Three Essays on Real-Financial Linkage in Dynamic Stochastic General Equilibrium Models

by

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Abstract

In this thesis, I contribute to the growing literature on the linkage between real and financial sectors, and study how financial frictions affect responses of important economic variables to different shocks in a New-Keynesian dynamic stochastic general equilibrium (DSGE) framework.

The first chapter of this thesis shows that financial frictions can mitigate an important puzzle in the literature related to investment-specific technology shocks. Evidence from estimated DSGE models and SVAR analysis suggests that investment shocks are an important source of business cycle fluctuation in the post-war U.S. economy. In most models that include investment shocks, consumption falls on impact, contradicting its observed comovement with investment, hours worked and output over the business cycle. This chapter shows that introducing financial frictions alongside endogenous capacity utilization in a New-Keynesian model can produce a positive consumption response to an investment shock. By attenuating the response of investment, the financial accelerator mechanism suppresses households’ intertemporal substitution towards savings and allows consumption to rise. Search frictions in the labour market introduce a wedge between marginal product of labour and wages determined through Nash bargaining and magnify the positive consumption response.

The second chapter of this thesis documents a new challenge for a class of models with binding borrowing constraints related to government spending shocks. We highlight that DSGE models with housing and collateralized borrowing predict a fall in
both house prices and consumption following positive government spending shocks. The quasi-constant shadow value of lenders’ housing and the negative wealth effect of future tax increases on their consumption are the key reasons for this result. By contrast, we show house prices and consumption in the U.S. rise after identified positive government spending shocks, using a structural vector autoregression methodology and accounting for anticipated effects. The counterfactual joint response of house prices and consumption poses a new challenge when using this class of models to address policy issues for the housing market which have come to fore due to the weak recovery after the 2008 financial crisis.

The final chapter of this thesis develops a search-theoretic banking model in a New Keynesian DSGE framework that can simultaneously explain cyclical movements in interest spreads and flows in gross loan creation and destruction, that were recently emphasized in the literature. The model features endogenous match separation, and allows bank loans for productive capital purchases to vary in both intensive and extensive margins. Search frictions in the banking sector generates a counter-cyclical interest spread that amplifies business-cycles. In addition, the model generates responses in gross loan destruction and net loan flows to a credit supply shock that can qualitatively match empirical responses estimated in a vector auto-regression framework.
To my daughters,

Mayabee Reza and Mohini Reza,

and to my younger sister,

Krishty Reza

May you find joy in learning and discovery.
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Chapter 1

Introduction

Until recently, the literature on business cycle fluctuations seldom considered financial markets as an important channel through which shocks affect the real economy. The Modigliani and Miller [1958] theorem, which states that under perfect financial markets, a firm’s value is independent of its source of funding, was largely interpreted to imply that financial or credit market conditions have negligible effect on the real economy. Consequently, notwithstanding a handful of notable exceptions (such as Bernanke et al. [1999] and Kiyotaki and Moore [1997]), workhorse Neoclassical or New-Keynesian Dynamic Stochastic General Equilibrium (DSGE) models used by academics and policy-makers to study the sources and transmission mechanisms of business cycles, abstracted from the details of financial markets.

The 2007 financial crisis, however, has highlighted that financial markets are an important channel through which shocks to the economy are transmitted and propagated, and has suggested that they may themselves be important sources of business cycle fluctuations. Consequently, a growing literature has emerged that studies the implications of financial frictions on the real economy, and develops DSGE models that introduce frictions in the financing arrangements of agents in the economy. In these models, the balance-sheet condition of firms or banks, or the presence of par-
ticular forms of financing constraints, have important implications for the cyclical dynamics of aggregate variables.

In this fast-growing literature, frictions are often introduced in financial markets as a wedge between interest rates paid for internally and externally sourced funds, or through binding constraints on the quantity of funding available. Such financial frictions amplify cyclical responses to shocks affecting aggregate demand, and attenuate responses to shocks affecting aggregate supply (Gerali et al. [2010], Christensen and Dib [2008]). Financial frictions have already been shown to allow general equilibrium macroeconomic models to achieve a better fit with data (Christensen and Dib [2008]), or explain a number of outstanding puzzles in the literature related to cyclical dynamics of macroeconomic variables.\footnote{Petrosky-Nadeau and Wasmer [2010] show that financial frictions aid in mitigating a volatility puzzle related to labour search models. Monacelli [2009] suggests that financial frictions can mitigate a co-movement puzzle for durable goods production in response to a monetary shock.} At the same time, a number of empirical studies have found that shocks originating within the financial sector have an important effect on the real economy, using both reduced-form estimation methods (Gilchrist and Zakrajsek [2012], Boivin et al. [2012]) as well as estimated DSGE models (Nolan and Thoenissen [2009]).

In this thesis, I contribute to the growing literature on the linkage between real and financial sectors, and study how financial frictions affect responses of important economic variables to different shocks in a DSGE framework. The first chapter of my thesis shows that financial frictions can mitigate an important puzzle in the literature related to investment-specific technology shocks, while the second chapter documents a new challenge for a class of models with binding borrowing constraints related to government spending shocks. In the third chapter, I develop a search-theoretic model of the banking sector that can simultaneously explain aggregate movements in interest spreads, as well as those in disaggregated flows in loan creation and destruction.
Recent research on the source of economic fluctuations in post-war U.S. find that shocks affecting the efficiency through which investment spending is transformed to productive capital, are an important driving force for business cycles (Justiniano et al. [2010]). In most models that include such investment-specific technology shocks, however, consumption falls on impact. This contradicts observed business cycle co-movements where consumption, output and investment move together. In the first chapter of my thesis, I show that introducing financial frictions alongside endogenous capacity utilization in a New-Keynesian model allows consumption to co-move with output. Intuitively, a shock that improves the marginal efficiency of investment also raises returns from investment. Under standard preferences, rational agents substitute consumption in favour of savings to take advantage of this temporary increase in returns. As a consequence, consumption always falls on impact. However, the increase in the supply of capital following such an investment-specific technology shock leads to a fall in the price of capital, and consequently, the net worth of firms who own this capital. In the presence of asymmetric information, Bernanke et al. [1999] show that a reduction in firm net worth, or a deterioration of the firm’s balance sheet condition, results in an increase in the spread between the interest rate paid by the firm and the risk-free rate prevalent in the economy. This increase in spreads attenuates investment response, and allows consumption to rise on impact. Search frictions in the labour market introduce a wedge between the marginal product of labour and wages through Nash bargaining, which also contribute to generating co-movement.

In the second chapter of my thesis, a joint work with my supervisor, Hashmat Khan, we highlight a new challenge for a class of models with binding borrowing constraints in transmitting government spending shocks. We show that house prices and consumption in the U.S. rise after an identified positive government spending shock, using a structural vector auto-regression methodology and accounting for anticipated
effects. In contrast, DSGE models with housing and collateralized borrowing similar to Iacoviello [2005] and Iacoviello and Neri [2010], predict a fall in both house prices and consumption following a positive government spending shock. The quasi-constant shadow value of lenders’ housing and the negative wealth effect of future tax increases on their consumption are the key reasons for this result. Introducing endogenous housing production, rigidities in house prices or wages, or allowing Greenwood et al. [1988] preferences do not overturn this result. The counterfactual joint response of house prices and consumption poses a new challenge when using this class of models to address policy issues related to the housing market, which have come to fore due to the weak recovery after the 2008 financial crisis.

In the third and final chapter of my thesis, I develop a banking model that serves both as a transmission channel for shocks, as well as a point of origin for recessions. Recently, a growing stream of literature has focused on developing quantitative business cycle models that emphasize the bank leverage ratio (the ratio between total bank assets and bank equity) as a channel for generating counter-cyclical interest spreads that amplify and prolong business cycles, and consider the banking sector as a possible source for cyclical movements. At the same time, a second stream of inquiry has used disaggregated bank lending data to reveal important patterns within the banking sector that are not captured at the aggregate level. In particular, Dell’Ariccia and Garibaldi [2005] and Contessi and Francis [2010] find that gross flows in new loan creation move pro-cyclically, while those in loan destruction move counter-cyclically in the U.S. However, currently available business-cycle models featuring a banking sector cannot account for these movements in disaggregated loan flows.

The third chapter of my thesis fills this gap in the literature by developing a

\footnote{See, among others, the work in Gertler and Karadi [2011], deWalque et al. [2010], Meh and Moran [2010], Gerali et al. [2010], and references therein.}
search-theoretic banking model that can simultaneously explain cyclical movements in interest spreads, as well as in disaggregated flows in loan creation and destruction. Search frictions generate a counter-cyclical wedge between loan and deposit rates for shocks that have a primary affect on both the supply and demand of loans. In this economy, loans are generated when a banking loan officer is matched with an unfunded project. Banks incur costs while searching for new funding opportunities, or in maintaining existing relationships. A negative shock to bank equity increases the leverage ratio in banks. To bring the leverage ratio back to its regulated steady-state, banks reduce their efforts in searching for new matches or in maintaining existing ones. On the other hand, when a negative technology, or monetary shock hits the economy, loan-demand falls, as does expected profits for banks. Banks again respond by reducing search efforts, and hence, new matches. In the benchmark calibration, the model produces a reduction in the supply of new matches large enough to generate a counter-cyclical rise in the spread between the policy interest rate determined by a Taylor-type rule and loan rates facing firms.

Moreover, the model generates responses in gross loan creation and destruction flows to a credit supply shock that qualitatively match empirical evidence. To show this, I estimate the effect of a credit supply shock on loan creation and destruction margins in a VAR framework. I then consider two shocks that can proxy the effects of a credit supply shock – a one time reduction in bank equity, and a one time increase in the match separation rate. I show that the shock to bank equity can produce impulse responses that qualitatively match evidence when investment is elastic to capital price. Overall, the chapter demonstrates that a search-theoretic banking model is a step in the right direction in jointly explaining movements in the interest rate spread and gross loan flows.
Bibliography


Chapter 2

Financial Frictions, Consumption Response and Investment Shocks

2.1 Introduction

Recent evidence from estimated Dynamic Stochastic General Equilibrium (DSGE) models (Justiniano et al. [2010], Christensen and Dib [2008]) and Structural Vector Auto-Regression (SVAR) analysis (Fisher [2006]) suggest that of all macroeconomic shocks, the one affecting investment technology accounts for the highest proportion of variation in output, hours and investment seen in post-war U.S. data.¹ A common shortfall for most models in the literature, however, is that consumption falls on impact for investment shocks. This is in stark contrast to the comovement of consumption with investment, output and hours observed over the business cycle. In

¹Justiniano et al. [2010] estimate a standard new-Keynesian DSGE model similar to Smets and Wouters [2007] using Bayesian techniques and find that more than three-quarters of the variation in investment, and half the variance in output in post-war U.S. economy can be explained by shocks to the marginal productivity of investment. Christensen and Dib [2008] use maximum likelihood to estimate a monetary DSGE model with financial frictions and also find investment shocks to be the main driver for variations in output and investment. Fisher [2006] differentiates between neutral and investment-specific technology shocks in a SVAR framework and find that the later account for two-thirds of the variation in output.
this chapter, I show that introducing financial frictions and capacity utilization in a
standard new-Keynesian framework can generate a positive consumption response on
impact for an investment-specific shock. I also show that introducing search frictions
in the labour market increases the magnitude of the positive consumption response,
as well as responses in output, investment and total hours.

The importance of shocks affecting the marginal productivity of investment in
generating business cycles was first emphasized by Keynes [1936]. In contrast to
such investment-specific technology shocks (or investment-shocks), neutral technology
shocks affect the productivity of both installed and new capital as well as that of
labour. Shocks to the marginal productivity of investment, however, are unable to
produce comovement of consumption, hours and output in the standard neoclassical
model. Barro and King [1984] show that under time-separable preferences, temporary
investment shocks increase the real rate of return from investment. In response,
optimizing households postpone consumption in favour of savings through the inter-
temporal substitution channel to take advantage of the temporary rise in returns.
For the same reason, households postpone leisure in favour of labour. Therefore,
consumption and wages move inversely with investment and hours in response to
investment shocks. Since then, the literature has highlighted three mechanisms that
can reduce the limitations of the neoclassical model in generating consumption co-
movement – (a) variable capacity utilization in production (Greenwood et al. [1988],
Khan and Tsoukalas [2011]), (b) wage and price rigidities (Justiniano et al. [2010]),
and (c) differences in the preference structure of households (Furlanetto and Seneca
[2010]).

Greenwood et al. [1988] shows that an endogenous increase in capacity utilization
following an investment shock boosts output, making it more likely to generate posi-
tive consumption co-movement. Khan and Tsoukalas [2011] argues that modelling the
cost of capacity utilization as a faster depreciation of installed capital, rather than a
direct reduction in output, is important for a model’s ability to generate consumption
co-movement. Justiniano et al. [2010] find that including both price and wage rigidities
significantly increase the variation in output to investment shocks. Greenwood
et al. [1988], Khan and Tsoukalas [2011] and Furlanetto and Seneca [2010] propose
that special preferences that restrict wealth effects on labour supply are helpful in
generating an increase in hours worked, and consumption, following an investment
shock.

Acknowledging the difficulty of generating a positive consumption co-movement
following a shock to the marginal productivity of investment, Christiano et al. [2010]
make a distinction between investment shocks affecting capital supply and those af-
fected demand. The later is able to generate positive co-movement of all important
variables, including consumption. Even in the presence of shocks affecting the demand
for capital, Christiano et al. [2010] finds that investment shocks that solely affect cap-
ital supply account for more than a third of the movement in investment for the U.S.
in business cycle frequencies. Consistent with the co-movement problem, however,
these shocks explain only a small proportion of the movement in consumption.

In this chapter, I propose financial frictions as a fourth mechanism that, along
with variable capacity utilization, can generate a positive consumption response. No
other restrictions on wealth effects, or wage rigidities are necessary for this result.
In addition, I show that search frictions in the labour market enhance the positive
consumption response afforded by financial frictions, but is not a necessary element.

Bernanke and Gertler [1989] show that under asymmetric information, the opti-
mal debt contract between a financer and a borrowing firm involves a variable loan-
premium on the cost of lending that depends on the borrowing firm’s balance sheet
position. The resulting financial accelerator mechanism amplifies the effects of de-
mand shocks, and attenuates the effects of supply shocks.\footnote{Iacoviello [2005] find a similar property for models using binding borrowing constraints to model financial frictions.} Christensen and Dib [2008] estimate a sticky-price DSGE model with financial frictions and find that the financial accelerator mechanism improves the response of macro variables to a variety of shocks other than to neutral technology. In particular, they also find investment shocks to be the most important driver of output and investment. However, consumption still responds negatively in their study. This chapter improves their result by adding capacity utilization, habit formation, and labour search, and generates a positive consumption response.\footnote{Note, however, that the main results presented in this chapter is robust to the inclusion of habit formation and search frictions in the labour market. A version of this chapter prepared for journal submission, therefore, dispenses of these two assumptions.}

Intuitively, a shock to the marginal productivity of investment increases the supply of capital in the economy, reducing its price, and consequently the net worth of firms that own capital. Because of the financial accelerator mechanism, firms with low net worth are then charged a higher interest rate for loans needed to finance new investment. This rise in funding cost for firms attenuates their investment activity. The resulting dampening of the response of savings allows households to increase their consumption.

The rest of the chapter is organized as follows: section 2.2 describes the benchmark model that includes financial frictions and labour search. Section 2.3 describes model calibration. Section 2.4 discusses the impulse responses to investment shock, offers explanations for the results, and identifying the contribution of search friction in the labour market. Section 2.5 provides concluding comments.
2.2 The Benchmark Model

The benchmark model adds capacity utilization and labour search to a closed-economy DSGE model with financial frictions as in Bernanke et al. [1999] and Christensen and Dib [2008]. The setup for capacity utilization in production technology follows Khan and Tsoukalas [2011], where the cost of increased capacity utilization is modeled as faster depreciation of installed capital. Search frictions in the labour market allow both hours (intensive margin) and employment (extensive margin) to vary across time.

The economy is populated by a representative household, a continuum of entrepreneurs or firms, capital producers, employment agencies, retailers, and a monetary authority. Entrepreneurs produce intermediate goods, invest in capital, and demand labour inputs. Employment agencies post vacancies to create jobs on behalf of entrepreneurs, and bargain for wages with workers. Although not explicitly modeled, the economy also includes financial intermediaries that take deposits from households, and converts them into business loans that entrepreneurs use to fund capital investment. Following Bernanke et al. [1999], I assume that asymmetric information between financiers and entrepreneurs leads to an optimal contract that imposes a loan premium or markup from financiers to entrepreneurs that depend on the financial position of entrepreneurs. A retail sector exists to motivate rigidities in price. Capital producers convert investment spending into productive capital used by entrepreneurs.

2.2.1 Households

The representative household can be thought of as an extended family containing a continuum of members indexed on the unit interval. In equilibrium, some members are employed, while some others are unemployed and searching for jobs. All members
are assumed to participate in the labour force. Each member has the following period utility function:

\[ U(c_t, h_t) = u(c_t) - g(h_t) \]
\[ u(c_t) = \ln (c_t - ac_{t-1}) \]
\[ g(h_t) = \frac{\Upsilon}{1 + \zeta} h_t^{1+\zeta} \]

where \( c_t \) is real per capita consumption of a final good and \( h_t \) is hours worked by an employed member. \( \zeta \) is the inverse of the Frisch elasticity of labour supply, \( \Upsilon \) is a structural parameter denoting the relative weight of leisure in utility, and \( a \) is the habit persistence parameter.

The representative household maximizes expected lifetime utility:

\[ E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - G_t] \]

subject to the following budget constraint:

\[ c_t + d_t \leq w_t l_t + R_t^d \frac{d_{t-1}}{\pi_t} \quad (2.1) \]

where \( G_t \) represents the family’s disutility of labour supply and includes the disutility from hours for only the employed members of the household.\(^4\) \( l_t \) represents total labour supplied by the household in return for wages, \( w_t \), and \( d_t \) represents real savings in financial institutions that earn a gross interest of \( R_t^d \). \( \pi_t = P_t/P_{t-1} \) is the rate of inflation. Following Merz [1995] and Andolfatto [1996], we assume that household members perfectly insure each other against fluctuations in consumption.

\(^4\)Since the level of hours worked is not a choice variable for households, but rather is determined as an outcome of bargaining between firms and workers, the family’s disutility of labour need not be specified further. See Trigari [2009] for details.
The solution to the household problem implies the following optimality conditions:

\[
\frac{1}{R^d_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \tag{2.2}
\]

\[
\lambda_t = \frac{1}{c_t - ac_{t-1}} - \beta E_t \frac{a}{c_{t+1} - ac_t} \tag{2.3}
\]

Equation 2.2 gives the intertemporal Euler condition, while 2.3 relates the shadow value of income, \( \lambda_t \), to current, past, and future consumption in the presence of habit formation (\( a > 0 \)).

### 2.2.2 Entrepreneurs

There is a continuum of entrepreneurs indexed by \( j \in (0, 1) \) that own capital \( k^j_t \), purchase labour services \( l^j_t \) from employment agencies, and determine capacity utilization \( u^j_t \) to produce a single intermediate good \( y^j_t \) that is sold in a competitive market at a common real price \( p^w_t \). Individual firms are subject to an idiosyncratic shock \( \omega^j_t \in (0, 1) \) that hits them after decisions on capital and labour services purchasing, and capacity utilization have been made. The firm’s constant returns to scale production function is given as:

\[
y^j_t = \omega^j_t \left( u^j_t k^j_t \right)^\alpha \left( l^j_t \right)^{1-\alpha} \tag{2.4}
\]

where \( \alpha \) is the share of capital in the production function. The idiosyncratic shock is normalized so that \( \int_0^1 \omega^j_t \, dj = 1 \) and \( E_t \omega_{t+1} = 1 \).

At the end of each period, entrepreneurs purchase capital \( k^j_{t+1} \) at price \( q_t \), which is used for production in the next period. Part of the funds for this capital acquisition is funded by the firm’s net worth, \( nw^j_t \), while the remainder is funded through loans from the financial intermediary.

The financial intermediary takes deposits from households at a gross interest rate
of \( R^d_t \) and provides loans to entrepreneurs for capital purchase at a gross rate of \( R^l_t \). The financial intermediary does not observe the idiosyncratic shock that hits individual entrepreneurs. If the idiosyncratic shock \( \omega^j_t \) falls above a threshold parameter \( \bar{\omega} \), the entrepreneur pays off the debt with interest to the intermediary and adds the remaining profits from production, net of labour service costs, to their net worth. If, however, the idiosyncratic shock falls below \( \bar{\omega} \), the entrepreneur claims bankruptcy. In this case, the financial intermediary retains all profits, and the entrepreneur gets nothing. Bankruptcy claims require financial intermediaries to conduct costly audits of the firms before settling claims. Bernanke et al. [1999] show that under this setup, the optimal contract between entrepreneurs and financiers imply a loan premium \( S(\cdot) \) that depends on the entrepreneur’s balance sheet position. Specifically, the loan premium is increasing in the firm’s leverage ratio, i.e., the ratio of assets to net worth. Underlying microeconomic parameter values, including parameters describing the threshold idiosyncratic shock and monitoring costs, determine the elasticity of the loan premium to firm’s leverage ratio.

The financial intermediary sets the gross loan rate as a markup on its expected opportunity costs of funding:

\[
R^l_t = E_t \left[ S \left( \frac{q_t k_{t+1} + nw_t}{nw_t} \right) \frac{R_t}{\pi_{t+1}} \right] \quad (2.5)
\]

where we assume that the deposit interest rate, \( R^d_t \), equals the risk-free rate, \( R_t \), by arbitrage. Letting the loan premium elasticity be \( \psi \), we assume that the loan premium takes the following functional form:

\[
S(\cdot) = \left( \frac{q_t k_{t+1} + nw_t}{nw_t} \right)^\psi \quad (2.6)
\]

Following Greenwood et al. [1988] and Khan and Tsoukalas [2011], I model the
cost of capacity utilization as faster depreciation of existing capital. In particular, I assume that the capital depreciation rate, \( \delta(\cdot) \), is increasing in capacity utilization, \( \delta'(u) > 0 \).

The firm’s optimal choice for labour, capital, and capacity utilization is given by the following conditions:

\[
\begin{align*}
    w_t &= (1 - \alpha) \varphi_t \frac{y_t}{l_t} \quad (2.7) \\
    R_t &= E_t \left[ \frac{\alpha \varphi_{t+1} \frac{y_{t+1}}{k_{t+1}} + q_{t+1} (1 - \delta (u_{t+1}))}{q_t} \right] \quad (2.8) \\
    \alpha \varphi_t \frac{y_t}{u_t} &= q_t \delta'(u_t) k_t \quad (2.9)
\end{align*}
\]

where \( \varphi_t \) is the firm’s marginal cost of production. Equation 2.7 equates the firm’s unit labour cost paid to the employment agency, \( w_t \), to the marginal productivity of labour. Equation 2.8 equates entrepreneurs’ marginal cost of loans, \( R_t \), to their marginal benefit. Loans secured in period \( t \) are used to purchase capital \( k_{t+1} \) at price \( q_t \). A unit of capital purchased at period \( t \) yields a flow benefit to entrepreneurs equal to the marginal product of capital in the following period, and can be sold at the prevailing price \( q_{t+1} \), net of depreciation. Finally, equation 2.9 equates the marginal benefit of increasing capacity utilization to its marginal cost of a faster depreciation of existing capital.

For firm \( j \) that survives bankruptcy this period, the difference between the ex-post contribution from capital and the ex-ante financing cost of capital available in the current period is added to its net worth.

\[
    nw_t^j = \left[ \alpha \varphi_t \frac{y_t^j}{k_t^j} + q_t \left( 1 - \delta \left( u_t^j \right) \right) \right] k_t^j - E_{t-1} \left[ R_{t-1}^j \left( q_{t-1} k_{t-1}^j - nw_{t-1}^j \right) \right]
\]

To ensure that entrepreneurial net worth is never enough to fully finance new
capital acquisition without requiring external financing, I follow Bernanke et al. [1999] and assume that only a fraction \( \nu \) of firms survive each period. The end-of-period net worth of the \( 1 - \nu \) proportion of firms that go out of business is transferred to new entrants as seed capital \( sc_t \). Aggregating across firms gives the following expression for economy-wide net worth:

\[
nw_t = \nu \left\{ \alpha \varphi_t \frac{y_t}{k_t} + q_t (1 - \delta (u_t)) \right\} k_t - E_{t-1} \left\{ R_{t-1} (q_{t-1} k_t - nw_{t-1}) \right\} \\
+ (1 - \nu) sc_t
\]  

(2.10)

where the ex-ante cost of capital, \( E_{t-1} R_{t-1} \) reflects the optimal loan rate determined by the financial intermediary, and given in equations 2.5 and 2.6.

### 2.2.3 Employment Agencies

To make the problems of labour market search and financial frictions more tractable, I separate hiring and vacancy posting activities from the entrepreneur’s problem. Following Christiano et al. [2011], I assume the existence of employment agencies that create vacancies and conduct hiring on behalf of entrepreneurs to meet firms’ labour demand. Employment agencies take the price of labour services paid by entrepreneurs, \( w^e_t \), as given revenue for each labour hour supplied. Final wages received by the household, \( w_t \), is determined through Nash bargaining between workers and the employment agencies. To allow labour inputs to vary both on the extensive (employment) margin as well as the intensive (hours) margin, I assume that each agency creates a single job, which can either be filled or vacant.

Vacancies, \( v_t \), are matched with job seekers, \( u_t \), to create new jobs, \( m_t \), in each period through a constant returns to scale matching function:

\[
m_t = \sigma_m u^\sigma_t v^{1-\sigma}_t
\]  

(2.11)
where $\sigma_m$ is a parameter reflecting the efficiency of the matching process. The probability that any open vacancy is matched with an job seeker at period $t$, $\varrho_t$, is given by:

$$\varrho_t = \frac{m_t}{v_t} \quad (2.12)$$

Similarly, the probability that a searching worker will be able to find a job at period $t$, $\varsigma_t$, is given by:

$$\varsigma_t = \frac{m_t}{u_t} \quad (2.13)$$

I assume that workers that are newly matched in period $t$ start producing in the next period. An existing match can be separated exogenously each period with probability $\rho$. Aggregate employment, $n_t$, therefore, equals the fraction of matched workers that survived separation, plus the newly formed matches from the last period that become productive in this period:

$$n_t = (1 - \rho) n_{t-1} + m_{t-1} \quad (2.14)$$

Workers separated from their jobs in period $t$ start looking for new jobs immediately. Since the labour force is normalized to one, the number of searching workers in period $t$, $u_t$, is given by:

$$u_t = 1 - (1 - \rho) n_t \quad (2.15)$$

**Bellman Equations**

The employment agency gets $w_t^e$ from entrepreneurs and pays $w_t$ to the household for each hour of labour supplied. The value of a filled and active job, $J_t$, includes current period’s profits from channeling $h_t$ hours of labour from the household to
entrepreneurs, plus the continuation value of the job:

\[
J_t = w^t h_t - w^t h_t + E_t \beta^{\lambda_{t+1}} (1 - \rho) J_{t+1}
\]  

(2.16)

where the probability of the match surviving till next period is given by \((1 - \rho)\).

Since employment agencies are owned by households, the continuation value of the
an active job is discounted by the same stochastic discount factor facing households.

Assume now that it costs an employment agency \(\kappa\) per period to keep a vacancy
open. With probability \(\varrho_t (1 - \rho)\), net of separation, a vacancy will be filled in this
period and become productive in the next. With probability \((1 - \varrho_t)\), a vacancy will
remain unfilled in the current period. The value of an open vacancy to an employment
agency, \(V_t\), expressed in terms of current consumption, is therefore given as:

\[
V_t = -\frac{\kappa}{\lambda_t} + E_t \beta^{\lambda_{t+1}} \left[ \varrho_t (1 - \rho) J_{t+1} + (1 - \varrho_t) V_{t+1} \right]
\]  

(2.17)

As long as the value of a vacancy is positive, new employment agencies will enter
the market and open new vacancies. Free entry ensures that in equilibrium, \(V_t = 0\)
at any period of time \(t\). The equilibrium condition for posting new vacancies is:

\[
\frac{\kappa}{\lambda_t \varrho_t} = E_t \beta^{\lambda_{t+1}} \left[ \varrho_t (1 - \rho) J_{t+1} \right]
\]  

(2.18)

Equation 2.18 implies that in equilibrium the cost of posting a vacancy is equal to
the expected benefit to the agency from a succesful match that becomes active in the
next period. Note that equation 2.18 can be re-written as:

\[
\frac{\kappa}{\lambda_t \varrho_t} = E_t \beta^{\lambda_{t+1}} \left[ w^t_{t+1} h_{t+1} - w^t_{t+1} h_{t+1} + \frac{\kappa}{\lambda_{t+1} \varrho_{t+1}} \right]
\]  

(2.19)

The value to the household of having one more member employed, \(W_t\), can be
written as:

\[ W_t = w_t h_t - \frac{g(h_t)}{\lambda_t} + E_t \beta \frac{\lambda_{t+1}}{\lambda_t} [(1 - \rho) W_{t+1} + \rho U_{t+1}] \tag{2.20} \]

where \( \frac{g(h_t)}{\lambda_t} \) is the disutility of labour expressed in terms of current consumption. The continuation value of employment takes into account the fact that with probability \( \rho \) the worker will lose the job and become unemployed in the next period.

