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LIMIT STATES DESIGN OF MASONRY:
AN ASSESSMENT OF PROPOSED NEW CANADIAN BUILDING CODE REQUIREMENTS

By
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Bachelor of Applied Science
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1954

A Thesis submitted to the Faculty of Graduate Studies in partial fulfilment of the requirements for the Degree of Master of Engineering
Department of Civil Engineering
Carleton University
Ottawa, 1983
The undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis

LIMIT STATES DESIGN OF MASONRY: AN ASSESSMENT OF PROPOSED NEW CANADIAN BUILDING CODE REQUIREMENTS

submitted by V. Charles Fenton, B.A.Sc., P.Eng.

in partial fulfilment of the requirements for the degree of Master of Engineering

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December 13, 1983
This study investigates a number of methods that have been proposed for masonry strength prediction for use in design.

A model section capable of describing in a simple manner the geometry for any hollow, partially grouted or solid masonry element with or without reinforcement was developed.

Resistance expressions based on this model section were derived for use in the computer programs and design problems.

The background theory for the moment magnifier method confirming its applicability to beam columns with transverse loads was found in the literature and the pertinent clauses extracted and included for information.

The methods were applied to a number of tested walls to confirm the validity of the derivations and consistency of the methods.

A number of typical structural masonry design problems were done to assess the relative ease of applicability of the methods and to clarify the potential of the methods for design aids.

A number of computer programs were written to perform the calculations using the various methods and providing numeric or graphic output of the iterative procedures. These were written in HP Basic for a Hewlett-Packard 85A desktop portable computer. Listings have been included in the appendix to assist in the development of future design aids and programs.

A wall test series has been suggested to fill an apparent gap in the available data.
ACKNOWLEDGEMENTS

The author would like to thank his research director, Professor Gary Suter, for helping to develop the concept and form of this paper and for the guidance provided during the work.

The work done by Professor Carl Turkenstra and his Task Group is very much appreciated and it is hoped that this paper will assist him in this important project.

The author would especially like to thank his longtime friend and partner Professor John Adjeleian for the encouragement to begin this long but extremely rewarding project.
# CONTENTS

Abstract .................................................. i  
Acknowledgements ......................................... ii  
Table of Contents ........................................ iii  
List of Figures ........................................... v  
Notation .................................................... vi  

