1 \), the load–displacement relationship exhibited a straight line shape starting from the second cycle for the case of smooth interface conditions. However, in the case of rough interface conditions, the load–displacement relationship in the reloading cycle exhibited a straight line shape starting from the first cycle. Referring to Figure (3.5.e) and Figure (3.5.g), for deep layer \( \frac{H}{D} = 2.5 \), the load–displacement (under the cen-
FIGURE 35g  EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT
RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.
FIGURE 3.5b EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.
FIGURE 3.5c EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.
FIGURE 3.5.d EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.
FIGURE 3.5: EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.
FIGURE 3.5.1 EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.

- $e = 0.1$
- $e = 1.5$ cm
- $B = 15.2$ cm
- $H = 15.2$ cm
- $H / B = 1$
- $\gamma = 17.4$ kN/m$^3$
- $\phi = 41^\circ$
- $\sigma_a = \frac{\delta_1 + \delta_2}{2}$

Range of 3 Tests
Figure 3.5-g: Effect of load cycling on load-settlement relationship for a rigid strip footing resting on a layer of sand.
FIGURE 35c EFFECT OF LOAD CYCLING ON LOAD-SETTLEMENT RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.
tre of the footing) relationship in the reloading cycle exhibited a shape of a curve until the tenth cycle in the case of smooth interface conditions. For rough interface conditions the load–displacement (under the centre of the footing) relationship of the reloading cycle exhibited a shape of a curve until the fourth cycle. Starting from the fifth cycle the relationship exhibited a straight line shape. For relatively shallow layer ($\frac{H}{b} = 1$) with rough interface conditions, the load–displacement relationship in the reloading cycle exhibited a straight line shape starting from the first cycle as shown in Figure (3.5.f).

In all of the tests performed, changing the interface conditions from smooth to rough caused the total and inelastic displacement to be smaller and the hysteresis loop to be narrower and the slopes of the load–displacement curves in the reloading cycle to be higher. Also, most of the inelastic deformation occurred in the first cycle of loading and unloading, and increased with additional cycles at a decreasing rate. The width of the hysteresis loop was also relatively large in the first cycle and decreases with subsequent cycles.

The relationship between the total and inelastic displacements and the number of cycles for the seven problem above are shown in Figure (3.6.a) to Figure (3.6.g). Regardless of the layer thickness or the interface conditions or load eccentricity, the change in the inelastic displacement with number of cycles is almost the same as that of the total displacement, which implies that the elastic displacement is constant with the number of cycles.

Figure (3.7.a) and Figure (3.7.b) show the percentage increase in the total displacement at the first cycle with the increase in the cycle number. As can be seen from these figures, the displacements continued increasing with increase of cycles at a decreasing rate with an indication that this increase will not stop until a few cycles, after the tenth cycles already performed. From Figure (3.7.a), it can be
Figure 3.7b: Increase in total displacement with increase in number of cycles.
FIGURE 3.6a RELATIONSHIP BETWEEN TOTAL AND INELASTIC DISPLACEMENTS AND NUMBER OF CYCLES
FIGURE 3.6 b RELATIONSHIP BETWEEN TOTAL AND INELASTIC DISPLACEMENTS AND NUMBER OF CYCLES
FIGURE 3.6.3 RELATIONSHIP BETWEEN TOTAL AND INELASTIC DISPLACEMENTS AND NUMBER OF CYCLES
FIGURE 3.6.1 RELATIONSHIP BETWEEN TOTAL AND INELASTIC DISPLACEMENTS AND NUMBER OF CYCLES.
FIGURE 3.6.9 RELATIONSHIP BETWEEN TOTAL AND INELASTIC DISPLACEMENTS AND NUMBER OF CYCLES.
**FIGURE 3.7.1** INCREASE IN TOTAL DISPLACEMENT WITH INCREASE IN NUMBER OF CYCLES.
FIGURE 3.7: Increase in Total Displacement with Increase in Number of Cycles.
noted that the percentage of increase in total displacement depends on the interface conditions but the load eccentricity seems to have a slight effect in the case of deep layer \( \frac{h}{B} = 2.5 \). For a relatively shallow layer \( \frac{h}{B} = 1 \), it can be seen from Figure (3.7.b) that the percentage of increase in total displacement depends on both interface conditions and load eccentricity. Also, it can be seen from Figure (3.7.a) and Figure (3.7.b) that the percentage of increase in total displacements depends on the layer thickness.

3.2.2 Influence of Load Cycling on the Slopes of Load–Displacement Curves

Figure (3.8.a) and Figure (3.8.b) represent the relationship between the slopes of the load–displacement curves in the reloading cycle and the number of cycles for the seven different problems mentioned in the previous section. The slope of the load–displacement curve in the reloading cycle increases with the increase of number of cycles regardless to the layer thickness, the interface conditions or load eccentricity. This increase was significant until about the fifth cycle and after that the rate of increase decreased noticeably.

Also, it can be seen from these figures, the slope of the load–displacement curve in the reloading cycle depends on the layer thickness. In the case of the deep layer \( \frac{h}{B} = 2.5 \), as shown in Figure (3.8.a), it can be noted that the load eccentricity and the interface conditions have only a slight effect on the slope of the load–displacement curve in the reloading cycle. In the case of shallow layer \( \frac{h}{B} = 1 \), it can be seen from Figure (3.8.b), the slope of the load–displacement curve in the reloading cycle depends on the load eccentricity, but the interface conditions seem to have a slight effect.
FIGURE 3.8.a. EFFECT OF NUMBER OF CYCLES ON SLOPE OF LOAD-DISPLACEMENT CURVE.
\( P \times 10^3 \)

\( \gamma B^2 \)

\( B = 15.2 \text{ cm} \)
\( H = 15.2 \text{ cm} \)
\( \gamma = 174 \text{ kN/m}^3 \)
\( H/B = 1 \)
\( \phi = 41^\circ \)

- Rough Interfaces, \( e/B = 0.1 \)
- Rough Interfaces, \( e/B = 0 \)
- Smooth Interfaces, \( e/B = 0 \)

No of Cycles

FIGURE 3.8b EFFECT OF NUMBER OF CYCLES ON SLOPE OF LOAD-DISPLACEMENT CURVE
3.2.3 Influence of Load Cycling on the Slopes of Unloading

Displacement Curves

Although the paths of the unload-displacement curves in all tests were not measured and consequently the exact slope of these curves were not known, reasonable values of these slopes could be predicted by assuming that these curves were straight lines as shown by dashed lines in Figure (3.5.a) to Figure (3.5.b).

This assumption is based on the fact that all load-displacement curves in the reloading cycle are almost straight lines at the first cycle and exactly straight lines during the rest of cycles. It was assumed that this straight line relationship held for the unloading-displacement curve and its slope was calculated. The relationship between this slope and the number of cycles is shown in Figure (3.9.a) and Figure (3.9.b). It can be noted that irrespective of the layer thickness, the interface conditions or the load eccentricity after the third cycle, the number of cycles have no influence on the slope of the unload-displacement.