The value to the household of having one more member unemployed, \( U_t \), is given by:

\[ U_t = b + E_t \beta \frac{\lambda_{t+1}}{\lambda_t} [\varsigma \{(1 - \rho) W_{t+1} + \rho U_{t+1}\} + (1 - \varsigma_t) U_{t+1}] \tag{2.21} \]

where \( b \) is a flow value of remaining unemployed, such as unemployment benefits. The continuation value of unemployment takes into account the probability of finding a job in the next period, net of separation, \( \varsigma_t (1 - \rho) \).

**Nash Bargaining**

Firms and workers negotiate hours worked and wages through a Nash bargaining process.\(^5\) The Nash bargained wage rate maximizes the following joint surplus from a successful match that accrue to workers and firms:

\[ \max_{\{w_t, h_t\}} (W_t - U_t)^\eta (J_t - V_t)^{(1 - \eta)} \]

where the first term in parentheses denotes the worker’s surplus from the match, the second denotes the firm’s surplus, and \( \eta \) denotes the relative bargaining power of the worker.

The optimal wage rate, given that in equilibrium, \( V_t = 0 \), satisfies the following

---

\(^5\)Trigari [2009] introduces two types of bargaining - an efficient bargaining where workers and employers negotiate to set both wages and hours, and right to manage bargaining, where wages are negotiated, but hours are unilaterally chosen by the firm and imposed on the worker. Our formulation follows the efficient bargaining assumption.
condition:

$$\eta J_t = (1 - \eta) (W_t - U_t)$$  \hfill (2.22)

Substituting in expressions for $J_t$, $W_t$ and $U_t$ into equation 2.22 and solving for $w_t$ yields the Nash bargained wage rate:

$$w_t h_t = \eta \left[ w_t c h_t + \frac{s_t}{\varrho_t} (1 - \rho) \frac{\kappa}{\lambda_t} \right] + (1 - \eta) \left[ b + \frac{g(h_t)}{\lambda_t} \right]$$  \hfill (2.23)

The wage equation implies that $\eta$ portion of the employment agency’s benefit from posting vacancies accrues to the worker. Workers are also compensated for a fraction $(1 - \eta)$ of the disutility of supplying hours and for the foregone flow benefit from unemployment. In the neoclassical model, the equilibrium wage rate equates the marginal rate of substitution between consumption and leisure to the marginal product of labour. In contrast, the Nash bargained wage rate includes a wedge between the two in the form of market tightness, expressed as the proportion of the two match probabilities:

$$\left( \frac{s_t}{\varrho_t} = \frac{u_t}{v_t} \right).$$

Note that equations 2.19 and 2.23, along with 2.14 describe the important movements pertaining to labour search, and can be expressed in terms of cyclical movements in market tightness. Define labour market tightness, $\theta_t$, as the ratio of vacancies to unemployment, $\theta_t = \frac{v_t}{u_t}$. Then, given the Cobb-Douglas matching function from equation 2.11, the probabilities of filling a vacancy and finding a job can be given, respectively, as follows:

$$\varrho_t = \theta_t^{-\sigma}$$  \hfill (2.24)

$$\varsigma_t = \theta_t^{1-\sigma}$$  \hfill (2.25)

Now, substituting in expressions for $\varrho_t$ from equation 2.24, $\varsigma_t$ from equation 2.25, and
from equation 2.15 into equations 2.14, 2.19 and 2.23, we get the key equations describing labour search in terms of $\theta_t$ as follows:

$$n_t = (1 - \rho)n_{t-1} + \theta_t^{-\sigma} [1 - (1 - \rho) n_{t-1}] \quad (2.26)$$

$$\frac{\kappa}{\lambda_t} \theta_t^\sigma = \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) \left[ w_{t+1}^e h_{t+1} - w_{t+1} h_{t+1} + \frac{\kappa}{\lambda_{t+1}} \theta_{t+1}^\sigma \right] \quad (2.27)$$

$$w_t h_t = (1 - \eta) \left[ b + \frac{g(h_t)}{\lambda_t} \right] + \eta \left[ w_t^e h_t + (1 - \rho) \frac{\kappa}{\lambda_t} \theta_t \right] \quad (2.28)$$

The first order condition of the Nash bargaining problem with respect to hours, $h_t$, gives:

$$(1 - \eta) (W_t - U_t) (w_t^e - w_t) = -\eta J_t \left[ w_t - \frac{g'(h_t)}{\lambda_t} \right] \quad (2.29)$$

Substituting in the optimality condition from equation 2.22, as well as the expression for the disutility of labour supply, and simplifying, we get the expression for equilibrium hours per worker:

$$w_t^e = \frac{\Upsilon}{\lambda_t} h_t^\xi \quad (2.30)$$

Note that given the firms’ labour demand from equation 2.7, the above condition equates marginal product of labour to the marginal rate of substitution between consumption and leisure, much like the neoclassical model. The friction in matching model imposed here creates a wedge in wages and employment, but the determination of hours per capita follows the same rule as the neoclassical model. In this sense, the method of determining per capita hours is efficient.

Finally, the aggregate labour market clearing condition is given by:

$$l_t = n_t h_t \quad (2.31)$$

which says that aggregate labour supply equals hours per worker times number of employed workers.
Competitive Labour Market

As mentioned earlier, the main results of this chapter holds in the absence of search frictions in the labour market. The discussion in the following sections, therefore, also include a version of the model with competitive labour markets. In that case, equations 2.11 through 2.29 are redundant, and the wages paid by firms, \( w_t \), and that received by households, \( \omega_t \), are the same. Since movements in the extensive margin of employment, \( n_t \), is now zero, we have that \( l_t = h_t \). The remaining calibrations presented below are the same across the two versions of the model.

2.2.4 Capital Producers

Capital producers purchase a fraction of final goods from retailers as investment to produce new capital using a linear technology, and sell this new capital to firms at price \( q_t \). The production of capital is subject to an investment-specific technology shock, \( \xi_t \), which is a shock to the marginal efficiency of investment, as in Greenwood et al. [1988]. The capital producers’ problem is to choose the quantity of investment, \( i_t \), to maximize their profits:

\[
\max_{i_t} E_t \left[ q_t \xi_t i_t - i_t - \frac{\chi I}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 i_t \right]
\]

which includes an investment adjustment cost.

The solution to the capital producers’ problem gives the following \( Q\)-type relationship:

\[
q_t \xi_t = E_t \left[ 1 + \chi I \left( \frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} + \frac{\chi I}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 - \chi I \left( \frac{i_{t+1}}{i_t} - 1 \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \right]
\]

(2.32)
The aggregate capital stock evolves according to:

\[ k_{t+1} = \xi_t i_t + (1 - \delta(u_t)) k_t \]  \hspace{1cm} (2.33)

and the investment specific shock \( \xi_t \) follows a first-order auto-regressive process:

\[ \log (\xi_t) = \rho \log (\xi_{t-1}) + \varepsilon_{\xi,t} \]  \hspace{1cm} (2.34)

### 2.2.5 Retailers

The purpose of the retail sector in this model is to introduce price rigidities in the economy. Retailers purchase wholesale goods from entrepreneurs at real price \( p_{w_t} \), equal to the entrepreneurs’ marginal cost \( \varphi_t \), and costlessly differentiate them into a continuum of final goods, \( y^j_t, j \in (0, 1) \), which they sell at price \( p^j_t \) in a monopolistically competitive market. The continuum of final goods make up a composite consumption good, \( y_t \), according to the Dixit-Stiglitz aggregator:

\[ y_t = \left( \int_0^1 y^j_t \frac{1}{1-\varepsilon} dj \right)^\frac{1}{\varepsilon}, \]

which implies an aggregate price of

\[ p_t = \left( \int_0^1 p^j_t \frac{1}{1-\varepsilon} dj \right)^\frac{1}{\varepsilon}. \]

Following Calvo [1983], I assume that only a fraction, \( 1 - \phi \), of retailers reoptimize their selling price at each period. This implies that with probability \( \phi \), the retailer keeps the price the same. These conditions lead to the following new-Keynesian Philips curve:

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \phi) (1 - \phi)}{\phi} \hat{\varphi}_t \]  \hspace{1cm} (2.35)

where hats indicate log-deviations for the variables from their respective steady-states.
2.2.6 Monetary Policy and Aggregation

In the aggregate, final goods are distributed between consumption and investment:

\[ y_t = c_t + i_t \]  \hspace{1cm} (2.36)

Finally, assume that the central bank adjusts the nominal interest rate, \( R_t \), in response to deviations in inflation, \( \pi_t \), and output, \( y_t \), according to the following standard Taylor rule:

\[ \frac{R_t - R}{R} = \left( \frac{\pi_t - \pi}{\pi} \right)^{\varrho_\pi} \left( \frac{y_t - y}{y} \right)^{\varrho_y} \]  \hspace{1cm} (2.37)

where \( R, \pi \) and \( y \) are steady-state values of the nominal risk-free interest rate, inflation and output respectively. Parameters \( \varrho_\pi \) and \( \varrho_y \) are response coefficients for inflation and the output gap.

2.3 Calibration

The calibration of parameters and steady-states in the benchmark model, summarized in Table 2.1, closely follows Christensen and Dib [2008] and Justiniano et al. [2009]. The former study estimates a model similar to the benchmark case described here using maximum likelihood, while the later study estimates a model without financial frictions using Bayesian techniques. Values for parameters related to labour search are taken from Trigari [2009].

A discount rate, \( \beta \), of 0.9928 imply an annual steady-state interest rate of 2.93%. An inverse Frisch elasticity of labour supply, \( \zeta \), of 3.79, matches the median value estimated in Justiniano et al. [2009].\(^6\) A value of 0.45 for the habit formation parameter, \( \varrho_y \), is often assumed in the macro literature. However, studies using micro data, such as Card [1991] and Chetty et al. [2011] suggest a much higher value. Trigari [2009] also uses a high inverse Frisch elasticity (\( \zeta = 10 \)) in the presence of labour search. The qualitative results presented in this paper is robust to the calibration of \( \zeta \).

\(^6\)A low value for \( \zeta \) is often assumed in the macro literature. However, studies using micro data, such as Card [1991] and Chetty et al. [2011] suggest a much higher value. Trigari [2009] also uses a high inverse Frisch elasticity (\( \zeta = 10 \)) in the presence of labour search. The qualitative results presented in this paper is robust to the calibration of \( \zeta \).
a, suggest a moderate degree of habit persistence.

The parameter $\varepsilon$, measuring the degree of the retail sector’s monopoly power, is set to 6, implying a steady-state price markup of 20%. The investment adjustment cost parameter, $\chi_I$, is set to 2.85. Both are consistent with median estimates from Justiniano et al. [2009]. The Calvo [1983] parameter describing the probability of keeping prices fixed, is set to 0.74, corresponding to the mean value estimated in Christensen and Dib [2008]. The response parameters in the Taylor-rule for inflation, $\varphi_\pi$, and output, $\varphi_y$, are set to 1.5 and 0.25 respectively, and are common in the literature.

The steady-state ratio of capital to net worth is set to 2, implying a ratio of debt to assets of 0.5. The survival rate of entrepreneurs, $\nu$, is set to 0.97, implying an expected working life of entrepreneurs of 36 years. The parameter determining the degree of financial acceleration, $\psi$, is set to 0.06, and is consistent with values estimated in Christensen and Dib [2008]. The steady-state value for $S$ implies a 3% annualized spread between loan and risk-free interest rates.

An endogenous job separation rate, $\rho$, of 0.08 is in line with values computed by Davis et al. [1998]. A steady-state employment rate of 0.8 is adopted from Trigari [2009]. The steady-state probability of finding a job, $\varsigma$, is set to 0.25, implying an average job searching spell of 4 quarters. A steady-state probability of filling a vacancy, $\varphi$, of 0.7, a flow benefit from unemployment, $b$, of 0.4, and a vacancy posting cost, $\kappa$, of 0.01, are common in the labour search literature and follow Trigari [2009].

The second derivative of endogenous depreciation with respect to capacity utilization, $\delta''(u)$ is set to 0.02, which is close to the mean value estimated by by Khan and Tsoukalas [2011] using Bayesian techniques.\footnote{In the log-linearized version of Khan and Tsoukalas [2011], this parameter determines the endogenous response of capacity utilization via the following equation: $\dot{u}_t = \frac{r^k}{\pi + \delta''(u)} (r^k_t - \hat{q}_t)$, where $r^k_t$ is the marginal product of capital, and $r^k$ is its steady-state value. In the current chapter, however, capacity utilization responds according to the following: $\dot{u}_t = \left(\frac{r^k}{\pi + \delta''(u)}\right) (\hat{r}^k_t - \hat{q}_t)$. Given a steady}
Table 2.1: Parameter and steady-state calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Habit formation parameter</td>
<td>0.45</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9928</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Inverse Frisch elasticity of labour</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in output</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
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</tr>
<tr>
<td>$\nu$</td>
<td>Entrepreneurial survival probability</td>
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<tr>
<td>$\chi_I$</td>
<td>Investment adjustment cost</td>
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</tr>
<tr>
<td>$\epsilon$</td>
<td>Dixit-Stiglitz elasticity of substitution</td>
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<tr>
<td>$\phi$</td>
<td>Calvo probability of prices remaining fixed</td>
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</tr>
<tr>
<td>$\psi$</td>
<td>Risk premium elasticity</td>
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</tr>
<tr>
<td>$\varrho_\pi$</td>
<td>Inflation coefficient of Taylor rule</td>
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</tr>
<tr>
<td>$\varrho_y$</td>
<td>Output gap coefficient for Taylor rule</td>
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</tr>
<tr>
<td>$\rho_\xi$</td>
<td>Persistence of investment shock</td>
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<tr>
<td>$\frac{k}{nw}$</td>
<td>Steady state capital to net-worth ratio</td>
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</tr>
<tr>
<td>$\frac{\xi}{y}$</td>
<td>Investment share in output</td>
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<tr>
<td>$S$</td>
<td>Steady-state risk premium</td>
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<td>$\delta''(u)$</td>
<td>Marginal depreciation cost of utilization</td>
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<td>$\rho$</td>
<td>Exogenous job separation rate</td>
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<td>$\sigma$</td>
<td>Elasticity in matching technology</td>
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<td>$\eta$</td>
<td>Workers’ bargaining power</td>
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<td>$\varrho$</td>
<td>Probability of filling vacancy</td>
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</tr>
<tr>
<td>$\varsigma$</td>
<td>Probability of finding job</td>
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<tr>
<td>$b$</td>
<td>Flow value of unemployment</td>
<td>0.4</td>
</tr>
<tr>
<td>$n$</td>
<td>Steady-state employment rate</td>
<td>0.8</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Vacancy posting cost</td>
<td>0.01</td>
</tr>
</tbody>
</table>
I solve the model in log-linear form using Dynare.

2.4 Results

Figure 2.1 shows impulse responses to an investment shock for three cases with search frictions in the labour market – the benchmark model with financial frictions (solid line), a model without financial frictions (dashed line), and one with constant capacity utilization (dotted line). Figure 2.2 shows the same impulse responses for three similar cases with competitive labour markets. The key finding is that regardless of frictions in the labour market, consumption is positive on impact only under the benchmark specification, i.e., in the presence of both variable capacity utilization and financial frictions. Both in the model with labour search and the one without, assuming a frictionless financial market or a constant rate of capacity utilization results in a decline in consumption following an investment shock.

In a standard neoclassical framework, temporary shocks to the marginal productivity of investment are unlikely to generate comovement of investment and consumption. Intuitively, an increase in the marginal productivity of investment increases the return to investment. This gives households an incentive to postpone consumption in favour of saving, and take advantage of this temporary and unexpected rise in returns. The same intertemporal substitution effect also provides incentives to postpone leisure. All else equal, the resulting increase in labour supply leads to an increase in hours and output and a decrease in wages. This way, investment and output moves inversely with consumption and wages in response to investment shocks. This result is shown by the dashed lines in figure 2.2.

To understand the inverse comovement issue in more detail, consider the equilibrium state value of $r^k$ equal to 0.035, my calibration of 0.02 is close to the mean value reported in Khan and Tsoukalas [2011].
Equilibrium condition in the standard neoclassical framework which equates the marginal rate of substitution (MRS) between consumption and labour to the marginal product of labour (MPL). With standard preferences and technology, the MRS depends positively on consumption and hours, while the MPL depends negatively on hours.

\[ MRS(C_+, L_+) = MPL(L_-) \]

Intertemporal substitution effects increase hours on impact. This reduces the MPL and increases the MRS. To remain in equilibrium, consumption must therefore necessarily decline.

Justiniano et al. [2010] highlight three margins through which frictions can be introduced in the equilibrium condition above to generate comovement in consumption. First, introducing habit formation in the utility function changes the form of the MRS. Households’ preference to smooth consumption prevents any rapid decline in consumption on impact of the shock. Second, endogenous capacity utilization shifts the MPL schedule outward. By increasing utilization of existing capital, investment shocks increase output, and hence MPL, on impact. This mechanism was first analyzed by Greenwood et al. [1988] and adopted in some form by all subsequent studies by researchers studying investment shocks. Finally, rigid prices and wages modelled through monopolistic competition in the new-Keynesian framework introduce counter-cyclical markups of price over marginal cost, and real wage over MRS, respectively. Subsequently, both price and wage rigidities have been used in the literature to increase the variation in output to investment shocks. However, as the dashed line in figure 2.2 shows, in the absence of financial frictions, the combination of habit formation, capacity utilization, and price rigidities is not enough to produce consumption co-movement in the benchmark calibration.8

8Wage rigidities are absent in this analysis. Note, however, that even in the presence of wage
Figure 2.1: Impulse responses to investment shock, search frictions in labour market.

Note: All responses from models with search frictions in the labour market. Solid line represents benchmark case with financial frictions and capacity utilization. Dashed line represents model without financial frictions. Dotted line represents model without capacity utilization.
The key result of this chapter is that financial frictions can generate consumption co-movement to investment shocks, without having to resort to restricting wealth effects in the labour supply, or assuming wage-rigidities. The financial accelerator mechanism adds another margin of frictions by directly attenuating the level of savings induced by investment shocks.

Christensen and Dib [2008] show that financial frictions magnify the response of shocks that work by affecting demand, such as consumption preference shocks or monetary policy shocks in their model, and attenuate the response of shocks that work through the supply side, such as technology and investment shocks. This characteristic of financial frictions is also supported by Iacoviello [2005], who considers a binding constraint on the level of debt determined by the value of collateral used to secure loans. It is precisely this attenuating effect that generates a positive wedge between the loan interest rate paid by entrepreneurs and the deposit rate received by households for their savings. The resulting higher loan rate dampens investment demand, allowing consumers to save less and consume more.

A positive shock to the marginal efficiency of investment increases the level of new capital produced per unit of investment. This increase in capital supply reduces the price of capital. In choosing capacity utilization, firms balance the benefit of increased output from higher utilization with the cost of higher depreciation of installed capital. Since there is no difference between new and installed capital in this framework, the value of installed capital falls on impact. From the firm’s point of view, depreciating installed capital at a higher rate is now less costly. Consequently, firms increase capacity utilization. This boosts output and raises marginal returns to capital. In the standard model, inter-temporal substitution effects induce households to postpone consumption in favor of savings to take advantage of this temporary higher return on rigidities, Justiniano et al. [2010] find that consumption falls following an investment shock.
Figure 2.2: Impulse responses to investment shock, competitive labour market.

Note: All responses from models with competitive labour market. Solid line represents benchmark case with financial frictions and capacity utilization. Dashed line represents model without financial frictions. Dotted line represents model without capacity utilization.
investment and capital, which, in turn, results in a fall in consumption on impact.

In the presence of the financial accelerator, however, the reduction in capital prices brought forth by a positive investment shock reduces the value of current capital holdings, and hence, firms’ net worth. With net worth reduced through lower capital prices, firms must now finance an even higher proportion of their investment activity through loans. Higher loans and lower net worth deteriorates the firms’ balance sheet positions, increasing the leverage ratio. The optimal financial contract now requires firms to pay a higher loan premium to compensate financiers for the increased riskiness of firms. As a consequence, the spread between the deposit and loan rate rises. This higher loan premium, in turn, attenuates investment activity. This way, financial friction abates the investment response to a positive shock. Since financial firms channel savings to investment, an attenuated investment response translates to a comparatively smaller increase in savings from households, allowing consumption to rise on impact. This result is demonstrated by the solid lines in figures 2.1 and 2.2, and is preserved regardless of whether the labour market is competitive or subject to search frictions.

The effect of investment shocks in the Neoclassical framework can also be explained as the combination to two conflicting effects on consumption. First, a substitution effect, which implies that to take advantage of the temporary rise in returns to capital, households substitute current consumption for savings. This effect reduces consumption. Second, an income effect, which implies that the rise in output and earning following an investment shock increases lifetime income of households. This effect increases consumption. In the model without financial frictions, the substitution effect dominates, and consumption falls. In the presence of financial frictions, however, a wedge is created between the cost of investment by entrepreneurs and the returns to investment received by households. This wedge reduces the substitution
effect of increased savings, allowing consumption to rise. Total output, however, may increase by less, depending on the parameterization and specification of labour market competitiveness. This, in turn, may also reduce the positive effect of the income effect identified above. However, the attenuation of the substitution effect due to the interest wedge is sufficient to generate a positive consumption response in this chapter.\textsuperscript{9}

Although consumption response is positive for both labour market specifications, the magnitude of consumption response depends on the dynamics of total hours. In the standard neoclassical model, the response of hours worked depend on both the intertemporal substitution effect as well as the wealth effect in labour supply decisions. Following an investment shock, intertemporal substitution effects in labour supply decisions direct households to postpone leisure in favour of labour. Through an increase in productive capital, the wealth effect, in contrast, pushes households to reduce labour supply. By completely shutting off the wealth effect channel in labour supply, Greenwood et al. [1988] preferences allow higher labour inputs in equilibrium, resulting in higher output and consumption. Empirical evidence, however, does not support such a stringent restriction. Using Bayesian techniques Khan and Tsoukalas [2011] estimate a model with Jaimovich and Rebelo [2009] preferences that allow for variable degrees of wealth effect, and find that post-war U.S. data support the presence of an intermediate degree of wealth effects in labour supply.

Instead of restricting wealth effects through special preferences, this paper introduces search frictions in the labour market, allowing labour inputs to vary both in

\textsuperscript{9}Note that the substitution effect is muted in a small open economy framework, where the home country’s real interest rate is equalized to the world interest rate. In this case, any rise in returns to capital in the home country will invite capital flows from the rest of the world. In this way, the burden on domestic savings to fulfill investment needs is lessened, as is the substitution effect on consumption. However, the empirical evidence cited in the first section of this chapter documenting the consumption co-movement puzzle is based on data from the U.S., which is thought to be closer to the closed economy, or at least large open economy, framework.
Figure 2.3: Impulse responses to investment shock, comparison of models with and without labour search.

Note: Solid line represents benchmark case with search frictions in labour market. Dashed line represents model with competitive labour market. Workers’ bargaining power, $\eta = 0.5$. Both models feature financial frictions and variable capacity utilization.
the intensive margin (hours per worker) as well as the extensive margin (number of workers). Figure 2.3 compares impulse responses for two models – the solid line representing the benchmark model with search frictions, and the broken line representing a model with competitive labour market. Both specifications include financial frictions and capacity utilization. In the case of search frictions, the response for total hours include movements in both the intensive and extensive margins. As the figure shows, search frictions allow for a higher initial response in consumption, investment, and output, stemming from a larger response in total hours worked.

Search frictions in the labour market introduce a pro-cyclical spread between the marginal productivity of labour and wages received by the household. As the Nash bargaining wage equation 2.28 specifies, part of this spread is due to movements in labour market tightness, $\theta_t$. From the employment agency’s perspective, a higher difference between the marginal productivity of labour and wages paid out to households create incentives to post more vacancies and hire more workers. This increases labour inputs in the extensive margin. The large movements in the extensive margin, in which household decisions to supply hours do not factor in, generate a large response in total hours, leading to higher output, investment and consumption. By separating total hours into intensive and extensive margins, search frictions reduce the amount of control households have in determining labour supply.\footnote{The amount of control households have on determining wages is represented by the households’ relative bargaining power, $\eta$. If households were to have complete power in the Nash bargain process, i.e., if $\eta = 1$, then impulse responses form the two cases would closely resemble each other.} Consequently, the negative influence of wealth effects on labour supply identified by Greenwood et al. [1988] is reduced under the presence of search frictions.
2.5 Conclusion

This paper shows that adding financial frictions and endogenous capacity utilization to a new-Keynesian DSGE model can generate a positive consumption response on impact for an investment shock. Financial frictions attenuate the effect of a shock to the marginal productivity of investment. A reduction in the price of installed capital lowers firms’ net worth, which in turn increases the wedge between the loan interest rate and the policy interest rate. A dampened response of investment results in a moderated response in household savings, allowing consumption to rise. This result is robust to the level of competitiveness in the labour market, and does not require special preference structures that restrict the wealth effect on labour supply, or wage stickiness.

Introducing search frictions in the labour market, however, contribute positively to the magnitude of consumption response, by allowing movements in both intensive and extensive margins of labour inputs. Nash bargained wages restrict the amount of control households have in determining labour supply. A pro-cyclical wedge between the marginal product of labour and wages paid to households provide incentives for employment agencies to post more vacancies and increase labour inputs in the extensive margin – enough so that output, investment and consumption increase compared to the competitive labour market case.

Note, however, that the solution to the consumption co-movement problem proposed in this paper hinges on (a) a counter-cyclical movement in the price of capital, which is characteristic of shocks affecting the supply of capital (Christiano et al. [2010]); and (b) the fact that financial frictions amplify the effects of demand shocks on investment, but dampen those of supply shocks (Christensen and Dib [2008]; Iacoviello [2005]). Despite this dampening effect, however, Christensen and Dib [2008] find that investment shocks explain the highest proportion of movements in invest-
ment for the U.S. Moreover, Christiano et al. [2010] show that even in the presence of shocks affecting the demand for capital, investment shocks that solely affect capital supply account for more than a third of the movement in investment for the U.S. in business cycle frequencies. Consistent with the co-movement problem, however, investment shocks explain only a small proportion of the movement in consumption in these studies. Adding both capacity utilization and financial frictions, as outlined in this paper, should bring consumption response in line with stylized facts, allowing for an improved analysis of the effects of investment shocks.
Bibliography


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Chapter 3

House Prices, Consumption, and Government Spending Shocks

3.1 Introduction

House price changes determine the amount of funds that financially constrained homeowners can borrow against the value of their homes for current consumption. If fiscal policies affect house prices, they can provide a channel for influencing private consumption, and hence, aggregate demand in the economy. This is important in the context of the U.S. economy for two reasons. First, the slow recovery following the 2008 financial crisis has coincided with a renewed interest in determining the effects of fiscal policy and a better understanding of its transmission mechanism.\(^1\) Second, the weakness in the housing market continues to be a major concern for economic recovery. Although the federal spending allotment of $14.7 billion under the American Recovery and Reinvestment Act (ARRA) of 2009 and housing policies under the Making Home Affordable Program may have slowed the decline in house prices, it is estimated that by mid-2012, in 22.3% of residential properties with mortgages, 

\(^1\)See, for example, Romer [2011].
borrowers owed more on their mortgages than the value of their homes.\textsuperscript{2} For both reasons, empirical evidence on the effects of fiscal policies on house prices and consumption can help inform policy on the housing market. At the same time, models used for policy analysis should reflect this evidence. Surprisingly, however, such evidence has not been adequately established.