1. Introduction ............................................ 1  
   1.1 Background ........................................... 1  
   1.2 Objective and Scope .................................. 2  
2. Limit States Design ..................................... 3  
   2.1 Definitions .......................................... 3  
   2.1.1 Limit States ....................................... 3  
   2.1.2 Serviceability Limit States ....................... 3  
   2.1.3 Ultimate Limit States ............................ 3  
   2.2 Ultimate Limit States Expressions ................. 3  
   2.3 Loads ................................................ 4  
   2.4 Effects .............................................. 4  
   2.5 Load Factors ........................................ 4  
   2.6 Section Resistance ................................... 4  
   2.7 Resistance Factors ................................... 5  
   2.8 Short Section Resistance Interaction Diagram .... 5  
3. Short Section Resistance Criteria ..................... 6  
   3.1 Strength Assumptions ................................ 6  
   3.1.1 Model Section ..................................... 7  
   3.2 Uncracked Section Resistance ....................... 8  
   3.3 Cracked Section Resistance ......................... 8  
   3.3.1 Plain Masonry ..................................... 8  
   3.3.2 Reinforced Masonry ................................ 8  
   3.3.2.1 Underreinforced Sections ...................... 9  
   3.3.2.2 Balanced Sections ................................ 9  
   3.3.2.3 Overreinforced Sections ....................... 11  
4. Slenderness Effects ................................... 12  
   4.1 Introduction ......................................... 12  
   4.2 Moment Magnifier Method, Background Theory ...... 13  
   4.2.1 Concept and Expressions for Proposed Method  18  
   4.2.2 Proposed Design Procedure ....................... 20  
   4.2.3 Modified Design Procedure ....................... 21  
   4.3 P-σ Method ........................................... 22  
   4.3.1 Concept and Expressions .......................... 22  
   4.3.2 Derivations and Sequence for Exact P-σ Method .. 23  
   4.4 Approximate P-σ Methods ............................ 26  
   4.4.1 Modified P-σ Method .............................. 26  
   4.4.2 Quadratic P-σ Method ............................. 26  
   4.4.2.1 Derivations and Expressions, Quadratic Method .. 26  
   4.4.2.2 Proposed Design Sequence ....................... 29
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>Application to Test Data</td>
<td>31</td>
</tr>
<tr>
<td>5.1</td>
<td>Summary of Methods</td>
<td>31</td>
</tr>
<tr>
<td>5.2</td>
<td>Test Series Analysed</td>
<td>31</td>
</tr>
<tr>
<td>5.3</td>
<td>Graphic Tabulation of Results</td>
<td>32</td>
</tr>
<tr>
<td>5.4</td>
<td>Discussion of Results</td>
<td>32</td>
</tr>
<tr>
<td>6.</td>
<td>Design Examples</td>
<td>40</td>
</tr>
<tr>
<td>6.1</td>
<td>Design Example 1</td>
<td>41</td>
</tr>
<tr>
<td>6.2</td>
<td>Design Example 2</td>
<td>50</td>
</tr>
<tr>
<td>6.3</td>
<td>Design Example 3</td>
<td>54</td>
</tr>
<tr>
<td>7.</td>
<td>Comments on Methods</td>
<td>61</td>
</tr>
<tr>
<td>7.1</td>
<td>General</td>
<td>61</td>
</tr>
<tr>
<td>7.2</td>
<td>Exact P-6 Method</td>
<td>61</td>
</tr>
<tr>
<td>7.3</td>
<td>Modified P-6 Method</td>
<td>61</td>
</tr>
<tr>
<td>7.4</td>
<td>Quadratic P-6 Method</td>
<td>61</td>
</tr>
<tr>
<td>7.5</td>
<td>Moment Magnifier Method</td>
<td>62</td>
</tr>
<tr>
<td>8.</td>
<td>Potential for Design Aids</td>
<td>62</td>
</tr>
<tr>
<td>8.1</td>
<td>Design Aids</td>
<td>63</td>
</tr>
<tr>
<td>9.</td>
<td>Conclusions</td>
<td>63</td>
</tr>
<tr>
<td>10.</td>
<td>Recommendations</td>
<td>64</td>
</tr>
<tr>
<td>11.</td>
<td>Bibliography</td>
<td>65</td>
</tr>
<tr>
<td>Appendix A</td>
<td>Test Series Output Tables</td>
<td>67</td>
</tr>
<tr>
<td>Appendix B</td>
<td>Computer Listings</td>
<td>71</td>
</tr>
<tr>
<td>Appendix C</td>
<td>Derivation of Effective Inertia</td>
<td>87</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Section resistance parameters</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Uncracked resistance</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Short section resistance diagram</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>Model section</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>Section forces</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>Underreinforced section forces</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>Balanced section forces</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>Cracked section geometry</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>Cracked section forces</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>Overreinforced section strains</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>P-E Effect</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>Loads and effects</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>Beam column with transverse load</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>Beam column with end moment</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>Beam column</td>
<td>17</td>
</tr>
<tr>
<td>16</td>
<td>Primary loads and effects</td>
<td>19</td>
</tr>
<tr>
<td>17</td>
<td>Section resistance diagram and magnified e line</td>
<td>20</td>
</tr>
<tr>
<td>18</td>
<td>Initialization of P and M</td>
<td>20</td>
</tr>
<tr>
<td>19</td>
<td>Resistance diagram and magnified e line, modified procedure</td>
<td>21</td>
</tr>
<tr>
<td>20</td>
<td>Convergence of tertiary displacements</td>
<td>22</td>
</tr>
<tr>
<td>21</td>
<td>10-segment deflection analysis</td>
<td>23</td>
</tr>
<tr>
<td>22</td>
<td>Initialize P</td>
<td>23</td>
</tr>
<tr>
<td>23</td>
<td>Tertiary effects</td>
<td>24</td>
</tr>
<tr>
<td>24</td>
<td>P-E iteration</td>
<td>25</td>
</tr>
<tr>
<td>25</td>
<td>Primary moments at mid height</td>
<td>26</td>
</tr>
<tr>
<td>26</td>
<td>Derivation of $f_{el}$ and $f_{e2}$</td>
<td>27</td>
</tr>
<tr>
<td>27</td>
<td>Interpolation of Mr for cracked transformed section</td>
<td>28</td>
</tr>
</tbody>
</table>