3.2.4 Influence of Load Cycling on the Footing Rotation

The loading and unloading curves for three different problems are shown in Figure (3.10.a) to Figure (3.10.c). These three problems, as mentioned in the previous section, are as follows. The first and the second concern a rigid strip footing resting on a layer of dense sand of thickness ratio $\frac{h_1}{h_0} = 2.5$ and $\frac{h_1}{h_0} = 1$ and subjected to eccentric line load with eccentricity equal to 10% of the width of the footing ($e = 1.5$ cm.). The interface conditions between the foundation and the sand layer and between the sand layer and the rigid base were rough. The third problem is similar to the first problem but with smooth interface conditions. As can be seen from the Figure (3.10.a) and Figure (3.10.b), the ratio between the layer thickness...
Figure 39a: Effect of number of cycles on slope of unload-displacement curve.
FIGURE 3.10 a EFFECT OF LOAD CYCLING ON LOAD-FOOTING
ROTATION (α) RELATIONSHIP FOR A RIGID
STRIP FOOTING RESTING ON A LAYER OF SAND

- B = 15.2 cm
- e = 1.5 cm
- H/B = 2.5
- γ = 17.4 kN/m³
- φ = 41°
- α = δ₁ - δ₂
- α = α x 10°

Range of 3 Tests
FIGURE 3.10.5 EFFECT OF LOAD CYCLING ON LOAD-FOOTING
ROTATION (\( \alpha \)) RELATIONSHIP FOR A RIGID
STRIP FOOTING RESTING ON A LAYER OF SAND.
FIGURE 3-10c EFFECT OF LOAD CYCLING ON LOAD-FOOTING ROTATION ($\alpha$) RELATIONSHIP FOR A RIGID STRIP FOOTING RESTING ON A LAYER OF SAND.

- $P_{fe}$: Rigid Footing
- $\phi$: 41°
- $\gamma$: 17.4 kN/m$^3$
- $B$: 15.2 cm
- $e$: 1.5 cm
- $H/B$: 2.5
- $\alpha$: $\frac{\delta_1 - \delta_2}{B}$
- $\alpha = \bar{\alpha} \times 10^4$
- Range of 3 Tests
to the footing width \((\frac{H}{B})\) has an influence on load-foothing rotation \((\alpha)\) relationship in the reloading cycle. For the deep layer \((\frac{H}{B} = 2.5)\), the relationship was in shape of a curve until the fifth cycle. Starting from the sixth cycle the relationship exhibited a straight line shape for both cases of smooth and rough interface conditions.

For relatively shallow layer \((\frac{H}{B} = 1)\), the load-foothing rotation \((\alpha)\) relationship in the reloading cycle exhibited a straight line shape starting from the second cycle as shown in Figure (3.10.b). Changing the interface conditions from smooth to rough decreased the total and inelastic footing rotation. In all of the tests performed, most of the inelastic footing rotation \((\alpha)\) occurred in the first cycle of loading and unloading, and increased with additional cycles at a decreasing rate. The width of the hysteresis loop in the first cycle is large compared to the subsequent cycles.

The relationship between the total and inelastic footing rotation and the number of cycles for the three problems mentioned above are shown in Figure (3.11.a) to Figure (3.11.c). For the three cases, the difference between the total and inelastic footing rotation is constant with the number of cycles. This means that the elastic footing rotation is constant with the number of cycles.

Figure (3.12) shows the percentage increase in the total footing rotation at first cycle with the increase in the cycles numbers. As can be seen from this figure, the total footing rotation \((\alpha)\) continued increasing with increase of cycles with an indication that this increase will not stop until a few cycles. The percentage increase in the total footing rotation depends on the layer thickness and the interface conditions as shown in Figure (3.12).
FIGURE 3.11a RELATIONSHIP BETWEEN TOTAL AND INELASTIC FOOTING ROTATION ($\alpha$) AND NUMBER OF CYCLES
FIGURE 3.11b RELATIONSHIP BETWEEN TOTAL AND INELASTIC FOOTING ROTATION ($\alpha$) AND NUMBER OF CYCLES
FIGURE 3.12 INCREASE IN TOTAL FOOTING ROTATION ($\alpha$) WITH INCREASE IN NUMBER OF CYCLES
Chapter 4

SETTLEMENT AND BEARING CAPACITY OF A
FOUNDATION ON A COMPOSITE SOIL REGION

This section examines the load-settlement behaviour of a rigid strip footing resting on a composite soil region. The composite soil region consisted of a dense soil region which is contained between two loose soil regions as shown in Figure 4.1. For example, when a site is excavated and the sand replaced using the same techniques mentioned in Section 2.3.2 as a result of a loose sand layer will be found. If a part of the replaced sand with a width (S) under the footing is compacted; therefore, a composite sand layer will be formed as shown previously in Figure (1.1). The influence of the width of the dense region on the settlement and the bearing capacity characteristics of a rigid strip foundation will be discussed in this chapter.

Four series of tests (A, B, C and D) were performed in order to investigate the effect of the dense region width, layer thickness and the interface conditions between the footing and the soil and between the soil and the rigid base on the settlement and bearing capacity characteristics of a rigid strip foundation.

In series A and C the interface conditions between the footing and the soil and between the soil and the rigid base were smooth. The ratio of the layer thickness to the footing width (t/L) was equal to 2.5 and 1.0. In series B and D, the interface conditions between the footing and the soil and between the soil and the rigid base
were rough and the ratio of the layer thickness to the footing width \( \frac{t}{b} \) was equal to 2.5 and 1.0.

In each series, the dense region width \( S \) was set equal to 0B (the loose state), 1.2B, 2B, 3B, 4B, 5B, 6B, 7B and 10B (the dense state). In the four series mentioned above, the load was applied until failure occurred. Details of each series are given in section 2.4.