The objectives of this chapter are twofold: First, to determine the effects of government spending shocks on house prices empirically, and second, to examine whether dynamic stochastic general equilibrium (DSGE) models with housing can account for these effects as these models are widely used in informing policy.\textsuperscript{3}

We employ the Blanchard and Perotti [2002] approach and identify government spending shocks using U.S. data, and examine their effects on house prices and consumption. As emphasized by Ramey [2011], however, this approach misses the timing of anticipated government spending and gives a result different from the narrative approach of Ramey and Shapiro [1998]. To account for anticipated effects in the identification of shocks, we follow Auerbach and Gorodnichenko [2012] and include forecasts and forecast errors for the growth rate of government spending in the VAR system. Our main empirical finding is that house prices rise in a persistent manner after a positive government spending shock.\textsuperscript{4} The increase in house prices is statistically significant and peaks between 5 and 8 quarters in the baseline specification that accounts for anticipation effects.

In sharp contrast to the empirical evidence, house prices \textit{fall} in a DSGE model.

\textsuperscript{2}CoreLogic Report (September 2012)and Sengupta and Tam [2009].

\textsuperscript{3}The nexus between the housing market and the macroeconomy has received renewed interest from both academics and policy makers. See Iacoviello [2010] for a recent perspective and Leung [2004] for an early review.

\textsuperscript{4}We are aware of only one previous study by Afonso and Sousa [2008] who examined the effects of government spending shocks on U.S. house prices. Although they do not control for expectations in the identification of shocks, our findings are still consistent with theirs. As it turns out, controlling for expectations has a big effect on the timing of the peak response of consumption to government spending shock. A second difference is that they do not explore the implications for DSGE models of housing which is one of the objectives of our paper.
with housing after a positive government spending shock. We highlight this counterfactual result relative to the SVAR evidence by introducing government spending shocks in the Iacoviello [2005] model of housing, with (patient) lenders and (impatient) borrowers. Both types of agents make housing purchases and receive utility from housing services. Housing also serves as a collateral in borrowing funds. The borrowers, however, face a collateral constraint which limits their borrowing to a certain fraction of the expected value of their housing stock. This framework is a natural starting point for studying the dynamic effects of shocks on house prices and consumption and has been widely used in the literature for this purpose. A recent example is Andres et al. [2012] who augment the Iacoviello [2005] model with search and matching frictions to study the size of fiscal multipliers in response to government spending shocks. But even in their more general model, house prices fall after a government spending shock.\(^5\) Since Andres et al. [2012] focus on studying fiscal multipliers, they do not examine whether the house price response to a positive government spending shock is consistent with empirical evidence as we do.

Why do house prices fall after positive government spending shocks in the model? The intuition follows from the approximately constant shadow value of housing for lenders. Housing is a long-lived good and provides a service-flow for many periods in the future. The property of near-constant shadow value of long-lived goods was first pointed out in Barsky et al. [2007] in the context of durable goods and permanent monetary policy shocks.\(^6\) In a lender-borrower DSGE model, the shadow value of housing for the lender, defined as the product of the relative price of housing and marginal utility of consumption, is determined by the expected infinite sum of

\(^5\) See Figures 2, 3, and 4 in Andres et al. [2012].

\(^6\) More recently, Sterk [2010] highlights the role of the quasi-constancy property to re-examine the extent to which credit frictions can resolve the lack of comovement between durable and non-durable consumption in New Keynesian models following a monetary tightening as studied by Monacelli [2009].
discounted marginal utility of housing. Two key features make the shadow value of housing approximately constant. First, the marginal utility of housing depends on the stock of housing. Housing flows do not contribute much to the variation in this stock and thus it remains close to its steady state. Second, temporary government spending shocks exert little influence on future marginal utility of housing. A positive government spending shock has a negative wealth effect on lenders as they expect an increase in future taxes. This causes them to cut current consumption, thereby raising the marginal utility of consumption. Since the shadow value of housing remains approximately constant, it follows that the relative price of housing must fall.

Aggregate consumption in the model also falls after a positive government spending shock. For lenders, the negative wealth effect that reduces their current consumption is reinforced by the fall in the value of the housing stock. For borrowers, the fall in the current and expected price of housing constrains consumption for two reasons. First, their initial housing stock is less valuable and, second, the expected real value of their collateral falls. These effects resemble the ones discussed in Callegari [2007] who shows that in the presence of durable goods, a positive government spending shock no longer leads to an increase in consumption as in a model with rule-of-thumb consumers considered in Galí et al. [2007].

Increasing either the proportion of impatient-borrowers or the loan-to-value ratio in the model continues to produce the joint decline in house prices and consumption to a government spending shock. Next, we consider a model with housing production and nominal wage rigidities as proposed by Iacoviello and Neri [2010], and introduce government spending shocks in that model. Even with housing production, both house prices and consumption fall upon impact after a government spending shock. This case confirms that the counterfactual movement in house prices in the benchmark model relative to the SVAR evidence is not driven by either the assumption of fixed
housing stock or the absence of wage rigidities in the labour market. The findings do not change when we consider price stickiness in housing production sector. Finally, we also consider non-separable preferences of the type in Greenwood et al. [1988] and show that the negative wealth effects of government spending shocks are not eliminated in the presence of housing. This suggests that the scope for aggregate consumption and real wages to rise under this type of preferences following a positive government spending shock is limited.

The implications are that existing DSGE models of housing do not agree with the joint response of house prices and consumption to identified government spending shocks. The counterfactual response of house prices and consumption poses a new challenge when using this class of models to assess the joint effects of discretionary fiscal policy for consumption and the housing market. Improving existing DSGE models of housing along this dimension is, therefore, a useful direction for future work.

The rest of the paper is organized as follows. Section 2 presents the empirical evidence. Section 3 presents a benchmark DSGE model of housing. Section 4 discusses the effects of government spending shocks on house prices, consumption, and other model variables, and presents extensions of the benchmark model. Section 5 concludes.

### 3.2 Empirical Evidence

The point of departure for our empirical analysis is the seminal paper by Blanchard and Perotti [2002], which examines the effect of fiscal shocks in a structural VAR with government spending, revenues and output. Changes in fiscal variables – government purchases and tax revenues – can result from discretionary policy action or automatic
responses to innovations in output.\textsuperscript{7} Fiscal shocks are then identified by assuming that discretionary fiscal responses do not occur within the same quarter as any innovation in output. By the time policy-makers realize that a shock has affected the economy, and go through the planning and legal processes of implementing an appropriate policy response, a quarter would have passed. Non-discretionary responses, on the other hand, can be identified through spending or revenue elasticities of output either estimated using institutional information or through auxiliary regressions. In this setting, any innovation to fiscal variables that are not predicted within the VAR system are interpreted as unexpected shocks to spending or revenues.

Since we are interested in estimating the effects of government spending shocks only (and not the effects of taxes on output), the timing assumption essentially reduces to a Cholesky-ordering of the VAR with government spending ordered first.\textsuperscript{8} Specifically, this implies that other shocks in the system do not affect government spending within a quarter, while government spending affects the remaining variables in the same quarter. This approach has been widely used (see Galí et al. [2007], Fatas and Mihov [2001]) in demonstrating that increases in government spending raises output, consumption and wages. We start by following these earlier studies and estimating a quarterly VAR in $X_t = \left[ G_t \ T_t \ Y_t \ C_t \ Q_t \right]'$ with four lags, a constant, and linear and quadratic time trends as follows:

$$X_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + A(L)X_{t-1} + e_t$$

Here, $A(L)$ is a lag polynomial of degree 4, $G_t$ is government consumption and gross investment, $T_t$ is government tax receipts less transfer payments, $Y_t$ is output, $C_t$

\textsuperscript{7}For example, an exogenous increase in output may result in an increase in total tax revenues if the tax base increases and tax rates remain the same.

\textsuperscript{8}For the effects of taxation on output, see Romer and Romer [2010] and Mertens and Ravn [2012], who provide evidence on the aggregate effects of tax shocks in the U.S. and Cloyne [2012] for the U.K.
is total private consumption less consumption of housing services and utilities, and \( Q_t \) is an index of the median price of new houses. All variables are deflated by the GDP deflator, and expressed in log per-capita terms where appropriate. Further descriptions and sources of the data is provided in Appendix ???. The data span from the third quarter of 1963 through the last quarter of 2007. The start date is limited by the availability of house prices, and the end date set to exclude the 2008 financial crisis. Spending shocks are then identified as a Cholesky-ordered innovation to \( G_t \). Figure 3.1 shows the impulse responses of a shock to government spending in this baseline VAR specification, along with one-standard deviation Monte Carlo confidence intervals. All impulse reponses are normalized their relative standard-deviations. Clearly, following a positive government spending shock, both consumption and house prices rise.

Ramey [2011], however, argues that if fiscal shocks are anticipated by private agents, the above identification scheme will be misleading. The timing of the shock plays a crucial role in identification. Alongside decision lags, there may be implementation lags in realizing fiscal policy. Often, governments announce their intended spending in advance, and the actual spending occurs in a staggered manner over a longer period of time. Private agents, then, would anticipate government spending well in advance and adjust their optimal consumption behaviour accordingly, while the econometrician would only see the effect of the policy when actual spending increases. If, contrary to the finding of Blanchard and Perotti [2002], private consumption were to decline upon the announcement of future increases in spending, a mis-timed VAR analysis would only capture the return of consumption to steady-

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9 The DSGE model presented in section 3.3 portrays a one-good economy where the numéraire is the final good. In particular, the real house price presented in the model is implicitly normalized by the final good price. It therefore makes sense to deflate the corresponding nominal variables in the empirical section by the GDP deflator.

10 E.g. a one standard deviation shock in government spending leads consumption to increase by around 0.26 standard deviations on impact.
Figure 3.1: Impulse responses of key variables to a government spending shock from a Cholesky-identified VAR

Notes: Dotted lines show the 16th and 84th percentile of the related distribution from 1000 Monte Carlo simulations.
state, and not the initial decline. Thus, the econometrician will mistakenly infer that consumption rises following a spending increase. Using narrative records of government accounts, Ramey and Shapiro [1998] and Ramey [2011] identify spending shocks as dates when a large amount of national defence spending was announced and find that consumption declines following a positive fiscal shock. The anticipation issue is emphasized in Ramey [2011] by showing that lagged defence spending dates Granger cause the VAR shocks identified by Blanchard and Perotti [2002], suggesting that their identification scheme misses information already available to private agents. Auerbach and Gorodnichenko [2012] also finds that a sizeable fraction of innovations identified by VAR is predictable.\footnote{Specifically, they find that residuals from projecting private sector forecasts of government spending growth on lags of variables included in the VAR is positively correlated with residuals from projecting actual government spending growth on the same variables. If the VAR innovations were unexpected, then the two residuals should be uncorrelated.}

The anticipation problem arises because private agents have access to more information than the econometrician, which allows them to develop a forecast of government spending that the econometrician does not observe. As discussed in Leeper et al. [2011], this issue can cause an invertibility problem, in that, it may not be possible to recover the structural shocks facing private agents from the identified shocks. Finding ways to address this issue is an area of ongoing research.\footnote{See, for example, Dupor and Han [2011], Forni and Gambetti [2011], and Sims [2012].} We consider two approaches that can help account for the anticipation effects and also mitigate the invertibility problem. First, we include variables containing private sector forecasts of future spending in the VAR specification. Following Auerbach and Gorodnichenko [2012], we control for the forecastable components by including in the VAR a variable that captures forecasted government spending from two sources: (i) the Survey of Professional Forecasters (SPF) and (ii) forecasts prepared by the Federal Reserve Board staff for the meetings of the Federal Open Market Committee.
(Greenbook). The SPF forecasts are available from 1982 onward, while the Greenbook forecasts are available from 1966 through 2004. We take the variable used in Auerbach and Gorodnichenko [2012] generated by splicing the two series to create a continuous forecast series starting in 1966.\footnote{We thank Yuriy Gorodnichenko for providing us with the data on government spending forecasts.} The variable contains forecasts made in period $t - 1$ for the period-$t$ spending value. We augment the baseline VAR by considering $\hat{X}_t = \begin{bmatrix} FE^G_t & G_t & T_t & Y_t & C_t & Q_t \end{bmatrix}'$, where $FE^G_t$ is the forecast error for the growth rate of government spending. The unanticipated shock, then, is identified as the innovation in the forecast error itself, rather than an innovation to $G_t$.

Second, following Forni and Gambetti [2011], we estimate a factor-augmented VAR (FAVAR) with 110 variables, 13 static factors, 4 lags and 6 structural shocks.\footnote{The dataset is almost identical to Forni and Gambetti [2011], with few exceptions, and is described in a separate appendix available upon request. All data are in quarterly frequencies and cover the period from 1963:1 through 2007:4.} We identify government spending shocks by imposing the following sign restrictions on the impulse response functions: total government spending, federal government spending, total government deficit, federal government deficit and output all increase in the fifth quarter following the shock. Imposing the restriction a few quarters after the impact period allows for anticipation effects. This strategy, however, does not allow us to differentiate between an expected and an unexpected shock.

Figure 3.2 shows the impulse responses of a shock to the one-step-ahead forecast error of government spending in the expectation-augmented VAR specification. The differences in the shape of the IRF’s suggest that expectations do play a role in determining the effects of government spending shocks. While in the baseline case (Figure 3.1), the largest effect of an increase in spending occurs in the first period, we see that the largest effect on output and consumption occurs about 5 quarters after the spending shock impact (Figure 3.2). Even after controlling for expectations, however, house prices clearly increase in a persistent manner following an unanticipated government
Figure 3.2: Impulse responses of key variables to a government spending shock, controlling for anticipation effects.

Notes: The VAR specification includes one-step-ahead forecast errors from private sector forecasts of government spending (Auerbach and Gorodnichenko [2012]). Government spending shock is identified as a Cholesky-ordered shock to the forecast error. Confidence bands show the 16th and 84th percentile of the related distribution from 1000 Monte Carlo simulations.
spending shock. Figure 3.3 shows the impulse responses for the FAVAR specification along with one-standard error boot-strapped confidence bands. The FAVAR specification also shows that house prices and consumption increase following a government spending shock.

3.3 A DSGE Model with Housing

In this section we describe a DSGE model with housing based on Iacoviello [2005] with an exogenous fixed supply of housing.\footnote{In section 3.4.1 we consider housing production.}

3.3.1 Households

There are two types of agents in the economy characterized by their different rates of time preference. The size of the total population is normalized to one. A fraction, $0 < \alpha < 1$, of the population denotes the proportion of impatient agents who discount the future at a rate higher than patient agents. Both agents receive utility from consuming a non-durable good, from the services of the stock of housing they own, and from leisure. In addition, only the patient agents hold government debt, and own physical capital which they rent out to the production sector. Both agents supply labour services to the production sector. In this setting, patient households are net lenders and impatient households are net borrowers in the steady state. Due to the presence of financial frictions, the borrowers face a constraint on the amount they can borrow in each period by using their stock of housing as collateral. As in Iacoviello [2005], the amount of uncertainty in the economy is small enough such that for borrowers, the effect of impatience on borrowing always dominates the precautionary motive for self-saving and consequently the collateral constraint is always binding in equilibrium.
Figure 3.3: Impulse responses of key variables to a government spending shock from a FAVAR specification.

Notes: Government spending shock identified using sign restrictions specifying an increase in total and federal government spending, deficits and real output on the 5th quarter following the shock. Bootstrapped confidence bands show the 16th and 84th percentile of the related distribution.
The optimization problems of patient-lenders and impatient-borrowers are to maximize the expected discounted lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^j_t \left[ \ln c^j_t + \Upsilon^j \ln h^j_t - \frac{1}{1 + \eta} (n^j_t)^{1+\eta} \right]$$

where $j = \ell$ for the patient-lenders and $j = b$ for the impatient-borrowers. The variables $c^j_t$, $h^j_t$, and $n^j_t$ denote consumption, housing, and labour supplied to the production sector, respectively. The parameters $\beta^j$, $\Upsilon^j$, and $\eta$ denote the discount factor, the weight of housing in the utility function, and the inverse Frisch elasticity of labour supply, respectively.

The budget constraint facing a patient-lender is

$$c^\ell_t + q_t h^\ell_t + i_t + b^\ell_t + b_t = w_t n^\ell_t + q_t h^\ell_{t-1} + r_t k_{t-1} + d^\ell_t + \frac{r^\ell_{t-1} b^\ell_{t-1}}{\pi_t} + \frac{r^\ell_{t-1} b_{t-1}}{\pi_t} - \tau^\ell_t$$

where $q_t$ is the relative price of housing stock, $k_t$ is capital rented out to the production sector, $r_t$ is the real rental return on capital, and $\delta$ is the capital depreciation rate. Alongside investing in capital, patient households own firms in the production sector from which they receive dividends, $d^\ell_t$, lend an amount $b_t$ (in real terms) to borrowers, and hold government debt $b^\ell_t$ (in real terms), both for the same rate of real gross return $r^\ell_{t-1}/\pi_t$, where $r^\ell_{t-1}$ is the nominal interest rate and $\pi_t$ is the inflation rate. Finally, $\tau^\ell_t$ is a lump-sum tax imposed by the government on patient-lenders. The capital accumulation process is given as

$$k_t = (1 - \delta) k_{t-1} + \phi \left( \frac{i_t}{k_{t-1}} \right) k_{t-1}$$

---

16 We have also experimented with a distortionary labour tax instead of a lump-sum tax, and the results presented in section 3.4 remain the same. We concentrate on the lump-sum formulation for ease of exposition and reference to relevant literature.
Where \( \phi(.) \) denotes capital adjustment costs which are increasing in the rate of investment \( i_t/k_{t-1} \), and which have the following properties: \( \phi'(.) > 0, \phi''(.) \leq 0, \phi'(i/k) = 1 \), and \( \phi(i/k) = i/k \), implying zero costs in the steady state. The budget constraint facing the impatient-borrowers is

\[
c_t^b + q_t h_t^b + \frac{r_{t-1}^n b_{t-1}}{\pi_t} = w_t n_t^b + q_t h_{t-1}^b + b_t - \tau_t^b
\]

(3.4)

where \( \tau_t^b \) is a lump-sum tax. The impatient-borrowers also face a collateral constraint

\[
b_t \leq m E_t \left\{ \frac{q_{t+1} h_{t+1}^b \pi_{t+1}}{r_t^n} \right\}
\]

(3.5)

which says that the real debt service due next period cannot exceed a fraction \( m \in [0, 1] \) of the expected real value of the housing stock held as collateral. Since only a fraction \( 0 < m < 1 \) of the expected discounted value of housing stock is available for borrowing, \( 1 - m \) can be interpreted as a down-payment requirement, and \( m \) the loan-to-value (LTV) ratio.

Denoting the Lagrange multipliers on the constraints (3.2) and (3.3) as \( \lambda_t^\ell_1 \) and \( \lambda_t^\ell_2 \), respectively, the first-order necessary conditions for the patient-lenders which characterize the optimal choices of their consumption, labour supply, housing, investment,
capital, lending and government bonds are as follows:

\[
\frac{1}{c_t} = \lambda_t^f \\
\frac{\left(n_t^f \right)^\eta}{w_t} = \lambda_t^f \\
\frac{\Upsilon_t}{h_t^f} = \lambda_t^f q_t - \beta_t E_t [\lambda_{t+1}^f q_{t+1}] \\
1 = \psi_t \phi' \left( \frac{i_t}{k_{t-1}} \right) \\
\psi_t = \beta_t E_t \left[ \frac{\lambda_{t+1}^f}{\lambda_t^f} \left( r_{t+1} + \psi_{t+1} \left( 1 - \delta \right) \phi \left( \frac{i_{t+1}}{k_t} \right) - \phi' \left( \frac{i_{t+1}}{k_t} \right) \frac{i_{t+1}}{k_t} \right) \right] \\
1 = \beta_t E_t \left[ \frac{\lambda_{t+1}^f n_t}{\lambda_t^f \pi_{t+1}} \right]
\]

where \( \psi_t \), defined as \( \lambda_2^f / \lambda_1^f \), represents the marginal value of capital in terms of the consumption, or the Tobin’s Q.

Denoting the Lagrange multipliers on the constraints (3.4) and (3.5) as \( \lambda^b_{1t} \) and \( \lambda^b_{2t} \), respectively, the first-order-conditions for the impatient-borrowers that characterize the optimal choices of their consumption, labour supply, housing, and borrowing are as follows:

\[
\frac{1}{c_t^b} = \lambda^b_{1t} \\
\frac{\left(n_t^b \right)^\eta}{w_t} = \lambda^b_{1t} \\
\frac{\Upsilon_t^b}{h_t^b} = \lambda^b_{1t} q_t - \beta_b E_t \left[ \lambda^b_{t+1} q_{t+1} \right] - \lambda^b_{2t} m E_t \left[ \frac{q_{t+1} \pi_{t+1}}{r_t^n} \right] \\
\lambda^b_{1t} = \beta_b E_t \left[ \frac{r_t^n}{\pi_{t+1}} \right] + \lambda^b_{2t}
\]
3.3.2 Firms

The production side in this model follows the standard New Keynesian approach which we briefly describe here. There is a perfectly competitive final good sector in which firms produce a non-durable consumption good, $y_t$, using a continuum of intermediate goods, $x_t(s)$ with $s \in [0, 1]$, and the Dixit-Stiglitz aggregation technology

$$y_t = \left( \int_0^1 x_t(s) \frac{\epsilon - 1}{\epsilon} ds \right)^{\frac{\epsilon}{\epsilon - 1}}, \quad \epsilon > 0$$

(3.6)

where $1/(\epsilon - 1) > 0$ is the constant elasticity of substitution between the intermediate goods. Profit maximization in the final goods sector, \(\max_{y_t(s)} \{ p_t y_t - \int_0^1 p_t(s)x_t(s)ds \} \) subject to (3.6) generates the demand for each type of intermediate good as

$$x_t(s) = \left( \frac{p_t(s)}{p_t} \right)^{-\epsilon} y_t, \quad s \in [0, 1]$$

where $p_t$ is the price of a unit of final good given by the zero-profit condition as

$$p_t = \left[ \int_0^1 \left( p_t(s) \right)^{1-\epsilon} ds \right]^{\frac{1}{1-\epsilon}}, \quad s \in [0, 1]$$

The firms in the intermediate goods sector operate under monopolistic competition. Each firm uses labour, capital, and a constant returns to scale technology to produce a differentiated good. The production function for a firm $s \in [0, 1]$ is

$$y_t(s) = k_{t-1}(s)^\gamma n_t(s)^\gamma, \quad 0 < \gamma < 1$$

where labour services, $n_t(s)$ are supplied by both agents while capital, $k_{t-1}(s)$ is supplied by lenders only.\footnote{Note that we normalize the productivity shock to 1 since we are interested in studying solely the responses to government spending shocks.} The factor markets are assumed to be perfectly competitive
and firms take the real wage and the real rental cost of capital as given. Cost minimization implies

\[
\begin{align*}
    r_t &= \gamma mc_t \frac{y_t}{k_{t-1}} \\
    w_t &= (1 - \gamma) mc_t \frac{y_t}{n_t} \\
    mc_t &= \gamma^{-\gamma}(1 - \gamma)^{-1} w_t^{\gamma} r_t^{1-\gamma}
\end{align*}
\]

where \( mc_t \) is the real marginal cost, same for all the firms.

Nominal prices of intermediate goods firms are assumed to be sticky. Following the standard Calvo [1983] approach to introduce nominal price stickiness and staggered price setting, only a fraction, \( 0 < (1 - \theta) < 1 \), of firms are assumed to re-set their prices optimally in a given period while the remaining fraction, \( \theta \), keep their prices unchanged. The optimization problem of a price-setting firm, \( s \in [0, 1] \), in period \( t \) is

\[
\max_{\tilde{p}_t} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}^{\ell} \theta^j [\tilde{p}_t y_{t+j}(s) - p_{t+j} mc_{t+j}]
\]

subject to a sequence of demand curves

\[
y_{t+j}(s) = x_{t+j}(s) = \left( \frac{\tilde{p}_t}{p_{t+j}} \right)^{-\epsilon} y_{t+j}
\]

The term \( \Lambda_{t,t+j}^{\ell} \equiv \beta^j \left( \frac{c_{t+j}}{c_{t+j} p_{t+j}} \frac{p_t}{p_{t+j}} \right) \) represents the nominal stochastic discount factor of the patient-lenders who are the owners of the firms in the intermediate goods sector.

\(^{18}\) The assumption of perfectly competitive factor markets allows us to highlight the basic mechanism behind the response of house prices to government spending shocks. Andres et al. [2012], for example, allow job search and unionized bargaining in the labour market to study the magnitude of fiscal multipliers.
The optimal price, $\tilde{p}_t$, satisfies the first-order condition

$$E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}^{\ell} \theta^j y_{t+j}(s) \left[ \tilde{p}_t - \frac{\epsilon}{\epsilon - 1} mc_{t+j} p_{t+j} \right] = 0$$

where

$$p_t = \left[ \theta p_{t-1}^{1-\epsilon} + (1 - \theta) \tilde{p}_{t}^{*(1-\epsilon)} \right]^{\frac{1}{1-\epsilon}}$$

is the aggregate price level.

### 3.3.3 Fiscal and Monetary Policies

We follow Galí et al. [2007] for the fiscal and monetary policy specifications. The government faces a budget constraint of the form (in real terms):

$$\tau_t + b^g_t \equiv \frac{r^n_t - \beta^{\tau}_{t-1}}{\pi_t} + G_t$$

where $\tau_t$ is lump-sum tax revenue (which equals $(1-\alpha)\tau^\ell_t + \alpha \tau^b_t$) and $G_t$ is government spending. The government sets taxes according to the following fiscal rule

$$\tilde{\tau}_t = \rho_b \tilde{b}^\ell_{t-1} + \rho_g \tilde{g}_t$$

where $\tilde{g}_t \equiv \frac{G_t - G}{Y}$, $\tilde{\tau}_t \equiv \frac{\tau_t - \tau}{Y}$ and $\tilde{b}_t \equiv \frac{B_t - B}{Y}$ are deviations of the fiscal variables from a steady state with zero debt and balanced primary budget (normalized by steady-state level of output). $\rho_b$ and $\rho_g$ are weights assigned by the fiscal authority on debt and current government spending. Note that government debt is not modelled as discountable bonds, and pays nominal gross interest, $r^n_t$, each period. This form of government debt makes it easier to compare intertemporal decisions of households.
across different saving instruments. Government purchases are assumed to follow an
exogenously determined auto-regressive process

\[ \tilde{g}_t = \rho_g \tilde{g}_{t-1} + \varepsilon_t \]

where \(0 < \rho_g < 1\) and \(\varepsilon_t\) is an i.i.d. government spending shock with variance \(\sigma_{\varepsilon}^2\).

Since our focus is to illustrate the effects of government spending shocks we assume
a simple monetary policy rule as in Gali et al. [2007] which determines the nominal
interest rate in the economy.\(^{19}\) This rule is given as

\[ \frac{\tilde{r}_n^n}{\hat{r}_n} = \left( \frac{\pi_t}{\pi} \right)^{\theta_x} \]

where \(\tilde{r}_n^n = r^n_t - 1\). And \(\tilde{r}_n^n\) and \(\pi\) are the steady state levels of the (net) nominal
interest rate and inflation, respectively.

### 3.3.4 Aggregation

Aggregate consumption, labor and housing (all denoted in upper case) are weighted
averages of the variables corresponding to patient-lenders and impatient-borrowers
and are given as

\[
\begin{align*}
C_t &= \alpha c^b_t + (1 - \alpha) c^\ell_t \\
N_t &= \alpha n^b_t + (1 - \alpha) n^\ell_t \\
H &= \alpha h^b_t + (1 - \alpha) h^\ell_t
\end{align*}
\]

\(^{19}\)This assumption can be further justified by Clarida et al. [2000]'s finding that the output gap
coefficient in an estimated policy reaction function for the Federal Reserve is statistically insignificant
in the post Volcker era.
Since capital is owned only by patient-lenders, aggregate investment and capital are given as

\[ I_t = (1 - \alpha)i_t \]
\[ K_t = (1 - \alpha)k_t \]

Finally, the aggregate resource constraint is given as

\[ C_t + I_t + \phi \left( \frac{I_t}{K_{t-1}} \right) K_{t-1} + G_t = Y_t \approx K_{t-1}^{\gamma} N_t^{1-\gamma} \]

where the aggregate production function holds up to a first-order approximation as shown in Woodford [2003]. The economy is in equilibrium when all the first-order necessary conditions are satisfied and all the goods and factor markets clear.

