LIST OF ABBREVIATIONS

ABBREVIATION and MEANING

A/D = Analog-to-Digital
AMP = Analog Matched Filter
BFSK = Binary Frequency Shift Keying
BLADES = Buffalo Laboratories Application of Digital Exact Spectra
BPF = Bandpass Filter
BPSK = Binary Phase Shift Keying
BTL = Bell Telephone Laboratories
CCD = Charged Coupled Device
CPK = Coherent Phase Shift Keying
CRC = Communication Research Centre
CW = Continuous Wave
D/A = Digital-to-Analog
DC = District of Columbia
DLTL = Delay Lock Tracking Loop
DMF = Digital Matched Filter
DS = Direct Sequence
FH = Frequency Hopping
FSK = Frequency-Shift-Keying
GPS = Global Positioning System
HF = High Frequency
HP = Hewlett Packard
HRRL = Hammon Radio Research Laboratory
IEEE = Institute of Electrical and Electronics Engineers
PERMISSION TO MICROFILM — AUTORISATION DE MICROFILMER

Full Name of Author — Nom complet de l'auteur

NORMAND J. A. COUTURIER

Date of Birth — Date de naissance
34 JANUARY 1950

Country of Birth — Lieu de naissance
CANADA

Permanent Address — Résidence fixe
1792 RAPWAY TERRACE
GLOUCESTER, ONT
K1C 3T2

Title of Thesis — Titre de la thèse
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Name of Supervisor — Nom du directeur de thèse
JAMES WIGHT

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Date
2 May 1985

Signature
Normand Couturier
A low-cost digital matched filter
for arbitrary constant envelope
spread spectrum waveforms

by

Normand J. A. Couturier, B. Sc.

A thesis submitted to the Faculty of Graduate Studies and Research in
partial fulfillment of the requirements for the degree of

Master of Engineering.

Faculty of Engineering
Department of Electronics
Carleton University
March 1985
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"A LOW-COST DIGITAL MATCHED FILTER FOR ARBITRARY CONSTANT ENVELOPE SPREAD SPECTRUM WAVEFORMS"

Submitted by Normand Couturier in partial fulfillment of the requirements for the degree of Master of Engineering

Prof. J.S. Wight
Thesis Supervisor

Prof. M.A. Copeland
Chairman
Department of Electronics

April 1985
ABSTRACT

The need for secure communication systems has increased dramatically in the last decade. The Spread Spectrum system engineer has had to respond quickly to new requirements and new threats. Whereas Spread Spectrum systems were solely designed for secure military communications in the late seventies, the list of potential users has since been growing at an amazing rate.

In secure communications, rapid acquisition of the transmitted signal's spreading code in a friendly or hostile environment is essential. To satisfy this crucial requirement, a matched filter is normally used.

This thesis presents the concepts, theoretical development and design implementation of a new breed of Digital Matched Filter which is designed to accommodate a family of Spread Spectrum waveforms rather than its predecessor which was matched to a single waveform. The performance of this Digital Matched Filter is investigated in a broadband noise jamming environment and in a tone jamming environment and compared to its digital and analog predecessors' performance.
ACKNOWLEDGEMENTS

The author would like to express his deep appreciation for the support and guidance his thesis supervisor, Professor James S. Wight has given him. Without his continued encouragement and understanding, the completion of this work would have been very difficult.

The author is indebted to Dr LeRoy Pearce for his guidance in selecting the topic, planning the experimental work and solving some of the initial design implementation problems. Additional thanks must also go to Dr Pearce, his staff and the Communications Research Centre of the Department of Communications for providing the use of their facilities and resources.

Sincere appreciation is extended to Mr Doug Lambert, Mr Robin Addison and technologists Al McEwen, Don Selin and Dave Sim for their guidance and help in designing the overall Digital Matched Filter. Their aid ensured the successful completion of the experimental work.

Finally, the author would like to sincerely thank his wife, Diane, for providing him the emotional support necessary to undertake this work.
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IP = Intermediate Frequency
ITT = International Telephone and Telegraph
JTIDS = Joint Tactical Information Distribution System
Kbps = Kilobit per second
KHz = KiloHertz
LO = Local Oscillator
LPF = Low Pass Filter
LSI = Large Scale Integration
Mbps = Megabit per second
MHz = MegaHertz
MIT = Massachusetts Institute of Technology
MLSR = Maximum Length Shift Register
MSK = Minimum Shift Keying
MTI = Moving Target Indicator
ns = nanosecond
OQPSK = Offset Quadrature Phase Shift Keying
PN = Pseudorandom
QPSK = Quadrature Phase Shift Keying
RC = Resistor Capacitor
RF = Radio Frequency
SAW = Surface Acoustic Wave
SNRi = Signal to Noise Ratio at the Input
SNRo = Signal to Noise Ratio at the Output
SS = Spread Spectrum
TDMA = Time Division Multiple Access
TH = Time Hopping
TOA = Time Of Arrival
TTL = Transistor-Transistor Logic
UHF = Ultra High Frequency
US = United States
USA = United States of America
VCO = Voltage Controlled Oscillator
VHF = Very High Frequency
VHSIC = Very High Speed Integrated Circuit
LIST OF SYMBOLS

SYMBOL and MEANING

A = Amplification factor
Bs = Spread bandwidth
C = Channel capacity
d = Delay
D = DMT performance degradation
dB = Decibel unit
E = Energy of transmitted signal
Eb = Energy per bit
E[.] = Expected value of
Fc = Chip rate
fs = Sampling frequency
G(.) = Filter transfer function
Gp = Processing gain or multiplicity factor
h = Matched filter impulse response
Im(.) = Imaginary part of (.)
j = \sqrt{-1}
L = Number of samples in the sampled data form approximation of V(t)
m(t) = Moment generating function
N = Noise power
nk = Noise sample
No = Noise density
\( n(t) \) = White gaussian noise  
\( p \) = probability  
\( P_i \) = Input power  
\( P_o \) = Output power  
\( \text{Re}(.) \) = Real part of (.)  
\( r(t) \) = Received signal  
\( S \) = Signal\# power  
\( S(.) \) = Input signal spectrum  
\( s(t) \) = Transmitted signal  
\( S(T) \) = Matched filter low pass waveform amplitude  
\( s(t) \) = Impulse response  
\( T \) = Time interval  
\( T_c \) = Chip interval  
\( u(t) \) = DMF low pass filter input  
\( \text{Var}(.). \) = Variance of [.]  
\( V(t) \) = Matched filter correlator output  
\( W \) = Bandwidth  
\( W(T) \) = Matched filter low pass components of the multiplier output  
\( \text{WT} \) = Bandwidth time product  
\( y(t) \) = Filter output  
\( Z \) = DMF output signal  
\( \delta \) = Delta function  
\( \phi_s \) = Unknown relative carrier phase  
\( \theta(t) \) = Signal phase fluctuation  
\( \theta \) = Phase quantization error
\phi(t) = \text{Noise phase fluctuation}
\phi = \text{Reference signal phase shift}
\sigma = \text{Standard deviation}
\mu = \text{mean}
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CHAPTER 1

INTRODUCTION
1.1 ORIGINS OF SPREAD SPECTRUM

In the last few years, the Spread Spectrum concept has emerged from its cloak of secrecy. To the modern communication engineer, Spread Spectrum seems to be a new technology that only saw life in the mid-seventies. Yet, the first work in this area was done by Mr. Marconi in 1899.

How can anyone explain this lack of impression? What happened during those seventy-five years? It seems that Spread Spectrum was solely used for military communications and kept "TOP SECRET". Any literature related to Spread Spectrum systems was not only classified but its distribution was also restricted. Furthermore, all personnel involved with these systems were also bounded by this secrecy which made public discussion of Spread Spectrum systems and their principles impossible.

Even today, although many papers have been written on Spread Spectrum in the last few years, only a very few comprehensive text books can be found in the open literature.

When did Spread Spectrum systems initially emerge? Although everyone cannot agree on the real "birth" of Spread Spectrum, there seems to be some agreement on the different stepping stones that led to modern Spread Spectrum systems as we know them today. It is generally recognized that modern Spread Spectrum systems have been developed since the early fifties [1]. Those systems were used in military anti-jamming tactical communications, guidance systems, experimental anti-multipath systems and a few other specialized applications [2].
In 1899, Guglielmo Marconi conducted several experiments on frequency-selective reception in response to worries about radio interference [3]. Although Marconi did not find any concrete solution, he laid the groundwork for further research on the matter. Nevertheless, his research led him to the invention of the first wireless radio.

The first system using some sort of Spread Spectrum (SS) signaling was invented by Benjamin F. Miessner in 1912 [3]. From the early 1900's, Miessner and John H. Hammond Jr. of the Hammond Radio Research Laboratory (HRRL) had been working at developing techniques for preventing enemy jamming of strategic and tactical communications. Their system was built and successfully tested, just before the end of World War I, by Major Edwin H. Armstrong in the face of powerful enemy interference. Frequency wobbling methods were used to transmit secret signals that sounded like some new kind of artificially produced static. They discovered that the quality of antijamming of their signal increased as the frequency deviation of the wobbling increased. The wide frequency shift reduced the amount of time that a narrow band jamming or interference could affect the intermediate frequency (IF) of the receiver.

It is also interesting to note that the Hammond Laboratory/US Army/US Navy experiments of 1920-1922 established points of interests with regards to the information theory, in particular when the interference greatly exceeded the signal, but of course could not be verified until 1957.

Although there were about thirty patents registered on SS at the end of World War II, only a few are worth mentioning. The German forces were the
first military force to implement a SS communication network. Using Paul Kotowski's and Kurt Dannehl's patent, Hitler used a wiring link and a VHF/UHF link to keep in contact with Rommel's forces in Derna, Libya. This system used a pair of irregularly slotted or saw-toothed disks turning at different speeds to generate a noise-like signal at the transmitter which was modulated by the voice signal. The receiver had matching disks which had to be synchronized by means of two transmitted tones, one above and one below the encrypted voice band.

Bell Telephone Laboratories (BTL) improved Kotowski's concept and in 1941 implemented the X system nicknamed the "Green Hornet". The system was placed under secrecy and was used daily for secure communications between the allied leaders during World War II. To insure its security of transmission, the prerecorded keys were changed daily [2].

In mid 1941, Hedy K. Markey (who later changed her name to Hedy Lamarr) and George Antheil applied for the first American patent using frequency hopping (FH) technique explicitly conceived for anti-jamming communications. It was also in 1941 that Claude E. Shannon graduated from MIT and joined BTL to begin work on his fundamental theory of communication within a statistical framework.

In 1947, Shannon published a paper in which he used the cardinal expansion to formulate the capacity of a communication channel to deliver information where the only perturbation was by additive white gaussian noise. Shannon demonstrated that the channel capacity was maximized when selective spreading of the signal's spectrum was done within a designated bandwidth and that the sum of the power spectral density of the signal with the
gaussian noise should remain stably low as possible while all the average transmitted power available would be used [2]. Analogous to Shannon's work, Norbert Wiener published a classified report in 1942 that laid the ground work for modern continuous parameter estimation theory.

On May 27, 1942, Henri Busignies of ITT's Paris Laboratories filed a patent at Lyon in France on the first time-wobbling system which was later earmarked an early relative of modern time hoping (TH) SS systems. This system was, of course, the famous radar moving target indicator (MTI) with intriguing antijam capabilities. If the jamming or interference signal was not transmitted as to follow the prescribed addition rule, the signal would appear as a bright background but would not prevent the real signal from being read.

Although there was a steady growth of the Spread Spectrum technology throughout and after World War II, the real push by the major American laboratories to design modern SS system started in the early fifties. This sudden interest was created by the development of secure guidance system for the new surface-to-surface missile. This period saw the discovery of many new modern techniques which are still the basis of SS system as known today.

On 4 February 1950, a research team from Sylvania, led by Norman Harvey, filed a patent on a missile guidance system that used correlation detection. Also at Sylvania, in 1957, a team led by Madison Nicholson built the first modern BPSK direct sequence (DS) SS system. Synchronization of this DS-SS signal was accomplished by an early-late gate tracking loop, coined "tau
tracker". This system was officially called the ARC-50. Nicholson also succeeded in creating an artificial Doppler effect using a tapped delay line. This was a breakthrough for all SS engineers. Nicholson's "linear modulator or cycle adder" was eventually used to slow down the time base in the ARC-50.

Jim Green, one of Nicholson's colleague, changed the DS-SS configuration of the ARC-50 to a FH-SS in the Buffalo Laboratories Application of Digital Exact Spectra (BLADES). BLADES was originally built to meet Admiral Raeburn's Polaris submarine communication requirements. FH-SS with FSK modulation was used to combat the serious distortion that multipath could cause in long range intercontinental or global HF communications. BLADES proved to be the only useful communication link for the flagship Mount McKinley over intercontinental distances (Mediterranean sea to Washington, DC) under intentional hostile jamming in a full scale operational test in 1963. Equipment optimization was the most important criteria in BLADES. In fact, error correcting codes, word interleaving and, integrate and dump filters were developed to meet this challenge. This is why BLADES represents the start of real world application of shift-register sequences to error correction coding.

There were several other systems invented and produced, but one would need to write a book to mention all of them and the object of this section is simply to introduce the origins of Spread Spectrum. In researching these origins, one readily noticed that the United States had a commanding role in SS systems development. There are at least two logical explanations for this stronghold. First of all, the theories of Wiener and especially Shannon, which propounded the properties of and motivated the use of random and
pseudorandom signals, were available in the USA before such basic theories were appreciated elsewhere in the world.

Secondly, SS development occurred towards the end and after World War II when most European countries had lost their technological power and their resources. USA, on the other hand, became the new home of many leading European scientists such as Henri Busignies and Wernher von Braun. Gustav Guanella of neutral Switzerland, was most probably the only exception, since Brown, Bonerl and company started developing a SS guidance system in 1950.

The earliest reference to SS systems in Russia was in 1957 in a proposal by Kharkevich on amplitude and frequency modulation of pure noise. Although it appears certain that SS technology exists within the USSR, little information is available.

1.2 THESIS OBJECTIVES

In SS systems, the main concern is with the acquisition of the received signal and the synchronization of the transmitted code with the local replica. In the last ten years, the technology has shown that when communications are bursty, frequent and rapid acquisition of short messages are required, matched filtering should be used as opposed to active correlators for signal's acquisition [4]. In the seventies, this fact had forced the industries to research new techniques. This is where the analog devices, such as charged-coupled-devices (CCD's) and surface acoustic wave (SAW) delay lines really benefited. On the other hand, digital matched filters (DMF's) remained unattractive because the large time-bandwidth product required for SS techniques could not be easily achieved with a few chips.
With the advent of large scale integration (LSI) and very high speed integrated circuits (VHSIC), DMF's received a second chance. In fact, SAW technology is presently criticized because of its high manufacturing cost, large size, temperature instability and large insertion loss. SAW devices don't offer operational flexibility since they cannot be easily reprogrammed. Similarly, CCD's suffer from relatively limited bandwidth, limited dynamic range (particularly at high temperature), a large amount of required support circuitry and a high development cost. CCD's have never been a popular device in SS systems operational implementation.

These severe limitations on analog matched filters (AMF's) have forced all manufacturers to completely stop research on CCD's. SAW's devices, on the other hand, were not totally discarded but a few manufacturers continue to build them for special purpose applications. In fact, companies, such as ITT and Singer, have switched from AMF using SAW technology to DMF technology for their new programs such as JTIDS [4].

In the mid-seventies when DMF's were still not attractive for use in operational systems, Turin showed that even though DMF suffered a performance degradation due to the quantization process, it could nevertheless outperformed an AMF in hostile environment. While the AMF was designed and manufactured for one frequency/phase and its associated waveform, Turin's DMF could be readily and quickly matched to a new frequency/phase anytime by the receiver. Turin's brainchild was primarily designed to operate with BPSK modulation, other modulations such as MSK and QPSK, could be used but his DMF would suffer a second performance degradation which would be substantial and unacceptable in operational systems.
Alfred Baiër proposed for the first time at the IEEE Military Conference of 1983 a new revolutionary breed of DMF's which were designed to operate with a family of waveforms or modulation schemes. This new DMF technique could theoretically outperform Turin's DMF by as much as 3 dB in a gaussian environment. This proposed DMF was not only very flexible but it could also be built inexpensively with off-the-shelf components. A single Baiër DMF could lend itself to several different applications.