4.1 Influence of the Rigid Base and Interface Conditions on the Settlement

Typical load-settlement curves obtained from series A, B, C and D are shown in Figure 4.1.a to Figure 4.1.j respectively. As can be seen from the figures, at a given load, the settlement decreases as the thickness of the sand layer, expressed by the ratio \( \frac{t}{b} \), decreases. In the shallow layer \( \left( \frac{t}{b} = 1.0 \right) \), the interface conditions have a significant effect on the settlement as shown in Figure 4.1.a to Figure 4.1.j, the settlement decreases as the interface conditions change from smooth to rough. But in the case of deep layer \( \left( \frac{t}{b} = 2.5 \right) \), changing the interface conditions from smooth to rough and the range of \( \frac{t}{b} \) from 0 to 7B, the interface conditions have no effect on the settlement. In the case of \( \frac{t}{b} = 10 \) (dense layer), changing the interface conditions from smooth to rough, the settlement decreases but with a small value; it actually appear that the interface conditions have only a slight effect on the settlement. It is also important to note that the influence of the layer density (changing from 16.1 kN/m\(^3\) to 17.4 kN/m\(^3\)) on the settlement is much greater than influence of the interface conditions and the layer thickness as shown from Figure 4.1.a and Figure 4.1.j.
FIGURE 4: EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON THE LOAD-SETTLEMENT AND THE ULTIMATE LOAD CAPACITY OF A RIGID STRIP FOOTING RESTING ON A COMPOSITE LAYER OF SAND.
\[
\begin{align*}
\gamma_d &= 17.4 \text{ kN/m}^3 \\
\gamma_l &= 16.1 \text{ kN/m}^3 \\
\gamma_{av} &= \frac{\gamma_d + \gamma_l}{2} \\
B &= 15.2 \text{ cm} \\
H/B &= 1 \\
S/B &= 1.2
\end{align*}
\]

**Figure 4.15** Effect of layer thickness and interface conditions on the load-settlement and the ultimate load capacity of a rigid strip footing resting on a composite layer of sand.
FIGURE 2.5 EFFECT OF THE WIDTH OF THE DENSE REGION ON THE LOAD-SETTLEMENT CURVES AND THE BEARING CAPACITY OF A RIGID STRIP FOOTING.
\[
\begin{align*}
\gamma_d &= 17.4 \text{ kN/m}^3 \\
\gamma_i &= 16.1 \text{ kN/m}^3 \\
\gamma_{av} &= \frac{\gamma_d + \gamma_i}{2} \\
B &= 15.2 \text{ cm} \\
H/B &= 2.5
\end{align*}
\]

FIGURE 4: EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON THE LOAD-SETTLEMENT AND THE ULTIMATE LOAD CAPACITY OF A RIGID STRIP FOOTING RESTING ON A COMPOSITE LAYER OF SAND
FIGURE: EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON THE LOAD-SETTLEMENT AND THE ULTIMATE LOAD CAPACITY OF A RIGID STRIP FOOTING RESTING ON A COMPOSITE LAYER OF SAND.
Figure 4.11 Effect of layer thickness and interface conditions on the load-settlement and the ultimate load capacity of a rigid strip footing resting on a composite layer of sand.
Figure 4.3: Effect of layer thickness and interface conditions on the load-settlement and the ultimate load capacity of a rigid strip footing resting on a composite layer of sand.

\[
\frac{P}{\gamma_{av} B^3} = \frac{\gamma_d = 17.4 \text{ kN/m}^3}{\gamma_l = 16.1 \text{ kN/m}^3} \quad \gamma_{av} = \frac{\gamma_d + \gamma_l}{2}
\]

\[B = 152 \text{ cm}
\]

\[S/B = 5
\]
FIGURE: EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON THE LOAD-SETTLEMENT AND THE ULTIMATE LOAD CAPACITY OF A RIGID STRIP FOOTING RESTING ON A COMPOSITE LAYER OF SAND.
FIGURE 4.1: EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON THE LOAD-SETTLEMENT AND THE ULTIMATE LOAD CAPACITY OF A RIGID STRIP FOOTING RESTING ON A COMPOSITE LAYER OF SAND.
FIGURE 4: EFFECT OF LAYER THICKNESS AND INTERFACE CONDITIONS ON THE LOAD-SETTLEMENT AND THE ULTIMATE LOAD CAPACITY OF A RIGID STRIP FOOTING RESTING ON A COMPOSITE LAYER OF SAND
4.2 Influence of the Width of the Dense region on the Settlement

The load-settlement curves obtained from series A, C and D are shown in Figure 4.2.a to Figure 4.2.d respectively. As can be observed from the figures, the width of the dense region has a significant effect on the settlement. In all series of tests, at a given load, the settlement decreases, as the width of the dense region \( \frac{S}{B} \) expressed in the ratio \( \frac{S}{B} \), increases.

The relationship between the settlement ratio \( \frac{S}{B} \), the width of the dense region ratio \( \frac{S}{B} \) and the ratio between a load on a layer of composite sand to the ultimate load of a layer of a dense sand which has the same thickness \( h \), width and interface conditions \( \left( \frac{P}{P_{u}} \right) \) are shown in Figures 4.3. The three different problems investigated in series A, C and D are shown in Figure 4.3.a to Figure 4.3.c. Figure 4.3.a illustrates the case of the deep layer (\( \frac{S}{B} = 2.5 \)) and the interface conditions were smooth. The results indicate that the settlement decreases as the width of the dense region increases and the ratio \( \frac{P}{P_{u}} \) decreases. The effect of the loose region on the settlement decreases as the ratio \( \frac{P}{P_{u}} \) decreases. Also it can be seen from Figure 4.3.a that the effect of the loose region depends on \( \frac{S}{B} \) and \( \frac{P}{P_{u}} \). For example, the effect of the loose region on the settlement can be neglected when the ratio of \( \frac{S}{B} = 7 \) and \( \frac{P}{P_{u}} \) changing from 0.1 to 0.3 and the composite layer can be considered as a completely dense layer.

Figure 4.3.b and Figure 4.3.c illustrate the case of the shallow layer (\( \frac{S}{B} = 1 \)) and the interface conditions between the footing and the sand layer and between the sand layer and the rigid base were smooth and rough respectively. The results indicate that the settlement decreases as the width of the dense region expressed
Figure 4.2a: Effect of the width of the dense region on the load-settlement curves and the bearing capacity of a rigid strip footing.
Figure 2-5 Effect of the width of the dense region on the load-settlement curves and the bearing capacity of a rigid strip footing.
FIGURE 4.2-c EFFECT OF THE WIDTH OF THE DENSE REGION ON THE LOAD-SETTLEMENT CURVES AND THE BEARING CAPACITY OF A RIGID STRIP FOOTING

\[ \frac{P}{\gamma_{av}B^2} \]

- \( S/B = 4 \)
- \( S/B = 3 \)
- \( S/B = 2 \)
- \( S/B = 1.2 \)

\[ \gamma_d = 17.4 \text{ kN/m}^3 \]
\[ \gamma_l = 16.1 \text{ kN/m}^3 \]
\[ \gamma_{av} = \frac{\gamma_d \times \gamma_l}{2} \]
\[ B = 15.2 \text{ cm} \]
\[ H/B = 1 \]

Range of 3 Tests
Figure 4.2.3: Effect of the width of the dense region on the load-settlement curves and the bearing capacity of a rigid strip footing.

\[
\frac{P}{\gamma_{av} B^2}
\]

\[
S/B = 7.10, \infty
\]

\[
S/B = 6
\]

\[
S/B = 5
\]