Test Series Graphic Tabulation of Results

Ptest / Pirtheory

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>S.C.P.I. Plain brick</td>
<td>34</td>
</tr>
<tr>
<td>29</td>
<td>Yokel Mathey and Dikkers Series 2</td>
<td>35</td>
</tr>
<tr>
<td>30</td>
<td>Hatzinikolas et al E1 to L4</td>
<td>36</td>
</tr>
<tr>
<td>31</td>
<td>Hatzinikolas et al L5 to K1</td>
<td>39</td>
</tr>
<tr>
<td>32</td>
<td>Hatzinikolas et al S1 to N1</td>
<td>38</td>
</tr>
<tr>
<td>33</td>
<td>Slender wall program</td>
<td>39</td>
</tr>
</tbody>
</table>

34 to 52 Design examples | 41 to 60
NOTATION

A, An 
Masonry net mortar bedded and grouted area
a
Depth of compression stress block
As, As1, As2
Reinforcing steel areas
Aseq, A1, A2, A3
Areas under M/EI diagram
-C For virtual work deflection analysis
C
Resultant compressive force
C1, C2, C3
Component compressive forces
Cm
Equivalent bending coefficient
for moment magnifier method
d
Depth from compressive face to centroid of tension steel
Dp, Ds, D
Flexural displacements
jd
Moment arm of moment resisting couple Ts&G
kd
Depth of compressive stress block (same as "a")
e, E, E2
Virtual eccentricity at a section M/P
e1, e2
Eccentricities of loading at wall ends
Em, Es
Masonry and steel moduli of elasticity
Em, Es
Masonry and steel strains
f'm
Factored masonry strength \( f'm \)
Fm
Factored masonry yield stress \( f'y \)
FY
Steel yield stress
Fs
Factored steel yield stress \( f's \)
H, h
Wall effective height between lateral supports
I, Io
Effective moment of inertia in
I1
Moment Magnifier Method
I1, I2
Cracked or uncracked moments of inertia
at wall ends 1 and 2 - the degree of
cracking depending on e1 and e2
Ieff
Effective moment of inertia of a wall
Icr
Cracked moment of inertia of a wall when
P=0 and MzMracking (transformed section)
L, L1, L2
Lengths of model section
Mp
Total primary moment at a section
M, M1, M2
Bending moments at section, end 1, end 2
M1
Moment capacity selected for initial trial
Mw
Moment due to lateral loading (wind)
M
The ordinate of the unit load moment diagram
at the centroid of the M/EI diagram - used
for virtual work deflection analysis
P
Axial load at section
P1, P2
Short section strength at ends 1 and 2
with eccentricities e1 and e2
Po
Crushing strength of short section with e=0
P1
Initial axial load selected for trial
Pcr
Euler buckling load
Pmax
Theoretical maximum load capacity
NOTATION (con't.)