The objective of the research work carried out for this thesis was to gather experimental data on this DMF approach. Such data is not yet available in the open literature. The DMF was implemented as economically as possible and tested with two different waveforms in two separate hostile environments. Its performance was analyzed using Turin's relationship of SNRi to SNRo and compared with the expected theoretical results. Secondary parameters analyzed were the dynamic range and the processing gain of the DMF in these experimental environments.

1.3 THESIS ORGANIZATION

This thesis is presented in the following order. After the historical review of Spread Spectrum in this chapter, chapter 2 discusses the basic concepts of Spread Spectrum. Definitions, requirements, attributes and characteristics of Spread Spectrum are presented at first. Shannon's postulate is reviewed prior to the discussion of the basic spreading techniques used in
modern Spread Spectrum systems. The synchronization problem and its remedy are studied next. Finally, the basic digital matched filter theory is introduced.

Chapter 3 presents Alfred Baier's four-phase digital matched filter and discusses its implementation in a low cost DMF designed for arbitrary constant envelope SS waveforms.

Chapter 4 illustrates the experimental evaluation of Baier's DMF theory in a broadband noise and a tone jamming environments with coherent PSK and MSK modulations.

Chapter 5 is dedicated to the conclusion and propositions for future work to be conducted in the same research area.
CHAPTER 2

THEORY
2.1 SPREADING TECHNIQUES

2.1.1 DEFINITIONS AND CHARACTERISTICS

What is Spread Spectrum?

Spread Spectrum is a means of communication transmission in which the signal generated occupies a bandwidth in excess of the minimum necessary to send the information. The band spread can be accomplished by the use of a fast code which is independent of the data. A synchronized reception with the same code at the receiver is used for despreading and subsequent data recovery [1].

The basic characteristics of a modern SS signal are as follows:

1) The carrier is a pseudorandom but unpredictable wideband low power spectral density signal. Pseudorandom (PN) refers to random in appearance but reproducible by deterministic means.

2) The bandwidth of the carrier is much larger than the bandwidth of the data modulation.

3) Reproduction of the intelligence of the signal is achieved by cross correlation of the received wideband signal with a synchronously generated replica of the wideband carrier.

4) While conventional communication systems have a multiplicity factor of about one, SS signals usually have multiplicity factors in the thousands. The multiplicity factor of the communication link is referred to as the number of possible orthogonal signaling formats.
5) The primary advantage of SS receivers is their ability to reject unintentional and intentional interference and in some cases the concealment of a user's signal.

2.1.2 ATTRIBUTES AND REQUIREMENTS

Why bother with Spread Spectrum? Is the cost worth it? Yes, for several reasons and in some particular cases, it is the only viable solution. Spread spectrum was initially used solely in military communications for the following reasons:

1) Antijamming,
2) Anti-interference,
3) Low probability of detection,
4) Low probability of intercept, and
5) Resistance to multipath interference.

Although the military milieu is still the biggest user of SS communication systems, there is a growing interest in its use for mobile radio network and satellite communications due to its multiple user random access communications with selective addressing capability. Finally, the second most important user is in the world of timing and positioning systems due to the high resolution ranging and accurate universal timing inherent with SS techniques. In these latter systems, such as Global Positioning System (GPS) or NAVSTAR, the sharp peak of the autocorrelation function for PN sequences is exploited to permit the detection of timing errors of the order of one nanosecond (ns).

The low probability of intercept attribute is achieved by using high processing gain and pseudorandom carrier signals. By spreading power thinly and
uniformly in the frequency domain, detection by a surveillance receiver is made very difficult. Typically, a SS system takes a baseband signal such as a voice channel of a few kilohertz (KHz) bandwidth and distributes it over several megahertz (MHz) wide. Since the signal received resembles noise, one would easily perceive the ongoing communication signal as random noise. With a Direct Sequency SS signal the faster the spreading code clock is, the wider the spread bandwidth is and consequently the lower the signal power spectral density is.

Antijam capability results from the use of a high processing gain and a pseudorandom carrier signal. Because of the large number of possible orthogonal signaling formats, the jammer cannot guess the multiplicity used and hence must jam all possibilities. This means significantly reducing his effective jamming power.

High resolution ranging and accurate universal timing are due to the high time resolution of SS systems, which is obtained by using wideband correlation detectors. Because differences in the time of arrival of the wideband signal (usually on the order of the reciprocal of the signal bandwidth) are detectable, multipath signals are suppressed and repeater jammers can be made less effective.

Multiple user random access communication is possible in SS due to the large number of potential orthogonal signaling formats. This means that transmitter-receiver pairs using independent random carriers can operate side by side in the same bandwidth with minimal cochannel interference. In fact, several networks, each using their own spreading code for example, can be established in the same geographical area.

SS systems also exhibit a resistance to multipath interference \([5,6]\). One way to see this is to consider the frequency domain representation
of a channel medium with multipath. The effect would be to greatly increase loss at frequencies where the delay between the direct and the reflected (multipath) signals results in destructive interference. For a wideband SS signal, such multipath spectrum nulls might remove only a portion of the total bandwidth, and thus only a portion of the total signal power. However, a narrowband signal falling in a multipath spectrum null might lose all or most of its energy, rendering it irrecoverable.

Multipath interference can also be viewed in terms of autocorrelation properties of the spreading code in the case of DS-SS. From the autocorrelation function theory [7], PN codes have a sharp correlation peak at zero delay, and a very small value for more than one chip delay or advance. Thus, a multipath signal arriving at the receiver delayed relative to the direct path signal will correlate very little with the code sequence synchronized to the direct path signal. In this way, the multipath interference will be rejected by the receiver.

While the use of Spread Spectrum means a wider bandwidth for each transmission and a lower spectral efficiency this can be easily compensated by the inherent interference reduction capability of SS techniques. Although this capability was postulated by Claude Shannon back in 1948, it wasn't until the sixties that it could be verified. Shannon showed that for a white gaussian noise channel and a fixed signal-to-noise density ratio, more efficient communication is possible as the bandwidth is increased. In fact, if the bandwidth is increase to infinity, reliable and error-free communication can be maintained at the channel capacity with an energy per bit to noise density ratio
(Eb/No) of -1.6 dB [5]. This value is popularly known as Shannon’s limit. Furthermore, Shannon also showed that when the data rate is less than the capacity of the particular channel, there exists several methods of signal modulation and data coding that will give an arbitrarily low probability of error upon reception and decoding. This explains why so much resources and research have been dedicated to design more efficient signal modulation and data coding.

2.1.3 SHANNON'S POSTULATE

The foundation of Spread Spectrum was expressed by Claude Shannon in the form of channel capacity.

\[
C = W \log_2(1+S/N) = 1.44W \log_e(1+S/N) \quad \{2.1\}
\]

where \( C \) = capacity in bits per second,
\( W \) = bandwidth in Hertz,
\( N \) = noise power in decibel, and
\( S \) = signal power in decibel.

Using only the logarithmic expansion, it can be readily shown that when \( S/N < 0.1 \), which is usually the case in Spread Spectrum, \( C \approx 1.44 \) \( WS/N \) and hence,

\[
W = NC/1.44S = NC/S \quad \{2.2\}
\]

Equation (2.2) states that for any given noise to signal ratio, one can have a low information error rate by increasing the bandwidth used to communicate the information. Usually the SS engineer is left with at most two variables since \( S \) is limited by the transmitter power and \( N \) is limited by the
receiver sensitivity. Depending on the application, a practical system might also be bandwidth limited. Therefore, new techniques are constantly sought to increase the multiplicity of the signal. It is now possible to decrease the bandwidth required for a certain multiplicity by using coding techniques and imbedding the information in the coded SS signal by several methods. Some systems, such as the Joint Tactical Information Distribution System (JTIDS), combine several spreading techniques as well as coding to increase the signal multiplicity to produce an extremely low information error rate [8].

2.1.4 DIRECT SEQUENCE (DS) SYSTEMS

Direct sequence (DS) or PN (for pseudorandom) spreading utilizes a binary sequence to achieve the desired signal bandwidth. This sequence, called the spreading code, is often either a periodic PN sequence, such as a maximal length linear shift register sequence, or some variations of such sequences. At the transmitter (fig. 2.1), the information signal is modulated with the code and the carrier, then transmitted. Although any of a number of modulation schemes could in theory be used, phase modulation is generally used in DS-SS.

A typical receiver would mix an identical, synchronized PN sequence with the incoming signal, thus despreading it. For the case of antipodal phase modulation, it can be easily seen that the spreading code is removed. Note that for such phase modulation, mixing two identical and aligned binary sequences
Fig. 2.1 Direct Sequence System

Fig. 2.2 Direct Sequence Spread Spectrum Signal
together results in a constant positive value. When the receiver's code is in exact alignment to the code that is spreading the desired signal, the mixing at the receiver removes the spreading code phase shifts leaving just the information signal.

For a PN code sequence generated by a maximal length shift register, the power spectrum is a series of impulses spaced at the PN sequence period frequency, with a \( (\sin x / x)^2 \) envelope. When such a PN code is used in a DS-SS transmitter, the spectrum of the information signal is repeated at each of these impulses and is scaled by the \( (\sin x / x)^2 \) envelope (Fig. 2.2). Figure 2.2 is a spectrum analyzer sweep of the DS-SS signal used in the experimental evaluation of the four-phase DMB of chapter 4.

The main lobe of this envelope has a two-sided bandwidth that is twice the PN sequence clock rate. Since the binary digits in the PN code sequence are called chips, this is referred to as the chip rate \( (F_c) \). Thus, the spread bandwidth, \( B_s \), is simply \( 2 F_c \). The processing gain or multiplicity factor is given by

\[
G_p = \frac{\text{data bit time}}{\text{chip time}} \quad \text{or} \quad G_p = 2 \frac{\text{PN code clock rate}}{\text{information rate}}. \tag{2.3}
\]

DS systems are renowned for their excellent time of arrival (TOA) resolution and are very efficient in power amplification due to the constant envelope. Maximal length shift registers (MLSR) are popular and easy to implement. Normally, several stages are needed to make the code long so as to reduce the chance of the code being "cracked". Normally a code can be cracked by solving simultaneously the set of linear equations representing the MLSR sequence if one can observe 2n clear code bits given that the period of the code
is $2^{n-1}$. Unfortunately, a long code can mean a long acquisition time, which is exactly what one wants to avoid at all cost in an hostile environment. So instead a Boolean function using a ROM can be used to produce a nonlinear, shorter and more "crackproof" code.

Whereas DS transmitting is elementary, DS receiving requires a carefully designed synchronization circuit. For best code correlation and proper demodulation, stringent timing and synchronization of both PN generators are imperative. Synchronization has to be within a fraction of the chip interval ($T_c$) or $1/W$ where $W$ is the signal spread bandwidth.

DS systems perform well in interference or jamming environments. In a jammer versus communicator relationship, unwanted signals, would be treated the same way as the desired signal. The jammer signal is multiplied by the reference PN code that maps the desired signal into the original carrier bandwidth. Any received signal not synchronous with the receiver's reference would be spread to a bandwidth equal to its own bandwidth plus the actual bandwidth of the PN reference code.

From stochastic processes theory, the receiver output signal variance is the covariance of its input signals. Since the synchronous signal is mapped into the bandwidth of the data signal, the receiver bandpass filter (BPF) can reject most of the power of the undesired signals. In fact, this is how the process gain is realized in DS systems. Using system information bandwidth, the postcorrelation filter bandwidth is set and the amount of power from any unsynchronized signal that reaches the demodulator can be determined by this filter. This is why the multiplication and filtering operation provides the DS signal with a definite advantage (i.e. process gain).
In the case of tone jamming, the situation becomes more complicated than the broadband noise jamming, which is pretty well treated as pure white noise. Tone jamming or single frequency jamming at the carrier frequency is the worst type of jamming a DS-SS system can be submitted to. The antijamming system performance depends on the location of the tone and if the period of the spreading sequence is at least as long as a data symbol. Depending on the conditions, the effect of a despread tone can usually be approximated as coming from an equivalent amount of gaussian noise [1]. There exists also swept-frequency jammers, pulse-burst jammers and repeat jammers. In recent years, several new adaptive receiver filters have been built and successfully tested. These new filters are designed to prefiltre the signal and cope with most type of jamming without suffering harmful degradation.

Generally, the RF bandwidth of a DS-SS system is defined as the null-to-null bandwidth of the main lobe in the power spectrum. It can be mathematically shown that 90% of the total signal power resides in the main lobe. Present technology can produce integrated circuits which allow limited code generation beyond 300 Megabits per second (Mbps). One must remember however that as the code rate increases, operating error rate must decrease accordingly. As an example, a 300 Mbps code generator with a $1 \times 10^{-8}$ error rate is expected to make one mistake per second. The alternative solution would be to lower the data rate by using coding techniques but here also, there is a limit, if one wants real time data transmission.

DS-SS systems have recently monopolized the timing application of SS systems while still remaining popular with the other applications discussed earlier.
2.1.5 FREQUENCY HOPPING (FH) SYSTEMS

In frequency hopping (FH), the carrier frequency (fig. 2.3) is shifted or hopped over discrete values throughout the desired spread bandwidth. This can be accomplished by using a synthesizer for carrier generation. A sequence of the frequency values is fed to the synthesizer. This sequence is some form of PN code or noise sequence, which enhances the signal's noisileike nature (fig. 2.4). At the receiver, another synthesizer is driven by an identical PN sequence and used to mix down the incoming SS signal to a non-hopped narrowband IF. Note that the spectrum of a frequency hop is roughly that of the information signal repeated at each of the discrete carrier frequencies. Thus, the spread bandwidth is independent of the rate at which the carrier is stepped and is equal to the number of steps times the spacing between adjacent carrier frequencies. If the carrier spacing is the same as the two-sided information bandwidth, then the processing gain, Gp, is simply equal to the number of steps or hops, or the hop time multiplied by the frequency range.

In general, the modulation is either binary or M-ary FSK. Studies have demonstrated that PSK modulation is more performance effective than FSK in a white gaussian noise channel, but maintaining phase coherence between hops is very difficult [5]. This is why FSK modulation with noncoherent detection is mostly used in FH-SS systems.

In BFSK, the modulator selects one of two frequencies which correspond to the transmission of either a one or a zero. The BFSK signal is translated in frequency by an amount that is governed by the PN generator and
**Fig. 2.3 FH system**

**Fig. 2.4 Example of FH pattern**
its output sequence. This PN generator output is used to control the frequency synthesizer. This frequency is applied to the mixer where it is mixed with the modulator's output.

The frequency hopping rate is usually selected to be either slower, equal to or faster than the symbol rate. If there are multiple hops per symbol, we have a fast-hopped signal. On the other hand, if the hopping is performed at or less than the symbol rate, we have a slow-hopped signal. Fast FH is employed in antijamming environments when it is necessary to prevent a type of jammer, called a follower jammer, from having sufficient time to intercept the frequency and retransmit it along with adjacent frequencies so as to create interfering signal components.

However, there is a penalty incurred in subdividing a signal into several frequency-hopped elements since the energy from these separate elements cannot be coherently recombined. Consequently the demodulator incurs a penalty in the form of a noncoherent combining loss.

In FH, synchronization requirements are looser than in DS. In fact, the chip interval is translated to the time spent in a particular frequency slot of bandwidth $B$ where $B < W$, so the fraction becomes $1/B$ which is much larger than $1/W$. This means that the timing requirements are not as stringent in FH as in DS. However, this also implies that TOA resolution of FH is coarser than DS. In today's military SS systems, if secure strategic and tactical communication is the main concern in a stand-alone system, FH-SS is often sought instead of DS-SS because as a rule wider bandwidth can be obtained.
2.1.6 TIME HOPPING (TH) SYSTEMS

In time hopping (TH), a time interval, which is selected to be much larger than the reciprocal of the information rate, is subdivided into a large number of time slots. The coded information symbols are transmitted (fig. 2.5) in a PN selected time slots as a block of one or more code words. PSK modulation is most often used to transmit the code bits. The processing gain, Gp, is simply the number of time slots used.