Rough Interfaces

\[
\gamma_l = 16.1 \text{ kN/m}^3
\]

\[
\gamma_d = 17.4 \text{ kN/m}^3
\]

\[
\gamma_{av} = \frac{\gamma_d + \gamma_l}{2}
\]

\[
B = 15.2 \text{ cm}
\]

\[
H/B = 1
\]

Range of 3 Tests
\[ \gamma_d = 17.4 \text{ kN/m}^3 \quad \gamma_1 = 16.1 \text{ kN/m}^3 \]

\[ \frac{\delta}{B} = 16 \]

\[ \text{H/B} = 2.5 \quad \text{Smooth Interfaces} \]

\[ |P_{\text{comp}}| = 0.78 \quad \frac{|P|}{P_{\text{dense}}} = 0.78 \]

\[ \ldots = 0.70 \]
\[ \ldots = 0.61 \]
\[ \ldots = 0.52 \]
\[ \ldots = 0.43 \]
\[ \ldots = 0.35 \]
\[ \ldots = 0.26 \]
\[ \ldots = 0.17 \]
\[ \ldots = 0.09 \]

**FIGURE 4.3 a RELATIONSHIP BETWEEN THE RATIO OF THE WIDTH OF THE DENSE REGION AND THE SETTLEMENT RATIO OF A RIGID STRIP FOOTING.**
Figure 4.3 shows the relationship between the ratio of the width of the dense region and the settlement ratio of a rigid strip footing.
Figure 43: Relationship between the ratio of the width of the dense region and the settlement ratio of a rigid strip footing.
by \( \frac{q}{b} \) increases and the ratio \( \left| \frac{F}{P_{sate}} \right| \) decreases. The effect of the loose region on the settlement decreases as the ratio \( \left| \frac{F}{P_{sate}} \right| \) decreases. For example, the effect of the loose region on the settlement can be neglected and the composite layer can be considered as a completely dense layer when the ratio of \( \frac{q}{b} = 7 \) and \( \left| \frac{F}{P_{sate}} \right| \) changing from 0.1 to 0.7 and from 0.05 to 0.6 in the cases of smooth and rough interface conditions respectively.

It can be concluded that the safe zone under any applied load can be determined. The effect of the loose region can be neglected and the composite layer can be considered as a dense layer \( (\delta_{\text{composite}} = \delta_{\text{dense}}) \). These safe zones are shown in Figure 4.3.a to Figure 4.3.c by a shaded area.

One of the major conclusions of this investigation is the establishment of effective widths of the dense sand region in a composite layer at which the behaviour of the soil layer can be represented by the dense equivalent.

Further more, it was observed that other factors must be considered in addition to the width of the dense region. Among these factors are the interface conditions (in the shallow layer), the layer thickness \( (H) \), the angle of internal friction of the soil \( (\phi) \) and the density of the sand layer.
4.3 Influence of the Width of the Dense Region on the Bearing Capacity

In the four series of tests, A, B, C and D mentioned previously, the load was increased until failure occurred. The load-settlement curves for the four different problems investigated in series A, B, C and D are shown in Figure 4.2.a to Figure 4.2.d. All load-settlement curves show a definite peak which indicates the ultimate load (shear failure as shown in Plate 5). However, the cases of loose sand \( \frac{h}{B} = 0 \) and the deep layer of composite sand \( \frac{h}{B} = 1 \) and 2) do not show a definite peak which demonstrates a punching failure as shown in Plate 6. It is observed from these figures that the ultimate load expressed in the term of the ratio \( \frac{P}{B^t} \) increased as the ratio of the width of the dense region expressed by \( \frac{h}{B} \), increased. Also it can be seen from Figure 4.1.a to Figure 4.1.j, the ratio of the layer thickness to the footing width \( \frac{h}{B} \) has a significant influence on the ultimate bearing capacity. It can be also important to note that the influence of the roughness or smoothness of the interface conditions between the footing and the sand layer and between the sand layer and the rigid base on the bearing capacity is much greater than the layer thickness that in the case of the shallow layer \( \frac{h}{B} = 1.0 \). But in the case of the deep layer \( \frac{h}{B} = 2.5 \), the interface conditions have no significant effect on the bearing capacity.

The effect of the width of the dense region on the ultimate bearing capacity can be described by the term \( \left( \frac{P_{ult}}{P_{ult,comp}} \right) \) which is defined as the ratio between the ultimate load of a composite layer \( P_{ult,comp} \) to the ultimate load of a dense sand layer where the thickness \( (H) \), footing width \( (B) \) and interface conditions are kept the same. The values of \( (P_{ult})_{dense} \) were taken to be corresponding to a spacing
ratio $\frac{s}{B} = 10$ under which condition the layer acts as a dense layer. The bearing capacity ratio $\frac{|P_{ult}|_{comp}}{|P_{ult}|_{dense}}$ is dependent on the ratio $(\frac{s}{B})$, as well as the width of the dense region ratio $(\frac{s}{B})$ and the interface conditions.

The relationship between $\frac{|P_{ult}|_{comp}}{|P_{ult}|_{dense}}$ and the ratio of the width of the dense region $\frac{s}{B}$ is shown in Figure 4.4. It can be seen from Figure 4.4, the relationship between $\frac{|P_{ult}|_{comp}}{|P_{ult}|_{dense}}$ and the ratio $\frac{s}{B}$ has a straight line shape regardless to the interface conditions or the layer thickness. Also it can be observed from that figure, in the case of the shallow layer $(\frac{s}{B} = 1)$, the effect of the loose region on the ultimate bearing capacity vanish at $\frac{s}{B} = 7$ and 7.7 in the case of rough and smooth interface conditions respectively. In the case of deep layer $(\frac{s}{B} = 2.5)$, the effect of the loose region on the ultimate bearing capacity vanish at $\frac{s}{B} = 9.1$ in the case of smooth interface conditions as shown in Figure 4.4.

As can be observed from Figure 4.4, it is possible to get the ratio between $\frac{|P_{ult}|_{comp}}{|P_{ult}|_{dense}}$ at different ratio of $\frac{s}{B}$. If the value of $|P_{ult}|_{dense}$ is calculated by using the general formula of bearing capacity or taken from the experimental results then the value of $|P_{ult}|_{comp}$ can be determined at different $\frac{s}{B}$ ratio.

It is also noted from Figure 4.4 that the ratio of the layer thickness to the footing width $(\frac{H}{B})$, the ratio of the width of the dense region $(\frac{s}{B})$ and the interface conditions (in the shallow layer) have a significant effect on the ultimate load ratio $\frac{|P_{ult}|_{comp}}{|P_{ult}|_{dense}}$. But in the case of the deep layer $(\frac{s}{B} = 2.5)$, the interface conditions have no significant effect on the ultimate load ratio.