3.3.5 Linearization, Calibration, and Model Solution

We log-linearize the first-order optimality conditions of the households and firms, and the aggregate market clearing conditions around a steady state. We use hats on variables to denote the percentage deviations from their steady-state values, respectively. We linearize the government budget constraint (3.7) around a steady state with zero debt and primary balanced budget.\(^{20}\)

The model is set in a quarterly frequency. The discount factors of the patient-lenders and the impatient-borrowers are set to 0.9925 and 0.97, respectively. Iacoviello and Neri [2010] and Iacoviello [2005] argue that this calibration value suffices to ensure that the borrowing constraint is binding in equilibrium. The capital share

\(^{20}\)Note that hatted variables are expressed in percentage deviations, i.e., deviations from their steady state values, normalized by their steady state values. Government variables marked with a tilde, on the other hand, are deviations from their steady state values, normalized by the steady-state level of output. In other words, \( \hat{X}_t = \ln X_t - \ln X \approx \frac{X_t}{X} \) and \( \tilde{X}_t = \frac{X_t}{Y} \) where \( Y \) is steady-state level of output.
of output, $\gamma$, is set to 0.33, and the depreciation rate $\delta$ is set to 0.025. We assume a steady-state price markup of 0.15, implying a steady-state marginal cost $mc$ of $\frac{1}{1.15} \approx 0.87$. The inverse of the Frisch-elasticity of labour supply, $\eta$, is set to 1. The elasticity of capital adjustment cost parameter, $\phi''(\frac{i}{K})$ is set to -14.25, to match the corresponding parameter estimated by Iacoviello and Neri [2010] using Bayesian methods. The benchmark value of $K/Y$ and $qHY$ is taken from Iacoviello and Neri [2010]. The later figure corresponds to the total value of household real estate assets in the U.S., as specified in the Flow of Funds Account (B.100 line 4). We set the Calvo price-adjustment frequency to 0.75, corresponding to an average price duration of one year, and the Taylor rule parameter measuring the response of the monetary authority to inflation, $\varrho_\pi$, to 1.5, a value commonly used in the literature. We match the benchmark values of the fiscal response parameters to those in Galí et al. [2007], and set the tax response to government spending, $\varrho_g$, to 0.1, the tax response to outstanding government debt, $\varrho_b$, to 0.33, and the persistence of government shock, $\rho_g$ to 0.9.

The dynamics presented in this paper depend importantly on the different optimal responses of patient-lenders and impatient-borrowers to a government spending shock. The exposition of these differences in dynamics become easier if we start off the two households with identical consumption and housing levels. As such, we set $c^\ell/Y = c^b/Y = C/Y = 0.5$ and $qh^\ell/Y = qh^b/Y = qHY/Y$. The first can be easily achieved by assuming different levels of steady-state lump-sum taxes. The later can be achieved by setting different values for the weight of housing in utility. Accordingly, we set $\Upsilon^\ell$ to 0.0816 and $\Upsilon^b$ to 0.1102. The benchmark loan-to-value ratio is set at 0.85, following Iacoviello and Neri [2010], and the implications of changing this value is explored in section

---

\[ \Upsilon^\ell = \frac{qHY}{C} (1 - \beta_\ell), \quad \text{and} \quad \Upsilon^b = \frac{qHY}{C} \left[ 1 - \beta_b - \left( 1 - \frac{\beta_b}{\beta_\ell} \right) m_\beta \right] \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Proportion of impatient-borrowers</td>
<td>0.5</td>
</tr>
<tr>
<td>$\beta_\ell$</td>
<td>Patient-lenders’ discount factor</td>
<td>0.9925</td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>Impatient-borrowers’ discount factor</td>
<td>0.97</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse Frisch-elasticity of labour supply</td>
<td>1</td>
</tr>
<tr>
<td>$\Upsilon_\ell$</td>
<td>Patient-lenders’ utility weight on housing</td>
<td>0.0816</td>
</tr>
<tr>
<td>$\Upsilon_b$</td>
<td>Impatient-borrowers’ utility weight on housing</td>
<td>0.1102</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Output elasticity of capital</td>
<td>0.33</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Calvo price-adjustment frequency</td>
<td>0.75</td>
</tr>
<tr>
<td>$\varrho\pi$</td>
<td>Taylor rule response parameter for inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\varrho_b$</td>
<td>Fiscal response parameter to outstanding government debt</td>
<td>0.33</td>
</tr>
<tr>
<td>$\varrho_g$</td>
<td>Fiscal response parameter to government spending</td>
<td>0.1</td>
</tr>
<tr>
<td>$m$</td>
<td>Loan-to-value ratio</td>
<td>0.85</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Persistence parameter for government shock</td>
<td>0.9</td>
</tr>
<tr>
<td>$\phi''(\frac{i}{k})$</td>
<td>Capital adjustment cost parameter 1</td>
<td>-14.25</td>
</tr>
<tr>
<td>$\phi'(\frac{i}{k})$</td>
<td>Capital adjustment cost parameter 2</td>
<td>1</td>
</tr>
<tr>
<td>$\phi(\frac{i}{k})$</td>
<td>Capital adjustment cost parameter 3</td>
<td>$\delta$</td>
</tr>
<tr>
<td>$R$</td>
<td>Steady-state interest rate</td>
<td>$\frac{1}{\beta_\ell}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Steady-state return on capital</td>
<td>$\frac{1}{\beta_\ell} - (1 - \delta)$</td>
</tr>
<tr>
<td>$mc$</td>
<td>Steady-state marginal cost</td>
<td>$\frac{1}{1.15}$</td>
</tr>
<tr>
<td>$wn$</td>
<td>Steady-state wage earnings to output ratio</td>
<td>$(1 - \gamma)mc$</td>
</tr>
<tr>
<td>$\frac{\bar{Y}}{\bar{C}}$</td>
<td>Steady-state consumption to output ratio</td>
<td>0.5</td>
</tr>
<tr>
<td>$\frac{\bar{Y}}{\bar{K}}$</td>
<td>Steady-state capital to output ratio</td>
<td>$2.05 \times 4$</td>
</tr>
<tr>
<td>$\frac{\bar{Y}}{qH}$</td>
<td>Steady-state housing value to output ratio</td>
<td>$1.36 \times 4$</td>
</tr>
<tr>
<td>$\frac{\bar{Y}}{b}$</td>
<td>Steady-state loans to output ratio</td>
<td>$m\beta_\ell \frac{qH}{\bar{Y}}$</td>
</tr>
</tbody>
</table>
3.4. Iacoviello and Neri [2010] estimate the proportion of borrowers $\alpha$ to be 0.21, and consequently, the proportion of savers to be 0.79. However, the rule-of-thumb literature often sets the proportion of non-Ricardian agents to 0.5. As argued below, the counterfactually negative response of housing prices stem from the particular manner in which savers value housing stock. A high proportion of savers lowers the response of housing price. We therefore set the benchmark value of $\alpha$ to 0.5 for ease of exposition. Table 3.1 summarizes the calibration values. Alternative values of $\alpha$ are considered in section 3.4. We use Dynare to solve the model.\footnote{See Adjemian et al. [2011] and http://www.dynare.org/}.

3.4 The Effects of Government Spending Shocks on House Prices and Consumption

Figure 3.4 presents the effects of a one standard deviation positive shock to government spending for the benchmark calibration of Table 3.1. The relative price of housing falls immediately after a positive government spending shock. This response is in sharp contrast to the evidence based on SVAR reported in section 3.2 and the key finding that we wish to highlight in this chapter.

Why does the relative price of housing fall after a positive government spending shock? The intuition follows from the approximately constant shadow value of housing for lenders. Housing is a long-lived good and provides a service-flow for many periods in the future. The property of near-constant shadow value of long-lived goods was first pointed out in Barsky et al. [2007] in the context of durable goods and temporary monetary policy shocks. We can define the shadow value of housing for the patient-lender as $v^\ell_t \equiv \lambda^\ell_t q_t$ and, using the first-order-condition for optimal housing,
Figure 3.4: Response to a positive government spending shock. Benchmark model with fixed housing.

Note: Dashed-dotted line (lender), Dotted line (borrower)
express it in log-linearized form as

\[ \hat{v}_t^\ell \equiv \hat{\lambda}_t^\ell + \hat{q}_t = (\beta_t - 1) \sum_{j=0}^{\infty} \beta_t^j \hat{h}_t^{\ell+j} \]

\[ \approx 0 \quad (3.8) \]

There are two key features which make the deviations of shadow value of housing from its steady state, \( \hat{v}_t^\ell \), approximately zero, as indicated in (3.8). First, the housing flows do not contribute much to the variation in the stock, which means that the marginal utility of housing remains close to its steady state (i.e., the \( \hat{h}_t^{\ell+j} \) terms are close to zero). Second, temporary government spending shocks have little influence on future marginal utility of housing (i.e., the \( \hat{h}_t^{\ell+j} = 0 \) as \( j \) increases). Now, a temporary positive government spending shock induces a negative wealth effect and causes the patient-lenders to reduce current consumption. This raises their marginal utility of consumption, \( \hat{\lambda}_t^\ell > 0 \). Since the shadow value of housing is approximately zero, it implies that the price of housing necessarily falls relative to its steady state value, i.e., \( \hat{q}_t < 0 \). As in Barsky et al. [2007], the quasi-constancy property here implies a near-infinite elasticity of inter-temporal substitution for the patient-lenders. The fall in the price of housing causes them to immediately increase their housing purchases. It is important to note that this result does not depend on the structure of the labour market. Indeed, Andres et al. [2012] introduce job search and unionized bargaining and provide a significant departure from the competitive labour market we consider here in an attempt to account for a positive consumption response following a government shock. Yet, even under that labour market structure they report that house prices fall. Thus, relative to the SVAR evidence reported in section 3.2, the counterfactual response of housing to a government spending shock arises not only in the benchmark model but also in variants with a richer labour market structure.
In contrast to the patient-lenders, the shadow value of housing for the impatient-borrowers rises after the government spending shock. This rise reflects the desire to increase housing to use it as collateral for future consumption. From (3.6), we define the shadow value of housing to the impatient-borrower as $v^b_t \equiv \lambda^b_t q_t$ and express it in log-linearized form (after simplifying the coefficients using steady state conditions) as

$$
\hat{v}^b_t \equiv \hat{\lambda}^b_{1t} + \hat{q}_t = (\beta_b - 1) E_t \sum_{j=0}^{\infty} \beta_b^j \hat{h}^b_{t+j} + m(\beta_t - \beta_b) E_t \sum_{j=0}^{\infty} \beta_b^j \left( \hat{\lambda}^b_{2t+j} + \hat{\pi}_{t+1+j} - \hat{\pi}_{t+j} + \hat{q}_{t+1+j} + \hat{h}^b_{t+j} \right)
$$

(3.9)

The increase in the shadow value of housing, $\hat{v}^b_t > 0$, is driven by the sharp tightening of the current and expected future collateral constraints $\hat{\lambda}^b_{2t+j} (> 0)$, as shown in Figure 3.4 (second row, third column). The coefficient $m(\beta_t - \beta_b)$ shows that the loan-to-value ratio and the relative impatience of the two agents together determine the gap $\hat{v}^b_t - \hat{v}^f_t$. While on impact, the housing demand of lenders decreases, they build up their stock of housing in view of increasing future borrowing, and hence consumption.

Turning to the response of consumption, for the patient-lenders, consumption always falls after a positive government spending shock. The negative wealth effects due to the lower present value of after-tax income, the reduced value of the housing stock due to a fall in the price of housing, and the reduced expected income from loans to borrowers all reinforce each other. Consumption of the impatient-borrowers also falls because the value of their collateral declines when house prices fall. This lowers their ability to borrow, which in turn, lowers consumption. This collateral constraint effect that limits borrowing is in addition to the negative wealth effect from the increased government spending, and works to lower the impatient-borrowers’
consumption. As borrowing slowly increases and returns to the steady state level, the housing stock of impatient-borrowers rises along with their consumption. Total consumption and investment are crowded out while output rises after the positive government spending shock. Note that the strong negative wealth effect on labour supply causes the real wage to fall. We discuss this point further in section 4.2 below.

We examine the robustness of the effects on house prices and consumption to changing the proportion of impatient-borrowers. Figure 3.5 shows the responses of house prices, shadow values of housing, consumption, and housing demand when the share of impatient-borrowers, $\alpha$, goes from zero to 80%. The responses are similar to the benchmark calibration. Both house prices and consumption fall after a positive government spending shock.

Figure 3.6 shows the results of relaxing the collateral constraint by increasing the loan-to-value ratio from zero to 0.95. House prices fall in all the cases. Note that consumption response of the impatient-borrowers is even more negative when $m = 0.95$ relative to the benchmark calibration of 0.85. On the one hand these agents are able to secure more loans but on the other the interest payments on the loans are also higher which constrains their consumption.

To demonstrate that housing assets are behind the negative consumption responses, we consider reducing the steady-state value of housing stock in the model. Note that this is akin to reducing the weight on housing in the utility function. In the benchmark case, where the housing stock-to-output ratio, $\frac{q^H}{Y}$, is set to 5.44 which is the total value of household real estate assets in the U.S. The timing of the model implicitly assumes that both households sell their housing assets at the end of each period, and use the proceeds from this wealth, along with other income to purchase

---

24These responses are consistent with those reported in Callegari [2007] who focuses on how in the presence of durable goods the response of consumption to government spending shock changes relative to when rule-of-thumb consumers are considered as in Gali et al. [2007].

70
Figure 3.5: Robustness to the proportion of impatient borrowers.

Note: Impulse responses to a positive government spending shock in the benchmark model with fixed housing. Dashed-dotted line (lender), Dotted line (borrower)
Figure 3.6: Robustness to the loan-to-value ratio. Benchmark model with fixed housing.

Note: Impulse responses to a positive government spending shock in the benchmark model with fixed housing. Dashed-dotted line (lender), Dotted line (borrower)
Figure 3.7: Robustness to the steady-state housing stock-to-output ratio.

Note: Impulse responses to a positive government spending shock in the benchmark model with fixed housing.
the optimal amount of housing and consumption goods in the next period. Calibrating to the total value of household real estate assets therefore makes sense. We, however, consider calibrations which produce a positive response of consumption. We first note that $q_{hH}$ also determines the steady-state collateralized loans-to-GDP ratio. The alternative calibration uses single family residential mortgages outstanding in U.S. commercial bank balance sheets. We get a ratio that is roughly a tenth of the benchmark value. This can be justified by noting that not all housing assets are traded every period as the model implies, and should not have a direct effect on consumption levels. To capitalize on any increase in house prices, however, a borrower must instead re-mortgage her current housing stock. As such, the mortgage assets are a better guide for calibrating this ratio. As shown in Figure 3.7, even under this alternative calibration of $q_{hH} = 0.54$, both consumption and house prices decline following a positive government spending shock. Moreover, the lower the steady-state housing value-to-GDP ratio, the bigger the drop in house prices. It turns out that for substantially smaller value 0.013 relative to the baseline calibration of 5.44 and a slightly larger share of borrowers ($\alpha = 0.6$), the consumption response turns positive. However, at this extreme calibration, the decline in house prices is also the largest.

3.4.1 Housing Production

In this section we relax the fixed housing stock assumption described in section 3.3, and consider housing production. Our starting point is the DSGE model with housing production developed by Iacoviello and Neri [2010]. We introduce government
spending shocks in this model. The households preferences are given as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c^j_t + \Upsilon^j \ln h^j_t - \frac{1}{1 + \eta} \left[ (n^j_t)^{1+\xi} + (n_h^j)^{1+\xi} \right]^{\frac{1+\eta}{\xi}} \right\}$$

where $j = \ell, b$ and $n_t^j$ denotes the labour supplied to the housing sector by type $j$-household. The aggregate housing stock evolves as

$$H_t = (1 - \delta^h)H_{t-1} + Y^h_t$$

where $\delta^h$ denotes the depreciation rate of housing, which is now assumed to be nonzero, and $Y^h_t$ is housing production. The allocation of aggregate housing to each type of household is decided in a competitive market with flexible house prices:

$$H_t = (1 - \alpha)h_t^\ell + \alpha h_t^b$$

Let the superscript ‘$h$’ denote a variable pertaining to the housing sector. The terms $q_t h^j_{t-1}$ in the budget constraints (3.2) and (3.4) are now modified to $(1 - \delta^h)q_t h^j_{t-1}$, respectively. The patient-lenders now also supply part of their capital stock to the housing production sector. Their budget constraint is

$$c^\ell_t + q_t h^\ell_t + i_t + i^h_t + b^g_t + b_t = \frac{w_t n^\ell_t}{X_t} + \frac{w_t n^h_t}{X^h_t} + (1 - \delta^h)q_t h^\ell_{t-1} + r_t k_{t-1} + r^h_h h^h_{t-1} + \delta^\ell_t$$

$$+ \frac{r^{n^h_h}_{t-1} h^h_{t-1}}{\pi_t} + \frac{R_{t-1} b_{t-1}}{\pi_t} - \tau^\ell_t$$

where $w^h_t$ and $r^h_t$ are the real wage rate and the real return to capital in the housing

\[\text{Note that Iacoviello and Neri [2010] consider the following set of shocks \{housing preference, monetary policy, housing technology, non-housing technology, investment-specific, cost-push, inflation target\}. They, however, do not consider government spending shocks. Since we are interested in studying the effects of government spending shocks only, we suppress the role of other shocks and certain features that Iacoviello and Neri [2010] add to estimate the model.}\]
production sector, respectively. The labour markets in the consumption and housing sectors are imperfectly competitive to allow for the possibility of nominal wage stickiness. The terms $X_t$ and $X_t^h$ denote the markup wedges between the wages received by the households and those paid by the firm to a labour market intermediary or union in the two sectors, respectively. The capital stock of the housing sector evolves as

$$k_t^h = (1 - \delta_h)k_{t-1}^h + \phi_h \left( \frac{i_t^h}{k_{t-1}^h} \right) k_{t-1}^h$$

The housing sector is perfectly competitive. A representative firm produces new housing using labour, capital and land ($\bar{L}$) via a Cobb-Douglas technology. Land is considered a fixed asset used to its capacity in equilibrium. New housing producing firms determine their optimal demand for factor inputs by solving the following optimization problem

$$\max_{K_{t-1}^h, N_t^h} \left[ q_t Y_t^h - r_t^h K_{t-1}^h - w_t^h N_t^h \right]$$

subject to

$$Y_t^h = (\bar{L})^\mu_t (K_{t-1}^h)^\mu (N_t^h)^{1-\mu_t-\mu}$$

Finally, the presence of nominal wage rigidities imply that wage dynamics follow the
standard wage-Phillips curves given as follows:

\[
\dot{\omega}_t = \beta^\ell \omega_{t+1} - \frac{(1 - \theta_w) (1 - \beta^\ell \theta_w)}{\theta_w} \dot{X}_t \\
\dot{\omega}_h^h = \beta^\ell \omega_{h+1}^h - \frac{(1 - \theta_{hw}) (1 - \beta^\ell \theta_{hw})}{\theta_{hw}} \dot{X}_t^h \\
\omega_t = \frac{w_t \pi_t}{w_{t-1}} \\
\omega_t^h = \frac{w_t^h \pi_t}{w_{t-1}^h}
\]

where \(\theta_w\) and \(\theta_{hw}\) are the probabilities of non-adjustment in a given period (Calvo [1983]). Finally, the additional aggregation conditions relative to the benchmark model are

\[
N_t^h = \alpha n_t^{hb} + (1 - \alpha) n_t^h \\
H_t = \alpha h_t^h + (1 - \alpha) h_t^\ell \\
I_t^h = (1 - \alpha)i_t^h \\
K_t^h = (1 - \alpha)k_t^h \\
Y_t^H = (L)^{\mu_l} (K_{t-1}^h)^{\mu} (N_t^h)^{1-\mu-\mu_l} = H_t - (1 - \delta_h) H_{t-1}
\]

The rest of the model is similar to the benchmark model.

Figure 3.8 displays the impulse responses for two variants of the model – one with nominal wage rigidities, and one without. The key point to note is that house prices fall upon impact after the government spending shock, as in the benchmark model. This case confirms that the counterfactual movement in house prices in the benchmark model relative to the VAR evidence is not driven by either the assumption of fixed housing stock or wage rigidities in the labour market. Finally, Figure 3.9 displays the impulse responses of the model when housing prices are assumed to be sticky. As evident, the conclusions do not change.
Figure 3.8: The effect of wage stickiness in model with housing production

Note: Response to a positive government spending shock in a DSGE model with housing production. Dashed line (flexible wage), solid line (sticky wages).
Note: Response to a positive government spending shock in a DSGE model with housing production. Dashed-dotted line (lender), Dotted line (borrower). Sticky housing prices.
3.4.2 Greenwood et al. [1988] Preferences

Greenwood et al. [1988] (GHH) propose a special case of non-separable preferences which eliminate the wealth effects on labour supply. The key property of GHH preferences

\[ U(c, n) = U(c - G(n)) \]

is that the marginal rate of substitution of consumption for leisure is independent of consumption which implies that the labour supply relation depends only on the real wage. In the context of government spending shocks, Monacelli and Perotti [2009] show that assuming GHH preferences and nominal price stickiness can lead real wage and consumption to increase after a positive government spending shock.\(^{26}\) As discussed above, the negative wealth effect on lenders which lowers their consumption is the reason why house prices fall in the model. Can GHH preferences and nominal price stickiness mitigate the negative wealth effect on lenders’ consumption to allow an increase in house prices following a positive government spending shock?

To answer this question, we consider GHH preferences over consumption, housing, and leisure\(^{27}\)

\[ U(c^j_t, h^j_t, n^j_t) = \frac{1}{1 - \gamma} \left[ x_t^{j(1-\gamma)} - 1 \right] \]  

\(^{26}\)Bilbiie [2009] provides a detailed general analysis of non-separable preferences and the conditions which allow for consumption to increase following a positive government spending shock. Cloyne [2011] considers the role of distortionary labour and capital taxes in the transmission of government spending shocks. Like Monacelli and Perotti [2009], both papers, however, consider only non-durable consumption. Kilponen [2012] considers non-separable preferences in the Iacoviello [2005] model to estimate a consumption Euler equation.

\(^{27}\)Two recent papers have examined the role of non-separable preferences in resolving the Barsky et al. [2007] puzzle of lack of comovement between non-durable and durable consumption following a monetary shock. Kim and Katayama [2012] consider general non-separable preferences and Dey and Tsai [2011] consider GHH preferences.
where

\[ x_t^j = (c_t^j)^{1-\psi_h} (h_t^j)_{\psi_h} - \frac{n_t^j (1+\eta)}{1+\eta} \]  

(3.11)

Parameter \( \psi_h \) is the weight of housing in the utility function. The labour supply relation is given as

\[ n_t^j = \left( (1 - \psi_h) w_t \left( \frac{h_t^j}{c_t^j} \right)^{\psi_h} \right)^{\frac{1}{\eta}} \]

Similar to Dey and Tsai [2011], the wealth effect on labour supply, \( n_t^j \), is not eliminated as long as \( \psi_h > 0 \) (positive weight on housing in the utility function). Since the negative wealth effect of government spending shock in the case of housing is even greater than in the case of non-durable consumption alone, the real wage and consumption fall immediately after the positive government shock. As can be seen in figure 3.10, the combination of GHH preferences with sticky nominal prices of consumption is insufficient to deliver a positive consumption and real wage response to a government spending shock. The quasi-constancy property then implies that house prices also fall.

### 3.5 Conclusion

We highlight that a broad class of DSGE models with housing and collateralized borrowing predict both house prices and consumption to fall after positive government spending shocks. The quasi-constant shadow value of lenders’ housing and the negative wealth effect of future tax increases on their consumption are the key reasons for this prediction. By contrast, we present evidence that house prices and consumption in the U.S. rise following positive government spending shocks, estimated using
Figure 3.10: Greenwood et al. [1988] preferences in benchmark model with fixed housing stock

a structural vector autoregression methodology that accounts for anticipated effects. The counterfactual joint response of house prices and consumption poses a new challenge when using DSGE models to address policy issues related to the housing market which have come to fore due to the weak recovery after the 2008 financial crisis. Our findings suggest scope for further model improvements.
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Chapter 4

Search Frictions, Bank Leverage, and Gross Loan Flows

4.1 Introduction

The financial crisis of 2007-08 has highlighted the importance of the banking sector in influencing business cycle dynamics. Consequently, a growing stream of literature has focused on developing quantitative business cycle models that emphasize the bank leverage ratio (the ratio between total bank assets and bank equity) as a channel for generating counter-cyclical interest spreads that amplify and prolong business cycles, and consider the banking sector as a possible source for cyclical movements.\(^1\) At the same time, a second stream of inquiry has used disaggregated bank lending data to reveal important patterns within the banking sector that are not captured at the aggregate level.\(^2\) First, Ivashina and Scharfstein [2010] documents that bank loans

\(^1\)See, among others, the work in Gertler and Karadi [2011], deWalque et al. [2010], Meh and Moran [2010], Gerali et al. [2010], and references therein.

\(^2\)At the onset of the 2008 recession, Chari et al. [2008] argued that although there was little doubt that the U.S. was undergoing a financial crisis, aggregate data on bank lending showed no signs of the downturn. In response, Cohen-Cole et al. [2008] argued that a deeper look at disaggregated lending data shows evidence of the credit crunch. In particular, aggregate lending data hides movements in new loan creation, which had collapsed early in the recession.
vary both at the intensive margin (amount of loans), as well as the extensive margin (number of loans). Second, Dell’Ariccia and Garibaldi [2005] and Contessi and Francis [2010] find that gross flows in new loan creation move pro-cyclically, while those in loan destruction move counter-cyclically in the U.S.\(^3\) Moreover, unlike other recent downturns in the U.S., those featuring an erosion of bank equity led to a decline in net loan flows. However, currently available business-cycle models featuring a banking sector cannot account for these movements in disaggregated loan flows.

This chapter fills this gap in the literature by developing a search-theoretic banking model that can simultaneously explain cyclical movements in interest spreads, as well as in disaggregated flows in loan creation and destruction. A search-theoretic banking model also provides a symmetric-information interpretation of the nature of frictions in the banking sector, and introduces a number of shocks that directly affect the supply of credit.

The model introduces costly search in a banking sector, a bank leverage ratio channel, and endogenous match separation to an otherwise standard New Keynesian DSGE model similar to Christiano et al. [2005] and Smets and Wouters [2007]. Bank loans are used by firms to finance productive capital purchases, and are allowed to vary both in extensive and intensive margins. I use the model to study responses to two shocks that affect credit supply – a one time decline in bank equity, and a one time increase in match separation. I also look at responses to traditional technology and monetary shocks, as well as a capital quality shock that has been shown by Gertler and Karadi [2011] to replicate well important movements related to the bank leverage channel during the crisis.\(^4\)

\(^3\)Gross loan flows calculated using both bank balance sheet data (Contessi and Francis [2010], Dell’Ariccia and Garibaldi [2005]) and firm-level compustat data (Herrera et al. [2011]) confirm this cyclical pattern.

\(^4\)The intuition for the capital quality shock is as follows. A negative shock to the quality of capital reduces the productivity of capital, and hence, its market price. This reduction in asset values is translated through the bank balance sheet identity to an erosion of bank equity, and a
First, I find that the model generates a counter-cyclical interest spread that amplifies and prolongs recessions for all shocks considered. This is similar to the results of Gertler and Karadi [2011] who find that compared to the standard New Keynesian case, a model with a banking sector amplifies and prolongs recessions for technology, monetary, and capital quality shocks by generating a counter-cyclical interest spread. Moreover, just like their model, the one presented in this chapter also generates a counter-cyclical spread and ensuing recession following a negative shock to bank equity.

To get an intuition for this result, consider first shocks that directly affect credit supply. In this economy, loans are generated when a banking loan officer is matched with an unfunded project. Banks incur costs while searching for new funding opportunities, or in maintaining existing relationships. A negative shock to bank equity increases the leverage ratio in banks. To bring the leverage ratio back to its regulated steady-state, banks reduce their efforts in searching for new matches or in maintaining existing ones. This reduces the supply of credit in the extensive margin, makes new matches scarcer, and increases the surplus from successful matches. Through Nash bargaining, this increase in match-surplus is translated to a higher loan interest rate.

Now, consider shocks that first affect credit demand. When a negative technology, or monetary shock hits the economy, loan-demand falls, as does expected profits for banks. Banks again respond by reducing search efforts, and hence, the supply of credit in the extensive margin. In the benchmark calibration, the model produces a reduction in the supply of new matches large enough to generate a counter-cyclical rise in the spread between the policy interest rate determined by a Taylor-type rule, consequent rise in the leverage ratio. This, in turn, reduces future lending and exacerbates the recession. Although a shock to capital quality leads to a recession on its own merit, it also provides a much larger secondary effect by generating a persistent decline in asset values in the bank balance sheet, as seen during the sub-prime crisis.

⁵In contrast, Gerali et al. [2010] finds that the presence of the banking sector amplifies responses to financial shocks, but attenuates the effects of technology shocks.
and loan rates facing firms. This counter-cyclical rise in the interest spread reduces loan demand even further following negative shocks, and generates deeper and more prolonged recessions compared to the standard New Keynesian setup.