Due to the burst characteristics of the transmitted signal, buffer storage must be provided at the transmitter. To ensure an uniform data stream to the user, a buffer should also be used at the receiver. Interleaving is usually added to the coding to provide greater protection against intentional partial time interference (i.e. pulse). TH is analogous to FH but is not very popular as a stand-alone system. This nonpopularity results from the fact that TH has the most stringent timing requirements of all SS systems. TH is mainly used in an hybrid system in time division multiple access (TDMA) networks.

Unfortunately, a simple TH system, as in fig. 2.5, offers nothing in the way of interference rejection and a continuous carrier at the signal center frequency can block communications very effectively. The only advantage of the TH system is its reduced duty cycle. Assuming the jammer doesn't know the timing code, the jammer would have to transmit continuously which means that the power required for the TH would be less than the jammer by a factor equal to the signal duty cycle. The main advantage of a time hopped system is that it makes the interception of the information signal more difficult.
Fig. 2.5 Time Hopping
2.1.7 CHIRP

Chirp or linear frequency modulation sweeps the carrier continuously over the desired spread bandwidth (fig. 2.6). The frequency-time relation need not be linear. A voltage controlled oscillator (VCO) can be driven by a ramp to generate such a signal. Another VCO used as the local oscillator at the receiver and operated to sweep with the received signal can be used to compress the signal and recover the transmitted information.

Since the two-sided bandwidth of a T second pulse is approximately 1/T, the processing gain of such a system would be roughly $G_p = T B_s$ or $W_{sweep}/W_{info}$. In pulse radars, a single chirp pulse received by the radar system is compressed by a matched filter, which requires special design.

Due to their non-continuous pulse nature, Chirp systems are mainly used in radar and seldom in communication. In radar applications, one looks at the compressed pulse for coarse information only. This chirp pulse has excellent duty cycle characteristics and, as a consequence, is difficult to jam completely. On the other hand, this pulse can be partially interfered without great expense which makes it unsuitable for communication systems in hostile environment, since fine information is required.

Radar chirp systems can be made very agile since the receiver has a priori knowledge of the expected return pulse frequency or frequency range. This implies that a radar transmitter can randomly select his chirp pulse frequency without any concern for the receiver. In communication systems, a
**Fig. 2.6 Chirp**

(a) TRANSMITTER  
(b) RECEIVER

**Fig. 2.7 DS Stepped Serial acquisition scheme**
bank of separate receivers would be required to monitor all possible frequencies used for a transmission, which would also make the overall system cumbersome.

2.1.8 HYBRID

In the military milieu, which is still the biggest user of SS systems, hybrid systems are preferred over stand-alone systems for their reliability and robustness. In an hybrid system, a combination of spreading techniques are used together for special purposes. The most popular combination is the DS-FH system where the signal is first DS spread and then hopped. This way one can achieve a larger processing gain and at the same time acquires a better security of communications. Where maximum protection is required, some SS systems such as JTIDS will use a TH-DS-FH hybrid. Other benefits can also be gained by using an hybrid system. Shorter PN codes can be used which usually means faster acquisition without risks of code cracking. Fewer hopping slots are required which means simpler synthesizers.

When selecting a SS system, priorities have to be establish early in the planning stage. These can be set by the system's requirements, its vulnerability and the expected operational environment. In addition, expansion capability and operational flexibility should never be overlooked.

2.2 SYNCHRONIZATION

The single most crucial phase of SS communications is the synchronization of the transmitter and receiver codes [8]. In SS systems,
synchronization is usually divided into two distinct phases, acquisition and tracking, which are often supported by their own separate circuitry. In the first phase, the acquisition circuit provides coarse synchronization of the received PN modulating sequence with the local PN replica to within half a chip or code bit. Fine acquisition or tracking is accomplished by a special tracking loop, since reliance on the clock stability to maintain accurate synchronization is generally not sufficient for proper transmission. This becomes more of a constraint in mobile communications. The Doppler frequency shift due to platform motion in aircraft and satellite applications imposes new and more stringent design requirements in today's SS communication systems.

2.2.1 ACQUISITION

For many years, acquisition has been obtained either by a stepped serial (active correlator) scheme or a matched filter. In 1981, C. A. Putman proposed a scheme which amalgamated the advantages of both former schemes [9].

In the stepped serial scheme (fig. 2.7, page 28), the timing epoch of the local PN code is set and the locally generated signal is correlated with the incoming signal. If, at the end of an examination interval the threshold is not exceeded, the search control sends a clock pulse to the PN generator so that the local code phase slips to the next cell, n cells per chip, and the process is repeated until the threshold is reached. The correlation threshold is usually set for a half-a-chip respective delay/advance of the two codes.
This scheme performs an active correlation with the received code sequence and has been able to provide very good detection capabilities even in adverse environments. The scheme is relatively simple to implement but its weakness is usually a long uncertainty region search time. Several approaches have been tried to improve this long acquisition time, such as using a shorter PN code or simultaneously searching over different parts of the time uncertainty (which requires additional synchronization detection circuits) or using special algorithms which give a priori knowledge to start the code search by setting the timing epoch, but nevertheless synchronization time was still found to be moderately long [8]. Shorter code also increases the chance that the code could be cracked. The matched filter, on the other hand, is a passive correlator and thus searches the incoming PN code in real time. As per figure 2.8, a sequence of \( M \) consecutive frequencies (in a FH application) is selected by the receiver to establish a code start epoch and the matched filter performs a near optimal noncoherent detection as the sequence is received [10].

Until the seventies, implementation of a matched filter for bandwidth-time (WT) products of several hundreds was impractical. With the development of SAW devices, it became possible to produce filters with WT product of 1000, i.e. providing integration over 1000 code chips. Matched filter detection requires a complex technology and hardware structure for similar performance. However, there is no doubt that searching the uncertainty region in real time offers a clear advantage where it performs adequately. Modern matched filtering techniques have considerably reduced the need for complex technology and today's matched filter implementation is as easy as implementing an active correlator.
Fig. 2.8 FH Matched Filter

Fig. 2.9 Two-level scheme
The two-level scheme (fig. 2.9) uses passive and active correlation techniques and combines the capability of searching the code in real time with integration over a large number of chips. A matched filter detects a relatively few number of hops (in a FH application) synchronization peaks and applies code start signals to a bank of active correlators. Each signal causes the next idle correlator to cycle through a fixed number of hops, at the end of which any correlator output exceeding the second threshold causes a synchronization indication, otherwise the correlator is again made available to the common bank.

It has been shown that the two-level scheme offers an improvement in performance in adverse environments [9]. It combines the advantages of the rapid search time of the matched filter with the detection reliability of an active correlator. Therefore, it would be most suitable in systems which do not rely on a time reference and which utilizes long codes but still require rapid and reliable acquisition in adverse environments.

The parallel bank of active correlators becomes a trump card in situations with impaired signal detectability due to interference and/or scatter. Even while some correlators may have been falsely engaged in operation, successful signal acquisition is still possible with one idle correlator. This parallelism is not available in the other two popular schemes but comes at the cost of hardware complexity.

2.2.2 TRACKING

Tracking of the PN code is accomplished by sensing any relative synchronization error and adjusting the frequency or the phase of the PN clock
to improve the synchronization. The tracking circuit is then a closed loop which employs feedback to reduce the tracking error to zero. There exists at least two broad categories of clock synchronizers, the early-late gate tracking loops (fig. 2.10) and the digital-data-transition or in-phase mid-phase tracking loops (fig. 2.11).

The heart of the early-late gate loop consists of a pair of gated integrators, each performing its integration over a time interval of T/2 second [11]. Integration by the early gate occurs in the T/2 second preceding the nominal location of data transitions, while the late gate integrates during the T/2 second immediately following the transitions. Gate intervals adjoin one another but do not overlap. If the timing error is zero, the data transitions fall exactly on the boundary between early and late gates.

If the timing error is not zero, a transition falls not on the boundary but within one or the other of the gates. Since signal polarity changes within the gate containing the transition, the associated integration reaches a lesser magnitude than when the transition is external to the gate. Comparison of magnitudes of the two integrations gives an indication of the timing error after each time interval.

The early-late gate loop acts to place the boundary between the two gates exactly at the transition instant. Departure from that timing generates the loop error signal. It is also possible to produce an error signal by using a single gate that straddles a transition. If the transition is exactly centered within this mid-phase gate, integration over the gate interval is zero. If the
Fig. 2.10 Early-Late gate synchronizer

Fig. 2.11 In-phase Mid-phase synchronizer
transition is not centered, then the integration produces a positive or negative error output.

Sense of the error must be determined according to the direction of the transition. Also, if there is no transition, no information has been presented and the integrator output must be ignored. Transition detection and direction sensing can be performed by conventional detection of the data bits and by digital logic operations on the current detected bit and its predecessor. It has been shown that the early-late gate family is superior and outperforms the in-phase mid-phase family by at least 1 dB [12, 13] in normal operational environment.

Within the early-late gate family, there exists several forms of tracking loops but the best performer in most environments is the delay lock tracking loop (DLTL) (fig. 2.12) [13]. In the DLTL, the signal is applied to the two multipliers where it is multiplied respectively by one of the two PN codes which are delayed relatively by an amount 2d. In fact, the product signals are the crosscorrelations, at two values of delay of the incoming signal and the PN code. When synchronization is not exact, one product will be greater than the other and the code clock will be advanced or delayed accordingly. At equilibrium, the crosscorrelations will be equally displaced, in a positive and negative sense, from the peak value. A PN code delayed by d will be exactly synchronized for message demodulation. Studies have shown that the DLTL outperforms its closest rival, the tau-dither tracking loop, by at least 1 dB for equal rms tracking jitter in normal environment and in adverse environment, this value would certainly increase [12, 13].
Fig. 2.12 Delay lock tracking loop (DLTL)
2.3 DIGITAL MATCHED FILTER (DMF)

2.3.1 ANALOG MATCHED FILTER (AMF)

When we deal with matched filters, the universal standard is the analog matched filter used either in a simple $M$-ary communication receiver (fig. 2.13) or the modern tapped delay line (fig. 2.14) [14, 15]. In the first case, the objective is not synchronization but demodulation of the received signal. We have $M$-ary possible solutions, so we store the $M$-ary filter impulse responses. The signal is received and is correlated in parallel with these impulse responses. The correlation output is submitted to a decision circuit which selects the most probable solution.

Why refer to impulse response? A signal, $s(t)$, of duration $T$ may be imagined to be generated by exciting a filter, whose impulse response is $s(\tau)$, with an unit impulse at time $\tau=0$. After all, an impulse generated by any transmitter or modulator is frequency or phase selective. In the case of an active correlator, the received signal $r(t)$ is crosscorrelated with the complex conjugate of each of the possible $M$ transmitted signals [16]. In the second case, the received signal $r(t)$ is passed through a parallel bank of $M$ filters, having equivalent low pass impulse responses [16]. The filters are said to be matched to the $M$ signaling waveforms, hence MATCHED FILTER.

The best example of an AMF is the tapped delay line (fig. 2.14). The delay line is tapped at several $d$ or time delays. Each tap is then correlated to the low pass impulse response of a particular symbol. The correlation results
Fig. 2.13 A simple M-ary communication receiver

Fig. 2.14 128 tap SAW Matched Filter
are summed for each time interval and fed to a decision circuit which selects the most probable solution or response.

2.3.2 TURIN'S RELATIONSHIP

Turin, a father of Matched Filter theory, defined the well-known characteristic relationship using the SNRi and SNRo of the digital matched filter for the coherent channel. This special and unique relationship, $SNRo = 2TW (SNRi) = 2E/No$, has become the only universal yardstick in matched filters. From this relationship, a matched filter shows an input-to-output SNR gain of $2TW$, which of course, has been responsible for thirty years of effort to achieve ever larger TW products. This is ironical since the second equality clearly states that all signals with a given energy perform equally well, irrespective of their TW products.

In fact, the requirement for large values of TW products stems more from practical reasons rather than theoretical ones. A few reasons will suffice to illustrate the point [15]:

1) the requirement for small peak and/or average signal power, which lead to the spreading of the energy $E$ over a large interval $T$,

2) the requirement for high time/velocity resolution in radar, timing and synchronizing circuits,

3) the desire to avoid detection or interception of one's signal in spreading techniques,

* See appendix A for derivation. $E = \int_0^T s^2(t) \, dt$. 
4) the need for high path resolvability in multipath systems, and

5) strategies to force spectral dispersion of a jammer's power.

Although the coherent channel model is feasible and can be implemented in a modern matched filter, it is more customary to use the noncoherent channel model in most practical cases for ease of implementation and less hardware complexity [17]. While both systems or models exhibit identical performance in terms of probability of error, the former will outperform the latter by a factor of four (i.e. 6 dB) in SNR₀ #, which subsequently becomes

\[ SNR₀ = \frac{TW}{2} (SNR_i) . \]

2.3.3 OPERATION OF DMF'S

While SAW AMF are realized using one bandpass matched filter, DMF are best realized using two low-pass filters, largely due to sampling rate considerations [15]. The digital implementation of figure 2.15 is best described by writing a succession of approximations to the convolution integrals which characterize these matched filters.

For the upper arm of figure 2.15, we have

\[ Vu(t) = \int_0^T S(t-T) Wu(t-T) \, dt . \]  

\[ \text{[2.4]} \]

# See appendix B for derivation.
Fig. 2.15 Low-pass realization of analog Matched filter

Fig. 2.16 Digital correlator
The first approximation consists of writing a sampled data form of (2.4),

$$V_{uk} = \delta \sum_{i=0}^{L-1} S_{l-i} W_{u,k-i}$$  \hspace{1cm} (2.5)

where $V_{uj} = V_{u(j\delta)}$, $W_{uj} = W_{u(j\delta)}$ and $S_j = S(j\delta)$ are samples of $V_{u(.)}$, $W_{u(.)}$ and $S(.)$ taken every $\delta$ s where $L\delta = T$. The sampling rate $1/\delta$ is an important parameter of the system and should be carefully considered. The minimum sampling rate should be the Nyquist rate determined by the bandwidth of the low-pass components of the multiplier output waveforms. In ranging or synchronization applications, greater or faster sampling is required to achieve finer resolution.

The second approximation consists in writing the binary expansions of the samples $W_{uj}$ and $S_j$ as follows,

$$W_{uj} = \sum_{m=0}^{\infty} W_{uj}^m 2^{-m}, \text{ and}$$  \hspace{1cm} (2.6)

$$S_j = \sum_{m=0}^{\infty} S_j^m 2^{-m}.$$  \hspace{1cm} (2.7)

In (2.6) and (2.7), $W_{uj}^m$ and $S_j^m$ can only take the values $+1$ or $-1$ and are the $m$th digits of the bipolar binary representations of the respective analog values. This implies that $W_{uj}$ and $S_j$ have been scaled so as to lie in the $-2$ to $+2$ amplitude range using an automatic gain control. If an analog value is greater than $+2$ (less than $-2$), its binary representation consists of all $+1$'s ($-1$'s). Next, the sums of (2.6) and (2.7) have to be approximated by truncating $W_{uj}$ to $M$ digits and $S_j$ to $N$ digits. After truncation, (2.5) becomes
\[
V_{uk} = \delta \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} 2^{-(m+n)} \sum_{i=0}^{L-1} S_{1-i} W_{u,k-i}. \quad (2.8)
\]

Equation (2.8) represents a digital realization of the upper arm of figure 2.15, and a similar equation (replacing subscript \( u \) by \( l \)) holds for the lower arm.

Therefore, to obtain an approximation to the sequence of samples \( W_u(.) \) of \( (2.4) \), we had to do the following [15],

1) scale \( W_u(.) \) and \( S(.) \) to the range \(-2 \) to \( +2 \),
2) sample \( W_u(.) \) and \( S(.) \) every \( \delta \) s,
3) digitize the samples into truncated binary forms,
4) correlate the \( m \)th digits of the binary expansions for the samples of \( S(.) \) against the \( n \)th digits of the binary expansions for \( W_u(.) \), thus performing \( MN \) correlations, and
5) combine the correlations according to (2.8), using the appropriate weights \( 2^{-(m+n)\delta} \).