S  Axial load in moment magnifier, theory derivations
T  Wall thickness
T1 Wall face shell thickness
Ts Steel tensile force
V Transverse weak or strong axis horizontal shear
1. INTRODUCTION

1.1 BACKGROUND

Limit states design has, in the last two decades, come to be considered as the best modern design philosophy. It provides nearly uniform safety indices over a wide range of loading conditions and applications. The basic concepts are simple and logical and are easily understood and applied by designers and code authorities.

Masonry design, which began thousands of years ago, has accumulated a legacy of semi-empirical design "rules of thumb" that in many cases defy logic and are therefore difficult to extrapolate for design of the many special cases which are commonplace in modern construction.

Much research and testing of masonry walls and other elements has been carried out over a long period of time, but most of the work was carried out in an uncoordinated manner with much duplication of effort and poorly recorded data.

Masonry is, by nature, a complex arrangement of elements partially interconnected by a variety of cementitious materials, metal ties, and anchors and involving many people with various skills, attitudes and shortcomings.

For these reasons, limit states design of masonry has lagged behind the other more easily controlled and manipulated materials structural steel and reinforced concrete which have enjoyed limit states design methods for many years.

C.S.A. Technical Committee S304 has established a task group headed by Professor Carl Turkstra of McGill University. This group has undertaken the following task leading to the creation of a Canadian Masonry Limit States Design Code.

(a) Accumulation of available test data and extraction of useful information.
(b) Organizing new test programs to fill gaps in the data
(c) Carrying out safety index analyses of existing code provisions and requirements
(d) Deriving new design philosophies, expressions and material performance factors reflecting workmanship, inspection and quality control parameters in plant and field
(e) Carrying out safety index analyses of the proposed new methods and "tuning" them to achieve an acceptable balance between economy and safety

This work has been largely completed and at the present time the second draft of the new standard C.S.A. S304-1 1985 "Limit States Design of Masonry" has just been issued for comment by committee members.
As the writer has had experience in code writing and in design, it was generally agreed that a valuable contribution at this formative stage of the new masonry code writing process would be an in-depth designer's assessment of the proposed methods and requirements, with particular emphasis on comprehensibility and ease of applicability in the design office.

Accordingly this paper will be concerned with:
(a) An assessment of the philosophies of the two proposed methods: the moment magnifier method and the load-deflection (P-E) methods.
(b) An assessment of the proposed design expressions, derivations and limited-case simplifications.
(c) Application of the methods to specific tested cases and assessment of the relation between predicted and tested strengths.
(d) Application of the methods to typical office design problems with exploration of potential for design aids and assessment of ease of comprehension and application.
(e) Recommendations for the selection of methods and expressions based on the foregoing assessments.
2. LIMIT STATES DESIGN

2.1 DEFINITIONS

2.1.1 LIMIT STATES are the conditions or events that produce impairment of use or performance of a structure or part of a structure. They are usually classified under the following two headings:

2.1.2 SERVICEABILITY LIMIT STATES are those states that affect normal use and occupancy and include deflection, vibration, appearance, weathering, corrosion and similar long term effects.

2.1.3 ULTIMATE LIMIT STATES are those that affect safety, are related to strength and stability and include overturning, crushing, yielding, splitting, buckling and other effects of overloading and overstressing the members and materials in the structure. Since building codes and authorities are primarily concerned with structural safety, this paper will deal with the ULTIMATE LIMIT STATES.

2.2 ULTIMATE LIMIT STATE EXPRESSIONS

\[ R \geq (P+M) + V \]  

This expression states the requirement of the Ultimate Limit State that the resistance \( R \) of a simple masonry wall element shall not be less than the applied loads and effects \( P, M \) and \( V \) defined as follows:

2.3 LOADS

2.3.1 VERTICAL IN-PLANE LOADS & EFFECTS

\[ P = F_d D + F_c (F_L L + F_q Q) \]  

where:

\( P \) is the total factored applied vertical in-plane (axial) compressive (or tensile) load on the element
\( D \) is gravity dead load
\( L \) is gravity live load
\( Q \) is a shearwall flange load effect caused by strong axis (in plane) shear and bending, usually due to lateral wind, earthquake or soil pressure loads transferred to the wall by floor or roof diaphragm
\( F_d, F_c, F_L \) and \( F_q \) are load and combination factors (see 2.5)
2.3.2 HORIZONTAL OUT-OF-PLANE LOADS

\[ W = F_L W_L \]

Where:

- \( W \) is the total applied horizontal out-of-plane uniform load on the element
- \( W_L \) is the specified load
- \( F_L \) is the specified load factor

2.4 MOMENT EFFECTS

2.4.1 \[ M = P_e + M_w + P_S \]

where:

- \( M \) is the total weak axis bending at a section along the span of the element
- \( P_e \) is the weak axis bending at the section induced by \( P_e1 \) & \( P_e2 \), the end moments due to load eccentricities top & bottom of the wall element
- \( M_w \) is the weak axis moment induced at the section by the out-of-plane load
- \( P_S \) is the weak-axis moment induced at the section by the curved path of the in-plane axial loads following the wall centroid which has been deflected out of plane by the applied weak axis moments

2.4.2 \( V \) is the in-plane (strong axis) or out-of-plane (weak axis) factored shear force applied at the section, usually caused by lateral loads.

2.5 LOAD FACTORS

The applied loads \( D, L, Q \) and \( W \) are specified by codes and are factored up (or down) to reduce to an acceptably low level the probability that a more severe load condition will occur during the life of the structure. Factors \( F_d, F_L, F_q \) and \( F_w \) are called load-factors, \( F_c \) is called a combination factor and the design loads which are called Factored Loads are obtained by multiplying the specified loads by the load factors and combination factors as expressed in equation (2).

\[ \text{Factored Loads} = \Phi \text{Specified Loads} \times \text{Load} \times \text{Combination Factors (if any)} \]

2.6 SECTION RESISTANCE

\( R \) is the section resistance or strength which is a function of the following parameters (illustrated in Figure (1) and Figure (2))

(a) the factored material strengths \( \Phi_{mf} \) and \( \Phi_{sfy} \)
(b) the section geometry
(c) the distribution of stress (eccentricity of loading)
(d) and the degree of cracking (which alters the geometry)

Figure (1) SECTION RESISTANCE PARAMETERS

2.7 RESISTANCE FACTORS

The material strengths $f'\text{m}$ and $F_y$ are factored down by factors $\phi\text{m}$ and $\phi_s$ to account for variation in material strength, workmanship, tolerance, etc.

2.8 SHORT SECTION RESISTANCE INTERACTION DIAGRAM:
(RESISTANCE DIAGRAM)

The strength or resistance of a given section for various eccentricities of loading is usually expressed as an "interaction diagram" (Figure (3)) which is obtained as follows:

(a) The upper portion of the diagram represents the wall capacity as the load eccentricity travels from the centroid to the kern. When the load is at the centroid the compressive stress is uniform across the section, the moment is zero and $P=\phi f'\text{m}An=Po$ where An is the net area of the section. When the load is at the kern, the stress at the face on the other side of the centroid is zero and the stress at the compression face is $\phi f'\text{m}$ therefore $P=Po/2$. These stress relationships are shown in Figure (2).
Figure (2) UNCRACKED RESISTANCE

This portion of the curve can be drawn straight line from 0, P to PoEk/2, Po/2 where Ek is the kern. eccentricity.

(b) The lower half of the curve can be obtained by stepping the neutral axis across the section from the tension face toward the compression face, calculating the values of P&M at each step. If the section is plain, the curve will pass through the origin. If the section is reinforced, the curve will pass through M0, 0 where M0 is the pure moment capacity of the section. The moment capacity may be governed by compression in the masonry if over-reinforced or by tension in the steel if under-reinforced.