In conclusion, it can be said that the influence of the width of the dense region on the ultimate load depends on many factors rather than on the width of the dense region alone. Among these factors are the interface conditions (in the shallow layer), layer thickness $(H)$, the angle of internal friction of the sand $(\phi)$ and the density of the sand layer.
FIGURE 4.4 EFFECT OF LAYER THICKNESS, INTERFACE CONDITIONS AND DENSE REGION WIDTH ON THE RATIO $\frac{|P_{\text{ull}}|_{\text{comp}}}{|P_{\text{ull}}|_{\text{dense}}}$
Chapter 5

NUMERICAL SOLUTION OF STRIP FOUNDATION
BY USING NONLINEAR FINITE ELEMENT ANALYSIS

5.1 Introduction

The finite element method provides a powerful technique for analysis of stresses and displacements in earth masses, and it has already been applied to a number of practical problems including embankment dams, open excavations, and a variety of soil-structure interaction problems.

The stress-strain characteristics of soils and rocks are influenced by a number of factors such as water content, density, stress history, etc. These factors make the stress-strain behaviour extremely complex, and the behaviour of the soil is nonlinear, inelastic, and highly depended on the magnitudes of the stresses in soils or rocks.

Using the finite element method, it is possible to approximate nonlinear behaviour in stress analysis. There are several such techniques (Duncan and Chang, 1970, Desai and Abel, 1972, Duncan and Wong, 1974, Desai and Christian, 1977, and Duncan and Wong, 1980).

Duncan and Wong (1980) presented a nonlinear finite element analysis of stresses and displacements in the soil mass. The parameters employed to represent nonlinear and stress-dependent, stress-strain and volume change behaviour were:

1) Tangent value of Young's modulus \( (E_t) \) which vary with confining pressure and
the percentage of strength mobilised.

2) Values of bulk modulus (B) which vary with confining pressure and which are independent of the percentage of strength mobilised.

Subsequent studies (Duncan and Wong, 1980) have shown that the volume change behaviour of the most soils can be modelled with equal accuracy by assuming that the bulk modulus of the soil varies with confining pressure, and is independent of the percentage on stress mobilised.

5.2 Stress-Strain Relationship

Duncan and Chang (1970) have developed the hyperbolic stress-strain relationship for use in nonlinear incremental analysis of soil deformation. In each increment of such analysis the stress-strain behaviour of the soil is treated as being linear and the relationship between stress and strain is assumed to be governed by the generalised Hooke's law for elastic deformations. For plane strain conditions, the incremental stress-strain law may be expressed as follows:

\[
\begin{bmatrix}
\Delta \sigma_x \\
\Delta \sigma_y \\
\Delta \tau_{xy}
\end{bmatrix}
= \frac{3B}{9B - E}
\begin{bmatrix}
3B + E & 3B - E & 0 \\
3B - E & 3B + E & 0 \\
0 & 0 & E
\end{bmatrix}
\begin{bmatrix}
\Delta \epsilon_x \\
\Delta \epsilon_y \\
\Delta \gamma_{xy}
\end{bmatrix}
\]

(5.1)

where

\( \Delta \sigma_x, \Delta \sigma_y \) = normal stress increments in X and Y direction respectively

\( \Delta \tau_{xy} \) = shear stress increment

\( \Delta \epsilon_x, \Delta \epsilon_y \) = normal strain increments in X and Y direction respectively

\( \Delta \gamma_{xy} \) = shear strain increment

\( E \) = Young's modulus

\( B \) = bulk modulus
By varying the values of Young's modulus and bulk modulus appropriately as the stresses vary within the soil, it is possible using the simple equation (5.1) to model three important characteristics of the stress-strain behaviour of soils, namely, nonlinearity, stress-dependency, and inelasticity. The procedure used to account for these characteristics are described in the following sections.

5.3 Nonlinear Stress-Strain Relationship Represented by Hyperbolas

One of these techniques which widely used in analysis of nonlinear problems in geotechnical engineering is due to Duncan and Chang (1970) and Duncan and Wong (1980). They have used the hyperbolic function proposed by Kondner (1963) and Kondner and Zelasko (1963) for simulation of stress-strain curves in finite element analysis. Kondner (1963) and Kondner and Zelasko (1963) have shown that the nonlinear stress-strain curves of both sand and clay can be represented by hyperbola to a reasonable degree of accuracy. Figure 5.1.a illustrates such as relationship which can be stated in equation form:

\[(\sigma_1 - \sigma_2) = \frac{\varepsilon_a}{a + b\varepsilon_a} \quad (5.2)\]

where

- \(\sigma_1\) and \(\sigma_2\) are respectively the major and the minor principal stresses,
- \(\varepsilon_a\) is the axial strain and
- \(a\) and \(b\) are constants.

Refering to Figure 5.1.a, it is indicated that, at very small strains, \(E_i = \frac{1}{a}\) = initial Young's modulus; and \((\sigma_1 - \sigma_3)_{ult} = \frac{1}{b}\) = asymptotic values of stress difference which the stress-strain curves approach at infinite strains.
FIGURE 5.1a HYPERBOLIC STRESS-STRAIN CURVE

FIGURE 5.1b TRANSFORMED HYPERBOLIC STRESS-STRAIN CURVE
The values of $E_i$ and $(\sigma_1 - \sigma_3)_{ult}$ for a given stress-strain curve can be determined easily. If the hyperbolic equation is transformed as shown in Figure (5.1.b), it presents a linear relationship between $\frac{\varepsilon_a}{(\sigma_1 - \sigma_3)}$ and $\varepsilon_a$ of the form:

$$\frac{\varepsilon_a}{(\sigma_1 - \sigma_3)} = \frac{1}{E_i} + \frac{\varepsilon_a}{(\sigma_1 - \sigma_3)_{ult}}$$

(5.3)

Thus, to determine the best-fit hyperbola for the stress-strain curve, the values of $\frac{\varepsilon_a}{(\sigma_1 - \sigma_3)}$ are calculated from the test data and plotted against $\varepsilon_a$. The best-fit straight line on this transformed plot corresponds to the best-fit hyperbola on the stress-strain plot. Duncan and Chang (1970) have shown that with several hundred stress-strain curves for well over a hundred different soils indicate that a good match is usually achieved by selecting the straight line so that it passes through the points where 70% and 90% of the strength mobilised. Thus, in practice, only two points for each stress-strain curve (the 70% point and the 95% point) are plotted on the transformed diagram.

The variation of $E_i$ with $\sigma_3$ is presented by an equation which was suggested by Jambu (1963):

$$E_i = KP_a\left(\frac{\sigma_3}{P_a}\right)^n$$

(5.4)

where

$K = $ the modulus number

$n = $ the modulus exponent

$P_a = $ atmospheric pressure

The value of $K$ and $n$ are the same for any system of units, and the units of $E_i$ are the same as the units of $P_a$. The variation of $E_i$ with $\sigma_3$ corresponding to Equation (5.4) is shown in Figure 5.2.