The scaling, sampling and digitizing of \( S(.) \) is done once and then the digital values are stored permanently in the receiver. The corresponding operations on \( W_u(.) \) are done, respectively, by an automatic gain control, a sampler and a \( N \) bit A/D converter. The correlation of step 4 is normally achieved by a digital correlator (fig. 2.16, page 42), while step 5 can be done using a D/A converter with appropriate weights.

The sequence of the \( n \)th digits of the expansions of the \( W_{uj} \) samples is loaded into the upper shift register from the left. At each sample clock pulse, the new sample's \( n \)th digits enters the leftmost stage, all previous
samples move one stage to the right and the rightmost sample is discarded. The reference shift register stores the \( m^{\text{th}} \) digits of the samples of \( S_j \). At each clock pulse, the contents of both shift registers are multiplied in pairs and the products are added.

Figure 2.17 represents a complete digital realization of the noncoherent matched filter of figure 2.15. It should be mentioned that, in this special case \( M = 1 \) and one reference register feeds all correlators since we are dealing with binary signaling. The major decision left to the designer is "where do you truncate \( N \)". Over the years, it has been showed that the biggest drawback of the DMF is its performance loss due to amplitude quantization [15, 18 and 19]. Depending on the type of interference suspected, several cures have been successfully tried [15, 18].

2.3.4 INTERFERENCE DEGRADATION

In the case of white gaussian noise, Turin showed that for a one bit quantization, a DMF suffers a SNR\( \text{Ro} \) degradation of \( \pi/2 \) or 1.96 dB * over its analog version [15]. Bruno and Cahn have successfully demonstrated that multilevel quantization can reduce this loss to 0.2 dB for three bit quantization, 0.1 dB for four bit and 0.05 for five bit [4, 19].

For constant amplitude interference at low SNR\( \text{i} \), DMF adaptive threshold biasing can lead to a higher output SNR\( \text{Ro} \) than is achievable with an

* See appendix C for derivation.
Fig. 2.17 A digital Matched Filter for binary signal
AMF [4, 15]. This can be easily implemented by using an RC integrator immediately after the digitizer and, the output of which biases the digitizer. This feedback technique allows the threshold to respond adaptively to deteriorating performance, independently of the type or strength of the interference [15].

In the case of malevolent controlled jamming where the jammer is, in theory, capable of successfully attacking any type of deterministic threshold biasing scheme, the receiver's best defense is to choose a random threshold strategy such as dithering [15, 18 and 19]. In dithering, random addition of noise at the receiver restores partial correlation by seizing partial control of the interference so as to make it less destructive than is the channel's interference.

Chang has shown that by sampling at higher rate without reducing the observation time or equivalently, by taking a large number of samples with fixed sampling rate, the error increase due to quantization, sampling and dithering can be decreased to a negligible value [20].

With today's IC technology, multilevel quantization and adaptive threshold biasing is no longer a physical problem. This suggests that DMF's can now replace AMF's and yield substantial benefits. DMF's can presently be built with off-the-shelf components and no longer require the expertise and technology of SAW devices. Yet, DMF's can offer an operational flexibility which is certainly unavailable with AMF's.
CHAPTER 3

BAIER'S DMF
3.1 BAIER'S DMF THEORY

3.1.1 INTRODUCTION

In the previous chapter, we showed that while the DMF suffered a small degradation over its analog counterpart, it offered several advantages. A SAW device used for AMF is frequency/phase and waveform matched at the factory and usually cannot be reprogrammed in the field or in an operational environment. A DMF, on the other hand, is extremely versatile and can be easily reprogrammed at anytime by the receiver.

Alfred Baier proposed in 1983-84 to extend Turin's noncoherent DMF technique with one bit quantization and designed a DMF which is matched or programmed for arbitrary constant envelope SS waveforms rather than BPSK only. This new DMF, called four-phase DMF, can outperform Turin's original DMF by as much as 3 dB when both DMF are subjected to waveforms having complex valued envelopes. The four-phase DMF is readily built with inexpensive off-the-shelf components and guarantees a performance very close to the upper bound.

3.1.2 NONCOHERENT TWO-PHASE DMF

To properly understand the reasons why Baier proposed his non-coherent four-phase DMF, it is best to start with Turin non-coherent two-phase DMF (fig. 3.1) and derive its output signal Z [15, 21]. The transmitted bandpass waveform is of duration T and band limited to the bandwidth W. W is chosen to be large enough such that the out-of-band power
Fig. 3.1 Noncoherent two-phase DMF

Fig. 3.2 Noncoherent four-phase DMF
of the original waveform is negligible and also, that the waveform is only slightly distorted. This way, the assumptions of finite duration and constant envelope are respected by the band limitation [22].

Sampling at Nyquist rate, at baseband $f_s = W$, the complex envelope $u_o(t)$, we get

$$N = f_s.T = WT \quad \{3.1\}$$

complex samples. We can write these samples as [16, 22]

$$u_{ok} = |u_{ok}|e^{j\phi_k}; \quad k = 1, 2, \ldots, N. \quad \{3.2\}$$

Similarly, we can look at the complex envelope of the interfered signal received at the input of the filter where [16, 22]

$$u_k = A.u_{ok}.e^{j\phi} + n_i + j.n_qk; \quad k = 1, 2, \ldots, N. \quad \{3.3\}$$

$A$ is the amplitude of the SS signal while $\phi$ is the unknown relative carrier phase with respect to the local receiver oscillator. It should be noted that $\phi$ is uniformly distributed on $[0, 2\pi]$ but assumed constant over each signal duration. The terms, $n_i$ and $n_q$, represents the superimposed stationary white gaussian noise samples divided into in-phase and quadrature components. The noise has a double-sided power spectral density $No/2$ and is band limited, as the signal, to $W$. This means that these samples are also gaussian with zero mean and along with the selected sampling rate, are statistically independent [7]. Either quadrature component has a variance $\sigma^2$ which is equal to the input noise power [7]

$$P_n = No.W = \sigma^2 \quad \{3.4\}$$

contained in the bandwidth $W$. As usual, the input signal-to-noise ratio is the quotient of the average signal power $P_s$ during signal duration $T$ and the noise
power $P_n$ or

$$SNR_i = \frac{P_s}{P_n} = A^2/2\sigma^2 \cdot \frac{1}{N} \sum_{k=1}^{N} |u_{ok}|^2 . \tag{3.5}$$

Using Turin's low-pass system representation, the analog time-discrete matched filter has the impulse response

$$h_k = K. u_r(N-k+1)^* \ ; \ k = 1, \ldots, N \tag{3.6}$$

which is matched to the reference signal

$$u_{rk} = u_{ok} e^{j\Psi} \ ; \ k = 1, \ldots, N . \tag{3.7}$$

$K$ in (3.6) is an arbitrary gain factor and will be taken as unity henceforth, for simplicity. It is important to recognize that the received signal (3.3) and the reference signal (3.7) must be sampled at the same instant. $\Psi$ is an arbitrary fixed phase shift of the reference but has no effect on the output of a noncoherent analog filter matched to $u_r$ [22].

When the maximum output signal-to-noise ratio (i.e. $SNR_o$) is reached, the corresponding output signal of the noncoherent square-law AMF is given by [15, 22]

$$Z = \left| \sum_{k=1}^{N} u_k.u_{rk}^* \right|^2 = (\sum_{k=1}^{N} u_{ik}.u_{irk} + \sum_{k=1}^{N} u_{qk}.u_{qrk})^2 +$$

$$+ (\sum_{k=1}^{N} u_{qk}.u_{irk} - \sum_{k=1}^{N} u_{ik}.u_{qrk})^2 . \tag{3.8}$$

where $i$ and $q$ denote, respectively, the in-phase and quadrature components of $u_k$
and urk. The second equation of (3.8) indicates that the best low-pass realization of a noncoherent matched filter should be done using four real filter channels. For BPSK signals, two channels suffice, because with $\psi = 0$ and phase values $\phi_{ok} = 0$ or 180, all samples $uqrk$ are zero.

This logic lead Baier to extend Turin's two-phase DMF (fig. 3.1) to an all new four-phase DMF by adding two cross-channels for the imaginary part $uqr$ of the reference signal (fig. 3.2, page 50).

3.1.3 NONCOHERENT FOUR-PHASE DMF

Baier, in [21] and [22], proposed a low cost implementation of the noncoherent four-phase DMF by means of sign detectors (fig. 3.2). When $\text{SNR}_0$ is maximum, this particular filter performs the modified correlation

$$Z = \left| \sum_{k=1}^{N} \left[ \text{sign}(uik) + j \cdot \text{sign}(uqk) \right] \cdot [\text{sign}(uirk) + j \cdot \text{sign}(uqrk)]^{*} \right|^2.$$  \hspace{1cm} (3.9)

As shown in figure 3.3, the binary quantization of the quadrature components represents a four-phase quantization in the complex plane where the four quadrants become the quantization regions and can only take on the complex values $\pm 1 \pm j$. As we'll see later, it is convenient to use polar coordinates to write the phasors $uqrk$ of the quantized reference signal as

$$uqrk = \text{sign}(uirk) + j \cdot \text{sign}(uqrk)$$

$$= \sqrt{2} \cdot e^{j \pi/4} \cdot e^{j m \pi/2}, \quad m \in \{0,1,2,3\}.$$  \hspace{1cm} (3.10)

Using (3.7) and (3.10), the phase quantization errors of the reference samples can be defined as
Fig. 3.3 Four-phase quantization
\[ \theta_k = \arg(\text{urk}) - \arg(\text{urk}) = (\phi_k + \psi) - (\pi/4 + mk\pi/2) \quad \{3.11\} \]

where each \( \theta_k \) lies in the interval \([\pi/4, \pi/4]\) and is also dependent on the phase shift \( \psi \) of the reference signal.

Just as Turin [15] derived the minimum degradation of the two-phase noncoherent DMF with BPSK signaling (appendix C), Baier [22] derived the minimum degradation of his four-phase noncoherent DMF for constant envelope SS waveforms (i.e. PN-BPSK, PN-QPSK, PN-OQPSK, PN-MSK, Chirp, PN-chirp and arbitrarily angle modulated). Therefore, we want to derive a general expression for the maximum

\[ \text{SNR}_0 = \left\{ \frac{\text{E}[\vert Z \vert^2]}{\text{E}[\vert Z \vert^2]} \right\}^2 \quad \text{Var}[\text{Z} \vert \text{S+N}] \quad \{3.12\} \]

of the four-phase DMF of figure 3.2. In our analysis, we will assume the general SS signal case where WT is large and SNRi \( \ll 1 \).

Before we proceed with the analysis, we have to investigate the effect of multiplying the quantized received signal by the quantized conjugate complex reference samples \( \text{urk}^* \). Therefore applying \( \{3.10\} \), \( Z \) given by \( \{3.9\} \) becomes

\[ Z = \sqrt{2} \cdot e^{j\pi/4} \cdot \sum_{k=1}^{N} e^{-jm\pi/2} \cdot \text{sign}[\text{Re}(\text{uk})] + j \cdot \text{sign}[\text{Im}(\text{uk})] \quad \{3.13\} \]

where the quadrature components \( \text{uik} \) and \( \text{uqk} \) are expressed respectively in terms of the real and imaginary part of \( \text{uk} \). A closer look at figure 3.3 reveals that rotating the quantized phasors of \( \text{uk} \) in \( \{3.13\} \) by an integer multiple of \( \pi/2 \) is equivalent to rotating the phasors \( \text{uk} \) themselves by the same angle before quantization takes place. Therefore, \( \{3.13\} \) becomes
\[ Z = \left| \sqrt{T} \sum_{k=1}^{N} \text{sign}[\text{Re}(u_k e^{-j mk\pi/2})] + j \text{sign}[\text{Im}(u_k e^{-j mk\pi/2})]\right|^2 \]

where the constant phase factor \( e^{-j \pi/4} \) has been omitted since it has no influence on the square envelope \( Z \). Given the received signal of \( u_k \) of (3.3), calculating the real and imaginary part of (3.14) leads to

\[ Z = V_i^2 + V_q^2 \]

where

\[ V_i = \sqrt{T} \sum_{k=1}^{N} \text{sign}[\text{A} \cdot |u_k| \cdot \cos(\phi_k + \phi - mk\pi/2) + n_i k] \]

\[ = \sqrt{T} \sum_{k=1}^{N} W_{ik} \quad \text{and} \]

\[ V_q = \sqrt{T} \sum_{k=1}^{N} \text{sign}[\text{A} \cdot |u_k| \cdot \sin(\phi_k + \phi - mk\pi/2) + n_q k] \]

\[ = \sqrt{T} \sum_{k=1}^{N} W_{qk} \].

It should be noted the \( V_i \) and \( V_q \) represented here are not the same as \( V_i' \) and \( V_q' \) of figure 3.2, because of the factor \( e^{-j \pi/4} \), but most important they yield the same square-law output \( Z \). Similarly, because we assumed that the bandpass noise was gaussian, \( n_i k' \) and \( n_q k' \) which arise from phase rotation display exactly the same statistics as \( n_i k \) and \( n_q k \) of (3.3) [7]. Most important is the fact that they are statistically independent from each other. Using the phase quantization errors \( \theta_k \) of (3.11) and a modified relative carrier phase \( \phi s' \) which is held constant over each signal duration as \( \phi s \), is, it is possible to express the argument of the trigonometric functions of (3.16) and (3.17) as follows

\[ \phi_k + \phi - mk\pi/2 = (\phi_k + \psi) - (\pi/4 + mk\pi/2) + (\phi - \phi + \pi/4) \]

\[ = \theta_k + \phi s' \].

\[ \{3.18\} \]
Wik and Wqk of (3.16) and (3.17) are random variables corresponding to simple Bernoulli trials which can only take the values +1 or -1. If we define the probabilities that Wik and Wqk equal -1 by pık and p$qk$ respectively, then the probabilities that they equal +1 are 1 - pık and 1 - p$qk$ respectively [7]. If we approximate the gaussian distributions of nik' and n$qk'$ by the condition SNRi << 1 and use (3.18), we create an analogy to (C.27) and (C.28) of appendix C where

\[
1 - 2\text{pık} = (2A_0 \text{uok} / \sqrt{2\pi} \sigma) \cos(\theta k + \phi s') \quad \text{and} \quad (3.19)
\]

\[
1 - 2\text{p$qk$} = (2A_0 \text{uok} / \sqrt{2\pi} \sigma) \sin(\theta k + \phi s') . \quad \text{(3.20)}
\]

The next step in our development of SNRo is the determination of the mean and the variance of Z. This is where the parallelism with Turin's development of SNRo for the two-phase DFM ends. Unlike Turin's case [15], the terms Wik and Wqk of (3.16) and (3.17) are not generally identically distributed for all k, which means that using the moment generating function would be too cumbersome. Instead we'll use the central limit theorem since the terms Wik and Wqk are independent and their variances never vanish [7]. The first condition comes from the fact that all noise samples nik' and n$qk'$ are statistically independent and the second, from the weak signal condition.

If we have a large WT, the number N of samples will also be large and the sums Vi and Vq will be approximately gaussian. Since we know the probability distributions of Wik and Wqk, we can determine the means and variances of Vi and Vq as follows

\[
E[Vi | S+N] = \sqrt{2} \sum_{k=1}^{N} E[\text{ Wik} ] = \sqrt{2} \sum_{k=1}^{N} (1-2\text{pık}) , \quad \text{(3.21)}
\]

\[
E[Vq | S+N] = \sqrt{2} \sum_{k=1}^{N} E[\text{ Wqk} ] = \sqrt{2} \sum_{k=1}^{N} (1-2\text{p$qk$}) , \quad \text{(3.22)}
\]
Fig. 3.6 Digital section (one of four)

Fig. 3.7 LPF response
In deriving these expressions we used the independence of $V_i^2$ and $V_q^2$, (3.15), (3.25) and (3.26). Since the variances of $V_i$ and $V_q$ are nearly identical, the output $Z$ is approximately chi-square distributed with two degrees of freedom [22]. This is rather interesting since from [15], the output $Z$ of the noncoherent square-law AMF is also chi-square distributed with two degrees of freedom. This means that, for $W T \gg 1$ and $SNR_i \ll 1$, the derived $SNR_o$ is a proper measure of performance comparisons between the four-phase DMF and the corresponding square-law AMF [15, 22].