In addition, I find that the search-theoretic framework is useful in generating movements in loan creation and destruction flows that qualitatively match empirical evidence. First, the model generates pro-cyclical movements in loan creation, and counter-cyclical movements in loan destruction. Second, the model generates impulse responses that qualitatively match estimated responses for an identified credit supply shock. To show this, I first estimate responses to gross loan creation and destruction margins to a credit supply shock in a VAR framework. I find that the interest rate spread and loan destruction margin rise, while loan creation falls after a credit supply shock. The model is able to generate responses for the capital quality and bank equity shocks that are qualitatively similar to empirical responses to credit supply shocks, when investment elasticity to capital price matches evidence from disaggregate data.

Early studies exploring the effect of search frictions on credit markets include den Haan et al. [2003], and Wasmer and Weil [2004]. However, none of these papers use search frictions to study business-cycle dynamics of bank loans. In that respect, the papers closest to this study are Dell’Ariccia and Garibaldi [2000] and Beaubrun-Diant and Tripier [2009]. The former considers a search model only to match the second moments of gross loan flows, and the later explores the ability of a costly search setup in generating a counter-cyclical interest spread for technology shocks in an RBC model. On top of considering a New-Keynesian setting, my model differs from Beaubrun-Diant and Tripier [2009] by allowing an endogenous bank leverage

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6Beaubrun-Diant and Tripier [2009] provides an analysis of the calibration necessary for search models to generate a counter-cyclical spread for a technology shock in an RBC model.

7The idea that conditions in the financial sector can be an independent source of business cycle fluctuations has recently been emphasized in the literature. Gilchrist and Zakrajsek [2012a] and Boivin et al. [2012] estimate the effect of a credit shock on U.S. data and find that a reduction in credit supply negatively affects the real economy.
channel and loans that are used for capital purchases to vary both in the intensive and extensive margins. In contrast, their model specifies loans used for intermediate good purchases, allows only movements in the extensive margin, and abstracts from both productive capital and bank equity.

To the best of my knowledge, this is the first study that considers a full-fledged banking model using search-frictions in a New-Keynesian setting, and explicitly models loan variation in both intensive and extensive margins. This is also the first study to consider both simulated and empirical impulse responses of gross loan flows to financial shocks.

The rest of the paper is organized as follows. Section 4.2 describes the benchmark model and calibration. Section 4.3 demonstrates the capacity of the model to amplify and propagate the effects of shocks to the economy, and explains the core search mechanism in generating a counter-cyclical interest spread. Section 4.4 determines the empirical responses for gross loan creation and destruction margins for a credit supply shock, and compares them with model responses. Section 4.5 provides concluding comments.

### 4.2 The Benchmark Model

In this section, I describe the benchmark model and calibration. The starting point is the workhorse New Keynesian DSGE model with nominal rigidities developed by Christiano et al. [2005] and Smets and Wouters [2007]. To this canonical framework, I add a banking sector that intermediates loanable funds between households and firms. Banks face a Mortensen and Pissarides [1994] type search friction that creates a wedge between the loan and deposit interest rates.\(^8\) Precisely, household deposits

\(^8\)The mechanism parallels the creation of a wedge between wages and the marginal product of labor in the job-search models of Pissarides [1985] and Andolfatto [1996].
are transferred to firms as loans for capital asset purchases when a banking loan officer is matched with a project that requires financing. The search process resulting in a matched pair of loan officer and project generates a countercyclical wedge between the loan interest rate charged to the firm and the deposit interest rate paid by the bank.

The model economy is explained graphically in figure 4.1. There are five types of private agents in the economy: households, banks, intermediate goods producers (referred to as firms here on), capital producers, and monopolistically competitive retailers. This setup closely follows Gertler and Karadi [2011], with the exception that the financing friction in this model is generated by search activity, rather than an agency problem stemming from information asymmetries between banks and depositors. Households consume, supply labour to firms, and save in bank deposits. They own banks, intermediate and capital goods producing firms, and retail institutions. Income from these institutions are repatriated to households in the form of dividends. Capital, however, is owned by firms, and can be bought only through funds secured from banks as loans. Firms use labour and capital to produce a homogenous intermediate good.

The representative firm is best thought of as the head office of a continuum of projects on a unit interval. The firm head office determines the amount of aggregate capital, and hence aggregate loans in each period, and distributes it evenly across active projects. Production and financing, however, occurs at the individual project level, and only if the project is matched with a loan officer who provides financing.

Similar to the firm, the representative bank is best thought of as the head office to

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9Consistent with the literature on banking sector models in DSGE settings, I assume that household savings and bank equity are the only sources for private sector credit. Note, however, that this assumption overlooks the growing importance of corporate savings, and abstracts from bond market and other sources of funding available to firms.

10This assumption is innocuous, given the free-movement of resources across projects, and is maintained for ease of tractability.
Figure 4.1: Model Economy
a collection of loan officers, who can either be actively financing an existing project, or searching for potential projects to finance. The bank head office determines the total number of loan officers in each period in accordance with a balance sheet constraint that equates total loans to total deposits plus bank equity, and a quadratic cost imposed on banks if the leverage ratio deviates from regulation. Following an aggregate shock, the bank head office determines how many new projects to finance, and how many of the currently existing relationships to keep. Separation of existing relationships, therefore, is endogenous, and modeled by introducing an idiosyncratic cost of continuing an existing match, imposed on the individual loan officer. Each period, new matches occur between loan officers looking for projects to finance, and unfunded projects. This generates an extensive margin for financing. Banks pay a deposit interest rate to households that equals the risk-free rate by arbitrage. Loan interest rates to firms are determined through Nash bargaining.

At the end of the period, worn out capital is replaced by the capital goods producing sector, which, following Bernanke et al. [1999] and Gertler and Karadi [2011], is considered separately to keep the problem of intermediate goods producers tractable in the face of financial frictions. A monopolistically competitive retail sector introduces nominal price rigidities by costlessly differentiating the homogeneous intermediate good into a continuum of differentiated goods, and repackaging them to produce a final good. A central bank conducts conventional monetary policy to control inflation. In the absence of financial frictions, the loan interest rate and the deposit rate are equal, and the question of matching is moot. In this case, the model reduces to the canonical New Keynesian model of Christiano et al. [2005] and Smets and Wouters [2007].

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11This is similar to the cost of managing a portfolio of clients and servicing their credit requirements in a traditional relationship-banking enterprise. This may also be seen as a reduced form way of modelling monitoring costs on existing clients.
The remainder of this section describes the benchmark model in detail. Appendix C.1 provides a summary of the timing of events.

4.2.1 Households

There is a representative household that consumes, supplies labour and saves in bank deposits to maximize the following discounted sum of utilities:

\[
\max E_t \sum_{i=0}^{\infty} \beta^i \left[ \ln \left( C_{t+i} - hC_{t+i-1} \right) - \frac{1}{1+\varphi} (L_{t+i}^H)^{1+\varphi} \right]
\]  

(4.1)

where \( C_t \) is consumption, \( L_t^H \) is labour supplied, \( 0 < \beta < 1 \) is the discount factor, \( 0 < \varphi \) is the inverse Frisch elasticity of labour supply, and \( 0 \leq h < 1 \) determines the degree of habit formation.

In each period, the household faces the following budget constraint

\[
C_t = W_t L_t^H + R_t B_{t-1}^H - B_t^H + Div_t
\]

(4.2)

where \( B_t^H \) is this period’s savings deposited in banks, which earn real gross interest rate \( R_t \) in each period. \( W_t \) is the real wage rate, and \( Div_t \) is the sum of dividends repatriated to households from private enterprises in each period. The superscript \( H \) on aggregate household savings and labour supply helps distinguish them from labour demanded by individual projects, and deposits used by individual loan officers to fund projects.

Let \( \lambda_t \) be the shadow value of income. The household’s first order conditions
governing labour supply and savings are as follows:

\[(I^H_t)^\varphi = \lambda_t W_t\]  \hspace{1cm} (4.3)

\[\lambda_t = (C_t - hC_t)^{-1} - h\beta E_t (C_{t+1} - hC_t)^{-1}\] \hspace{1cm} (4.4)

\[1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} R_t\] \hspace{1cm} (4.5)

Equation 4.3 gives an expression for the household’s optimal labour supply. Equation 4.5 gives the intertemporal Euler equation, where the shadow value of income is given in 4.4.

### 4.2.2 Matching

As mentioned earlier, the banking sector is a collection of individual loan officers, while the firm is a collection of projects. A project can either be funded and productive, or looking for funding. Each period, the bank head office determines the number of loan officers, \(v_t\), that are actively searching for projects to fund. Let \(u_t\) be the number of projects looking for funding. At the end of each period, after production takes place, and all factor payments and loan repayment commitments are completed, new matches, \(m_t\), are made between searching loan officers and unfunded projects according to the following Cobb-Douglas matching function:

\[m_t = \bar{m} u_t^\chi v_t^{1-\chi}\] \hspace{1cm} (4.6)

where \(\bar{m}\) is a parameter capturing the productivity of new match formation, and \(0 \leq \chi \leq 1\) is the match elasticity.\(^{12}\) A pair of loan officer and project matched in period \(t\) becomes active in period \(t+1\).

\(^{12}\)A time-varying specification for match productivity, \(m_t\), would allow for shocks to be introduced in the supply of matches. In the current chapter, however, we abstract from such assumptions, and normalize \(\bar{m}\) to one.
Let \( q_t \) be the probability with which an individual searching loan officer finds a match in period \( t \), and \( p_t \) be the probability of an unmatched project finding a match. Then we have the following relationships:

\[
q_t = \frac{m_t}{v_t} = m\theta^{-x}_t \quad (4.7)
\]
\[
p_t = \frac{m_t}{u_t} = m\theta^{1-x}_t \quad (4.8)
\]
\[
\theta_t = \frac{v_t}{u_t} = \frac{p_t}{q_t} \quad (4.9)
\]

where \( \theta_t \) captures the tightness in the credit market. A low value for \( \theta_t \) makes it more difficult for an unmatched project to secure funding.

### 4.2.3 Banks

A bank head office manages a continuum of loan officers. Although aggregate funding allocation decisions are taken by the head office, matching, lending and loan collection is undertaken by individual loan officers. The timing is as follows: the bank enters period \( t \) with \( n_{t-1} \) matched projects to fund, and household savings deposits of \( B_{t-1}^H \).

At the beginning of the period, the bank head office allocates a portion of deposits, \( B_{j,t-1} \), and net worth to each matched loan officer \( j \). The active loan officer then lends out an amount \( S_{j,t} \) to the project with which it is matched, collects gross real loan interest rate \( R_{l,t} \) and repays the head office \( R_t \) for use of deposit funds, which is then repaid to the household as deposit interest rate.

After all loan commitments have been fulfilled and loan payments collected, an active loan officer is hit by an idiosyncratic ‘relationship shock’, \( \omega_{j,t} \in [\omega, \overline{\omega}] \), representing the cost of continuing an existing relationship. The loan officer can either pay a cost \( \Upsilon \omega_{j,t} \), and continue the relationship, or pay a fixed amount \( T \), and separate the existing relationship. After separation takes place, the bank posts new loan vacancies.
at a unit cost of \( c_v \), each of which finds a match with probability \( q_t \). The pool of matched projects \( n_t \) is then funded in the next period.

In a symmetric equilibrium, all loan officers receive the same amount of deposits, \( B_t = B_{j,t} \), and give out the same amount of loans, \( S_t = S_{j,t} \). Since the shock hits the loan officer after all loan obligations are cleared, profits vary across loan officers ex-post. Let \( \Pi_t^{b,c} \) be the flow profit of a loan officer if the existing relationship continues to the next period, and \( \Pi_t^{b,x} \) be the flow profit if the relationship separates. Then, in a symmetric equilibrium, we have that:

\[
\Pi_t^{b,c}(\omega_j) = \left[ R_t^l S_t - R_t B_{t-1} \right] - \Upsilon \omega_{j,t} \\
\Pi_t^{b,x}(\omega_j) = \left[ R_t^l S_t - R_t B_{t-1} \right] - T
\]

Let \( \bar{\omega}_t \) be the reservation continuation cost, such that the existing relationship survives if \( \omega_{j,t} \leq \bar{\omega}_t \) and separates if \( \omega_{j,t} > \bar{\omega}_t \). The flow profit of the average loan officer, \( \Pi_t^b \), is then given by:

\[
\Pi_t^b = \left[ R_t^l S_t - R_t B_{t-1} \right] - \Upsilon \int_{\bar{\omega}_t}^{\hat{\omega}_t} \omega dG(\omega) - \int_{\bar{\omega}_t}^{\hat{\omega}_t} T dG(\omega)
\]

where \( G(\omega) \) is the distribution function for the idiosyncratic shock \( \omega_{j,t} \).

Two conditions restrict the total amount of loans given out by the banking sector. First, a balance sheet constraint equates total loans to the sum of deposits and bank equity:

\[
n_{t-1} S_t = n_{t-1} B_{t-1} + N_t^b
\]

where \( n_{t-1} B_{t-1} = B_t^H \) is the amount of savings deposit available to the bank at period \( t \), and \( N_t^b \) is bank equity. Second, a regulatory authority imposes a capital
adequacy constraint on the bank. Specifically, the bank incurs a quadratic cost if its leverage ratio, defined as the ratio between bank loans (assets) and bank equity, exceeds a maximum regulated value, \( \bar{\kappa} \). Precisely, the regulator wants \( \frac{n_{t-1}S_t}{N_t^b} \leq \bar{\kappa} \), and banks pay a deviation cost otherwise.

Given the one-period lag between the time a match is created and loans are disbursed, the bank can only influence the number of matches that will become active in the next period. The bank searches for new matches by posting a loan vacancy at a fixed marginal cost of \( c_v \) per unit, and finds one with probability \( q_t \) each period. The head office also determines the number of existing matches that are separated. In every period, the aggregate banking sector receives an average profit of \( \Pi_t^b \) from each of the \( n_{t-1} \) active matches, and pays a loan vacancy posting cost of \( c_v v_t \) as well as a quadratic penalty if it deviates from the regulated leverage ratio of \( \bar{\kappa} \). The bank’s job is to choose the number of loan vacancies, \( v_t \), and the reservation cost of continuation, \( \tilde{\omega}_t \), in order maximize the following value function

\[
\max_{\{v_t, \tilde{\omega}_t\}} V^B(n_{t-1}) = n_{t-1} \Pi_t^b - c_v v_t - \frac{\Psi}{2} \left[ \frac{n_{t-1}S_t}{N_t^b} - \bar{\kappa} \right]^2 \frac{N_t^b}{S_t} + \beta E_t \Lambda_{t,t+1} V(n_t)
\]

subject to the law of motion for active matches

\[
n_t = n_{t-1} \int_{\tilde{\omega}_t} dG(\omega) + q_t v_t
\]

\[
= x_t n_{t-1} + q_t v_t
\]

where the endogenous survival rate, \( x_t \), is related to the threshold continuation shock by

\[
x_t = \int_{\tilde{\omega}_t} dG(\omega)
\]
and the match separation rate is given as $1 - x_t$. The optimality conditions with respect to $v_t, n_t, \tilde{\omega}_t$, and the envelope condition are as follows:

\begin{align*}
    c_v &= \lambda^b_{q_t} \quad (4.14) \\
    \beta E_t \Lambda_{t,t+1} \frac{\partial V_B(n_t)}{\partial n_t} &= \lambda^b_t \quad (4.15) \\
    \Upsilon \tilde{\omega}_t &= T + \lambda^b_t \quad (4.16) \\
    \frac{\partial V_B(n_{t-1})}{\partial n_{t-1}} &= \Pi^b_t - \Psi \left[ \frac{n_{t-1}S_t}{N_t^b} - \overline{\kappa} \right] + \lambda^b_t \int_{\omega} \tilde{\omega}_t dG(\omega) \quad (4.17)
\end{align*}

where $\lambda^b_t$ is the Lagrange multiplier for the constraint (4.13). The envelope condition (4.17) provides an expression for the value to the bank of an additional match, and represents the match surplus.

Combining equations (4.14), (4.15) and the envelope condition (4.17), we get the optimal rule for posting new loan vacancies:

\begin{align*}
    \frac{c_v}{q_t} &= \beta E_t \Lambda_{t,t+1} \left\{ R_{t+1} S_{t+1} - R_{t+1} B_t - \Upsilon \int_{\omega} \tilde{\omega}_{t+1} \omega dG(\omega) - (1 - x_{t+1})T ight. \\
    &\left. - \Psi \left[ \frac{n_{t}S_{t+1}}{N_{t+1}^b} - \overline{\kappa} \right] + \frac{c_v}{q_{t+1}} x_{t+1} \right\} \quad (4.18)
\end{align*}

Recall that $q_t = m_t/v_t$. Equation (4.18) then implies that an expected increase in future profit will prompt the bank to post more loan vacancies in order to increase the number of active loans. Similarly, an expected increase in the actual leverage ratio from the regulated level of $\overline{\kappa}$, and a subsequent penalty for deviation, will induce the bank to reduce the number of active loans. The optimal separation rule follows a similar logic and is found by combining equations (4.14) and (4.16):

\begin{align*}
    \Upsilon \tilde{\omega}_t &= T + \frac{c_v}{q_t} \quad (4.19)
\end{align*}

The bank adds its profits net of vacancy posting and regulatory costs to its capital.
base. Each period, the bank repatriates a fixed proportion $\delta^n$ of its net worth to households as dividends. The law of motion determining bank equity is therefore given by:

$$N^b_t = n_{t-1}\Pi^b_t - c_vv_t - \frac{\Psi}{2} \left[ \frac{n_{t-1}S_t}{N^b_t} - \bar{\kappa} \right]^2 \frac{N^b_t}{S_t} + (1 - \delta^n)N^b_{t-1} - \varepsilon^n_t \tag{4.20}$$

where $\varepsilon^n_t$ is a direct shock to level of bank equity, and is meant to capture a financial shock that erodes the equity base of the banking sector.$^{13}$ In this model, we abstract from capital issuance from the banking sector.

### 4.2.4 Intermediate Goods Firm

The representative firm manages a continuum of projects in the unit interval, and produces a homogeneous intermediate good that is sold to retailers at price $P^m_t$. Capital and labour allocation decisions are taken by the firm head office, while search, matching and production activities take place at the level of individual projects. I assume perfect capital and labour mobility across projects.$^{14}$ This implies that capital and labour allocation decisions by the firm head office does not depend on the number of firms that are matched. Capital allocation across projects is conducted in a competitive market, at the prevailing market price.

The timing of events is as follows: the firm enters period $t$ with $n_{t-1}$ matched projects, and $K^Y_{t-1}$ units of aggregate capital available for production. Each matched project $i$ takes a loan of $S_{i,t}$ from the bank at loan rate $R^l_t$, and purchases $K_{i,t-1}$ units of capital at price $Q_t$. The head office hires labour at wage $W_t$ and allocates $L_{i,t}$.

$^{13}$This is similar to the net worth shock considered in Gertler and Karadi [2011]. A similar shock for entrepreneur net worth has been shown to be more important in explaining output fluctuations in post-war U.S. data than monetary shocks by Nolan and Thoenissen [2009].

$^{14}$This is similar to the assumption of consumption sharing across family members in models of labour search, such as Merz [1995] and Monacelli et al. [2010]. With this assumption, there is no need to keep track of the match state of individual projects.
amount per project. Production occurs only in matched projects according to the following Cobb-Douglas function:

\[ Y_{i,t} = z_t (\xi_t K_{i,t-1})^\alpha (L_{i,t})^{1-\alpha} \] (4.21)

where \(z_t\) is a standard technology shock and \(\xi_t\) is a capital quality shock common to all projects, similar to Gertler and Karadi [2011], making \(\xi_t K_{i,t-1}\) the effective quantity of productive capital at time \(t\). While \(z_t\) affects total factor productivity, \(\xi_t\) affects solely the productivity of capital. Unlike the former shock, \(\xi_t\) therefore influences the sales price of capital, which, in turn, affects the balance sheet and leverage of banks. The capital share of output is given by \(\alpha\). Once production is complete, project \(i\) repays the loan and interest to the bank, and sells the capital back to the market at price \(Q_t\) for reallocation in the next period amongst projects that will be in production.

Depending on the match-specific continuation cost \(\omega_{i,t}\) that each project’s loan officer faces, the project either continues the match into the next period (if \(\omega_{i,t} < \tilde{\omega}_t\)) or separates (otherwise). Separated projects start looking for another match in the same period. The number of unmatched projects looking for funding at the end of period \(t\) is denoted by \(u_t\) and defined as:

\[ u_t = 1 - \int_{\omega}^{\tilde{\omega}_t} n_{t-1} dG(\omega) \] (4.22)

Each period, a proportion \(\delta\) of capital is depreciated, and is replaced at unit cost. Firms do not face capital adjustment costs.\(^{15}\)

The flow benefit of a matched project, \(\Pi_{i,t}^e\) takes into account the marginal product of capital, the value of capital stock sold back to the market after production, and

\(^{15}\)Following Gertler and Karadi [2011], adjustment costs occur on net rather than gross investment, and is borne by capital goods producers.
the loan cost:

\[
\Pi_{i,t} = \left[ \alpha \frac{P_i}{\xi K_{i,t-1}} Y_{i,t} + (Q_t - \delta) \right] \xi_t K_{i,t-1} - R_t S_{i,t} \tag{4.23}
\]

where the loan amount is spend only on purchasing capital:

\[
S_{i,t} = Q_t K_{i,t-1} \tag{4.24}
\]

Let \( H^c_t(\omega_{i,t}) \) be the value function of a project that continues its match, and \( H^x_t(\omega_{i,t}) \) be the value function of a project that separates after production in period \( t \). Then we have that:

\[
H^c_t(\omega_{i,t}) = \Pi_{i,t} + \beta E_t \Lambda_{t,t+1} H_{t+1} \tag{4.25}
\]

\[
H^x_t(\omega_{i,t}) = \Pi_{i,t} + U_t \tag{4.26}
\]

where \( H_{t+1} \) is the ex-ante or average match value in period \( t + 1 \), and \( U_t \) is the value function of an unmatched project. An unmatched project finds a match with probability \( p_t \), and remains unmatched with probability \( 1 - p_t \). The value of an unmatched project is then given by:

\[
U_t = p_t \beta E_t \Lambda_{t,t+1} H_{t+1} + (1 - p_t) \beta E_t \Lambda_{t,t+1} U_{t+1} \tag{4.27}
\]

In a symmetric equilibrium, all projects receive the same amount of labour, \( L_t = L_{i,t} \), and capital inputs, \( K_{t-1} = K_{i,t-1} \), and consequently produce the same amount
of output $Y = Y_{t,t}$. Defining the surplus from the match as $V^F_t$, we have that:

$$V^F_t = \int_\omega H_t(\omega_t) dG(\omega) - U_t$$

$$= \left[ p^m \alpha \frac{Y_t}{K_{t-1}} + Q_t(1 - \delta) \xi_t \right] K_{t-1} - R^l_t S_t + x_t (1 - p_t) \beta E_t \Lambda_{t,t+1} V^F_{t+1} \quad (4.28)$$

where the surplus from the match rises if the marginal product of capital increases, or if the resale value of capital rises. In contrast, an increase in loan interest rate or an increase in the probability of finding a new match reduces surplus to firms.

Since the firm head office makes the labour allocation decision, the optimality condition for labour demand is standard:

$$W_t = (1 - \alpha) p^m \frac{Y_t}{L_t} \quad (4.29)$$

where $W_t$ is the real wage rate. Capital is owned by the firm head office, and is transferred inter-temporally. The optimal condition for capital demand equates the return of capital to the cost of financing:

$$R^l_t = E_t \left[ \frac{\alpha p^m Y_t}{K_{t-1}} + (Q_{t+1} - \delta) \xi_t \right] \frac{Q_t}{Q_t} \quad (4.30)$$

Note that the value of capital stock left over after production takes into account the unit replacement cost of depreciated capital, and is given by $E_t(Q_{t+1} - \delta) \xi_t K_{t-1}$. The capital quality shock $\xi_t$ generates variation in the return to capital, as well as in the price of capital $Q_t$.

### 4.2.5 Nash Bargaining

At the beginning of the period, the loan interest rate $R^l_t$ is determined via Nash bargaining by maximizing the joint match surpluses accruing to the bank and the
firm:

$$\max_{\{R_t^i\}} (V_t^F)^\eta \left( \frac{\partial V(n_{t-1})}{\partial n_{t-1}} \right)^{1-\eta}$$

where $0 \leq \eta \leq 1$ is the firm’s bargaining power. The optimality condition is given by

$$(1 - \eta) V_t^F = \frac{\partial V(n_{t-1})}{\partial n_{t-1}} \tag{4.31}$$

Replacing the expressions for the match surpluses from equations (4.28) and (4.17), and simplifying, we get the following condition for the loan interest rate:

$$R_t^i S_t = (1 - \eta) \left[ P_t^m \alpha Y_t^{\frac{K_{t-1}}{K_{t-1}}} + (Q_t - \delta) \xi_t \right] K_{t-1} + \eta R_t B_{t-1} + \eta \gamma \int_{\omega} \tilde{\omega} \omega dG(\omega) \tag{4.32}$$

$$+ \eta (1 - x_t) T + \eta \Psi \left[ \frac{n_{t-1} S_t}{N_t^b} - \kappa \right] - \eta c_v x_t \frac{p_t}{q_t}$$

where the following equality has been taken into account:

$$(1 - \eta) \beta E_t \Lambda_{t,t+1} S_{t+1}^e = \eta \beta E_t \Lambda_{t,t+1} \frac{\partial V(n_t)}{\partial n_t} = \frac{c_v}{q_t}$$

Equation (4.32) specifies that the loan interest rate, $R_t^i$, increases when firm profits go up, or if the cost to the bank for servicing the loan goes up. In particular, an increase in the cost of funding for banks, $R_t$, or in the leverage ratio, $\frac{n_{t-1} S_t}{N_t^b}$, will be (at least partially) transferred to firms through higher loan rates. Finally, a tighter credit market, implied by a fall in $\theta_t = \frac{v_t}{q_t}$, will prompt banks to charge a higher rate to firms.
4.2.6 Capital Producing Firms

At the end of period $t$, capital producing firms buy capital from intermediate goods producing firms, repair depreciated capital at unit cost, build new capital and sell it at price $Q_t$. Following Gertler and Karadi [2011], I assume that the cost of refurbishing depreciated capital is unity. Accordingly, there is no adjustment cost associated with refurbishing depreciated capital, but there are flow adjustment costs with producing new capital. These adjustment costs are imposed on net investment flow, which, by definition, excludes depreciated capital, rather than gross investment flow that include them. The capital producing firms’ problem is to chose net investment to maximize the following sum of discounted profits:

$$\max_{\{I^n_t\}} E_t \sum_{\tau=0}^{\infty} \beta^{t+\tau} \Lambda_{t,t+\tau} \left[ Q_{t+\tau} I^n_{t+\tau} - I^n_{t+\tau} - f \left( \frac{I^n_{t+\tau} + \bar{I}}{I^n_{t+\tau-1} + \bar{I}} \right) (I^n_{t+\tau} + \bar{I}) \right]$$

where $I^n_t$ is net capital created, and $\bar{I}$ is steady-state investment. The adjustment cost function satisfies $f(1) = f'(1) = 0, f''(1) > 0$.