The variation of $(\sigma_1 - \sigma_3)_{ult}$ with $\sigma_3$ is accounted by relating $(\sigma_1 - \sigma_3)_{ult}$ to the compressive strength or stress difference at failure, $(\sigma_1 - \sigma_3)_f$, and by using
the Mohr-Coulomb strength equation to relate \((\sigma_1 - \sigma_3)_f\) to \(\sigma_3\). The values of 
\((\sigma_1 - \sigma_3)_{ult}\) and \((\sigma_1 - \sigma_3)_f\) are related by:

\[
(\sigma_1 - \sigma_3)_f = R_f (\sigma_1 - \sigma_3)_{ult}
\]  

where

\(R_f\) = the failure ratio

Because \((\sigma_1 - \sigma_3)_f\) is always smaller than \((\sigma_1 - \sigma_3)_{ult}\), the value of \(R_f\) is always smaller than unity, and varies from 0.5 to 0.9 for most of soils (Duncan and Wong, 1980).

The variation of \((\sigma_1 - \sigma_3)_f\) with \(\sigma_3\) is represented by familiar Mohr-Coulomb strength relationship, which can be expressed as follows:

\[
(\sigma_1 - \sigma_3)_f = \frac{2\sigma_3 \sin \phi + 2c \cos \phi}{1 - \sin \phi}
\]  

By differentiating Equation (5.3) with respect to \(\epsilon_3\) and substituting the expressions of Equation (5.4), (5.5) and (5.6) into the resulting expression for \(E_t\), the following equation can be derived:

\[
E_t = E_i \left[ 1 - \frac{R_f (1 - \sin \phi)(\sigma_1 - \sigma_3)}{2c \cos \phi + 2\sigma_3 \sin \phi} \right]^2 KP_a \left( \frac{\sigma_3}{P_a} \right)^n
\]  

or

\[
E_t = E_i \left[ 1 - \frac{R_f (1 - \sin \phi)(\sigma_1 - \sigma_3)}{2c \cos \phi + 2\sigma_3 \sin \phi} \right]^2
\]  

This equation can be used to calculate the appropriate value of tangent modulus for any stress conditions \((\sigma_3\) and \((\sigma_1 - \sigma_3))\) if the values of the parameters \(K, n, c, \phi\) and \(R_f\) are known.

In the hyperbolic stress-strain relationships, the same value of unloading-reloading modulus, \(E_{ur}\), is used for both unloading and reloading. The value of
Figure 3: Transformed Stress-Strain Plot for Ottawa Sand
Figure 5.2 Variation of Initial Tangent Modulus with Confining Pressure

\[ \log_{10}\left(\frac{E}{\sigma_0}\right) \]

\[ E = K_0 \frac{\sigma_0^n}{\sigma_0^m} \]

Figure 5.3 Variation of Bulk Modulus with Confining Pressure
The values of $\phi'$ can be determined from each triaxial test, assuming the envelope for that circle passes through the origin of stress, by using the formula:

$$
\phi' = \sin^{-1} \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}
$$

(5.11)

The values of $\phi'$ can be also determined by drawing envelopes for each of the circles of stress, each envelope passing through the origin. When this is done, it is found that the values of $\phi'$ decrease in proportion with the logarithm of the confining pressure (Duncan and Wong, 1980). Duncan and Wong (1980) presented this variation by an equation of the form:

$$
\phi = \phi_o - \Delta\phi \log \left( \frac{\sigma_3}{P_o} \right)
$$

(5.12)

where

$\phi' = \phi_o - \Delta\phi$ can be determined from each triaxial test

$\phi_o$ = is the value of $\phi$ at $\sigma_3$ equal to $P_o$

$\Delta\phi$ = is the reduction in $\phi$ for a 10-fold increase in $\sigma_3$

Equation (5.12) can be used to evaluate the friction angle appropriate for any confining pressure within the range of pressures encompassed by the test results.

5.4 Input Parameters

It has been found that the parameters that effect the nonlinear stress-strain behaviour of the soil significantly are $K$, $n$, $c$, $K_{sw}$, $\phi$, $\Delta\phi$, and $R_f$ (Duncan and Wong, 1980). For the purpose of evaluating these parameters, a series of triaxial compression tests were carried out by Rabbaa (1981), using the Ottawa sand (see Chapter 2).

The value of $\Delta\phi$ can be determined by using Equation (5.11) and from Figure 5.4. Two steps are involved in evaluating the modulus parameters $K$ and $n$. The
Figure 5: Variation of friction angle with confining pressure for Ottawa sand.
Figure 5.6 Variation of initial tangent modulus with confining pressure for Ottawa sand.

\[ K = 1750 \]
\[ n = \tan \beta = 0.70 \]
first is to determine the value of $E_i$ of each test, and the second is to plot these values against $\sigma_3$ (on log-log scales) to determine the values of $K$ and $n$ as shown in Figure 5.5 and Figure 5.6. Table 5.1 gives a summary of stress-strain parameters for Ottawa sand (dense state) used in the present study. These parameters are characterised in terms of the material parameters used in the FEADAM program.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\Delta \phi$</th>
<th>$K$</th>
<th>$n$</th>
<th>$K_{uv}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>11</td>
<td>1150</td>
<td>0.70</td>
<td>1380</td>
</tr>
</tbody>
</table>

As for the bulk modulus parameters, $K_s$ and $m$ incorporated in Equation (5.10), they can be determined from triaxial test results by using a hyperbolic relationship between the axial strain and volume change to that given in Equation (5.2) and by following the same procedure described above. But since these parameters for Ottawa sand were determined or can be calculated from the results given by many research results (see, e.g. Kulhsawy, et al., 1966; Selvadurai, et al., 1979; Duncan and Wong, 1980 and Rabbas, 1981) the values given in these references were used in the present analysis and no effort has been made to establish them. These values of $K_s$ and $m$ were taken as 300 and 0.5 respectively which resulted in an initial Poisson's ratio of value 0.33.

5.5 The Computer Program "FEADAM"

The computer program used in the present analysis is called "FEADAM". This program was developed by Duncan and Wong (1980) and published as a report distributed by the University of California, Berkeley. The program employs the hy-
FIGURE 57a RIGID STRIP FOUNDATION RESTING ON A LAYER OF SAND ON TOP OF A RIGID BASE

FIGURE 57b FINITE ELEMENT MODEL (SMOOTH INTERFACE)

FIGURE 57c FINITE ELEMENT MODEL (POUGH INTERFACE)
the rigidity of the foundation. The horizontal displacements (x-displacements) of the nodal points beneath the foundation and on the bottom boundary in contact with the rigid base, can be allowed zero or free movement thereby simulating interface conditions with full friction or zero friction respectively. The nodal points along the vertical boundaries are to be constrained to move only in the vertical direction.