Using equations (3.19) to (3.22) on (3.27), we get

$$E^2[V_i | S+N] + E^2[V_q | S+N] = 2 \cdot \frac{(4A^2/2\pi)^2 \cdot \left( \sum_{k=1}^{N} \left| u_{ok} \cos(\theta_k + \phi_s') \right|^2 + \frac{\left( \sum_{k=1}^{N} \left| u_{ok} \right| \cdot \sin(\theta_k + \phi_s') \right)^2}{\sum_{k=1}^{N} \left| u_{ok} \right|^2} \right)}{N \cdot (SNR_i)}$$

$$= 4 \cdot \frac{d \cdot N \cdot N \cdot (SNR_i)}{\pi} \cdot \frac{1}{N \cdot \sum_{k=1}^{N} \left| u_{ok} \right|^2} \cdot \left| u_{ok} \right|^2$$

where $SNR_i$ is as given in (3.5) and

$$d = 2/\pi \cdot \left( \sum_{k=1}^{N} \left| u_{ok} \cdot e^{j\theta_k} \right|^2 / N \cdot \sum_{k=1}^{N} \left| u_{ok} \right|^2 \right)$$

(3.31)
From Schwarz inequality, \( d \) is less than or equal to \( 2/\pi \). Also, \( \{3.30\} \) and \( \{3.31\} \) are independent of the unknown relative carrier phase \( \phi_s' \) [22]. Similarly,

\[
\text{Var}[V_i | S+N] + \text{Var}[V_q | S+N] - \text{Var}[V_i | N] - \text{Var}[V_q | N] = \\
-2 \sum_{k=1}^{N} (1-2\pi k)^2 - 2 \sum_{k=1}^{N} (1-2\pi q k)^2 \\
= -2.4A^2/2\pi^2 \sum_{k=1}^{N} |u_{ok}|^2 \\
= -4.2/\pi.N.(SNR_i). \quad \{3.32\}
\]

Here we have used \( \{3.23\}, \{3.24\}, \{3.19\}, \{3.20\} \) and \( \{3.5\} \) respectively. Substituting the results of \( \{3.30\} \) and \( \{3.32\} \) into definition \( \{3.12\} \), we get

\[
\text{SNR}_{\text{Ro}}(4\phi-\text{dmf}) = [(d.N-2/\pi)(SNR_i)]^2 / (2.d.N.(SNR_i) + 1) \quad \{3.33\}
\]

for \( N >> 1 \) and \( \text{SNR}_i << 1 \).

Baier has shown that \( d \) is not very much smaller than \( 2/\pi \) for all constant envelope signals and many other types of signals such as MSK and chirp [21]. As \( N \) is large and \( d \) is approximately \( 2/\pi \), the term \( 2/\pi \) in the numerator of \( \{3.33\} \) can be neglected against \( d.N \) to yield
SNRo(4φ-dmf) = \[d \cdot N \cdot (SNR_i)\]^2 / (2 \cdot d \cdot N \cdot (SNR_i) + 1) \tag{3.34}

for \( N \gg 1, SNR_i \ll 1 \) and \( d \gg 1/N \). From \([15]\),

\[SNRo(\text{amf}) = [N \cdot (SNR_i)]^2 / (2N \cdot (SNR_i) + 1) \tag{3.35}\]

where, as in (3.34), \( N = WT \). Therefore, the performance degradation of the four-phase DMF with reference to the corresponding AMF is

\[D(\text{dB}) = 10 \log_{10} \left( 1/d \right) \geq 1.96 \text{ dB} \]. \tag{3.36}

3.1.4 OPTIMIZATION OF THE FOUR-PHASE DMF

In the previous section, we showed that Bauier's four-phase DMF has the same upper bound as Turin's two-phase DMF. But unlike its two-phase counterpart, which can only reach this limit with BPSK signaling, the four-phase DMF can reach this upper bound with all constant envelope signals and many other signals. In fact, Bauier derived a lower bound for arbitrary constant envelope waveforms for his DMF \([22]\). In this particular case, \(3.31\) becomes

\[d \geq 2/\pi \cdot 0.8/\pi^2 \cdot \sum_{k=1}^{N} |u_{ok}|^2 / (N \cdot \sum_{k=1}^{N} |u_{ok}|^2) \tag{3.37}\]

and using Schwarz inequality, the maximum degradation becomes

\[D(ce) \leq 10 \log_{10} \left( \pi^3 / 16 \right) \text{ dB} = 2.87 \text{ dB} \]. \tag{3.38}

Therefore, when the four-phase DMF is properly matched, an arbitrary constant envelope waveform is bounded as follows

\[1.96 \text{ dB} \leq D(ce) \leq 2.87 \text{ dB} \]. \tag{3.39}
For a two-phase DMF, for an arbitrary constant envelope waveform, the maximum degradation is [22]

\[
D(ce)' \leq 10 \log_{10} \left( \frac{\pi^2}{8} \right) \text{ dB} = 5.88 \text{ dB}
\]

or is bounded by

\[
1.96 \text{ dB} \leq D(ce)' \leq 5.88 \text{ dB}
\]

which means the four-phase DMF can outperform its two-phase cousin by as much as 3 dB in this situation.

To reach the minimum degradation of 1.96 dB, the four-phase DMF must be optimized. This condition can only be achieved if all amplitude values \(|u_{ok}|\) as well as all phase quantization errors \(\theta_k\) are identical [21]. Therefore, we have two conditions to fulfill. First, the optimum waveforms must be of the constant envelope type and secondly, the arguments \(\phi_{ok}\) of their samples \(u_{ok}\) must only differ by integer multiples of \(\pi/2\). The SS waveforms that meet these two conditions are the PN-BPSK, PN-QPSK and PN-OQPSK. PN-MSK can also fulfill these conditions if it is sampled once per chip at the chip boundaries or chip centers or if it is sampled twice per chip, but \(f_0\) is the signaling frequency rather than the center frequency [21, 22]. To highlight the difference between the two-phase and the four-phase DMF, Baier [22] derived the upper bound for several waveforms and are presented here in table 3.1.
TABLE 3.1

Degradation for constant envelope SS waveforms (dB)

<table>
<thead>
<tr>
<th>Waveform</th>
<th>4-Phase DMF</th>
<th>2-Phase DMF</th>
</tr>
</thead>
<tbody>
<tr>
<td>PN-BPSK</td>
<td>1.96</td>
<td>1.96</td>
</tr>
<tr>
<td>PN-QPSK</td>
<td>1.96</td>
<td>4.97</td>
</tr>
<tr>
<td>PN-OQPSK</td>
<td>1.96</td>
<td>4.97</td>
</tr>
<tr>
<td>PN-MSK (1 sample/chip)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fo = center freq</td>
<td>1.96</td>
<td>4.97</td>
</tr>
<tr>
<td>fo = signaling freq</td>
<td>1.96</td>
<td>1.96</td>
</tr>
<tr>
<td>PN-MSK (2 samples/chip)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fo = center freq</td>
<td>2.65</td>
<td>5.66</td>
</tr>
<tr>
<td>fo = signaling freq</td>
<td>1.96</td>
<td>4.00</td>
</tr>
<tr>
<td>Chirp &amp;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chirp-PN-PSK</td>
<td>2.87</td>
<td>5.88</td>
</tr>
<tr>
<td>Arbitrarily modulated</td>
<td>1.96...2.87</td>
<td>1.96...5.88</td>
</tr>
</tbody>
</table>

When Baier propounded his four-phase DMF theory [21] and proposed a low cost DMF for arbitrary constant envelope SS waveforms [22], his hardware project was under construction and he had no experimental data to support his theory. Intrigued by this new breed of DMF and its performance, the author decided to build a four-phase DMF and investigate its performance with two different waveforms in two separate environments.
3.2 IMPLEMENTATION OF BAIER'S DMF

3.2.1 INTRODUCTION

Upon deciding to build and test Baier's DMF, a few initial criteria were established. The implementation of figure 3.4 would be accomplished using only off-the-shelf components to minimize the cost. Secondly, optimization of performance would dictate the final selection of components. Thirdly, operating frequencies would be the normal or most commonly used in today's operational systems. Fourthly, wirewrapping techniques would be used as the connecting medium.

The first criteria also meant that no lead time were required for components. The second criteria meant that every component characteristics, such as operating bandwidth, noise immunity, power requirement, dynamic range, etc., would all have to be scrutinized. The top two or three candidate components would then be tested in the experimental circuit and the best performer would be selected.

In order to be able to conduct a careful operational investigation of Baier's DMF, the four-phase DMF would have to be implemented using two arms, a control arm and a reference arm. The reference arm would be hardwired to the transmitter and would replace the receiver's generation of the waveforms to be matched. The control arm would have an injection port permitting a controlled addition of noise or interference. For implementation purposes, the DMF was divided into three sections, the RF, the digital and the correlator sections.
Fig. 3.4 Noncoherent digital Matched Filter (4-phase DMF)

Fig. 3.5 RF section
3.2.2 THE RF SECTION

Mainly concerned with SS voice communications, the author selected a Hewlett-Packard (HP) model 1645A data error analyzer to supply the information to the modulator at a 19.2 kbps rate (fig. 3.5) with a 2047 bit repeating pattern. A highly stable Rockland model 5600 frequency synthesizer was chosen for the 70 MHz reference signal to the modulator. The modulator or modem, a home-made Communication Research Centre (CRC) model, could modulate one of eight data rates (75 to 19.2 kbps) at 70 MHz and generate one of four modulated signal (coherent PSK or CPSK, coded CPSK, MSK and coded MSK). A HP model 3760A data generator fed with a 2.5 MHz reference clock was used to generate a $2^{15}$ PN code and spread the modulated signal. The 5 MHz oscillator's pulse was cleaned up with a Schmitt trigger (Motorola 74LS14). A quad D flip-flop (Motorola 74LS175) provided the 2.5 MHz reference clock by dividing the oscillator's pulse frequency by two.

The DS-SS 70 MHz signal, with a 5 MHz bandwidth, was split to produce the two arms required for the experimental study. A 3dB attenuator pad was added to the reference arm to compensate for the injection port loss in the control arm. Each arm's signal was further separated into two low-pass components I and Q. The carrier was removed using a 70 MHz local oscillator (LO) signal generated by a Rockland frequency synthesizer and the Q arms were phase shifted as required.

The selected mixers were Watkins-Johnson M1's with a 0.2 - 500 MHz bandwidth capability. The phase shifters used to create the required 90 degree
phase shift between the I and Q arms were Anzac JH-10-4 with an operational bandwidth capability of 20 - 140 MHz. The injection port was implemented using an Olektron O-HJ-302 power combiner which had an operational bandwidth of 2 - 400 MHz. The power dividers selected to split the signal were Olektron O-HJ-402 devices with a 2 - 100 MHz range. The power divider used to split the frequency synthesizer output into two signals (i.e., modem's carrier and CW jamming signals) was an Olektron B-HJ-303H with a 1 - 100 MHz operating range.

In selecting all RF components, the designer purposely opted for wide bandwidth devices to enhance the flexibility of the experimental DMF. In other words, the system was designed to be able to operate over a wide range of frequencies with effortless readjustment capability if need be. In fact, all one had to do was to select another synthesizer's frequency.

3.2.3 THE DIGITAL SECTION

Figure 3.6 displays the digital section. Considerable effort was spend designing and testing low-pass filters. Chebyshev and Butterworth passive and active LPF's were designed using tables [24] and implemented using wirewrapping techniques. A few problems quickly developed; the desired sharp rolloff could not be obtained using a few sections, acceptable noise/interference immunity could not be obtained using wirewrapping and a good active filter was difficult to implement because of the bandwidth of the signal. The 2.5 and 5.0 MHz clock pulses produced tones which were picked up by the wirewrapping posts and were much stronger than the received signal. These could not be completely attenuated using wirewrapping techniques.
Fig. 3.6 Digital section (one of four)

Fig. 3.7 LPF response
After much investigation, there appeared to be only one solution, a four section m-derived \((m=0.6)\) shielded LPF [25]. This LPF was enclosed in a Pomona RF shielded box within which each section was also individually shielded. The result was a passive LPF with an implementation loss of less than 0.5 dB and a ripple less than 1.2 dB in the passband (fig. 3.7, page 65). As customary in DMF, the cut-off frequency \(f_{co}\) was selected as half the sampling rate [15, 22]. At \(f_{co}\), the loss was about 1.8 dB and at 3.6 MHz, the signal was attenuated to approximately \(-85\) dB. As a result, a crisp, clean signal with no overtones was produced and the shielded lines and receptacles provided excellent noise immunity.

The location of the amplification stage required some careful planning. At first, it was assumed that amplification should be performed before heterodyning. After trials and research, this suggestion was rejected for at least two reasons [26]. The mixers employed were current limited devices and hence amplification would have to be limited. Secondly, mixer theory requires the LO signal to be much stronger than the received signal for superior intermodulation suppression. Theoretically, the LO signal should be 15 to 20 dB higher than the received signal. The experimental LO signal was set to approximately 3 dBm while the experimental received signal was limited to about \(-14.25\) dBm. Therefore, RF amplification was discarded and baseband amplification was implemented.

The filtered signal had an amplitude of \(+/- 60\) mvolts and required amplification before sampling. Wirewrapping techniques were used from the output of the LPF to the output of the DMF. A wide bandwidth high speed operational
amplifier (Fairchild uA715) with a high slew rate was selected to drive the non-inverting amplifier (A = 90). Its output amplitude ranged from -5.5 to +5.5 volts. A quad analog switch (Motorola MC14066B) controlled by the 5 MHz clock sampled the waveform twice per chip to achieve finer resolution. Each chip was sampled near its boundary and near its centre. A zero-crossing detector using a high speed dual comparator (National Semiconductor LM319) was designed as the one bit digitizer.

To ensure proper synchronization of both PN codes, a delay network was designed to compensate for the different path lengths of the two arms and the injection port delay of the control arm. Digital delay lines (Digital devices DDU-4) tapped at each 50 or 100 ns intervals were used together with an hexadecimal coded switch (Grayhill 79B16) and two 8-input multiplexers (Motorola 74LS251) cascaded to provide a four chip variable delay to the control arm. The reference arm was given a fixed two chip delay using the same type of digital delay lines. This delay network also enabled us to briefly look at the correlation triangle of the two PN codes.

To achieve good synchronization, a relative time advance of about 50 ns was required in the control arm for both waveforms (CPSK and MSK). With no advance, the correlation only reached approximately 92% of its maximum value. With 50 ns advance, it climbed the correlation triangle to 97% on the average. Since the digital delay lines available had a 50 ns delay as the smallest interval, it was impossible to determine if slightly less or more than 50 ns would be required for perfect synchronization. With a delay or an advance of half a chip (ie. 200 ns), the autocorrelation dropped to around 50% in both cases. In fact, when the delay or advance was more than a chip, the correlation
results dropped to about zero which agreed with the autocorrelation theory of PN
codes [7]. A 16 channel logic state analyzer (HP model 1600A) was used to
simultaneously observe the correlation results of two correlators.

While the digital delay line network solved the synchronization
problem, it spoiled the signal's waveform. The network's output signal had only
an amplitude of about 3 volts with a 10% high frequency ripple. To restore the
original waveform and give good noise immunity to the system, a Schmitt trigger
(Motorola 74LS14) was used. The trigger yielded a sharply defined, jitter-free
output signal TTL compatible with an excellent noise margin. A quadruple
complementary-output element chip (Texas Instruments SN74265) provided each
arm's correlators with symmetrical complementary inputs as required by figure
3.4 and restored the correct polarity of the signal.

3.2.4 THE CORRELATOR SECTION

The heart of the four-phase DMF is the 64 bit digital output
correlator (TRW model TDC1023). This new and first-of-its-kind monolithic device
(fig. 3.8) consists of three 64 bit independently clocked shift registers, one
64 bit reference holding latch and a 64 bit independently clocked digital
summing network. The manufacturer cites that this device is capable of a 17 MHz
parallel correlation rate. The 7 bit threshold register allows the user to
preload a binary number 0 to 64. Whenever the correlation is equal to or greater
than that number, the threshold flag goes high to indicate acquisition or coarse
synchronization. In our experiments, the 64 bit mask shift register
(M register), which allows the user to mask or selectively choose "no compare"
bit positions, was not used.
Fig. 3.8 Digital correlator (TRW TDC 1023)
The reference word (i.e. reference signal) is serially shifted into the B register while the control word (i.e. control signal) is serially shifted in the A register. The R latch is clocked to allow the data to be parallel-loaded into the R reference latch. This allows the user to serially load a new reference word into the B register while correlation is taking place between the A register and the R latch. The two words are continually compared bit-for-bit by exclusive-OR circuits. Each exclusive-OR provides one bit to the digital summer. The correlator output is a 7-bit word representing the sum of positions which agree at any one time between the A register and R latch. A control provides either true or inverted binary output formats.