The definition of net new capital and the law of motion of capital stock is given by:

$$I^n_t \equiv I_t - \delta \xi_t K^Y_{t-1} \quad (4.33)$$
$$K^Y_t = \xi_t K^Y_{t-1} + I^n_t \quad (4.34)$$

where $I_t$ is gross capital created, and aggregate capital, $K^Y_t = n_t K_t$. Any profit from the capital producing firms is rebated back to the household. The optimal condition for the capital producing sector gives the following relation for net investment:

$$Q_t = 1 + f(\cdot) + \frac{I^n_t + \bar{I}}{I^n_{t-1} + \bar{I}} f'(\cdot) - E_t \beta \Lambda_{t,t+1} \left( \frac{I^n_{t+1} + \bar{I}}{I^n_t + \bar{I}} \right)^2 f'(\cdot) \quad (4.35)$$
4.2.7 Retail Sector

Following Bernanke et al. [1999] and Gertler and Karadi [2011], I model the retail sector in two steps. First, a continuum of monopolistically competitive retail firms indicated by $i \in [0, 1]$ purchases the homogeneous intermediate good at price $P^m_t$ and costlessly differentiates them to a continuum of retail goods. Second, these differentiated goods $Y_{i,t}$ are combined into a composite final good $Y^F_t$ by a packaging firm using a Dixit-Stiglitz aggregating technology:

$$Y^F_t = \left[ \int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right] \frac{1}{\epsilon}, \epsilon > 0$$

where $1/(\epsilon - 1) > 0$ is the constant elasticity of substitution between differentiated retail goods. Cost minimization by the final good packer gives the following demand for each type of retail good

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y^F_t$$

where $P_t$ is the price of a unit of final good determined by the zero profit condition:

$$P_t = \left[ \int_0^1 P_{i,t}^{1-\epsilon} di \right]^{1/(1-\epsilon)}$$

The marginal cost faced by the monopolistically competitive retail firms is the price, $P^m_t$, of intermediate goods. Following Calvo [1983], nominal price stickiness is introduced by assuming staggered price setting by the retail firm. Each period a fraction, $0 < 1 - \Theta < 1$, of retail firms re-sets their sales price optimally, while the remaining fraction $\Theta$ keeps their prices unchanged. The retail firm’s problem is to chose the
optimal sales price, $P^*_t$, given the staggered price setting mechanism

$$
\max_{P_t} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \Theta^j \left[ P^*_t Y_{t,t+j} - P_{t+j}^m P_{t+j}^m \right]
$$

subject to a sequence of demand curves

$$
Y_{t,t+j} = \left( \frac{P^*_t}{P_{t+j}} \right)^{-\epsilon} Y_{t+j}^F
$$

The optimal price, $P^*_t$, satisfies the first-order condition

$$
\mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \Theta^j Y_{t,t+j} \left[ P^*_t - \frac{\epsilon}{\epsilon - 1} P_{t+j}^m P_{t+j}^m \right] = 0 \quad (4.36)
$$

where the aggregate price level is given by

$$
P_t = \left[ \Theta P_{t-1}^{1-\epsilon} + (1 - \Theta) P_t^{\epsilon(1-\epsilon)} \right]^\frac{1}{\epsilon} \quad (4.37)
$$

Log-linearizing equations (4.36) and (4.37) around a steady-state inflation rate of 1, we get the following expression for the New-Keynesian Philips curve

$$
\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1 - \beta \Theta)(1 - \Theta)}{\Theta} \hat{P}_t^m \quad (4.38)
$$

where $\pi_t = P_t/P_{t-1}$ is the inflation rate, and hats represent variables that are log-linearized around their respective steady-states.

### 4.2.8 Resource Constraint and Policy

Final output is the sum of output from each individual project, and is divided between consumption, investment, adjustment costs, and costs incurred by banks for
separation and search for new projects.

\[ Y_t^F \equiv n_{t-1}Y_t = C_t + I_t + f \left( \frac{I^n_t + \bar{I}}{I^n_{t-1} + \bar{I}} \right) (I^n_t + \bar{I}) + c_t v_t + n_{t-1} \int_{\omega} \omega dG(\omega) \]

\[ + n_{t-1} \int_{\omega} TdG(\omega) + \Psi \left[ \frac{n_{t-1}S_t}{\bar{N}_t^b} - \bar{K} \right] \]  \hspace{1cm} (4.39)

A central bank determines monetary policy by setting the nominal gross interest rate \( i_t \), according to a Taylor rule with interest rate smoothing, expressed in log-linearized form

\[ \hat{i}_t \equiv \hat{R}_t + E_t \hat{\pi}_{t+1} = \varphi_r \hat{i}_{t-1} + (1 - \varphi_r) \left( \varphi_n \hat{\pi}_t + \varphi_Y \hat{Y}^F_t \right) + \varepsilon^n_m \]  \hspace{1cm} (4.40)

where the smoothing parameter, \( \varphi_r \), lies between zero and unity, \( \varepsilon^n_m \) is an i.i.d. shock to the nominal interest rate, and the link between nominal and real interest rates follow a Fisher relationship, and the real interest rate equals the real deposit rate, \( R_t \), by arbitrage. \( \varphi_n \) and \( \varphi_Y \) are the Taylor response coefficients for inflation and the output gap, respectively.

This concludes the description of the model. Appendix C.2 summarizes the list of non-linear equations defining the model.

4.2.9 Calibration

I log-linearize the model around its steady-state\(^{16}\) and solve using Dynare.\(^{17}\) The system of log-linearized equations is presented in appendix C.3. Tables 4.1 and 4.2 summarize the choice of parameter values and steady-state ratios for the benchmark model. Calibration values of the discount factor, \( \beta \), the depreciation rate, \( \delta \), and

\(^{16}\)Note that the steady-state value of net investment \( I^n_t \) is zero. I linearize the equations containing \( I^n_t \) and normalize by output. All log-linearized variables presented in appendix C.3 are denoted by a hat, and linearized variables denoted by a tilde.

\(^{17}\)See Adjemian et al. [2011] and http://www.dynare.org/.
the capital share of income, \( \alpha \), are standard. The depreciation rate, \( \delta \), and the steady-state capital to output ratio, \( \frac{K}{Y} \), implies a steady-state investment to output ratio, \( \frac{I}{Y} \), of 0.2. I set the habit parameter to 0 in the benchmark, and the inverse Frisch elasticity of labour supply to 1. The elasticity of net investment to capital price parameter is set to 1.728, following Gertler and Karadi [2011]. The elasticity of substitution among retail goods, \( \epsilon \), is chosen to match a markup of 15 per cent. The Calvo parameter for the frequency of price adjustment is set to 0.8, which is common in the literature. Following Gertler and Karadi [2011], I set the interest smoothing parameter in the Taylor rule, \( \varrho_r \), to 0.8, the inflation response parameter, \( \varrho_{\pi} \), to 1.5, and the output response parameter, \( \varrho_Y \), to 0.5/4.

There are no standard values prescribed in the literature for the parameters and steady-state values pertaining to the matching variables. I follow Beaubrun-Diant and Tripier [2009] and set the match probability of searching projects, \( p \), to 0.4 and the loan market tightness, \( \theta \), to 0.6. The former value implies an average search duration of two-and-half quarter for projects. The match elasticity parameter, \( \chi \), is set to 0.5 and the bargaining power of the firm, \( \eta \), is equated to this value in order to observe the Hosios [1990] condition of match efficiency. The steady state annual gross loan interest rate is set to 1.05, implying a 3 percent spread between loan and deposit interest rates.

Using disaggregated quarterly data from the U.S. banking system from 1999 through 2008, Contessi and Francis [2010] calculate a net growth rate in gross loans \((NET)\) of 4.67 percent, and in loan reductions \((NEG)\) of 2.42 per cent. To account for the trend growth in gross loans, I set the loan survival rate, \( x \), to match the following

\[
x = \frac{1 - NEG}{1 + NET}
\]
Table 4.1: Calibration of parameters and steady-state ratios

<table>
<thead>
<tr>
<th>Category</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td>β</td>
<td>Discount rate</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>h</td>
<td>Habit parameter</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>ϕ</td>
<td>Inverse Frisch elasticity of labour supply</td>
<td>1</td>
</tr>
<tr>
<td><strong>Intermediate Firms</strong></td>
<td>α</td>
<td>Capital share of output</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>δ</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td><strong>Capital Producing Firms</strong></td>
<td>f''(·)</td>
<td>Elasticity of net investment to capital price</td>
<td>1.728</td>
</tr>
<tr>
<td><strong>Retailers</strong></td>
<td>Θ</td>
<td>Calvo price setting parameter</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>ε</td>
<td>Elasticity of substitution</td>
<td>7.5</td>
</tr>
<tr>
<td><strong>Aggregation and Policy</strong></td>
<td>C</td>
<td>Consumption to output ratio</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>Capital to output ratio</td>
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</tr>
<tr>
<td><strong>Shocks</strong></td>
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<td>Persistence of technology shock</td>
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<tr>
<td></td>
<td>ρξ</td>
<td>Persistence of capital quality shock</td>
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Table 4.2: Calibration of parameters and steady-state ratios *contd.*

<table>
<thead>
<tr>
<th>Matching</th>
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<th>Banks</th>
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<tbody>
<tr>
<td>$p$</td>
<td>Match probability of searching projects</td>
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</tr>
<tr>
<td>$\theta$</td>
<td>Loan market tightness</td>
<td>0.6</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Elasticity of match</td>
<td>0.5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Bargaining power of firm</td>
<td>0.5</td>
</tr>
<tr>
<td>$R_l$</td>
<td>SS loan interest rate</td>
<td>$1.05^{1/4}$</td>
</tr>
<tr>
<td>$R$</td>
<td>SS deposit interest rate</td>
<td>$\frac{1}{\beta}$</td>
</tr>
<tr>
<td>$x$</td>
<td>SS match survival rate</td>
<td>0.95</td>
</tr>
<tr>
<td>$\tilde{\omega}$</td>
<td>Threshold continuation shock</td>
<td>$x/2$</td>
</tr>
<tr>
<td>$n$</td>
<td>SS number of matches</td>
<td>$\frac{1}{1 - x(1 - p)}$</td>
</tr>
<tr>
<td>$\bar{\kappa}$</td>
<td>SS bank loan to capital ratio</td>
<td>0.08</td>
</tr>
<tr>
<td>$\delta^n$</td>
<td>Bank dividend payout rate</td>
<td>0.37</td>
</tr>
<tr>
<td>$S$</td>
<td>SS loan to output ratio</td>
<td>$\frac{1}{n \times \frac{K}{Y}}$</td>
</tr>
<tr>
<td>$\bar{Y} \frac{N^b}{N^b}$</td>
<td>SS bank capital to output ratio</td>
<td>$\frac{1}{\bar{\kappa} \times \frac{S}{Y}}$</td>
</tr>
<tr>
<td>$n\frac{c_v}{\bar{Y}}$</td>
<td>Elasticity of bank equity to vacancy posting</td>
<td>0.1470</td>
</tr>
<tr>
<td>$n\frac{1-x}{\bar{Y}}$</td>
<td>Elasticity of bank equity to separating existing match</td>
<td>0.3921</td>
</tr>
<tr>
<td>$n\frac{\tilde{\omega}x}{\bar{Y}}$</td>
<td>Elasticity of bank equity to continuing existing match</td>
<td>6.1582</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Quadratic cost for deviating from capital adequacy</td>
<td>0.1</td>
</tr>
</tbody>
</table>
This gives a separation rate of 0.95. I assume a uniform distribution for the ‘relationship shock’ on a unit spread (i.e., $\bar{\omega} - \omega = 1$), and set the threshold value for $\tilde{\omega}$ such that total cost to maintaining loan relationships in the steady-state equals zero.\(^{18}\)

I set the steady-state capital price to equal 1, and the total loan to output ratio to equal the capital to output ratio. The parameter $\bar{\kappa}$ is set such that bank equity is equal to 8 per cent of total assets. This matches the requirements set out in the 1988 Basel-I accord. In the steady-state, bank equity matches this requirement. I set the bank dividend payout rate, $\delta^n$, such that dividends equal net profits of the banking sector, including interest income, loan vacancy posting cost, and separation cost.\(^{19}\)

The dynamics of the search mechanism depends importantly on the loan vacancy posting cost, $c_v$, and the parameters determining the cost of continuing or separating existing relationships, $\Upsilon$ and $T$. Since the literature is silent on prescribing values for these parameters, I choose benchmark values to give reasonable impulse responses to the four types of shocks considered. In particular, the values are chosen to give a non-negative interest spread response on impact for technology and monetary shocks. The benchmark value for $c_v$ implies an elasticity of bank equity to vacancies of 0.147 percent. The benchmark value for $T$ implies an elasticity of bank equity to the match separation rate of 0.392. Finally, the benchmark value for $\Upsilon$ implies an elasticity of bank equity to the match continuation rate of 6.158. I check the sensitivity of model results to the calibration of these parameters in section 4.3.1.

\(^{18}\)A uniform distribution implies $\int_{\omega}^{\tilde{\omega}} \omega dG(\omega) = \frac{\tilde{\omega}^2 - \omega^2}{2(\bar{\omega} - \omega)}$. So, $\tilde{\omega}$ must follow the following two conditions $x = \frac{\tilde{\omega} - \omega}{\bar{\omega} - \omega}$ and $0 = \frac{\tilde{\omega}^2 - \omega^2}{2(\bar{\omega} - \omega)}$.

\(^{19}\)Precisely, $\delta^n$ is set such that $\delta^n \frac{N_b}{Y} = n \left[ R^t - R \left( 1 - \frac{1}{\bar{\kappa}} \right) \right] \frac{S}{Y} - n(1 - x) \frac{T}{Y} - v \frac{c_v}{Y}$.
4.3 Counter-cyclical Interest Spread

In this section, I analyze the responses of the benchmark model to three different shocks that have a primary effect on credit demand – a technology shock, a monetary shock, and a shock to productive capital quality. I show the results for shocks that directly affect credit supply to the next section.

First, I show that the banking model amplifies and propagates the responses to all three shocks. The amplification is due to a counter-cyclical spread between the loan rate and the deposit rate. I then explain the core mechanism of the search framework that generates the interest spread. Finally, I consider alternate calibration values for a few key parameters. I frame my results below in terms of contractionary shocks in an attempt to make the discussion relevant to current times. The magnitude and direction of the impulse responses, however, are symmetric around the steady state, and depend on the sign of the exogenous shock.

Figure 4.2 compares impulse responses from the benchmark model with those from a standard New Keynesian one. The New Keynesian specification is constructed by eliminating the banking sector from the benchmark model, and equating the loan interest rate $R^l_t$ to the deposit rate $R_t$. The remaining sections and calibrations are the same across the two specifications. The first three columns show responses for a one percent shock to technology, interest rate and capital quality, respectively. The persistence parameter of the technology shock, $\rho_z$, is set to 0.8, and that of the capital shock, $\rho_\xi$, is set to 0.66, following Gertler and Karadi [2011]. Figure 4.3 shows responses of selected financial market variables to the same shocks, presented in the same order.

The presence of a banking sector amplifies and propagates the responses to all shocks. In contrast, using exogenous borrowing constraints in a monopolistically competitive banking model, Gerali et al. [2010] find that the presence of banks at-
tenuates the economy’s responses to monetary and technology shocks. Moreover, all negative shocks generate a counter-cyclical interest rate spread and a persistent recession.

Following Gertler and Karadi [2011], I consider a negative shock to capital quality in an attempt to capture the effects of a financial crisis. A decline in capital quality reduces the productivity of capital and, in itself, can generate a recession. However, the resulting decline in capital prices also induces a persistent decline in the value of bank assets, as seen during the sub-prime crisis. It is this secondary effect working through the financial system that is of interest here. Figure 4.2 shows that a reduction in the effectiveness of capital induces a modest recession in the NK model. In the presence of a banking sector, however, this decline is greatly amplified. The fall in capital value reduces the size of banking assets. This loss in assets is translated to a strong and persistent reduction in bank equity through the balance sheet identity. Interest spreads rise on impact, further reducing the demand for capital and investment by the firm. Capital prices fall even further, exacerbating the balance sheet position of banks. As a result, the overall contraction in the economy is magnified.

4.3.1 Explanation of the Core Mechanism

Search frictions generate a counter-cyclical wedge between loan and deposit interest rates that amplifies business cycles. The mechanism can be explained by considering equations 4.18 and 4.32. They are reproduced below for ease of reference, where the match finding probabilities $p_t$ and $q_t$ have been expressed in terms of the market tightness variable $\theta_t$, and the expression for average bank profit from loan officers,
Figure 4.2: Comparison of the benchmark model with search frictions in the banking sector and the standard new Keynesian model.

Notes: Solid lines represent impulse responses from the benchmark model with a banking sector, broken line represents a standard New Keynesian model. The first column represents impulse responses to a negative technology shock, the second to a contractionary monetary shock, and the third to a negative capital quality shock.
Figure 4.3: Impulse responses for selected banking sector variables.

Notes: The first column represents impulse responses to a negative technology shock, the second to a contractionary monetary shock, the third to a negative capital quality shock.
\( \Pi_t^b \) is given in equation 4.10.

\[
\frac{c_v}{m} \theta_t^x = \beta E_t \Lambda_{t,t+1} \left\{ \Pi_{t+1}^b - \Psi \left[ \frac{n_t S_{t+1}}{N_t^{b}} - \bar{\kappa} \right] + \theta_{t+1}^{N} \int_{\omega} \omega c_v dG(\omega) \right\} \quad (4.41)
\]

\[
R_t^l S_t = (1 - \eta) \left[ \alpha P_t^m \frac{Y_t}{K_{t-1}} + (Q_t - \delta) \xi_t \right] K_{t-1} + \eta R_t \left( S_t - \frac{N_t^{b}}{n_{t-1}} \right) + \eta \int_{\omega} \omega dG(\omega) + \eta \int_{\omega} T dG(\omega) + \eta \Psi \left[ \frac{n_{t-1} S_t}{N_t^{b}} - \bar{\kappa} \right] \quad (4.42)
\]

We know that whenever a negative technology shock reduces output, \( Y_t \), a contractionary monetary shock increases interest rate, \( R_t \), or a negative capital quality shock, \( \xi_t \), hits the economy, the firm’s demand for loans, and consequently, its surplus from an active match declines. Through Nash bargaining, this reduction in the firm’s surplus calls for a reduction in the loan rate, \( R_t^l \). Despite this downward pressure on \( R_t^l \) from reduced loan demand, search frictions imply a counter-cyclical rise in the spread between loan and deposit interest rates.

The intuition is as follows. Lower loan demand means less profit for banks. Faced with lower expected profits, banks reduce their effort in searching for new matches, or in maintaining existing matches. Consequently, matches become scarcer, and the market tightness variable, \( \theta_t = \frac{v_t}{u_t} \), decreases. This can be readily seen in the vacancy creation condition, equation 4.41. Holding all else constant, a reduction in \( \theta_t \) provides upward pressure on the loan rate through the Nash bargaining condition in equation 4.42. In the benchmark calibration, this upward pressure from \( \theta_t \) is enough to generate a counter-cyclical spread between \( R_t^l \) and \( R_t^r \).

\(^{20}\) Beaubrun-Diant and Tripier (2009) provides an analysis of the calibration necessary to produce a counter-cyclical rise in spreads for technology and monetary shocks in an RBC model in the absence of an endogenous bank leverage ratio.
in equilibrium. Consequently, investment and output fall by more than the standard New Keynesian case.

Two additional channels in the benchmark model aid in generating a counter-cyclical interest spread. First, through the balance sheet identity, a reduction in bank profits is translated to a fall in bank equity, which raises the bank leverage ratio counter-cyclically. Since banks incur a cost for deviating from a regulated leverage ratio, they respond by reducing the number of loan vacancies. Consequently, $\theta_t$ in equation 4.41 falls, and $R_l^t$ rises through equation 4.42.

Second, endogenous match separation in the benchmark model is triggered by the same mechanism as search efforts. Since banks are required to pay an idiosyncratic servicing cost to maintain existing matches, a reduction in expected profits for banks prompts them to reduce the number of existing relationships. A reduction in the match survival rate $\int_{\tilde{\omega}} \tilde{\omega} dG(\omega)$ (or an increase in the separation rate) increases the number of unmatched firms searching for matches, further reduces $\theta_t$, and consequently further enhances the accelerator effect described above.

Therefore, for technology, monetary, or capital quality shocks, a counter-cyclical rise in interest spread is triggered by the banking sector’s decision to reduce credit supply in the extensive margin in response to the decline in credit demand from the firm. In contrast, shocks to bank equity works by directly reducing the supply of credit. In particular, a fall in bank equity leads to a rise in the leverage ratio. To bring the leverage ratio back to the steady-state, banks reduce vacancy postings, and consequently, the loan rate, $R_l^t$, rises relative to the deposit rate, $R_d^t$. The effects of a supply-side shock to credit is shown in the next section.

Finally, the vacancy posting cost $c_v$, and the cost of separating an existing match, $T$, are important in determining the dynamics of interest spreads. Figure 4.4 provides sensitivity analysis for the elasticity of bank net worth to loan vacancy posting. If
Figure 4.4: Sensitivity to the cost of posting loan vacancies.

Note: Solid line represents benchmark value for the elasticity of bank net worth to loan vacancy posting, $n_{Nv} = 0.15$, broken line represents a bigger value of $n_{Nv} = 0.3$, and dotted line represents a lower value of $n_{Nv} = 0.08$. The first column represents impulse responses to a negative technology shock, the second to a contractionary monetary shock, and the third to a negative capital quality shock.
Figure 4.5: Sensitivity to the cost of separating existing loans.

Note: Solid line represents benchmark value for the elasticity of bank net worth to separating existing loans, $n \frac{T}{N} (1 - x) = 0.39$, broken line represents a bigger value of $n \frac{T}{N} (1 - x) = 0.52$, and dotted line represents a lower value of $n \frac{T}{N} (1 - x) = 0.26$. The first column represents impulse responses to a negative technology shock, the second to a contractionary monetary shock, and the third to a negative capital quality shock.
the cost of loan vacancy posting is higher, the response in credit market tightness is subdued. In this case, the effect of technology and monetary shocks on the Nash-bargained loan rate is not overturned by the reduction in the extensive margin of loans by the banking sector. As a result, the interest spread falls on impact for these two shocks. In contrast, a lower cost of loan vacancy posting allows for a higher response in spreads.

Figure 4.5 provides sensitivity analysis for the elasticity of bank net worth to separating existing loans. Similar to the explanation above, if the cost of separating existing loans is higher, the movement in credit market tightness is subdued, and the negative effect of technology and monetary shocks on the interest spread becomes stronger. This produces a lower initial credit spread response. In contrast, a lower cost of separation increases the volatility of the tightness variable, and allows for a higher initial response in spreads. However, this higher response in spreads is followed by a large correction, which is difficult to imagine in real life. A careful analysis on the empirical response of credit spreads to technology, monetary and capital quality shocks would spread light on the appropriate calibration warranted in this model.

4.4 Credit Supply Shock and Gross Loan Flows

In this section, I show that the model produces empirically plausible responses in loan creation and destruction flows for a credit supply shock. First, I estimate the empirical response of gross loan flows and loan levels to an orthogonal credit supply shock in a VAR framework. Second, I show that a negative shock to bank equity generates counter-cyclical spreads, and movements in loan creation and destruction margins that qualitatively match VAR results when investment is elastic to capital price. However, quantitatively, the model generates insufficient variations in gross loan flows compared to data.
Gilchrist and Zakrajsek [2012a] and Boivin et al. [2012] find that a shock to credit supply has a substantial effect on the real economy. Gilchrist and Zakrajsek [2012b] finds that a negative shock to credit supply reduces aggregate bank lending to firms using a VAR framework. However, whether this decline in aggregate loans is carried through by a reduction in new loan creation, or by destroying existing lending relations has not been studied.

Creation and destruction margins may respond differently to shocks affecting the economy for several reasons. First, extending new loans and retiring existing ones are two distinct activities involving different costs. The former may involve the cost of information acquisition, searching for new clients, or evaluating new projects. On the other hand, contractions in loans may depend on the liquidity of borrowers or on the legal steps involved in ensuring repayment or separation of relationships.

Second, loan expansion and contraction decisions may vary differently across banks. Regional and sectoral differences in both borrowers and lenders may result in overall loan contraction in some banks and expansion in others. Moreover, these idiosyncratic differences may depend on the type of shock affecting the economy. The 2008-09 recession was brought forth by a shock to the entire financial system. All banks were more-or-less negatively affected by the crisis.\(^{21}\) This is not necessarily the case for other recessions.

Finally, Contessi and Francis [2010] finds that gross loan flows followed a unique pattern during the 2008 and 1990-91 recessions – both of which coincide with an erosion of bank equity. During most recessions, loan destruction increases and creation decreases. However, the magnitude of the changes are such that net loans, defined as expansion minus contraction, usually remains a small positive in level terms. Moreover, loan destruction does not usually persist beyond the initial quarter of the recession.

\(^{21}\)See Brunnermeier [2009] for a summary of the economic mechanisms that characterized the financial crisis.
sion. In contrast, during the two recessions involving stress on the financial system, loan destruction exceeded creation so that the net loan flow became negative. This suggests that credit shocks may have a unique effect on the dynamics of these two variables. The goal of this section is to capture the pattern of response from a credit shock and to see whether the benchmark model is able to match it.

In the remainder of the section, I first provide a brief description of the data and estimation. I then describe the empirical findings and compare them with model results.

4.4.1 Data and Estimation

I use quarterly data series on gross loan flows constructed using bank-level balance sheet information for 1979 through 1999 by Dell’Ariccia and Garibaldi [2005] and extended from 1999 through 2008 by Contessi and Francis [2010]. Loan creation, \( POS_t \), is defined as the weighted sum of changes in loans for banks that increased loans in any given quarter. Loan destruction, \( NEG_t \), is defined as the absolute value of the weighted sum of changes in loans for all banks that decreased their loans since the previous quarter.

Note that the data represents all types of loans extended by commercial banks, including mortgages, lines of credit, and commercial and industrial loans. The model considered in the remainder of the paper, however, considers only commercial and industrial loans. Although this difference may be non-trivial for shocks to technology

\[22\] The two gross loan series are constructed using quarterly bank-level balance sheet information publicly available in the Reports of Condition and Income database (commonly called Call Report Files). Consolidation, entry and exits of banks during the time period is corrected for by matching the data with the National Information Center’s (NIC) transformation table to avoid double counting. See Contessi and Francis [2010] for a full description of the data construction methodology. I join the series calculated by Contessi and Francis [2010] to that from Dell’Ariccia and Garibaldi [2005] and consider an unbroken series running from 1979:Q2 through 2008:Q2.

\[23\] This approach of constructing gross loan flows mirrors the calculation of gross job flows in Davis et al. [1998], where net employment changes in the economy is decomposed into the two components of job creation and job destruction.
or monetary policy, it is not unreasonable to expect the general response of aggregate
loans to carry through for other loan categories for a financial shock that affects bank
balance sheets on a systemic scale.

Following Gilchrist and Zakrajsek [2012b], I estimate a VAR with two lags, and
identify a bank credit supply shock as an orthogonal shock to the ‘GZ excess bond
premium’. The VAR includes the following variables, ordered accordingly: (1) the
log-difference in real investment, (2) the log-difference in real GDP, (3) log-difference
in the GDP deflator, (4) the excess bond-premium, (5) the log-difference in real busi-
ness loans outstanding in all commercial banks, (6) the effective (nominal) federal
funds rate, (7) real loan creation (POS) and (8) real loan destruction (NEG). This
ordering allows the first three variables, which are traditionally viewed as ‘slow mov-
ing’, to respond to a shock in credit supply only after a quarter. The variables ordered
after the excess bond-premium, however, are allowed to respond contemporaneously
to a shock in credit supply. The data are in quarterly frequencies, and span the
period from 1979:Q2 through 2008:Q2, for which information on loan creation and
destruction are available.

Figure 4.6 shows the impulse responses to a Cholesky-ordered shock to the excess
bond-premium, along with their 68th and 95th percentile Monte Carlo confidence
bands. An increase in the excess bond-premium generates a recession driven by a

\footnote{This variable is constructed by Gilchrist and Zakrajsek [2012a] to measure the deviation in the
pricing of corporate debt claims relative to the expected default risk of the issuer, and captures shifts
in risk attitudes of financial intermediaries. Using a large panel of corporate bonds issued by non-
financial firms, credit spreads associated with individual firms are decomposed into two components:
one capturing the risk of default, and another capturing the cyclical-fluctuations of the relationship
between default risk and credit spreads. Gilchrist and Zakrajsek [2012a] and Gilchrist and Zakrajsek
[2012b] show that the majority of the information contained in credit spreads commonly used in the
literature is attributable to movements in the excess bond premium, and that shocks to the excess
bond premium that are orthogonal to current macroeconomic conditions have significant effects on
the real economy, as well as on outstanding bank loans. Accordingly, I take the lag structure of the
VAR and the Cholesky-ordered identification scheme as given in Gilchrist and Zakrajsek [2012a] and
Gilchrist and Zakrajsek [2012b].}

\footnote{Real values of aggregate business loans, and disaggregated flows in loan creation and
destruction are calculated by dividing the relevant variables with the GDP deflator.}
Figure 4.6: VAR Impulse responses to an increase in the excess bond premium.

Note: Dashed lines represent 68th percentile and dotted lines represent 95th percentile Monte Carlo confidence bands.
Figure 4.7: Net percentage of banks tightening lending and the excess bond premium.

Note: Solid line represents the net percentage of banks reported as tightening lending standards for commercial and industrial loans to large and medium firms. Marked line represents the Gilchrist and Zakrajsek [2012a] excess bond premium.
Figure 4.8: VAR impulse responses to an increase in the net percentage of banks tightening lending standards.