Figure (3) SHORT SECTION RESISTANCE DIAGRAM

3. SHORT SECTION RESISTANCE CRITERIA

Ojiñaga and Turkstra (1979) studied the effects of linear and non-linear section stress-strain assumptions and concluded that a linear assumption with triangular stress block and ultimate masonry strain between .001 and .002 were conservative and appropriate until better information becomes available.
3.1 STRENGTH ASSUMPTIONS
(a) Modulus of Elasticity
\[ E_m = 1000 f'm < 20,000 \text{MPa} \text{ for concrete block masonry} \]
\[ E_m = 850 f'm < 20,000 \text{MPa} \text{ for brick masonry} \]
\[ E_{\text{steel}} = 200,000 \text{MPa} \]

(b) Distribution of Strain across Section: Linear

(c) Ultimate Strain: 0.002

(d) Tensile Strength in Masonry: Zero

(e) Ultimate Compressive Stress in Masonry: \( f'm > 0.002 E_m \)

(f) Steel Compressive Stress: Zero

(g) Ultimate Steel Tensile Stress: \( f_{sFy} \)

(h) Reinforced Cracked Section Resistance: based on Transformed Section

3.1.1. MODEL SECTION

Figure (4) shows the geometry and notation which has been adopted for describing a masonry wall section.

![Model Section Diagram](image)

**Figure (4) MODEL SECTION**

In this Figure, \( L_1 \) is the sum of the thicknesses of mortar bedded webs and grouted cores.

The section forces are illustrated in Figure (5).

![Section Forces Diagram](image)

**Figure (5) SECTION FORCES**
3.2 UNCRACKED SECTION RESISTANCE

As the eccentricity of loading moves from the section centroid toward the kern eccentricity the interaction line is straight between points $0, Po$ & $PoEk/2, Po/2$. This relationship can be expressed as follows:

$$Mr=(Po-P)Ek$$

where $Ek=S/A$ and $S=2I/T$

3.3 CRACKED SECTION RESISTANCE

As the eccentricity of loading moves past the kern point, the section begins to crack but the section compressive capacity initially decreases more slowly than the eccentricity is increasing and the moment capacity, which is the product of these two, increases until $E=0.25T$ in a plain solid section and at this point of maximum moment $P/Po=4$. However, for simplicity this point will be taken to occur at $Po/2$.

3.3.1 PLAIN MASONRY

As the eccentricity of load moves from the kern toward the compression face, the moment capacity and load capacity both decrease and the curve finally passes through the origin. Reference (5) recommends the following expression for plain masonry with $P/Po<.5$.

$$Mr=(1-gP/Po)PT/2$$

in which $g=2(1-4I/AT^2)$

3.3.2 REINFORCED MASONRY

A reinforced section behaves like a plain section (because compression in the steel is ignored) until the flexural cracking extends past the first layer of reinforcement causing it to exert a new tensile force. This slows the cracking process, steepening the slope of the interaction curve which eventually passes through the point $Mo,0$ in which $Mo$ is the pure moment (no axial load) capacity of the section. This portion of the curve can conservatively be taken as a straight line between points $PoEk/2, Po/2$ and $Mo,0$.

$Mo$, the section pure moment capacity is governed by the tensile capacity of the steel in underreinforced sections and by the compressive capacity of the masonry in overreinforced sections.
3.3.2.1 UNDERREINFORCED SECTIONS

In these sections the tensile reinforcement has reached yield stress ($\Re_f \sigma_y$) and elongates until the neutral axis has moved to a point where the stress in the masonry reaches the ultimate strength $f_m$ and a secondary compression failure occurs. At this point the moment arm $jd$ is measured from the centroid of the compression block to the centroid of the steel and the resisting moment $Mo = Ts \cdot jd$

For a rectangular section we can express this as follows: (Fig. 6)

\[
\begin{align*}
Fs &= \Re_f \sigma_y \\
Fm &= \Re_f f_m \\
n &= \frac{Es}{Em} \\
C &= Fm a L / 2 \\
Ts &= Fs As \\
C &= Ts \text{ (if } P=0) \\
\end{align*}
\]

Figure (6) UNDERREINFORCED SECTION FORCES

a = $2FsAs / Fm L$ \hspace{1cm} jd = $d - a / 3$ \hspace{1cm} $Mo = Fs As jd.$ \hspace{1cm} (6)

The depth of the compression zone "a" and the location of the resultant "C" for a hollow or flanged section can be found iteratively by increasing "a" in small increments until $C = Ts$.