The shift registers, the exclusive-OR gates and the summer fulfill the three functions of correlation: time delay, multiplication and integration, respectively. This digital correlator can perform both correlation and convolution, operating according to the discrete summation equations.

The digital correlator is a very flexible and powerful device. It can be cascaded to provide extended length correlation (i.e. 128, 192, 256 bits, etc.) or can be arranged to either provide a multi-bit word by one bit word correlation or two multi-bit word correlation. In all these three cases, an external digital summer is required.

From figure 3.4, the implementation of the correlator section required four digital correlators. Both I and Q arms' symmetrical complementary output elements (i.e. a real and an imaginary channel) each fed one correlator.
The correlation results of the two cross-channels were added to the other two channels, as shown in figure 3.4, using cascaded 4 bit binary full adder with fast carry (National Semiconductor 74LS283). The square law devices (i.e. envelope detectors) were implemented with LSI nXn bit parallel multipliers (TRW model MPY-12AJ). Adding the two outputs of the square law devices yield Z as a 16 bit word.

It should be noted again that figure 2.2 is the actual spectrum of the DS-SS experimental signal and figure 3.7 is the actual experimental LPF frequency response.
CHAPTER 4

EXPERIMENTAL EVALUATION OF BAIER'S DMF
4.1 INTRODUCTION

The experimental evaluation consisted of broadband noise jamming and tone jamming trials at the carrier frequency of the DS-SS signal with CPSK and MSK modulations. To satisfy equation (3.12), a large number of samples of $Z$ were taken using a logic state analyzer (HP model 1600A). This 16 channel logic state analyzer had a 32 channel storage capacity (i.e. 512 bits) which meant that $Z$ could be sampled twice and each thirty-two 16 bit digital word would be stored. The thirty-two samples would be recorded and the process would be repeated.

Determining the required sample size without a priori knowledge of the population required careful trials. Using the Empirical Rule [27], a sample size of approximately 1000 would yield a high confidence probability (i.e. about 95%) that our estimate ($\mu$) would lie within $2\sigma$ of the population mean. Several trials were conducted using a sample size of 63 sets of 16 samples (i.e. 1008 samples) and two observations were made. The mean, $\mu$, of $Z$ (signal plus noise) tended to stabilize after 500 to 600 samples. However, the remaining 400 or 500 samples were required to reduce and stabilize its variance. Due to the random properties of noise, the mean of $Z$ (noise only) required at least 900 samples to stabilize. Therefore, at each selected SNRi, 1008 samples would be recorded for each condition (signal plus noise and noise only). Using equation (3.12), SNR0 could be calculated for each SNRi.
4.2 BROADBAND NOISE JAMMING

A broadband noise source was created by terminating an RF amplifier (Aventek UTO523) input with a 50 ohm impedance. Four stages of amplification were cascaded to produce the dynamic range required of the power spectral density. A variable power control was added before the last amplification stage. As defined in [1], the jamming signal was bandlimited to occupy approximately the total RF bandwidth of the DS-SS system.

Using a power meter (HP model 435A), the signal power was measured and the broadband noise power was set to meet a particular SNRi. Next, the 1008 samples of Z (signal plus noise) were taken and recorded. For the samples of Z (noise), the signal arm of the interference injection port was disconnected and capped. As shown in table 4.1, several SNRi's were tried to evaluate the four-phase DMF's response and determine its dynamic range. For experimental purposes, the dynamic range was defined as the range over which the four-phase DMF showed a performance degradation of approximately less than 1 dB from the upper bound.

As customary in DS-SS systems, the broadband noise jamming signal was modelled as a zero-mean wide sense stationary gaussian noise process with a flat power spectral density over the RF bandwidth of interest [1, 15]. From chapter 3 and appendix C, the upper bound becomes

\[
\text{SNR}_o = (2/\pi)(TW/2)\text{SNR}_i = (2/\pi)((N/2)\text{SNR}_i)
\]

\{(4.1)\}

The results for the DS-SS CPSK and MSK modulations submitted to broadband noise jamming can be examined in tables 4.1 and 4.2, and graphs 4.1 and 4.2. In both cases, the observed degradation in the dynamic range was
minimal and the four-phase DMF's performance was very close to the upper bound. The dynamic range was approximately 10 dB and roughly the same for both waveforms. When the first results were tabulated, the fact that the dynamic range began around -7 dB raised a few questions. After much research and more trials, the researcher was pleased to confirm what Turin and others had postulated; "The one-bit DMF suffers only a 1.96 dB degradation over its analog rival or reaches its upper bound for gaussian interference only when SNRi << 1 and SNRb >> 1" [15, 18, and 19]. While this verification might have been expected with CPSK, it was a pleasant surprise to confirm it also held with MSK. Table 4.2 and graph 4.2 prove that Baier's DMF 'outperformed' Turin's DMF by more than 2 dB (see Table 3.1) with DS-MSK signaling. With DS-CPSK, it performed equally well as Turin's DMF with the added versatility of being capable to receive several waveforms.

From equations (3.1) and (4.1), one can predict what the processing gain of the experimental DMF should be. The predicted gain should be (N/2) since (2/\pi) is the quantization loss. Since the experimental DMF has four 64-bit correlators, the number of samples processed is N = 256 and the DMF processing gain should be 21.07 dB when SNRi << 1 and SNRb >> 1 [15, 22]. From Tables 4.1 and 4.2, one can readily confirm that such gain was attained in the dynamic range since this gain is simply (SNRb - SNRi + 1.96 dB).

The DMF's processing gain should not be confused with the DS-SS system's processing gain, Gp. From equation (2.3), the latter is simply Gp = 2(3.5 MHz) / (19.2 KHz) = 24.16 dB. The DMF's processing gain is governed by the environment in which it operates, i.e. in a gaussian environment it has a 21.07
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<th>SNR0</th>
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Table 4.2

BROADBAND NOISE JAMMING (MSK)

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BROADBAND NOISE JAMMING (MSK)

GRAPH 4.2

SNR\(r_o\) (dB) vs. SNR\(i\) (dB)

- Upper bound
- Experimental points

Dynamic range
dB gain while in a tone jamming situation, it offers no processing gain. The processing gain of the DS-SS system, on the other hand, is by definition equation (2.3) regardless of the environment in which it operates.

4.3 TONE JAMMING

To an unprepared or unaware DS-SS system, the worst type of jamming is usually a single frequency sine wave or tone at the carrier frequency. Whereas the DMF had a processing gain of N/2 in the broadband noise jamming, it offers no gain against tone jamming. The effect of capture by the coherent constant amplitude interference can be very severe. When SNRi is around 0 dB, the interference cannot destroy both of the digital correlations Vi' and Vq' (fig. 3.2). When SNRi becomes much less than 0 dB, the interference can capture the DMF and annihilate both correlations Vi' and Vq', in which case, the performance deteriorates rapidly [15].

The interfering or jamming tone used was provided by the same frequency synthesizer that fed the modem. Transmitting at exactly the same frequency and maintaining quasi-coherence ensured the worst possible situation for this DS-SS system. The power density of the 70 MHz CW tone was varied by adding appropriate attenuator pads.

Turin derived an upper bound for the DMF in a tone jamming environment[15]. This upper bound, which is derived in appendix D, is

$$\text{SNRo} = \left(\frac{8}{\pi^2}\right)(\text{SNRi})^{0.5} \quad \{4.2\}$$

This upper bound also holds for Baier's DMF since the TW product has no effect on the outcome. One should remember that Baier's addition of two correlators gave a 3 dB advantage to his DMF over Turin's DMF in a
gaussian environment for most complex valued waveforms. Since there is no processing gain in the tone jamming situation, as clearly stated by equation \(4.2\), Turin's upper bound in a tone jamming still holds for CPSK. The advantage of Baier's DMF, as seen by the author, should be in its ability to be able to receive a weaker signal or ultimately, resist the tone jamming to a lower SNRi than its rival. As stated earlier, Turin's DMF degrades quickly in performance when SNRi \(<\!< 1\). This is very important since SS receivers usually operate in a SNRi \(<\!< 1\) environment. For MSK, the situation is a little different. From available literature, it seems Turin's DMF has never been used in tone jamming with MSK signaling due to suspected large degradation in performance (at least 2 dB) from the upper bound. Baier's DMF, on the other hand, should theoretically perform equally well with MSK and CPSK signaling because of its added cross-channels.

The results of the DS-SS CPSK and MSK signals submitted to tone jamming are presented in tables 4.3 and 4.4, and on graphs 4.3 and 4.4. As in the two previous experimental studies, the observed degradation in the dynamic range is minimal and the four-phase DMF performance approaches the upper bound. Again, the dynamic range was approximately 10 dB and roughly the same for both waveforms. As expected, Baier's DMF responded well to MSK signaling and showed no performance degradation when switched from CPSK to MSK modulation scheme.

When compared to broadband noise jamming, the dynamic range of the tone jamming moved up by about 5 dB. This phenomena agreed well with Turin's capture theory of the tone jamming [15].
### Table 4.3

**TONE JAMMING (CPSK)**

<table>
<thead>
<tr>
<th>SNRi (dB)</th>
<th>( \mu_{\text{noise}} )</th>
<th>( \mu_{\text{S+N}} )</th>
<th>( \sigma )</th>
<th>SNRo 4\Phi-DMF Bound (dB)</th>
<th>SNRo (dB)</th>
<th>Diff (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8218.1729</td>
<td>8160.7770</td>
<td>80.8722</td>
<td>-2.98</td>
<td>-0.91</td>
<td>2.07</td>
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<td>-1.35</td>
<td>8222.3988</td>
<td>8212.0156</td>
<td>14.0751</td>
<td>-2.64</td>
<td>-1.59</td>
<td>1.05</td>
</tr>
<tr>
<td>-2.58</td>
<td>8219.5213</td>
<td>8208.1904</td>
<td>15.3480</td>
<td>-2.64</td>
<td>-2.20</td>
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<td>-3.16</td>
<td>8224.5079</td>
<td>8211.9167</td>
<td>17.3347</td>
<td>-2.78</td>
<td>-2.49</td>
<td>0.29</td>
</tr>
<tr>
<td>-3.89</td>
<td>8227.1794</td>
<td>8205.0363</td>
<td>31.5179</td>
<td>-3.07</td>
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<td>0.21</td>
</tr>
<tr>
<td>-4.93</td>
<td>8209.3591</td>
<td>8201.8016</td>
<td>11.3204</td>
<td>-3.51</td>
<td>-3.38</td>
<td>0.13</td>
</tr>
<tr>
<td>-5.80</td>
<td>8218.8849</td>
<td>8209.5794</td>
<td>14.9374</td>
<td>-4.11</td>
<td>-3.81</td>
<td>0.30</td>
</tr>
<tr>
<td>-6.70</td>
<td>8236.4922</td>
<td>8224.8450</td>
<td>19.5791</td>
<td>-4.51</td>
<td>-4.28</td>
<td>0.23</td>
</tr>
<tr>
<td>-8.13</td>
<td>8243.5393</td>
<td>8231.0933</td>
<td>23.0248</td>
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<td>0.36</td>
</tr>
<tr>
<td>-9.50</td>
<td>8242.0076</td>
<td>8228.8909</td>
<td>25.9291</td>
<td>-5.92</td>
<td>-5.66</td>
<td>0.26</td>
</tr>
<tr>
<td>-10.85</td>
<td>8244.5938</td>
<td>8233.4577</td>
<td>23.3522</td>
<td>-6.43</td>
<td>-6.34</td>
<td>0.09</td>
</tr>
<tr>
<td>-11.46</td>
<td>8241.4683</td>
<td>8229.4167</td>
<td>29.0389</td>
<td>-7.64</td>
<td>-6.64</td>
<td>1.00</td>
</tr>
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<td>-12.55</td>
<td>8254.0952</td>
<td>8241.4613</td>
<td>32.7426</td>
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<td>-7.19</td>
<td>1.08</td>
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<td>-13.52</td>
<td>8253.2567</td>
<td>8242.9134</td>
<td>34.6805</td>
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<td>-7.67</td>
<td>2.84</td>
</tr>
<tr>
<td>-14.97</td>
<td>8216.1675</td>
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<td>-8.40</td>
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</tr>
<tr>
<td>-15.94</td>
<td>8222.0937</td>
<td>8231.2242</td>
<td>38.8153</td>
<td>-12.57</td>
<td>-8.88</td>
<td>3.69</td>
</tr>
</tbody>
</table>
TONE JAMMING (CPSK)

GRAPH 4.3

SNR_o (dB) vs SNR_i (dB) graph with experimental points and an upper bound. Dynamic range indicated.

- Dynamic range
- SNR_o (dB) axis
- SNR_i (dB) axis
Table 4.4

**TONE JAMMING (MSK)**

<table>
<thead>
<tr>
<th>SNRi (dB)</th>
<th>μ(noise)</th>
<th>μ(S+N)</th>
<th>σ</th>
<th>SNR0 4ϕ-DMF upper bound (dB)</th>
<th>SNR0 bound (dB)</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9263.6558</td>
<td>9977.4911</td>
<td>934.2666</td>
<td>-2.34</td>
<td>-0.91</td>
<td>1.43</td>
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<tr>
<td>-1.25</td>
<td>8977.1409</td>
<td>9581.8621</td>
<td>794.9441</td>
<td>-2.38</td>
<td>-1.54</td>
<td>0.84</td>
</tr>
<tr>
<td>-2.50</td>
<td>9045.9403</td>
<td>9709.2996</td>
<td>879.3997</td>
<td>-2.45</td>
<td>-2.16</td>
<td>0.29</td>
</tr>
<tr>
<td>-3.77</td>
<td>8701.9246</td>
<td>9201.1188</td>
<td>718.4254</td>
<td>-3.16</td>
<td>-2.80</td>
<td>0.36</td>
</tr>
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<td>-5.12</td>
<td>8627.3581</td>
<td>9072.6637</td>
<td>688.5517</td>
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<td>-3.47</td>
<td>0.32</td>
</tr>
<tr>
<td>-6.35</td>
<td>8588.6425</td>
<td>8990.2153</td>
<td>703.1849</td>
<td>-4.87</td>
<td>-4.09</td>
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</tr>
<tr>
<td>-7.87</td>
<td>8434.4189</td>
<td>8748.5268</td>
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<td>0.68</td>
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<td>582.3873</td>
<td>-11.69</td>
<td>-8.79</td>
<td>2.90</td>
</tr>
</tbody>
</table>
CHAPTER 5

CONCLUSION
5.1 SUMMARY

This thesis contains a comprehensive treatment of Digital Matched Filters used in Direct Sequence Spread Spectrum systems for synchronization's purposes. The study was carried out at both the theoretical and experimental levels with very good agreement.

First, a thorough study of Spread Spectrum theory was presented. Two systems stood out among the different types of Spread Spectrum, the Direct Sequence and the Frequency Hopping systems. To date in modern Spread Spectrum systems, Direct Sequence has been the most popular type despite its stringent timing requirements. With better and cheaper frequency synthesizers being developed, Frequency Hopping is quickly evolving. Frequency Hopping requires relaxed timing requirements and can offer better communication security and reliability in a hostile environment. In secure military systems, the new trend is to select a hybrid system where Direct Sequence Spreading and Frequency Hopping are performed successively.

Second, the author focused on the most difficult problem of all Spread Spectrum systems, synchronization of the transmitter and receiver pseudorandom codes. The synchronization problem is always separated into two phases, acquisition and tracking. In Spread Spectrum, several studies have shown that Matched Filters provide faster acquisition and Early-Late Gate Tracking Loops provide better tracking in friendly and hostile environments.