Note: Dashed lines represent 68th percentile and dotted lines represent 95th percentile Monte Carlo confidence bands.
reduction in investment. Aggregate commercial and industrial loans outstanding at all banks decline. Most importantly, loan creation, \( POS_t \), falls and loan destruction, \( NEG_t \), rises a few quarters after the initial shock. Therefore, in so far as the excess bond-premium captures information about the supply of bank credit, we can say that a negative supply shock increases loan destruction, and reduces loan creation.

A more direct measure of bank credit supply, however, can be gathered from survey results on banks’ willingness to lend. The Federal Reserve Board’s Senior Loan Officer Opinion Survey on Banking Lending Practices, conducted each quarter on 60 U.S. banks, asks senior officers whether banks have changed their credit standards over the past three months. The data, available from 1990:Q2, is plotted in figure 4.7, along with the excess bond premium.\(^{26}\) The solid line represents the net percentage of banks that reported tightening their credit standards on commercial and industrial loans to large and medium-sized firms. The net percentage value is calculated as the percentage of banks that report tightening less the percentage that report not tightening lending standards.\(^{27}\) As the figure shows, banks tighten their lending standards counter-cyclically, and at the height of the financial crisis, almost all banks reported doing so.

With this arguably more direct measure of bank credit supply, I run a VAR with two lags that includes the following variables, ordered accordingly: (1) the log-difference in real investment, (2) the log-difference in real GDP, (3) net percentage of banks tightening lending standards, (4) the excess bond-premium, (5) the log-difference in real business loans outstanding in all commercial banks, (6) the effective (nominal) federal funds rate, (7) real loan creation (POS) and (8) real loan destruc-
tion (NEG). The impulse responses from an orthogonalized shock to bank lending standards is reported in figure 4.8. A tightening of lending standards increases the excess bond premium on impact. Consequently, outstanding commercial and industrial bank loans fall gradually, as do investment and output. Loan creation declines with a lag, and loan destruction rises on impact.

Overall, I take the above evidence to suggest that a reduction in bank credit supply has a negative effect on the real economy, driven by a rise in interest spreads, and a consequent fall in loans to firms. Moreover, following a credit supply shock, disaggregated loan destruction rises and loan creation falls.

4.4.2 Model Analysis

In this subsection, I compare model results on loan creation and destruction flows from a credit supply shock with empirical evidence outlined above. Note that the empirical definition of these flows include changes in loans from both the intensive and extensive margins. To conform with data, I derive the following parallel in the benchmark model:

\[ POS_t = \frac{m_{t-1}S_t}{n_{t-2}S_{t-1}} \]
\[ NEG_t = -x_t \frac{S_t}{S_{t-1}} \]
\[ POS_t - NEG_t = \frac{n_{t-1}S_t}{n_{t-2}S_{t-1}} \]

where \( POS_t - NEG_t \) equals the change in total loans from period \( t - 1 \) to period \( t \).

I find that the model can generate impulse responses for loan creation and destruction flows to a credit supply shock that qualitatively match evidence, if investment is allowed to be less sluggish than usually assumed in the literature. To capture the concept of a shock to credit supply, I consider an i.i.d. shock to bank equity, as well
as an i.i.d. increase to the match separation rate. The former shock is specified in the law of motion of bank equity (equation 4.20), while the later is modelled as an exogenous drop in the survival rate, $x_t$. The optimal separation condition (equation 4.19) is then changed to

$$\Upsilon x_t \varepsilon_{t} = (T - \Upsilon \omega) + \frac{c_v}{q_t}$$

where $\varepsilon_{t}^{x}$ is an i.i.d. shock to the survival rate.

Figure 4.9 shows impulse responses for both these shocks for the benchmark calibration. For comparison, the ‘crisis experiment’ case of a negative capital quality shock is also included in the figure. We see that all three shocks result in a counter-cyclical rise in the interest spread, and an ensuing recession. However, apart from the rise on impact for NEG, and the eventual decline in POS for the capital quality shock, the response for disaggregated flows do not conform with data.

To see why this may be the case, note that net loan flows, which equal loan creation less loan destruction, captures changes in loan levels. For each of the shocks considered, we see a large temporary drop in capital prices, followed by a gradual decline in capital stock. Since loans are used in this model to purchase capital, the impulse response of total real loans reflect both the large initial drop in real capital prices as well as the small gradual decline in capital levels. As such, the impulse response for loans does not match the smooth response seen in the data. Since responses in disaggregated loan flows capture the curvature of the response in loan levels, we see the counter-factual result shown in figure 4.9.

If however, investment were allowed to be more responsive to capital prices, we may generate loan responses of the shape seen in the data. In the benchmark model, the parameter capturing the inverse of the elasticity of net investment to the price of capital was set to 1.728, as adopted by Gertler and Karadi [2011]. This is repre-
Figure 4.9: Benchmark model impulse responses to credit supply shock

Note: First column represents a negative shock to productive capital quality, second column represents an i.i.d. reduction in bank equity, and the third column represents an i.i.d. increase in the separation rate. Benchmark calibration implies an inverse investment elasticity to capital price of 1.728.
Figure 4.10: Comparison of impulse responses to credit supply shock for different investment elasticities

Note: First column represents a negative shock to productive capital quality, second column represents an i.i.d. reduction in bank equity, and the third column represents an i.i.d. increase in the separation rate. Dashed lines represent the benchmark calibration of an inverse investment elasticity to capital price of 1.728. Solid line represents an inverse investment elasticity to capital price of 0.11.
Figure 4.11: Model impulse responses to credit supply shock for elastic investment

Note: First column represents a negative shock to productive capital quality, second column represents an i.i.d. reduction in bank equity, and the third column represents an i.i.d. increase in the separation rate. The inverse investment elasticity to capital price is set to 0.11.
sentative of the values commonly assumed in the macro literature. However, using disaggregated data from 18 U.S. manufacturing industries, Groth and Khan [2010] estimate the weighted average of the elasticity of investment to the current shadow price of capital to be 15.2, suggesting a value of the inverse elasticity parameter of around 0.067. Figure 4.10 shows impulse responses of investment, aggregate loans, real capital price, and capital levels for credit supply shocks for two calibrations. The dashed line represents the benchmark calibration with a high inverse elasticity parameter, and the solid line represents a calibration of 0.11, a value between the two extremes mentioned above. We see that with a more elastic investment, the impulse response for loans are smoother, with a much higher drop in investment for a credit supply shock.

Figure 4.11 shows the responses for output, spreads, and both aggregate loans and disaggregated loan flows to the same three shocks for the highly elastic investment case. We see now that both for the bank equity shock, as well as the capital quality shock, loan destruction, $NEG_t$, increases on impact, and loan creation, $POS_t$, falls persistently. This result is more in tune with the empirical results seen in figures 4.6 and 4.8. A one-time reduction in the separation rate, however, does not generate impulse responses of disaggregated loan flows that match evidence, despite the smooth reaction of investment. Loan destruction, $NEG_t$, rises due to the shock to the separation rate. However, in light of an increased interest spread, and in the absence of a shock affecting loan demand, banks have no reason to continue reducing loans. Rather, banks increase their efforts to find new matches, which in turn, increases loan creation. This result suggests that a more meaningful representation of credit supply shocks is needed to match movements in disaggregated loan flows seen in the data. In this manner, the search-theoretic banking model proves useful in separating plausible explanations of supply-side shocks from implausible ones.
From a quantitative perspective, I find that the model generates cyclical co-movements in gross loan flows that are comparable to empirical findings. The data calculated from bank balance sheet information shows that loan creation is pro-cyclical, while loan destruction is counter-cyclical. Herrera et al. [2011] calculate similar flows using non-financial firm-level data from S&P full coverage Compustat tapes, and find cyclical correlations of a similar magnitude. Table 4.3 compares the correlation of gross loan flows with output from the Dell’Ariccia and Garibaldi [2005] and Contessi and Francis [2010] dataset and those derived from model simulations for each of the shocks considered in the model for the benchmark calibration. For all shocks considered, the correlation of gross loan flows with output are comparable to evidence.

Table 4.3: Correlation of gross loan flows with output

<table>
<thead>
<tr>
<th>Variables</th>
<th>data</th>
<th>technology</th>
<th>monetary</th>
<th>capital quality</th>
<th>bank equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>POS</td>
<td>0.20</td>
<td>0.17</td>
<td>0.31</td>
<td>0.47</td>
<td>0.35</td>
</tr>
<tr>
<td>NEG</td>
<td>-0.21</td>
<td>-0.19</td>
<td>-0.34</td>
<td>-0.12</td>
<td>-0.24</td>
</tr>
<tr>
<td>NET</td>
<td>0.20</td>
<td>0.38</td>
<td>0.37</td>
<td>0.51</td>
<td>0.30</td>
</tr>
</tbody>
</table>

The model fails, however, to generate volatilities in gross loan flows comparable to evidence. Dell’Ariccia and Garibaldi [2005] find that both variations in loan destruction and loan creation are larger in magnitude than variations in output. Moreover, destruction is more volatile than creation of loans. Table 4.4 summarizes the standard deviations of creation and destruction flows relative to output for each of the shocks considered in the model for the benchmark calibration, along with the relevant empirical values for aggregate loans.

Table 4.4: Standard deviation of gross loan flows relative to output

<table>
<thead>
<tr>
<th>Variables</th>
<th>data</th>
<th>technology</th>
<th>monetary</th>
<th>capital quality</th>
<th>bank capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>POS</td>
<td>23.85</td>
<td>0.10</td>
<td>0.11</td>
<td>0.18</td>
<td>0.11</td>
</tr>
<tr>
<td>NEG</td>
<td>26.71</td>
<td>0.17</td>
<td>0.25</td>
<td>0.35</td>
<td>0.33</td>
</tr>
</tbody>
</table>

\(^{28}\)i.e. the relative standard deviation if the economy were subject to one type of shock only.
We see that for each shock, model generated flows in loan creation and destruction are less volatile than output. The values for the corresponding relative standard deviations in the high investment elasticity case are comparable with the benchmark calibration of low investment elasticity shown here. This suggests, if anything, that the model requires an even stronger variation in endogenous separation.

4.5 Conclusion

In this chapter, I develop a search-theoretic banking model in a New-Keynesian DSGE framework that can simultaneously explain movements in gross flows in loan creation and destruction, and the interest rate spread. The model demonstrates that the presence of the banking sector amplifies and propagates the economy’s response of a number of shocks by generating a counter-cyclical interest rate spread. Shocks that disproportionately affect the balance sheets of banks are particularly amplified. Moreover, shocks originating within the banking sector also lead to prolonged recessions.

I show that the model generates responses in gross loan creation and destruction flows to a credit supply shock that qualitatively match empirical evidence. I estimate the effect of a credit supply shock on loan creation and destruction margins in a VAR framework. I then consider two shocks that can proxy the effects of a credit supply shock – a one time reduction in bank equity, and a one time increase in the match separation rate. I also consider a negative shock to the quality of productive capital, which works through the bank balance sheet identity and reproduces well important movements related to the bank leverage channel observed during the recent crisis. I show that both the capital quality shock, as well as the shock to bank equity can produce impulse responses that qualitatively match evidence, if investment is elastic to capital price. An exogenous increase in the separation rate, however, does not produce empirically plausible responses for disaggregated loan flows.
A limitation of the model, however, is that it cannot quantitatively match the business cycle volatilities of gross loan creation and destruction flows. In particular, volatilities of these two margins are far lower than seen in the data. Overall, however, the paper demonstrates that a search-theoretic banking model is a step in the right direction in jointly explaining movements in the interest rate spread and gross loan flows.
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Appendix A

Appendix for Chapter 2

A.1 Non-linear Equations for Benchmark Model

Household

\[
\lambda_t = \frac{1}{c_t - ac_{t-1}} - \beta E_t \frac{a}{c_{t+1} - ac_t} \quad (A.1)
\]

\[
\frac{1}{R^d_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \quad (A.2)
\]
Entrepreneurs

\[ y_t = (u_t k_t)^\alpha (z_t l_t)^{1-\alpha} \]  
\[ w_t^e = (1 - \alpha) \varphi_t \frac{y_t}{l_t} \]  
\[ r_t^k = \varphi_t \alpha \frac{y_t}{k_t} \]  
\[ r_t^k = q_t \delta' (u_t) u_t \]  
\[ E_t \{ R_{t+1} \} = E_t \left\{ R_{t+1} \left( \frac{q_t k_{t+1}}{n w_t} \right)^{\psi} \frac{1}{\pi_{t+1}} \right\} \]  
\[ E_t \{ R_{t+1} \} = E_t \left[ \frac{r_{t+1}^k + q_{t+1} (1 - \delta (u_t))}{q_t} \right] \]  
\[ n w_t = \nu \left[ \{ r_t^k + q_t (1 - \delta (u_{t-1})) \} k_t - E_{t-1} R_t (q_{t-1} k_t - n w_{t-1}) \right] \]  
\[ + (1 - \nu) s c_t \]

\[ q_t \xi_t = 1 + \chi_t \left( \frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} + \frac{\chi_t}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \]  
\[ - E_t \chi_t \left( \frac{i_{t+1}}{i_t} - 1 \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \]  
\[ k_{t+1} = \xi_t i_t + (1 - \delta (u_t)) k_t \]  

Capital Producers

\[ q_t \xi_t = 1 + \chi_t \left( \frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} + \frac{\chi_t}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \]  
\[ - E_t \chi_t \left( \frac{i_{t+1}}{i_t} - 1 \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \]  
\[ k_{t+1} = \xi_t i_t + (1 - \delta (u_t)) k_t \]
Employment Agencies

\[ m_t = \sigma_m u_t v_t^{1-\sigma} \] (A.12)
\[ \varrho_t = \frac{m_t}{v_t} \] (A.13)
\[ \varsigma_t = \frac{m_t}{u_t} \] (A.14)
\[ n_t = (1 - \rho) n_{t-1} + m_{t-1} \] (A.15)
\[ u_t = 1 - (1 - \rho) n_t \] (A.16)
\[ \frac{\kappa}{\lambda_t \varrho_t} = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) \left[ w_t^e h_t - w_t h_t + \frac{\kappa}{\lambda_{t+1} \varrho_{t+1}} \right] \] (A.17)
\[ w_t h_t = \eta \left[ w_t^e h_t + \varsigma_t (1 - \rho_t) \frac{\kappa}{\lambda_t} \right] + (1 - \eta) \left[ b + \frac{g(h_t)}{\lambda_t} \right] \] (A.18)
\[ w_t^e = \frac{\Upsilon}{\lambda_t} h_t^\xi \] (A.19)
\[ l_t = n_t h_t \] (A.20)

Model Closing

\[ \frac{R_t}{R} = \left( \frac{\pi_t}{\pi} \right)^{\varphi_y} \left( \frac{y_t}{y} \right)^{\varphi_y} \exp (\varepsilon_{R,t}) \] (A.21)
\[ \pi_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \phi) (1 - \phi)}{\phi} \hat{\varphi}_t \] (A.22)
\[ y_t = c_t + i_t \] (A.23)

A.2 Log-linearized Equations for Benchmark Model

Household

\[ \hat{\lambda}_t = \frac{1}{(1 - a)(1 - a\beta)} \left[ \hat{c}_t - a\hat{c}_{t-1} \right] - \frac{a\beta}{(1 - a)(1 - a\beta)} \left[ E_t \hat{c}_{t+1} - a\hat{c}_t \right] \] (A.24)
\[ \hat{R}_t = \hat{\lambda}_t + E_t \hat{\pi}_{t+1} - E_t \hat{\lambda}_{t+1} \] (A.25)
Entrepreneurs

\[ \hat{y}_t = \alpha \hat{u}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{z}_t + (1 - \alpha) \hat{l}_t \]  \hspace{1cm} (A.26)

\[ \hat{u}_t^e = \hat{\varphi}_t + \hat{y}_t - \hat{l}_t \]  \hspace{1cm} (A.27)

\[ \hat{r}_t^k = \hat{\varphi}_t + \hat{y}_t - \hat{k}_t \]  \hspace{1cm} (A.28)

\[ \hat{u}_t = \left( \frac{r^k}{r^k + \delta''(u)} \right) (\hat{r}_t^k - \hat{q}_t) \]  \hspace{1cm} (A.29)

\[ E_t \hat{R}_{t+1}^l = E_t \left[ \hat{R}_{t+1} + \psi \left( \hat{q}_t + \hat{k}_{t+1} - n \hat{\omega}_t \right) - \hat{n}_{t+1} \right] \]  \hspace{1cm} (A.30)

\[ E_t \hat{R}_{t+1}^k = \left[ \frac{r^k}{r^k + (1 - \delta)} \right] E_t \hat{r}_t^k + \left[ \frac{(1 - \delta)}{r^k + (1 - \delta)} \right] E_t \hat{q}_t - \hat{q}_t \]  \hspace{1cm} (A.31)

\[ \frac{n \hat{\omega}_t}{\nu} = \left( \frac{k}{nw} \right) \left[ r^k \hat{r}_t^k + (1 - \delta) \hat{q}_t - R^l \hat{q}_{t-1} - r^k \hat{u}_t \right] \]  \hspace{1cm} (A.32)

\[ - \psi \left( \frac{k}{nw} - 1 \right) R^l \left[ \hat{k}_t + \hat{q}_{t-1} \right] - \left( \frac{k}{nw} - 1 \right) R^l \left( \hat{R}_t - \hat{n}_t \right) + R^l \left[ \psi \left( \frac{k}{nw} - 1 \right) + 1 \right] n \hat{\omega}_{t-1} \]

Capital Producers

\[ \hat{q}_t = 2 \chi \hat{l}_t - \chi \hat{l}_{t-1} - \chi \hat{l} E_t \hat{u}_{t+1} - \hat{\xi}_t \]  \hspace{1cm} (A.33)

\[ E_t \hat{k}_{t+1} = \delta \hat{\xi}_t + \delta \hat{l}_t + (1 - \delta) \hat{k}_t - r^k \hat{u}_t \]  \hspace{1cm} (A.34)
Employment Agencies

\[ \hat{m}_t = \sigma \hat{u}_t + (1 - \sigma) \hat{\nu}_t \]  
\[ \hat{g}_t = \hat{m}_t - \hat{\nu}_t \]  
\[ \hat{\varsigma}_t = \hat{m}_t - \hat{u}_t \]  
\[ \hat{n}_t = (1 - \rho) \hat{n}_{t-1} + \rho \hat{m}_{t-1} \]  
\[ \hat{u}_t = -\frac{n}{1 - (1 - \rho) n} (1 - \rho) \hat{n}_t \]  
\[ \hat{\omega}_t = \hat{\rho} - \hat{\lambda}_t \]  
\[ \hat{\rho}_t = \beta \lambda (1 - \rho) \left[ \left( \frac{w h \lambda \varrho}{\kappa} \right) E_t (\hat{\omega}_{t+t} - \hat{\omega}_{t+t+1}) + E_t \hat{\omega}_{t+1} \right] \]  
\[ \hat{\omega}_t = \eta \hat{\omega}_t + \left[ \eta + (1 - \eta) \frac{\Upsilon h \varsigma}{\lambda w} - 1 \right] \hat{h}_t \]  
\[ - \frac{1}{w h} \left[ \eta (1 - \rho) \frac{\kappa \varsigma}{\lambda \varrho} + (1 - \eta) \frac{\varrho}{\lambda} \right] \hat{\lambda}_t \]  
\[ + \frac{\eta (1 - \rho) \kappa \varsigma}{w h \lambda \varrho} (\hat{\varsigma}_t - \hat{\omega}_t) \]  
\[ \hat{\lambda}_t = \hat{n}_t + \hat{h}_t \]  

Model Closing

\[ \hat{R}_t = \varphi \hat{\pi}_t + \varphi_y \hat{y}_t + \varepsilon_{R,t} \]  
\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \phi) (1 - \phi)}{\phi} \hat{\varphi}_t \]  
\[ \hat{y}_t = \frac{c}{y} \hat{\varsigma}_t + i \hat{y}_t \]  
\[ \hat{\xi}_t = \rho_x \hat{\xi}_{t-1} + \varepsilon_{\xi,t} \]
Important Steady-States

\[
\frac{w^e h}{Y} = (1 - \alpha) \frac{\phi}{n}
\]

\[
\lambda Y = \left[ \frac{1 - a \beta}{1 - a} \right] \frac{Y}{c}
\]

\[
\frac{wh}{Y} = \frac{w^e h}{Y} - \frac{\kappa \theta \sigma}{\lambda Y} \left[ \frac{1 - \beta (1 - \rho)}{\beta (1 - \rho)} \right]
\]

\[
g(h) \frac{\lambda Y}{\lambda Y} = \frac{\gamma h^{1 + \zeta}}{1 + \zeta} = \frac{w^e h}{Y(1 + \zeta)}
\]

\[
g'(h)h \frac{\lambda Y}{\lambda Y} = \frac{\gamma h^{1 + \zeta}}{\lambda Y} = \frac{w^e h}{Y}
\]
Appendix B

Appendix for Chapter 3

B.1 Data for VAR Analysis

**Tax Revenue**: Current tax receipts + Income receipts on assets + Current transfer receipts - Current transfer payments - Interest payments - Subsidies. Source: Table 3.1. Government Current Receipts and Expenditures, Bureau of Economic Analysis.


**Output**: Gross domestic product. Source: Table 1.1.5. Gross Domestic Product, Bureau of Economic Analysis.

**Consumption**: Nondurable goods (Personal Consumption) + Services (Personal Consumption minus housing and utilities services consumption). Source: Table 1.1.5. Gross Domestic Product, Bureau of Economic Analysis.


The data are seasonally adjusted at annual rates. We transformed this into log
real per capita terms by first normalizing the original data by Total Population: All Ages including Armed Forces Overseas (Quarterly Average, Source: Monthly National Population Estimates, US Department of Commerce: Census Bureau) and the GDP implicit price deflator (Seasonally adjusted, 2005=100, Source: Table 1.1.9. Implicit Price Deflators for Gross Domestic Product, Bureau of Economic Analysis) and then taking logarithm.

B.2 Factor Augmented VAR

The mismatch in information sets between the econometrician and private agents, and the resulting anticipation issues, may cause a deeper problem than the one described in section 3.2. Leeper et al. [2011] show that in the presence of fiscal foresight, the MA representation of a limited set of variables can be non-fundamental, and VAR techniques would fail to recover the structural shocks. To understand the issue of fundamentalness, consider an MA representation of a solution from a DSGE model:

\[ X_t = B(L)u_t \]

Here, \( X \) is an \( n \)-vector of macroeconomic variables of interest and \( u \) is the \( q \)-vector structural shock, with \( n \geq q \). This representation is fundamental if \( X_t \) lies in the space spanned by present and past values of \( u_t \) and, conversely, \( u_t \) lies in the space spanned by present and past values of \( X_t \). In this case, observing \( X_t \) is enough to recover \( u_t \). The second part of the above definition, however, holds generically for tall systems when \( n > q \), but not for square systems where \( n = q \).

If the econometrician wants to estimate \( B(L) \) and \( u_t \) by running an \( n \)-dimensional VAR on \( X_t \), we would end up with \( n \) linearly independent shocks, more than the number of structural shocks given by theory. In this case, the econometrician may opt
to estimate a VAR on a square $q$-dimensional subsystem $A(L)\hat{X}_t = \epsilon_t$ and attempt to recover the structural shocks by imposing identifying restriction on a matrix $D$ such that $\epsilon_t = Du_t$. If the square subsystem is non-fundamental, however, the space spanned by $\hat{X}_t$ will not contain the structural shocks, and the VAR will not be able to recover them, regardless of the identification scheme used to determine $D$.

Leeper et al. [2011] analytically shows how expectations about future tax changes in a simple model can misalign the information sets available to private agents, who receive news about the tax changes well in advance, and the econometrician who sees the changes only when they occur. Without correctly modelling the flow of information, the smaller subset of information available to the econometrician gives rise to non-fundamentalness in VAR models. In this situation, the Blanchard and Perotti [2002] scheme of identifying tax shocks give different results from the case where information flow is taken into consideration.

Forni and Gambetti [2011] suggest that one way of overcoming the limitations of the reduced rank VAR model is to consider a factor model, where information present in a large number of variables is reduced to a fundamental rectangular system by taking linear combinations of the variables. In a factor model, each macroeconomic variable $x_{it}$ is represented as a sum of a common component $\chi_{it}$ and an idiosyncratic component $\xi_{it}$. The common components are in turn assumed to be linear combinations of a relatively small number $r$ of factors $\chi_{it} = a_i f_t$. The factors, in turn, are responsible for most of the co-movement between macroeconomic variables, and are assumed to have a fundamental MA representation:

$$f_t = N(L)u_t$$

A reduced rank estimate of the dynamic factors would then recover the structural shocks $u_t$. Identification, however, becomes less straightforward as the factors them-
selves lose a direct economic interpretation. In this case, identification can be achieved via sign restrictions on the final macroeconomic variables.

Following Forni and Gambetti [2011], we estimate a factor-augmented VAR with 110 variables, 13 static factors, 4 lags and 6 structural shocks. The dataset is almost identical to Forni and Gambetti [2011], with few exceptions. All data are in quarterly frequencies and cover the period from 1963:1 through 2007:4.

### B.3 Sign Restriction

To understand the method of identification through sign restrictions, consider the MA representation of a VAR:

\[ y_t = D(l) e_t \]

where \( y_t \) is a vector of economic variables of interest, \( D(l) \) is a matrix of lag operators, and \( e_t \sim (0, \Sigma_e) \) is the vector of reduced form errors. The problem of identification arises precisely because there is no direct structural interpretation of the reduced form errors, which is assumed to be a linear combination of the orthogonal structural shocks. If, however, we can find a matrix \( P \), such that \( PP' = \Sigma_e \), we can generate impulse response functions to orthogonal shocks from the following MA representation:

\[ y_t = D(l) PP^{-1} e_t = \tilde{D}(l) \epsilon_t \]

where \( \epsilon_t = P^{-1} e_t \) are orthogonal shocks such that \( \epsilon_t \sim (0, P^{-1}\Sigma_e P^{-1'} = I) \). The Cholesky (lower triangular) factorization of the estimated error covariance \( \Sigma_e \) is one way of generating such a matrix \( P \).

Note, however that this factorization of \( \Sigma_e \) is not unique. In particular, for any
orthogonal matrix \(Q\), such that \(QQ' = I\), we can have

\[
y_t = D(l) PQQ'P^{-1} \varepsilon_t = \tilde{D}(l) \varepsilon_t
\]

where \(\varepsilon_t \sim (0, Q'P^{-1}\Sigma_eP^{-1}Q = I)\). So, for any orthogonal matrix \(Q\), we can generate impulse responses of orthogonal shocks by identifying \(\varepsilon_t = Q'P^{-1}\varepsilon_t\). The method of sign restricted identification considers those \(Q\) matrices that result in impulse responses that conform to a pre-defined scheme derived from minimal theory.