As for the discretizations, there are two possible elements that can be used. There are either 4 noded rectangular elements or 3 noded triangular elements. Typical discretizations of the two types of elements are shown in Figure 5.8.a and Figure 5.8.b. Investigations carried out indicate that both finite element meshes tend to give almost the same results. The finite element mesh with rectangular elements was used throughout this investigation because it is simpler and more economical than that of triangular elements.

5.7 Calibration of the Program "FEADAM"

To obtain some confidence in results obtained from the finite element program "FEADAM" used in the present analysis, the program was used to solve three problems with known results. The first problem concerns the plane strain load-displacements of a rigid strip footing on the surface of compressible layer of finite depth which rests on a rigid base (Rabbaa, 1981). Rabbaa (1981) solved the problem by using experimental model and a nonlinear finite element technique (ISBILD program by Duncan and Osawa, 1973). As can be seen from Figure 5.9, the result of the FEADAM program is in reasonable agreement with the experimental results. The second and the third problem concern the plane strain load-displacements embankment on a rigid foundation and embankment in a compressible foundation,
Figure 5.8a Finite element discretization for an elastic layer of finite depth (rectangular elements)
**Figure 5.8b**: Finite element discretization for an elastic layer of finite depth (triangular elements)
Figure 5.3 Comparison between the load-settlement curves given by Rabbaa (1981) and those obtained by using the program "FEADAM"
FIGURE 5.10a FINITE ELEMENT DISCRETIZATIONS USED TO CALIBRATE THE PROGRAM FEADAM.
(AFTER DUNCAN AND WONG, 1980)
Figure 5.10.b Finite element discretizations used to calibrate the program FEADAM (after Duncan and Wong, 1980).
with loads applied after construction as examined by Duncan, Wong and Osawa (1980) using a nonlinear finite element technique (FEADAM program). The two problems are shown in Figure 5.10.a and Figure 5.10.b. The results of the two problems are the same as given by Duncan, Wong and Osawa (1980).

5.8 Comparison Between Experimental and Theoretical

Results

The computed settlements from the program FEADAM are compared with the experimental results obtained from the test model. For example, three different problems are investigated. The first problem has a foundation resting on a layer of dense sand of ratio $\frac{H}{D} = 2.5$ and $\epsilon = 0$. The interface conditions between the footing and the sand layer and between the sand layer and the rigid base were smooth. The second problem was similar to the first problem but with rough interface conditions. The third problem was similar to the second problem but $\frac{H}{D} = 1.0$.

The results of the three problems are shown in Figure 5.11.a to Figure 5.11.c together with theoretical results obtained from the nonlinear finite element analysis. The theoretical results are reasonable agreement with the experimental curves. It should be mentioned that the computer results gave larger settlements in the case of smooth interface conditions than those actually observed from experimental model. In the case of rough interface conditions, the computer results gave smaller settlements than those actually observed from experimental model as shown in Figure 5.11.a to Figure 5.11.c.
FIGURE 5: LOAD-DISPLACEMENT CURVE - COMPARISON BETWEEN EXPERIMENT AND NONLINEAR FINITE ELEMENT ANALYSIS
\[ e = 0 \]
\[ B = 15.2 \text{ cm} \]
\[ H = 38.0 \text{ cm} \]
\[ \gamma = 17.4 \times \text{N/m}^3 \]
\[ H/B = 2.5 \]
\[ \phi = 41^\circ \]

Figure 5.13 Load-Displacement Curve: Comparison between Experiment and Nonlinear Finite Element Analysis.
Figure 5.10: Load-Displacement Curve - Comparison between Experiment and Nonlinear Finite Element Analysis

- Rigid Footing
- Rough Interfaces

- $P = 0$
- $B = 15.2\, \text{cm}$
- $H = 15.2\, \text{cm}$
- $\gamma = 17.4\, \text{kN/m}^3$
- $\phi = 41^\circ$
- $H/B = 1$

---

FE Analysis

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Range of Tests

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Chapter 6

Conclusions and Recommendations

6.1 Summary and Conclusions

This thesis presents a general summary of the behaviour of a rigid strip footing resting on the surface of a layer of sand underlain by a rigid base. The foundation is subjected to central line load or eccentric line load with an eccentricity of 0.1 B (ie. \( e = 1.5 \text{ cm} \)).

In Chapter 1, a literature review of the influence of a rigid base on the settlement and the bearing capacity of foundation was presented. In addition, several factors which affect the behaviour of a rigid strip foundation were identified. These factors were:

(i) Layer thickness (H)
(ii) Interface conditions
(iii) Sand density
(iv) Composite sand layer
(v) Central and eccentric load
(vi) Loading and unloading cycles

Results of the literature review pointed out the need for a better understanding of the aforesaid problem.

Chapter 2, described the plane strain apparatus, selection of the rigid model footing, the loading devices and the general properties of Ottawa sand used in the
experimental investigations.

The experimental program developed for the purpose of studying the influence of a rigid base on both, the settlement and the bearing capacity of foundation under the identified factors was described in Chapter 3 and Chapter 4.

A numerical solution to the problem of a rigid strip footing resting on the surface of a dense sand layer underlain by a rigid base was presented in Chapter 5. The numerical solution was based on the nonlinear finite element technique developed at the University of California, Berkeley (FEADAM program). The input parameters were determined from the results of a series of triaxial tests on Ottawa sand; the material used in the experiments.

Despite some shortcomings which have been mentioned in previous chapters, the experimental and numerical results indicated the following for the dense, loose and composite sand layers.

(a) The Dense Sand Layer

1) The settlement is significantly influenced by the layer thickness (H) and the interface conditions between the footing and the sand layer and between the sand layer and the rigid base. At a given load, the settlement decreases as the thickness of the sand layer, expressed by the ratio $H$, decreases. The settlement also decreases when the interface conditions are changed from smooth to rough. But the effect of the interface conditions is more significant in the case where $H = 1$. In the case where $H = 2.5$, the interface conditions have only a slight effect on the settlement.

2) In the case of deep layer ($H = 2.5$), the eccentric line load ($e = 10\%$ B) has no significant effect on the settlement under the centre of the footing in both cases of interface conditions (smooth and rough). But in the case of shallow
decreases. In the case of the shallow layer \((\frac{H}{D} = 1)\), the settlement decreases when the interface conditions are changed from smooth to rough. But in the case of deep layer \((\frac{H}{D} = 2.5)\), the interface conditions have no effect on the settlement.

(c) The Composite Sand Layer

1) In the shallow layer \((\frac{H}{D} = 1)\), the interface conditions have a significant effect on the settlement, the settlement decreases as the interface conditions change from smooth to rough. But in the deep layer \((\frac{H}{D} = 2.5)\), the interface conditions have no effect on the settlement.