Third, the dilemma of Analog versus Digital matched filtering was studied. The analog implementation had a definite advantage in the last decade mainly because of its reliable bandpass implementation with Surface Acoustic
Wave devices. Until the advent of Large Scale Integration and Very High Speed Integrated Circuits, a low-pass digital implementation using a few chips was impossible. Large time-bandwidth products required by Spread Spectrum systems is now achievable in compact digital format.

Analog devices have two important disadvantages, their inherent technological cost and their inflexibility of frequency/phase programming. Digital devices, on the other hand, suffer quantization loss when multi-bit digitization is not used.

Finally, the concept of real versus complex digital matched filtering was presented. Turin's original Digital Matched Filter was designed to be used with waveforms having purely real valued envelopes such as BPSK. While this concept was acceptable in the mid-seventies, optimization of bandwidth and spectrum efficiency has forced modern Spread Spectrum systems engineers to resort to complex valued envelopes such as QPSK, OQPSK, MSK, PN-chirp, chirp and other coherent FSK or continuous phase modulation schemes.

Baier [22] demonstrated that a Digital Matched Filter designed for these complex valued envelopes would require four real filter channels instead of Turin's two [15]. This newly proposed Digital Matched Filter was obtained by using Turin's original two-phase filter and adding two cross channels for the imaginary part of the reference signal to create a four-phase Digital Matched Filter. Unfortunately, Baier had no experimental data to support his low cost Digital Matched Filter implementation for arbitrary constant envelope Spread Spectrum waveforms.
To experimentally verify Baier's low cost Digital Matched Filter for arbitrary constant envelope Spread Spectrum waveforms, the author built the four-phase Digital Matched Filter using off-the-shelf components and tested it using a real valued and a complex valued waveform in two hostile environments, broadband noise and tone jamming.

In all four experiments, Baier's four-phase Digital Matched Filter displayed an excellent near-upper bound performance meeting the theoretical expectations. With pseudorandom Spread Spectrum binary coherent phase shift keying modulation, it performed as well as Turin's two-phase Digital Matched Filter in both broadband noise and tone jamming. With pseudorandom Spread Spectrum minimum shift keying data modulation, it outperformed Turin's Digital Matched Filter by over 2 dB in both types of hostile environments.

The achieved 10 dB dynamic range of the experimental set-up was less than the 24 dB dynamic range achievable for a Digital Matched Filter as suggested by Rappaport[4]. This limitation could be due to the hostile environments in which the Digital Matched Filter was tested and/or by the zero crossing detector which was used as the one bit digitizer. The 21 dB processing gain suggested by Rappaport [4] for Digital Matched Filter was consistently achieved.

Superb operation of Baier's four-phase Digital Matched Filter for arbitrary constant envelope waveforms theory was demonstrated by the author's experimental data. The four experiments revealed that Baier's Digital Matched Filter has the suggested flexibility of signaling schemes, is very reliable in hostile environments and can be easily implemented using inexpensive
off-the-shelf components. Baier's filter is not only phase discriminant like Turin's filter, but also frequency discriminant since Minimum Shift Keying is a special case of Continuous Phase Frequency Shift Keying. With the growing customer market and the demilitarization of Spread Spectrum systems, Baier's Digital Matched Filter should become a very popular alternative to Analog Matched Filters and to Turin's two-phase Digital Matched Filters.

5.2 FUTURE WORK

Baier's four-phase Digital Matched Filter has created a new field in matched filtering and the author feels several areas should be investigated.

The use of Quadrature Phase Shift Keying and chirp should be investigated. However, since this Digital Matched Filter performs near optimally in an hostile environment with phase and frequency signaling schemes, it shouldn't encounter any performance degradation in a friendly environment with these two signaling schemes. Since user density is the only criteria affecting service quality in this application, an investigation on user density management and its related subsequent performance degradation would be welcome.

The limited dynamic range experienced in the four experimental evaluations must be thoroughly studied and remedied. Any future studies on the dynamic range of Baier's four-phase Digital Matched Filter should concentrate on two areas, signal quantization and prefiltering. It should be experimentally verified that multilevel or multibit quantization not only improves the performance by reducing the quantization degradation loss but also
extends the dynamic range significantly. In addition new adaptive filters proposed and tested during the past year have shown that they can suppress most malignant intentional interference. Such filters, accompanied by an automatic gain control, could be tried as a prefilter or whitening filter to Baier’s four-phase Digital Matched Filter.

Since the primary function of a DMF in SS systems is signal acquisition, a synchronization study on Baier’s DMF would be most welcome, now that the four-phase DMF theory has been verified.
LIST OF REFERENCES


8. N. G. Davies, "Performance and Synchronization Considerations", AGARD Lecture, Series No. 58, Chap. 4.


A.1 DERIVATION OF TURIN'S SNR0/SNRI RELATIONSHIP

A.1.1 LINEAR SYSTEM GENERALIZATION

To derive Turin's relationship or criteria, it is best to look at one filter, rather than a bank of filters, in terms of maximization of SNR.

Suppose we have, as in figure A.1, received a waveform, \( r(t) \), which consists either solely of a white gaussian noise, \( n(t) \), of power density \( N_0/2 \) watts/cps\(^3\) (double-sided), or of \( n(t) \) plus a signal, \( s(t) \), of known form.

Assuming that \( n(t) \) is a stationary process, the average power of \( n(t) \) at any moment is the integrated power under the noise power density spectrum at the output of the filter [17]. If the transfer function of the filter is \( G(j2\pi f) \), the output noise power density is simply \( (N_0/2)|G(j2\pi f)|^2 \) [7]. This means that the output noise power is

\[
N_0/2 \int_{-\infty}^{\infty} |G(j2\pi f)|^2 df. \tag{A.1}
\]

If the input signal spectrum is \( S(j2\pi f) \), then the output signal spectrum is \( S(j2\pi f)G(j2\pi f) \), and its inverse Fourier transform \( Y_s(t) \), evaluated at \( t = t_1 \) is [7]

\[
Y_s(t_1) = \int_{-\infty}^{\infty} S(j2\pi f) G(j2\pi f) e^{j2\pi ft_1} df. \tag{A.2}
\]

Here we wish to maximize the power ratio, which is the same as the ratio of the square of \( \{A.2\} \) to \( \{A.1\} \), or

\[
\text{SNR} = 2\left[ \int_{-\infty}^{\infty} S(j2\pi f) G(j2\pi f) e^{j2\pi ft_1} df \right]^2 / N_0 \int_{-\infty}^{\infty} |G(j2\pi f)|^2 df . \tag{A.3}
\]
Fig. A.1 Linear system
Substituting \( f(x) \) for \( G(j2\pi f) \) and \( g(x) \) for \( S(j2\pi f) e^{j2\pi ft} \) and recognizing that the integral in the numerator is real, one can use Schwarz's inequality as follows,

\[
\left| \int f(x) g(x) dx \right|^2 \leq \int |f(x)|^2 dx \int |g(x)|^2 dx \quad \text{(A.4)}
\]

to obtain

\[
\text{SNR} \leq \frac{2}{\text{No}} \int_{-\infty}^{\infty} |S(j2\pi f)|^2 \, df. \quad \text{(A.5)}
\]

But \( |S(j2\pi f)|^2 \) is simply the energy density spectrum of \( s(t) \), therefore the integral in (A.5) is the total energy, \( E \), in \( s(t) \) and

\[
\text{SNR} \leq \frac{2E}{\text{No}}. \quad \text{(A.6)}
\]

A.1.2 SNR MAXIMIZATION

Since we wish to maximize the SNR at the output of the filter, (A.6) becomes

\[
\text{SNR}_0 = \frac{2E}{\text{No}} \quad \text{(A.7)}
\]

where \( \text{SNR}_0 \) depends on the signal through its energy only and is independent of the shape of the signal spectrum.

Let the noise bandwidth of the matched filter, which can be seen as the bandwidth of a rectangular band filter with the same maximum gain, and which would have the same output noise power as the matched filter, be represented by \( W \). The amount of input noise power within the matched filter band is simply \( \text{Ni} = W\text{No} \). If the average signal power at the filter input is \( \text{Pin} = \frac{E}{T} \), where \( T \) is the effective duration of the signal, then \( \text{SNR}_i = \frac{\text{Pin}}{\text{Ni}} \), and (A.7) becomes
\[ SN_{Ro} = 2TW(SN_{Ri}) \] \hspace{1cm} (A.8)

Combining (A.7) and (A.8) yields

\[ SN_{Ro} = 2TW(SN_{Ri}) = 2E/No \] \hspace{1cm} (A.9)

as required.
APPENDIX B
APPENDIX B

B.1 NONCOHERENT CHANNEL WITH ENVELOPE DETECTOR

B.1.1 BANDPASS FORMULATION

In the noncoherent channel, the matched filter is followed by an envelope detector rather than a phase coherence restoration circuit. Using the familiar bandpass form [15, 16], the matched filter and envelope detector combination can best be analyzed by writing \( s(.) \) and \( n(.) \) terms as follows

\[
s(t) = S(t) \cos(\omega_0 t + \Theta(t)), \quad \text{and} \quad \{B.1\}
\]
\[
n(t) = N(t) \cos(\omega_0 t + \phi(t)) \quad \{B.2\}
\]

where \( \omega_0 \) is the center (carrier) frequency, \( S(.) \), \( N(.) \), \( \Theta(.) \) and \( \phi(.) \) are the low-pass waveforms representing the amplitude and phase fluctuations. It should also be noted that the phase shift imparted to \( s(.) \) by the channel is also included in \( \Theta(.) \).

B.1.2 COHERENT CHANNEL MODEL

Recalling the coherent channel model, the filter's output is given by

\[
Y(t) = \int_0^T s(T-t) s(t-t_1) dt_1 + \int_0^T s(T-t) n(t-t_1) dt_1 \quad \{B.3\}
\]

where the impulse response of the matched filter is defined to be

\[
h(t_1) = \{ks(T-t_1), \quad 0 \leq t_1 \leq T\}
\]

\[
\{0, \text{ elsewhere} \}
\]

where \( k \) is an arbitrary gain factor, usually taken as unity [14].
Applying the customary trigonometric identities and ignoring the integrals involving double-frequency terms, and substituting \( B.1 \) and \( B.2 \) into \( B.3 \) yields

\[
Y(t) = \frac{1}{2} \left( \int_0^T S(t)S(t-a)\cos(\theta(t)-\theta(t-a)) \, dt \right.
+ \int_0^T S(t)N(t-a)\cos(\theta(t)-\phi(t-a)) \, dt \right) \cos(W_o a)
- \frac{1}{2} \left( \int_0^T S(t)S(t-a)\sin(\theta(t)-\theta(t-a)) \, dt \right.
+ \int_0^T S(t)N(t-a)\sin(\theta(t)-\phi(t-a)) \, dt \right) \sin(W_o a)
\]

\[B.4\]

where \( a = T-t \).

If we denote the four integrals in \( B.4 \) by \( I_1, I_2, I_3 \) and \( I_4 \), respectively, and define the envelope of \( Y(t) \) as follows \([16]\)

\[
Ye(t) = \frac{1}{2} \left( [I_1(a)+I_2(a)]^2 + [I_3(a)+I_4(a)]^2 \right)^{1/2}
\]

\[B.5\]

Then when \( t = T \) or \( a = 0 \), \( B.5 \) becomes

\[
Ye(T) = \frac{1}{2} \left( [I_1(0)+I_2(0)]^2 + I_4(0)^2 \right)^{1/2}
\]

\[B.6\]

since \( I_3(0) = 0 \), ie. \( \sin(0) = 0 \).

\[\]

B.1.3 SNRo DEFINITION

If, on the other hand, SNRo is large (ie. \( \text{SNRo} \gg 1 \)), then \( I_2(0) \) and \( I_4(0) \) are very small compared to \( I_1(0) \), and

\[
Ye(T) = \frac{1}{2}[I_1(0)+I_2(0)] = Y(T)
\]

\[B.7\]
since this approximation recognizes that, for SNR_o \gg 1, I_1(0) + I_2(0) is positive with very high probability. In fact, (B.7) clearly shows that for a large SNR_o, the performance of the matched filter and envelope detector combination is very similar to that of the matched filter alone. Furthermore, when the noise is of gaussian nature and SNR_o \gg 1, equation (A.9) still holds very closely for the noncoherent arrangement [15].

As it will be seen in the implementation of Baier's DMF, it is more convenient to implement a square-law envelope detector, i.e. Z = Y^2, rather than Y. Since Z and Y are related in a one-to-one manner, no information should be lost nor degradation of performance realized in terms of probability of error in making a hard decision based on Z, due to this transformation. This expectation is supported by Turin's analysis [17].

In terms of probability theory and stochastic processes [7], SNR_o can be defined as

\[
\text{SNR}_o = \left\{ \text{E}[Y(T)] - \text{E}[\tilde{Y}(T)] \right\}^2 / \text{Var} s[Y(T)]
\]

(B.8)

where E[.] is the mean value, Var[.] is the variance, the subscripts s and ns denote the conditions "signal present" and "signal absent" respectively.

Since the statistics of Z are different from those of Y, the SNR_o obtained by applying (B.8) to Z rather than to Y will be affected. Using (B.7) and its conditions, and assuming that N(.) in (B.4) has zero mean, we get

\[
\text{E}[Y] = 1/2 \cdot I_1(0), \quad \text{E}[\tilde{Y}] = 0, \quad \text{Var}[Y] = 1/4 \cdot I_2^2(0)
\]

(B.9) \hspace{1cm} (B.10) \hspace{1cm} (B.11)
where the overbar denotes the mean value. Therefore, (B.8) becomes
\[ \overline{\text{SNRo}}(Ye) = \frac{I_0^2(0)}{I_2^2(0)}. \]  \hspace{1cm} \text{[B.12]}

From (B.6), we can also write [5]
\[ Z = Ye^2(T) = \frac{1}{4} \left[ I_2^2(0) + 2I_1(0)I_2(0) + I_2^2(0) + I_4^2(0) \right] \]  \hspace{1cm} \text{[B.13]}
\[ \] which leads to
\[ E_{\text{sn}}(Z) = \frac{1}{4} \left[ I_2^2(0) + I_4^2(0) \right] \]  \hspace{1cm} \text{[B.14]}
\[ E_{\text{ns}}(Z) = \frac{1}{4} \left[ I_2^2(0) + I_4^2(0) \right], \text{ and} \]  \hspace{1cm} \text{[B.15]}
\[ \text{Var}_{\text{ns}}(Z) = \frac{1}{16} I_2^2(0)I_2^2(0)(4+kI_2^2(0)/I_2^2(0)) \]  \hspace{1cm} \text{[B.16]}
where \( k \) is defined by [15]
\[ (kI_2^2(0))^2 = 4I_1(0)[I_2^2(0) + I_2(0)I_4^2(0) + I_2^2(0) - I_4^2(0)^2 + I_2(0)I_4^2(0) - I_2^2(0)I_4^2(0) - I_2^2(0)]. \]  \hspace{1cm} \text{[B.17]}

Using (B.14) to (B.16) with (B.12) into (B.8) for \( Z \) leads to
\[ \text{SNRo}(Z) = \text{SNRo}(Ye)/4 + [k/\text{SNRo}(Ye)] \]  \hspace{1cm} \text{[B.18]}

When \( \text{SNRo}(Ye) >> 1 \), then (B.18) becomes
\[ \text{SNRo}(Z) = 1/4 \text{ SNRo}(Ye). \]  \hspace{1cm} \text{[B.19]}

**B.1.4 CONCLUSION**

(B.19) implies that, for white gaussian noise, the noncoherent version of (A.9) becomes
\[ \text{SNRo}(Z) = E/2\text{No} = TW(\text{SNRi})/2, \]  \hspace{1cm} \text{[B.20]}
and a difference of 6 dB is noted. It should also be noticed that the value of \( k \) in (B.18) can be easily evaluated if \( I_2(0) \) and \( I_4(0) \) are jointly gaussian. For gaussian interference, (B.17) reduces to \( k = 4 \) and for coherent constant amplitude interference, \( k = 3 \) [15].
C.1 UPPER BOUND FOR NONCOHERENT TWO-PHASE DMF IN GAUSSIAN ENVIRONMENT

C.1.1 INTRODUCTION

Just as \((B.20)\) was calculated for a noncoherent analog matched filter at \(t = T\), we will now compute the SNR\(_o\) for the two-phase DMF at \(k = L\) or for the output sample \(\{15\}\),

\[
Z = Y e^{2} (L) = V u L^2 + W L^2 = \left\{ \sum_{i=0}^{L-1} (S_L^{-i} W_u, L^{-i})^2 \right. \\
+ \left. \sum_{i=0}^{L-1} (S_L^{-i} W_L, L^{-i})^2 \right\} \tag{C.1}
\]

in a gaussian noise environment for a one bit (ie. binary) digitizer with binary signals.