In this study, we identify government spending shocks by imposing the following sign restrictions on the impulse response functions: total government spending, federal government spending, total government deficit, federal government deficit and output all increase in the 5 quarter following the shock. Imposing the restriction on further from the impact period allows for anticipation effects. This strategy, however, does not allow us to differentiate between an expected and an unexpected shock.
B.4 Non-linear Equations for Benchmark Model

Patient household

\[ c^\ell_t + q^\ell_t h^\ell_t + i_t = w^\ell_t n^\ell_t + q^\ell_t h^\ell_{t-1} + r^\ell_t k^\ell_{t-1} + d^\ell_t + \frac{r^\ell_{t-1} b^\ell_{t-1}}{\pi^\ell_t} + \frac{R^\ell_{t-1} b^\ell_{t-1}}{\pi^\ell_t} \]  \hspace{1cm} (B.1)

\[ k^\ell_t = (1 - \delta) k^\ell_{t-1} + \phi \left( \frac{i^\ell_t}{k^\ell_{t-1}} \right) k^\ell_{t-1} \]  \hspace{1cm} (B.2)

\[ \frac{1}{c^\ell_t} = \lambda^\ell_{1t} \]  \hspace{1cm} (B.3)

\[ \frac{(n^\ell_t)^\eta}{w^\ell_t} = \lambda^\ell_{1t} \]  \hspace{1cm} (B.4)

\[ \frac{\Upsilon^\ell}{h^\ell_t} = \lambda^\ell_{1t} q^\ell_t - \beta^\ell E^\ell_t \lambda^\ell_{1t+1} q^\ell_{t+1} \]  \hspace{1cm} (B.5)

\[ 1 = \psi^\ell \phi^\ell \left( \frac{i^\ell_t}{k^\ell_{t-1}} \right) \]  \hspace{1cm} (B.6)

\[ \psi^\ell E^\ell_t \frac{\lambda^\ell_{1t}}{\lambda^\ell_{1t+1}} = \beta^\ell E^\ell_t \left[ r^\ell_{t+1} + \psi^\ell_{t+1} \left\{ (1 - \delta) + \phi \left( \frac{i^\ell_{t+1}}{k^\ell_t} \right) - \phi^\ell \left( \frac{i^\ell_{t+1}}{k^\ell_t} \right) \frac{i^\ell_{t+1}}{k^\ell_t} \right\} \right] \]  \hspace{1cm} (B.7)

\[ 1 = \beta^\ell E^\ell_t \left[ \frac{\lambda^\ell_{1t+1} r^\ell_{t}}{\lambda^\ell_{1t} \pi^\ell_{t+1}} \right] \]  \hspace{1cm} (B.8)

Borrowing Households

\[ c^b_t + q^b_t h^b_t + \frac{r^\ell_t b^\ell_{t-1}}{\pi^\ell_t} = w^b_t n^b_t + q^b_t h^b_{t-1} + b_t - \tau^b_t \]  \hspace{1cm} (B.9)

\[ b_t = m E^\ell_t \left\{ \frac{q^b_{t+1} h^b_t \pi^\ell_{t+1}}{R^\ell_t} \right\} \]  \hspace{1cm} (B.10)

\[ \frac{1}{c^b_t} = \lambda^b_{1t} \]  \hspace{1cm} (B.11)

\[ \frac{(n^b_t)^\eta}{w^b_t} = \lambda^b_{1t} \]  \hspace{1cm} (B.12)

\[ \frac{\Upsilon^b}{h^b_t} = \lambda^b_{1t} q^b_t - \beta^b E^\ell_t \lambda^b_{1t+1} q^b_{t+1} - \lambda^b_{2t} E^\ell_t \left[ \frac{m q_{t+1} \pi^\ell_{t+1}}{R^\ell_t} \right] \]  \hspace{1cm} (B.13)

\[ \lambda^b_{1t} = \beta^b E^\ell_t \left[ \frac{\lambda^b_{1t+1} R^\ell_t}{\pi^\ell_{t+1}} \right] + \lambda^b_{2t} \]  \hspace{1cm} (B.14)
Firms

\begin{align*}
y_t &= k_{t-1}^\gamma n_t^{1-\gamma} \\
r_t &= \gamma mc_t \frac{y_t}{k_{t-1}} \\
w_t &= (1 - \gamma) mc_t \frac{y_t}{n_t} \\
mc_t &= \gamma^{-\gamma}(1 - \gamma)^{1-\gamma} w_t^\gamma r_t^{1-\gamma}
\end{align*}

Nominal Rigidities and Monetary Policy

\begin{align*}
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \theta_\pi)(1 - \beta_t \theta_\pi)}{\theta_\pi} \hat{mc}_t \\
\frac{\tilde{r}_t}{\tilde{r}^n} &= \left(\frac{\pi_t}{\pi}\right)^{\theta_\pi}
\end{align*}

Government

\begin{align*}
\tau_t + b_t^g &= \frac{R_t^g b_{t-1}^g}{\pi_t} + G_t \\
\tilde{\tau}_t &= \rho b_{t-1}^g + \rho_g \tilde{g}_t \\
\tilde{g}_t &= \rho_g \tilde{g}_{t-1} + \varepsilon_t^g
\end{align*}

Aggregation

\begin{align*}
C_t &= \alpha c_t^b + (1 - \alpha) c_t^l \\
N_t &= \alpha n_t^b + (1 - \alpha) n_t^l \\
H_t &= \alpha h_t^b + (1 - \alpha) h_t^l \\
I_t &= (1 - \alpha) i_t \\
K_t &= (1 - \alpha) k_t \\
Y_t &\approx K_{t-1}^\gamma N_t^{1-\gamma} = C_t + \phi \left(\frac{I_t}{K_{t-1}}\right) K_{t-1} + G_t
\end{align*}
B.5 Non-linear Equations for Model with Housing Production

Patient Households

\[ c^\ell_t + i_t + i^h_t = \frac{w^\ell_t n^\ell_t}{X^\ell_t} + \frac{w^h_t n^h_t}{X^h_t} + (1 - \delta^h) q_t h^\ell_{t-1} + r_t k_{t-1} + r^h_t k^h_{t-1} + d^\ell_t \]

\[ + \frac{r^h_{t-1} b^q_{t-1}}{\pi_t} + \frac{R_{t-1} b^t_{t-1}}{\pi_t} - b^q_t - b_t - q_t h^\ell_t - r^\ell_t \quad (B.30) \]

\[ k_t = (1 - \delta) k_{t-1} + \phi \left( \frac{i_t}{k_{t-1}} \right) k_{t-1} \quad (B.31) \]

\[ k^h_t = (1 - \delta_h) k^h_{t-1} + i^h_t - \phi_h \left( \frac{i^h_t}{k^h_{t-1}} \right) k^h_{t-1} \quad (B.32) \]

\[ \frac{1}{c^\ell_t} = \lambda^\ell_{t+1} \quad (B.33) \]

\[ \frac{\gamma^\ell_t}{h^\ell_t} = \lambda^\ell_{t+1} q_t - \beta^\ell E_t \lambda^\ell_{t+1} q_{t+1} \quad (B.34) \]

\[ \lambda^\ell_t w^c_t = \left[ \left( n^{cp}_{t+1} \right)^{1+\xi} + \left( n^{hp}_{t+1} \right)^{1+\xi} \right]^{\frac{1+\xi}{1+\xi} - 1} \left( n^{cp}_t \right)^{\xi} \quad (B.35) \]

\[ \lambda^\ell_t w^h_t = \left[ \left( n^{cp}_{t+1} \right)^{1+\xi} + \left( n^{hp}_{t+1} \right)^{1+\xi} \right]^{\frac{1+\xi}{1+\xi} - 1} \left( n^{hp}_t \right)^{\xi} \quad (B.36) \]

\[ 1 = \psi_t \phi \left( \frac{i_t}{k_{t-1}} \right) \quad (B.37) \]

\[ \psi_t E_t \frac{\lambda^\ell_t}{\lambda^\ell_{t+1}} = \beta^\ell E_t \left[ r_{t+1} + \psi_{t+1} \left\{ (1 - \delta) + \phi \left( \frac{i_{t+1}}{k_{t+1}} \right) - \phi' \left( \frac{i_{t+1}}{k_{t+1}} \right) \frac{i_{t+1}}{k_{t+1}} \right\} \right] \quad (B.38) \]

\[ 1 = \psi^h_t \phi_h \left( \frac{i^h_t}{k^h_{t-1}} \right) \quad (B.39) \]

\[ \psi^h_t E_t \frac{\lambda^\ell_t}{\lambda^\ell_{t+1}} = \beta^\ell E_t \left[ r^h_{t+1} + \psi^h_{t+1} \left\{ (1 - \delta^h) + \phi_h \left( \frac{i^h_{t+1}}{k^h_{t+1}} \right) - \phi'_h \left( \frac{i^h_{t+1}}{k^h_{t+1}} \right) \frac{i^h_{t+1}}{k^h_{t+1}} \right\} \right] \quad (B.40) \]

\[ 1 = \beta^\ell E_t \left[ \frac{\lambda^\ell_{t+1} r^\ell_t}{\lambda^\ell_t \pi_{t+1}} \right] \quad (B.41) \]
Impatient Households

\[ c_t^b + q_t h_t^b + \frac{n_{t-1}^{\pi} b_{t-1}}{\pi_t} = w_t n_t^b + q_t h_{t-1}^b + b_t - \tau_t^b \]  

(B.42)

\[ b_t = m E_t \left\{ \frac{q_{t+1} h_t^b \pi_{t+1}}{\pi_t} \right\} \]  

(B.43)

\[ \frac{1}{c_t^b} = \lambda_{tt}^b \]  

(B.44)

\[ \frac{Y^b}{h^b_t} = \lambda_{tt}^b q_t - \beta_b E_t \left[ \lambda_{tt+1}^b q_{t+1} \right] - \lambda_{2tt}^b E_t \left[ \frac{m q_{t+1} \pi_{t+1}}{R_t} \right] \]  

(B.45)

\[ \lambda_{1tt}^b = \beta_b E_t \left[ \lambda_{tt+1}^b \frac{R_t}{\pi_{t+1}} \right] + \lambda_{2tt}^b \]  

(B.46)

\[ \lambda_t^w c = \left[ (n_t^{cp})^{1+\xi} + (n_t^{hp})^{1+\xi} \right] ^{\frac{1+\eta}{1+\xi} - 1} (n_t^{cp})^\xi \]  

(B.47)

\[ \lambda_t^w h = \left[ (n_t^{cp})^{1+\xi} + (n_t^{hp})^{1+\xi} \right] ^{\frac{1+\eta}{1+\xi} - 1} (n_t^{hp})^\xi \]  

(B.48)

Firms

\[ Y_t^h = (L)^{\mu} (K_t^{h-1})^{\mu} (N_t^{h})^{1-\mu} \]  

(B.49)

\[ Y_t = K_t^{h-1} N_t^{1-\gamma} \]  

(B.50)

\[ r_t = \gamma m c_t \frac{Y_t}{K_{t-1}} \]  

(B.51)

\[ w_t = (1 - \gamma) m c_t \frac{Y_t}{N_t} \]  

(B.52)

\[ m c_t = \gamma^{-\gamma} (1 - \gamma) \gamma^{-1} w_t^\gamma r_t^{1-\gamma} \]  

(B.53)

\[ \gamma_t^h = \mu X_t^h \frac{Y_t^h}{K_t^{h-1}} \]  

(B.54)

\[ w_t^h = (1 - \mu - \mu_t) X_t^h \frac{Y_t^h}{N_t^h} \]  

(B.55)
Nominal Rigidities and Monetary Policy

\[
\dot{\omega}_t = \beta^t \omega_{t+1} - \frac{(1 - \theta_w) (1 - \beta^t \theta_w)}{\theta_w} \dot{X}_t \tag{B.56}
\]

\[
\dot{\omega}_t^h = \beta^t \omega_{t+1}^h - \frac{(1 - \theta_{hw}) (1 - \beta^t \theta_{hw})}{\theta_{hw}} \dot{X}_t^h \tag{B.57}
\]

\[
\omega_t = \frac{w_t \pi_t}{w_{t-1}} \tag{B.58}
\]

\[
\omega_t^h = \frac{w_t^h \pi_t}{w_{t-1}^h} \tag{B.59}
\]

\[
\dot{\pi}_t = \beta_t \dot{\pi}_{t+1} + \frac{(1 - \theta_{\pi}) (1 - \beta_t \theta_{\pi})}{\theta_{\pi}} \dot{m}c_t \tag{B.60}
\]

\[
\hat{r}_t^n = \rho_{\pi} \dot{\pi}_t + \epsilon_t^n \tag{B.61}
\]

Government

\[
T_t + b_t^g = \frac{R_t^g b_{t-1}^g}{\pi_t} + G_t \tag{B.62}
\]

\[
\frac{T_t - T}{Y} = \rho_b \left[ \frac{b_t^g - b}{Y} \right] + \rho_g \left[ \frac{G_t - G}{Y} \right] \tag{B.63}
\]

\[
\left[ \frac{G_t - G}{Y} \right] = \rho_g \left[ \frac{G_{t-1} - G}{Y} \right] + \epsilon_t^g \tag{B.64}
\]
Aggregation

\[ C_t = \alpha c_t^b + (1 - \alpha) c_t^\ell \]  \hspace{1cm} (B.65)

\[ N_t = \alpha n_t^b + (1 - \alpha) n_t^\ell \]  \hspace{1cm} (B.66)

\[ N_t^h = \alpha n_t^{hb} + (1 - \alpha) n_t^{h\ell} \]  \hspace{1cm} (B.67)

\[ H_t = \alpha h_t^b + (1 - \alpha) h_t^\ell \]  \hspace{1cm} (B.68)

\[ I_t = (1 - \alpha)i_t \]  \hspace{1cm} (B.69)

\[ K_t = (1 - \alpha)k_t \]  \hspace{1cm} (B.70)

\[ I_t^h = (1 - \alpha)i_t^h \]  \hspace{1cm} (B.71)

\[ K_t^h = (1 - \alpha)k_t^h \]  \hspace{1cm} (B.72)

\[ Y_t = C_t + \phi \left( \frac{I_t}{K_{t-1}} \right) K_{t-1} + \phi_h \left( \frac{I_t^h}{K_{t-1}^h} \right) K_{t-1}^h + G_t \] \hspace{1cm} (B.73)

\[ Y_t^H = H_t - (1 - \delta_h) H_{t-1} \] \hspace{1cm} (B.74)

\textbf{B.6 GHH Preferences}

Lending Households

\[ x_t^\ell = (c_t^\ell)^{1-\psi_h} (h_t^\ell)^{\psi_h} - \frac{n_t^\ell (1+\eta)}{1+\eta} \] \hspace{1cm} (B.75)

\[ \lambda_{1t}^\ell = (1 - \psi_h) x_t^{\ell(-\gamma)} \left( \frac{h_t^\ell}{c_t^\ell} \right)^{\psi_h} \] \hspace{1cm} (B.76)

\[ x_t^{\ell(-\gamma)} n_t^\ell \eta = \lambda_{1t}^\ell w_t \] \hspace{1cm} (B.77)

\[ \psi_h x_t^{\ell(-\gamma)} \left( \frac{c_t^\ell}{h_t^\ell} \right)^{1-\psi_h} = \lambda_{1t}^\ell q_t - \beta \lambda_{1,t+1}^\ell q_{t+1} \] \hspace{1cm} (B.78)
Borrowing Households

\[ x_t^b = (c_t^b)^{1-\psi_h} (h_t^b)^{\psi_h} - \frac{n_t^b (1+\eta)}{1+\eta} \]  \hspace{1cm} (B.79)

\[ \lambda_{1t} = (1-\psi_h) x_t^{b(-\gamma)} \left( \frac{h_t^b}{c_t^b} \right)^{\psi_h} \]  \hspace{1cm} (B.80)

\[ x_t^{b(-\gamma)} n_t^{b\eta} = \lambda_{1t}^b w_t \]  \hspace{1cm} (B.81)

\[ \psi_h x_t^{b(-\gamma)} \left( \frac{c_t^b}{h_t^b} \right)^{1-\psi_h} = \lambda_{1t}^b q_t - \beta \lambda_{1,t+1}^b q_{t+1} - \lambda_{2,t} \frac{q_{t+1} \pi_{t+1}}{r_t^n} \]  \hspace{1cm} (B.82)

Bibliography


Appendix C

Appendix for Chapter 4

C.1 Timing

State Variables:

- \( n_{t-1} \): number of projects that receive funding in period \( t \).
- \( K_{t-1} \): symmetric equilibrium capital used for production in each project in period \( t \).
- \( N_{t-1}^b \): bank capital at the beginning of period \( t \).
- \( B_{t-1} \): amount of deposits saved in the banking sector the end of period \( t - 1 \) by households that make up loans for the purchase of capital in period \( t \).

Timing:

- Time \( t \) begins with the number of funded projects, total productive capital quantity, and bank deposits to be used in period \( t \) as given.
- Aggregate shocks hit the economy.
- Firms and Banks determine loan interest rate \( R_t^l \) through Nash bargaining.
• Firms chose symmetric equilibrium loan level $S_t = Q_t K_{t-1}$ for each funded project, given the loan interest rate $R^l_t$. Each funded project receives capital $K_{t-1}$ purchased at price $Q_t$ for production this period.

• Labour demand / supply and wage setting takes place.

• Production occurs in funded projects according to $Y_t = z_t (\xi_t K_{t-1})^\alpha L_1^{1-\alpha}$.

• Firms repay $R^l_t S_t$ to banks.

• After loans are repaid for the period, each project / loan officer relationship suffers an idiosyncratic ‘relationship shock’ $\omega_t$ from a uniform distribution. This is a cost to maintaining the relationship from the bank’s perspective. If $\omega \leq \tilde{\omega}$, the relationship survives, banks pay $\omega_t$ and keep the match. If $\omega > \tilde{\omega}$, banks pay a fixed penalty amount $T$ in lieu of the $\omega$ relationship cost and severe the relationship.

• Banks determine how many currently active relationships to sever (separation rate $x_t$) and how many new credit relationships to open. Accordingly, they hire loan officers at a cost of $c_v$ per hire and look for new relationships, keeping in mind the penalty to be paid if the capital adequacy ratio is not met.

• Match between unfunded projects and open credit lines occur, and the number of funded projects for $t + 1$ is determined.

• Firms sell worn out capital to the capital producing sector, where new capital is created for use in $t + 1$.

• Firms sell intermediate goods to the retail sector. Prices are determined. The monetary authority reacts to inflation by setting the nominal policy rate $R_t + \pi_{t+1}$.
• Banks repay households for their deposits \( B_{t-1} \) at real interest rate \( R_t \), and pay any penalties for deviating from the capital adequacy ratio \( \bar{\kappa} \). Aggregate bank profits are realized, dividends \( \delta^N N_t^b \) are paid out, and Bank net worth is determined.

• Period \( t \) ends.

### C.2 Non-linear Equations for Benchmark Model

**Household**

\[
C_t = W_t L^Y_t + R_t (n_{t-1} S_t - N_t^b) - (n_t S_{t+1} - N_t^b) + Div_t \tag{C.1}
\]

\[
(L^Y)_t = \lambda_t W_t \tag{C.2}
\]

\[
\lambda_t = \frac{1}{(C_t - aC_t)} - \beta E_t \frac{a}{(C_{t+1} - aC_t)} \tag{C.3}
\]

\[
1 = \beta E_t \Lambda_{t,t+1} R_t \tag{C.4}
\]

\[
E_t \Lambda_{t,t+1} = E_t \frac{\lambda_{t+1}}{\lambda_t} \tag{C.5}
\]
Bank

\[ n_{t-1}S_t = n_{t-1}B_{t-1}^b + N_t^b \]  \hspace{1cm} (C.6)

\[ S_t = Q_tK_{t-1} \]  \hspace{1cm} (C.7)

\[ x_t = \frac{\bar{\omega}_t - \bar{\omega}}{\bar{\omega} - \bar{\omega}} \]  \hspace{1cm} (C.8)

\[ \Upsilon\bar{\omega}_t = T + \frac{c_v}{q_t} \]  \hspace{1cm} (C.9)

\[ N_t^b = n_{t-1}\Pi_t^b - c_vv_t - \frac{\Psi}{2} \left[ \frac{n_{t-1}S_t}{N_t^b} - \bar{\kappa} \right]^2 \frac{N_t^b}{S_t} + (1 - \delta^n)N_{t-1}^b - \epsilon_t^n \]  \hspace{1cm} (C.10)

\[ n_{t-1}S_t = n_{t-1}B_{t-1} + N_t^b \]  \hspace{1cm} (C.11)

\[ \frac{c_v}{\bar{m}_t} \theta_t^* = \beta E_t\Lambda_{t,t+1} \left\{ \Pi_{t+1}^b - \Psi \left[ \frac{n_tS_{t+1}}{N_{t+1}^b} - \bar{\kappa} \right] + \frac{\theta_{t+1}}{\bar{m}} \int_\omega \frac{c_v}{\bar{m}} dG(\omega) \right\} \]  \hspace{1cm} (C.12)

\[ R_t^iS_t = (1 - \eta) \left[ \alpha P_t^m \frac{Y_t}{K_{t-1}} + (Q_t - \delta)\xi_t \right] K_{t-1} + \eta R_t \left( S_t - \frac{N_t^b}{n_{t-1}} \right) \]

\[ + \eta \Upsilon \int_\omega \bar{\omega} dG(\omega) + \eta \int_\omega TdG(\omega) + \eta \Psi \left[ \frac{n_{t-1}S_t}{N_t^b} - \bar{\kappa} \right] \]

\[ - \eta \theta_t \int_\omega c_v dG(\omega) \]  \hspace{1cm} (C.13)

Firm

\[ Y_t = z_t (\xi_tK_{t-1})^\alpha (L_t)^{1-\alpha} \]  \hspace{1cm} (C.14)

\[ W_t = (1 - \alpha)P_t^m \frac{Y_t}{L_t} \]  \hspace{1cm} (C.15)

\[ R_t^i = \frac{\alpha P_t^m \frac{Y_t}{K_{t-1}} + (Q_{t+1} - \delta)\xi_t}{Q_t} \]  \hspace{1cm} (C.16)
Matching

\[ n_t = x_t n_{t-1} + q_t v_t \] (C.17)
\[ u_t = 1 - x_t n_{t-1} \] (C.18)
\[ v_t = \theta_t [1 - x_t n_{t-1}] \] (C.19)
\[ q_t = \bar{m} \theta_i^{-x} \] (C.20)
\[ p_t = \bar{m} \theta_1^{-x} \] (C.21)

Capital

\[ I^n_t \equiv I_t - \delta \xi_t K^Y_{t-1} \] (C.22)
\[ K^Y_t = \xi_t K^Y_{t-1} + I^n_t \] (C.23)
\[ Q_t = 1 + f(\cdot) + \frac{I^n_t + \bar{I}}{I^n_{t-1} + \bar{I}} f'(\cdot) - E_t \beta \Lambda_{t+1} \left( \frac{I^n_{t+1} + \bar{I}}{I^n_t + \bar{I}} \right)^2 f'(\cdot) \] (C.24)

Model Closing

\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \hat{P}_t^n \] (C.25)
\[ \hat{i}_t = \hat{R}_t + E_t \hat{\pi}_{t+1} = \theta \hat{\pi}_t + \theta \hat{Y}_t^Y + \epsilon_t^m \] (C.26)
\[ Y^Y_t = C_t + I_t + f \left( \frac{I^n_{t+\tau} + \bar{I}}{I^n_{t+\tau-1} + \bar{I}} \right) (I^n_{t+\tau} + \bar{I}) + c_v v_t + n_{t-1} T \int_0^\omega \omega dG(\omega) + n_{t-1} T dG(\omega) \] (C.27)
C.3 Log-linearized Equations for Benchmark Model

Household

\[
\frac{C}{Y} = \frac{WLY}{Y} \left( \hat{W}_t + \hat{n}_{t-1} + \hat{L}_t \right) + R \left[ \frac{n}{Y} - \frac{N^b}{Y} \right] \hat{R}_t
\]
\[+ n \frac{S}{Y} \left( \hat{n}_{t-1} + \hat{S}_t \right) - R \frac{N^b}{Y} \hat{N}_t - n \frac{S}{Y} \left( \hat{n}_t + \hat{S}_{t+1} \right)
\]
\[+ \frac{N^b}{Y} \hat{N}_{t+1} + \delta_n \frac{N^b}{Y} \hat{N}_{t-1} \quad (C.28)
\]
\[
\varphi(\hat{n}_{t-1} + \hat{L}_t) = \hat{W}_t - \hat{C}_t \quad (C.29)
\]
\[
\hat{C}_{t+1} - \hat{C}_t = \hat{R}_t \quad (C.30)
\]
\[
\hat{N}_t = \hat{C}_t - \hat{C}_{t+1} \quad (C.31)
\]

Firm

\[
\hat{Y}_t = \hat{z}_t + \alpha(\hat{\xi}_t + \hat{K}_{t-1}) + (1 - \alpha) \hat{L}_t \quad (C.32)
\]
\[
\hat{W}_t = \hat{P}_t^m + \hat{Y}_t - \hat{L}_t \quad (C.33)
\]
\begin{align*}
\dot{x}_t &= \frac{\hat{\omega}}{x} \left[ \frac{1}{\hat{\omega} - \omega} \right] \hat{x}_t \tag{C.34} \\
x \frac{Y}{Y} \dot{x}_t &= \frac{\chi}{q} \frac{c_v}{Y} \hat{\theta}_t \tag{C.35} \\
\dot{n}_t &= x(1-p)(\dot{x}_t + \hat{n}_{t-1}) + (1 - \chi) \frac{p(1-xn)}{n} \hat{\theta}_t \tag{C.36} \\
\dot{S}_t &= \dot{\theta} t + \dot{K}_t \tag{C.37} \\
\frac{N^b}{Y} \hat{N}^b_t &= n R^l \frac{S}{Y} \hat{R}^l_t + R \left[ \frac{N^b}{Y} - n \frac{S}{Y} \right] \hat{R}_t + n x \left[ \frac{T}{Y} + \theta \frac{c_v}{Y} - \frac{Y}{Y} \hat{\omega} \right] \hat{x}_t + n \left[ (R^l - R) \frac{S}{Y} - (1 - x) \frac{T}{Y} + x \theta \frac{c_v}{Y} \right] \hat{n}_{t-1} + (1 - \delta^n) \frac{N^b}{Y} \hat{N}^b_{t-1} - \epsilon^n \tag{C.38} \\
\frac{\chi}{q} \frac{c_v}{Y} \hat{\theta}_t &= \frac{1}{q} \frac{c_v}{Y} \hat{\Lambda}_t + \beta R^l \frac{S}{Y} \hat{R}^l_{t+1} + \beta R \left[ \frac{1}{n} \frac{S}{Y} - \frac{S}{Y} \right] \hat{R}_{t+1} + \beta \left[ \frac{S}{Y} \left( R^l - R - n \frac{\Psi}{N^b} \right) \hat{S}_{t+1} + \beta x \left[ \frac{T}{Y} + \frac{c_v}{Y} \frac{1}{q} - \frac{Y}{Y} \hat{\omega} \right] \hat{x}_{t+1} + \beta \left[ n \frac{S}{Y} - n \frac{\Psi}{N^b} \right] \hat{n}_{t-1} + \beta \frac{S}{Y} \hat{\kappa}_{t+1} + \beta x \frac{c_v}{q} \hat{\theta}_{t+1} \tag{C.39} \\
R^l \frac{S}{Y} \left( \hat{R}^l_t + \hat{S}_t \right) &= (1 - \eta) \alpha P^m \left( \hat{P}^m_t + \hat{Y}_t \right) + (1 - \eta)(1 - \delta) \frac{K}{Y} \left( \hat{K}_{t-1} + \hat{\xi}_t \right) + (1 - \eta) \frac{K}{Y} \hat{Q}_t + \eta \left[ \frac{R}{n} \frac{\Psi}{N^b} + n \frac{\Psi}{Y} \frac{S}{N^b} \right] \hat{n}_{t-1} + \eta \left[ R + n \frac{\Psi}{N^b} \right] \frac{S}{Y} \hat{S}_t - \eta \left[ \frac{R}{n} + n \frac{\Psi}{N^b} \frac{S}{N^b} \right] \frac{N^b}{Y} \hat{N}^b_t + \eta R \left[ \frac{S}{Y} - \frac{1}{n} \frac{N^b}{Y} \right] \hat{R}_t - \eta x \left[ \frac{T}{Y} + \theta \frac{c_v}{Y} - \frac{Y}{Y} \hat{\omega} \right] \hat{x}_t - \eta x \theta \frac{c_v}{Y} \hat{\theta}_t \tag{C.40} \end{align*}
\begin{align*}
\hat{Q}_t &= f''(\cdot) \left\{ (1 + \beta)\bar{I}_t^n - \bar{I}_{t-1}^n - \beta \bar{I}_{t+1}^n \right\} \quad (C.41) \\
\bar{I}_t^n &= \delta \frac{K}{Y} \left( \hat{I}_t - \hat{\xi}_t - \hat{K}_{t-1} \right) \quad (C.42) \\
\hat{K}_t &= (1 - \delta)(\hat{K}_{t-1} + \hat{\xi}_t) + \delta \hat{I}_t \quad (C.43)
\end{align*}

Model Closing

\begin{align*}
\hat{\pi}_t &= \beta \hat{\pi}_{t+1} + \frac{(1 - \beta \theta)(1 - \theta)}{\theta} \hat{P}_t^m \quad (C.44) \\
\hat{R}_t + \hat{\pi}_{t+1} &= \theta_\pi \hat{\pi}_t + \theta_Y \hat{Y}_t + \epsilon_t^\pi \quad (C.45) \\
\hat{Y}_t &= \frac{C}{Y} \hat{C}_t + \delta \frac{K}{Y} \hat{I}_t \quad (C.46) \\
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \epsilon_t^z \quad (C.47) \\
\hat{\xi}_t &= \rho_\xi \hat{\xi}_{t-1} + \epsilon_t^\xi \quad (C.48)
\end{align*}

POS and NEG:

\begin{align*}
P\hat{OS}_t &= \hat{m}_{t-1} + \hat{S}_t - \hat{\eta}_{t-2} - \hat{S}_{t-1} \quad (C.49) \\
N\hat{EG}_t &= \hat{x}_{t-1} + \hat{S}_t - \hat{S}_{t-1} \quad (C.50) \\
N\hat{ET}_t &= \frac{m}{n} P\hat{OS}_t + x N\hat{EG}_t \quad (C.51)
\end{align*}