2) The settlement decreases as the width of the dense region expressed by \(\frac{S}{D}\) increases and the ratio \(\frac{\left|\frac{P}{P_{ui}}\right|_{max}}{\left|\frac{P}{P_{ui}}\right|_{max}}\) decreases.

3) The relationship between \(\frac{\left|\frac{P}{P_{ui}}\right|_{max}}{\left|\frac{P}{P_{ui}}\right|_{max}}\) and the ratio of the width of the dense region \(\frac{S}{D}\) has a straight line shape regardless to the interface conditions or the layer thickness.

4) One of the major conclusion in this research is the establishment of effective widths of the dense sand region in a composite layer at which the behaviour of the soil layer can be represented by the dense equivalent.
6.2 Recommendations for Future Work

Further research work can be extended to study the following:

1) Research carried out to date in this area is mainly concerned with the problem of a single footing or interference between two footings. It is suggested to investigate the influence of the interference between a group of more than two footings.

2) The investigation of a composite sand layer should be continued by changing the relative density of the composite layer. Also the investigation should be directed to study the case of composite cohesive layer.

3) It is recommended to extend the current investigation to study the effect of changing the eccentric line load distance (e) on the settlement and bearing capacity of foundation.

4) The influence of a rigid base on the settlement and bearing capacity of a compressible layer was found to be strongly dependent on the ratio of the layer thickness to the foundation width as well as the roughness conditions of the interface between the foundation and the soil and the interface conditions between the soil and the rigid base. More research is needed to investigate the interactions between these parameters. For example in the current research the interface conditions were similar at both interfaces (i.e. smooth / smooth or rough / rough). Other interface conditions such as smooth / rough or rough / smooth should be investigated.
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Appendix A

Study of Friction on the Interior Walls of the Test Tank

(After Rabbana, 1981)

In order to study the effect of friction of the interior walls of the test tank, let us assume that:

\[ P_o = \text{the ultimate load obtained with both interior walls of the test tank lined with sheets of stainless steel of angle of friction } \lambda_o. \]

\[ P_r = \text{the ultimate load obtained with both interior walls of the test tank lined with rough sheets of angle of friction } \lambda_r. \]

\[ P_{or} = \text{the ultimate load obtained with one of the interior walls lined with a stainless steel sheet and the other lined with a rough sheet.} \]

\[ P_o = \text{the ultimate load which is supposed to be obtained with both interior walls perfectly smooth.} \]

\[ \Delta P_o = \text{the increase in the ultimate load due to the friction on one side of the test tank lined with a sheet of stainless steel and} \]

\[ \Delta P_r = \text{the increase in the ultimate load due to the friction on one side of the test tank lined with a sheet roughened with sand.} \]

By definition:

\[ P_o = P_o + 2\Delta P_o \]  
(1)

\[ P_r = P_o + 2\Delta P_r \]  
(2)

\[ P_{or} = P_o + \Delta P_o + \Delta P_r \]  
(3)

The values of \( P_o \), \( P_r \), and \( P_{or} \) as obtained from experiments and expressed in terms of \( \gamma B^2 \) are 255, 319 and 286 respectively (see Figure 2.7). Equations (1) to (3) can be
rewritten as follows

\[
P_o + 2 \Delta P_r = 255.0 \tag{4}
\]

\[
P_o + 2 \Delta P_r = 319.0 \tag{5}
\]

\[
P_o + \Delta P_r + \Delta P_r^* = 286.0 \tag{6}
\]

but

\[
\Delta P_r = \tan \lambda_r \\
\Delta P_r^* = \tan \lambda_r
\]

from the results of the pull out test, equation (2.6) is as follows (see section 2.2.2)

\[
\tan \lambda_r = 0.133 \tag{8}
\]

\[
\Delta P_r = 0.133 \tag{9}
\]

From equation (4) and equation (5),

\[
\Delta P_r - \Delta P_r = 32.0 \tag{10}
\]

and from equation (9) and (10)

\[
\Delta P_r = 0.135 \times \Delta P_r = 32.0
\]

or

\[
\Delta P_r = 36.9 \quad \text{and} \quad \Delta P_r = 4.9
\]

and from equation (4)

\[
p_o = 255 - 2 \Delta P_r = 255 - 2 \times 4.9 = 245.2
\]

As a check from equation (6):

\[
P_{sr} = P_o + \Delta P_r + \Delta P_r^*
\]

\[
P_{sr} = 245.2 + 4.9 + 36.9 = 287.1
\]

compared to 286 from experiments.

Now the percentage of increase in the ultimate load due to a friction angle \( \lambda_r \) of the stainless steel is:

\[
\frac{2 \Delta P_r}{P_o} \times 100 = \frac{2 \times 4.9 \times 100}{245.2} = 4\%
\]
Appendix B

Rigidity of the Aluminium Plate

A) The differential displacement of the aluminium plate can be obtained by simplifying the problem to the following configuration:

The ultimate bearing capacity of a strip foundation resting on the surface of a cohesionless layer, the following equation proposed by Terzaghi (1943) can be used:

$$q_{ult} = 0.5\gamma B N_\gamma$$

for a layer of dense sand $\phi = 41^\circ$; $N_\gamma = 126$

$$q_{ult} = 0.50 \times 17.4 \times 0.152 \times 126 = 166.6 \text{ kPa}$$

assume the maximum applied load is equal to $q_{ult}$ and assuming that:

$$E_{adam} = 70 \times 10^6 \text{ kPa}$$

$$I = \frac{t^3}{12}$$

$$I = 11 \times 10^{-6} \text{ m}^4$$

At the end of the plate, the maximum displacement is:
\[ \delta_{\text{max}} = \frac{ql^4}{8EI} \]

\[ \delta_{\text{max}} = 9 \times 10^{-6} \text{ m.} \]

Relative to the total plate settlement of 5 cm. The differential displacement is small and the plate can be considered to settle uniformly.

B) Relative Rigidity, \( K_r \)

According to Borowiczka (1939):

\[ K_r = \frac{4(1 - \nu_s^2)E_{\text{alum}}}{3(1 - \nu_{\text{alum}}^2)E_s} \left( \frac{t}{B} \right)^3 \]  \( (1) \)

For a layer of dense sand:

\( \nu_s = 0.25 \)

\( E_s = 30000 \text{ kPa} \)

For aluminium plate:

\( \nu_{\text{alum}} = 0.30 \)

\( E_{\text{alum}} = 70 \times 10^6 \text{ kPa} \)

\( t = 0.051 \text{ m.} \)

\( B = 0.152 \text{ m.} \)

From equation (1) and Figure B.1:

\( K_r = 121 \)

for that, it appears that the plate can be considered to be rigid.
Figure 8.1 Relationship between the relative rigidity of soil-footing and the footing width.
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