To be able to use these results in chapter 4, we will restrict our derivation to phase shift keying, where \(S(t) = +/- A (A > 0)\) for all \(t\), and we assume that a new value of \(S(.)\) is transmitted once every \(\delta s\) or chip.

C.1.2 SAMPLES FORMULATION

Using \(s(.)\) and \(n(.)\) of \(\{B.1\}\) and \(\{B.2\}\) as inputs to figure 2.17, and tracing through to the output of the samplers, the samples taken during the \(j\)th chip are

\[
W_u j = S_j/2 \cos n + nuj, \quad \text{and} \tag{C.2}
\]

\[
W_l j = -S_j/2 \sin n + nlj \tag{C.3}
\]

where \(S_j = +/- A\) and

\[
nuj = N(j \delta)/2 \cos[\phi(j \delta) + n], \quad \text{and} \tag{C.4}
\]
\[ n_lj = -N(j\delta)/2 \sin[\phi(j\delta)+\eta]. \tag{C.5} \]

Because we are dealing with an unbiased one bit digitizer which is characterized by \( y = \text{sgn} \, x \), where \( x \) and \( y \) are respectively the input and the output of the digitizer and

\[
\text{sgn} \, x = \begin{cases} 
+1, & x > 0 \\
-1, & x < 0 
\end{cases} \tag{C.6}
\]

We can rewrite \( V\text{uL} \) and \( V\text{lL} \) of (C.1) as followed

\[
V\text{uL} = \sum_{j=1}^{L} S_j^0 \, \text{sgn}[S_j/2 \cos \eta + nuj], \quad \text{and} \tag{C.7}
\]

\[
V\text{lL} = \sum_{j=1}^{L} S_j^0 \, \text{sgn}[-S_j/2 \sin \eta + nlj]. \tag{C.8}
\]

Let's look at the summation of (C.7) and suppose that \( S_j = +A \), such that \( S_j^0 = +1 \). To be able to have \( \text{sgn}[A/2 \cos \eta + nuj] = -1 \), one must have \( nuj < -A/2 \cos \eta \).

\[
\text{This is the case when the digitized transmitted and received samples } S_j^0 \text{ and } Wuj^0 \text{ don't agree. Conjointly, if } S_j = -A; \text{ a disagreement will occur if} \tag{C.9}
\]

\[ nuj > A/2 \cos \eta. \tag{C.10} \]

Suppose that \( nuj \) is a symmetric random variable such that the probability of (C.9) and (C.10) are equal and let this probability be \( pu(\eta) \) where

\[ pu(\eta) = Pr[aju < -A/2 \cos \eta]. \tag{C.11} \]

In fact \( pu \) is the probability that a term in (C.7) equals \(-1\) and \((1-pu)\) is the probability that it equals \(+1\). If the \( nuj \)'s are independent and identically distributed, the terms in (C.7) are also independent and identically distributed, taking on the values \(+/-1\), which means that \( V\text{uL} \) is binomially distributed.
C.1.3 Moments

From [7, 15], the moment generating function of \( VUL \) is simply

\[
m(t) = [(1-pu) e^t + pu e^{-t}]^L \quad \text{[C.12]}
\]

and the moments of \( VUL \) are

\[
E[VUL^k] = \frac{d^k m}{dt^k}|_{t=0}. \quad \text{[C.13]}
\]

From this, we can get, after some rearrangements, the second and fourth moments,

\[
E[VUL^2] = L[(L-1)(1-2pu)^2 + 1] \quad \text{and} \quad \text{[C.14]}
\]

\[
E[VUL^4] = L[(L-1)(L-2)(L-3)(1-2pu)^4 + 2(3L-4)(1-2pu)^2 + (3L-2)^2]. \quad \text{[C.15]}
\]

From this result, we get

\[
\text{Var}(VUL^2) = 2L(L-1)((3-2L)(1-2pu)^4 + 2(L-2)(1-2pu)^2 + 1). \quad \text{[C.16]}
\]

Similar equations can be derived for the moments of \( VUL \) by replacing \( pu \) by

\[
pl(n) = \Pr[nlj < A/2 \sin \eta]. \quad \text{[C.17]}
\]

Since the set of \( nuj \)'s and the set of \( nlj \)'s are independent, the mean and the variance of \( Z \) of \( \{C.1\} \) can be found,

\[
E[Z|\eta] = L[(L-1)((1-2pu)^2 + (1-2pl)^2) + 2], \text{and} \quad \text{[C.18]}
\]

\[
\text{Var}[Z|\eta] = 2L(L-1)((3-2L)((1-2pu)^4 + (1-2pl)^4) + 2(L-2)((1-2pu)^2 + (1-2pl)^2) + 2). \quad \text{[C.19]}
\]

where the condition on \( \eta \) recognizes the dependence of \( pu \) and \( pl \) on \( \eta \). Since we are considering a noncoherent system, \( \eta \) is a random variable which is uniformly
distributed between 0 and $2\pi$, we have

\[
E[Z] = \frac{1}{2} \int_{0}^{2\pi} E[Z|\eta] d\eta, \quad \text{and} \quad \{C.20\}
\]

\[
\text{Var}[Z] = \frac{1}{2} \int_{0}^{2\pi} \text{Var}[Z|\eta] d\eta + \frac{1}{2} \int_{0}^{2\pi} (E[Z|\eta] - E[Z])^2 d\eta. \quad \{C.21\}
\]

Using (C.20) and (C.21), Turin wrote the SNR as [15]

\[
\text{SNR} = \frac{(E_s[Z] - E_{ns}[Z])^2}{\text{Var}[Z]} \quad \{C.22\}
\]

where the subscripts denote the same conditions as in (B.8).

### C.1.4 Gaussian Channel

Let's now apply those results to a system where the channel interference is white and gaussian, with double-sided spectral density $N_0/2$ as usual. From Turin's work, $\nu_1, \ldots, \nu_L, n_1, \ldots, n_{1L}$ of (C.4) and (C.5) are zero mean gaussian random variables, approximately independent, symmetric and with variance $\sigma^2 = N_0 W/4$ where $W$ is, as defined in appendix B, the noise bandwidth of the Digital Matched Filter receiver's front-end. In fact, it is equal to the noise bandwidth of $s(.)$ and that of the associated Analog Matched Filter.

\[
\nu(u) = \frac{1}{2} - 1/(2\pi) \int_{-\infty}^{(A/2) \cos \eta} \exp(-x^2/2\sigma^2) dx, \quad \text{and} \quad \{C.23\}
\]

\[
\nu(u) = \frac{1}{2} + 1/(2\pi) \int_{(A/2) \sin \eta}^{\infty} \exp(-x^2/2\sigma^2) dx. \quad \{C.24\}
\]
C.1.5 SPREAD SPECTRUM CASE

Let's concentrate our effort on the most familiar case in Spread Spectrum, i.e. where the matched filter is normally used, when SNRi is small but SNR0 is large. From appendix A and equation (B.1), SNRi = Pi/Ni,

\[ E = \int_{0}^{T} s^2(t)dt \text{ and } Pi = E/T, \text{ we can say that} \]

\[ Pi = \frac{1}{2T} \int_{0}^{T} S^2(t)dt = A^2/2. \] \quad \text{(C.25)}

Since \( W \) is now the receiver's front end noise bandwidth, the equation \( Nin = WNo \) from appendix A still holds for \( Ni \), and therefore

\[ SNRi = A^2/2WNo = A^2/8\sigma^2. \] \quad \text{(C.26)}

When we have \( SNRi \ll 1, \text{ as in our case, the integrands of (C.23) and (C.24) are pretty well constant and equal to unity over the intervals of integration, which means that} \]

\[ 1 - 2pu(\eta) = A\cos\eta/(2\pi) \] \quad \text{(C.27)}

\[ 1 - 2p1(\eta) = -A\sin\eta/(2\pi) \] \quad \text{(C.28)}

If we substitute these values in (C.18) and (C.19), then apply (C.20) and (C.21) and, finally, use (C.22) we get

\[ SNR0 = L(L-1)(A^2/2\pi\sigma^2)^2 / 2[(3-2L)(3A^4/32\pi^2\sigma^4) + 2(1-2)(A^2/2\pi\sigma^2) + 2]. \] \quad \text{(C.29)}

In the region we are interested, we have \( SNRi = A^2/8\sigma^2 \ll 1 \text{ and } SNR0 \gg 1, \text{ which means that (C.29) rightfully implies that } L \gg 1 \text{ and therefore reduces to} \]

\[ SNR0 = L/2 A^2/4\pi\sigma^2. \] \quad \text{(C.30)}
For a phase shift keyed signal, the noise bandwidth is the reciprocal of the chip rate, i.e. \( W = 1/5 = L/T \), and (C.30) becomes

\[
SNR_o = 2/\pi(TW/2)(SNR_i) \tag{C.31}
\]

C.1.6 CONCLUSION

The result of (C.31) agrees with [19] and means that, when \( SNR_i \ll 1 \), \( SNR_o \gg 1 \), and the channel interference is white and gaussian, a one bit Digital Matched Filter suffers about 1.96 dB of degradation compared to its associated Analog Matched Filter. (C.31) is also referred to as the upper bound.
APPENDIX D
D.1 UPPER BOUND FOR NONCOHERENT DMF IN CW JAMMING ENVIRONMENT

D.1.1 STABLE JAMMING SIGNAL

We now look at the constant interference samples case which can be the result of a sine wave jammer at or near the carrier frequency. The signal is stable such that its phase remains essentially constant relative to the receiver phase \( \eta \) over \( L \) chips but unstable enough to see its relative phase fluctuate at random from \( L \) chip transmission to \( L \) chip transmission.

Since the interference samples \( n_{uj} \) and \( n_{lj} \) are random variables which are constant from chip to chip, we have \( n_{u1}=n_{u2}=\ldots=n_{uL} \) and \( n_{l1}=n_{l2}=\ldots=n_{lL} \). Equations (C.4) and (C.5) become

\[
\begin{align*}
    n_{uj} &= J/2 \cos(\phi+\eta), \quad \text{and} \\
    n_{lj} &= -J/2 \sin(\phi+\eta)
\end{align*}
\]

where \( \phi \) is an uniformly distributed random variable independent of \( j \).

D.1.2 HOSTILE ENVIRONMENT

The first case of interest is when \( \text{SNR}_i=A^2/J^2<1 \). It is useful to use phasors to picture the situation. For our case, the phasors

\[
W_j = W_j +/ - = A/2 e^{-i\eta} + J/2 e^{-i(\phi+\eta)}
\]

are shown in figure D.1. The real and imaginary parts of \( W_j \) are, respectively, \( u_{uj} \) and \( l_{lj} \) of (C.2) and (C.3), with \( S_j = +/-A \) while \( n_{uj} \) and \( n_{lj} \) are as in (D.1) and (D.2).
Fig. D.1 Signal and interference phasors for the coherent constant amplitude interference case.
From the geometry shown, it is interesting to note that, as the code SL, ..., SL goes through its sequence of +/- A's, the resulting phasor W_j flips back and forth between W_j+ and W_j- but always remains in the fourth quadrant. Therefore, as j changes, W_j and W_lj never change sign. For this particular geometry, the digitized samples become

\[ W_j^0 = \text{sgn}(\text{Re } W_j) = +1, \text{ all } j \quad \text{and} \quad W_lj^0 = \text{sgn}(\text{Im } W_j) = -1, \text{ all } j. \quad \text{(D.4)} \]

If we imagine that the excess of the number of +1's over the number of -1's in our sequence \( \{S_j^0 = \text{sgn}S_j\} \) is Q, (i.e. \( Q = +1/-1 \) for a pseudorandom sequence, according to whether L is even or odd), then from equation (C.7) and (C.8) we get

\[ V_{UL} = Q, \text{ and} \quad V_{IL} = -Q \quad \text{(D.6)} \]

where \( Z = V_{UL}^2 + V_{IL}^2 = 2Q^2 \) is the D.M.F output at \( k=L \) [15].

These results hold whenever the tip of the phasor \( J/2 \ e^{-i(\phi+\eta)} \) lies on one of the darkened arcs of the circle of figure D.1 for a given value of \( \eta \). From simple trigonometry, the probability that \( \phi \) has a value such that this is true is

\[ \alpha(\eta) = 1 - 2/\left(\sin^{-1}\left|\frac{A}{J}\sin\eta\right| + \sin^{-1}\left|\frac{A}{J}\cos\eta\right|\right) \quad \text{(D.8)} \]

Similarly, the tip of the phasor \( J/2 \ e^{-(\phi+\eta)} \) lies on the undarkened line of the circle with probability \( 1-\alpha(\eta) \), which means that either

\[ W_j^0 = +S_j^0 \text{ or } W_j^0 = -S_j^0, \text{ for all } j, \text{ or} \quad \text{(D.9)} \]

\[ W_lj^0 = +S_j^0 \text{, or } W_lj^0 = -S_j^0 \text{ for all } j. \quad \text{(D.10)} \]
D.1.3 MOMENTS

(D.9) and (D.10) imply that either $W_{j}^{0}$ or $W_{-j}^{0}$ agrees perfectly with $S_{j}^{0}$ or $-S_{j}^{0}$ for every $j$ and the sign depends on $\eta$. We can easily see from figure D.1 that it is impossible for both $W_{j}^{0}$ and $W_{-j}^{0}$ to agree perfectly with the digitized sample. If one did, then the other must assume a constant value of $+1$ or $-1$ for all $j$ (cf. (D.4) and (D.5)). Therefore, $Z = V_{u}L_{2}^{2} + V_{l}L_{2}^{2} = L_{2}^{2} + Q_{2}$ with probability $1-\alpha(\eta)$. This leads to

\begin{align*}
\mathbb{E}_\eta(Z) &= 2Q^{2}\alpha(\eta) + (L_{2}^{2} + Q_{2})(1 - \alpha(\eta)), \quad (D.11) \\
\mathbb{E}n_{\eta}(Z) &= 2Q^{2}, \quad \text{and} \quad (D.12) \\
\text{Var}(Z|\eta) &= (L_{2}^{2} - Q_{2})\alpha(\eta)(1 - \alpha(\eta)). \quad (D.13)
\end{align*}

If we average these over $\eta$ and apply equations (C.20), (C.21) and (C.22), after cancellations we are left with

\begin{equation}
\text{SNR}_0 = (1 - \bar{\alpha})/\bar{\alpha}, \quad (D.14)
\end{equation}

where

\begin{equation}
\bar{\alpha} = 2/\pi \int_{0}^{\pi/2} \alpha(\eta) \, d\eta. \quad (D.15)
\end{equation}

It is important to realize that (D.14) does not depend on $Q$, which suggests that $Q$ could change randomly from transmission to transmission. Let's now look at the most familiar case in Spread Spectrum where $A/J << 1$. In this case, equation (D.8) simplifies to

\begin{equation}
\alpha(\eta) = 1 - 2/\pi |A/J| \left( |\sin\eta| + |\cos\eta| \right), \quad (D.16)
\end{equation}

and equation (D.14) reduces to

\begin{equation}
\text{SNR}_0 = 8/\pi^2 \ A/J = 8/\pi^2 \ (\text{SNR}_j)^{\frac{1}{2}}. \quad (D.17)
\end{equation}
D.1.4 CONCLUSION

(D.17) is known as the upper bound for a noncoherent DMF in a CW jamming environment. One witnesses the phenomena of "capture" in this situation and the DMF does not produce any processing gain such as \((TW/2)\) in a white gaussian environment. It should be noticed that these results only hold for \(\text{SNR}_i \leq 1\), since for \(\text{SNR}_i = A_i^2/J^2 > 1\), it is impossible for both \(\text{VuL}\) and \(\text{VII}\) to be reduced in magnitude from \(L\) to \(Q\) as they were in (D.6) and (D.7). Only one such reduction could be possible [15].
END

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FIN