3.3.2.2 BALANCED SECTIONS

A balanced section reaches yield stress in the steel and ultimate compressive stress at the compression face at the same time. Using transformed section conventions we can express this as follows:
(a) For a rectangular section: (Fig. (7))

\[ F_m = \phi m f'm = \varepsilon m E_m \quad F_s = \phi s F_y = \varepsilon s E_s \]

\[ n = \varepsilon_s / E_m \quad k = \varepsilon_m / (\varepsilon_s + \varepsilon_m) = F_m / (F_s / n + F_m) \]

\[ a = kd \quad j_d = d - a / 3 \]

\[ C = F_m a L / 2 \quad T_s = F_s A_s \]

\[ M_o = M_b a L = C j d \]

\[ T_s = C \text{ therefore } \]

\[ A_s b a l = C / F_s \quad (7) \]

**Figure (7) BALANCED SECTION FORCES**

Similarly for a flanged section (using model section); Fig. (8) and Fig. (9)

**Figure (8) CRACKED SECTION GEOMETRY**

Note that if \( kd < T_1 \) the section capacity can be calculated as a rectangular section.

\[ F_s = \phi s F_y \quad \eta = \varepsilon_s / E_m \]

\[ F_m = \phi m f' m / 0.002 E_m \]

\[ k = F_m / (F_s / n + F_m) \]

\[ A_s b a l = C / \phi s F_y = (C_1 + C_2 + C_3) / F_s \quad (8) \]

\[ C = T_s \quad \text{Figure (9) CRACKED SECTION FORCES} \]
The primary function of the balanced section is to indicate the transition point between under-reinforced and over-reinforced sections.

3.3.2.3 OVERREINFORCED SECTIONS

In these sections the steel is not at yield and the section fails when the steel force Ts equals the maximum compressive force C that the masonry can develop. Since both Ts and C are dependent on the location of the neutral axis, the problem involves the solution of a quadratic equation or an iterative approach in which the neutral axis is stepped across the section until Ts=C.

For a solid overreinforced section the solution can be derived as follows: (Fig.(10))

\[ C = Ts = \phi f' m k d L / 2 \]
\[ Ts = A_s \varepsilon_s E_s \]
\[ F_m = \phi f' m + 0.002 E_m \]
\[ A_s \varepsilon_s E_s = F_m L d E_m / 2 (E_m + \varepsilon_s) \]

Multiply both sides by \((E_m + \varepsilon_s)\)

\[ \varepsilon_s^2 (A_s \varepsilon_s) + \varepsilon_s (A_s \varepsilon_s E_m) - F_m L d E_m / 2 = 0 \]

Which is a quadratic equation the solution to which is:

\[ \varepsilon_s = \frac{E_m}{2} \left( \frac{1 + 2 F_m L d}{A_s \varepsilon_s E_m} \right) \]

\[ M_p = A_s \varepsilon_s E_s d \left( 1 - \frac{E_m}{3(E_m + \varepsilon_s)} \right) \]

For a hollow section the derivation of a rigorous solution does not seem practical and the iterative approach is more attractive.
4. SLENDERNESS EFFECTS

4.1 INTRODUCTION

If a slender wall becomes curved and is subjected to an axial load, a moment is induced at each section that is the product of the axial load $P$ and the displacement of the centroid of the section from a centroidal plane through the wall ends. This moment is called the $P-6$ effect and is usually referred to as a secondary effect and is illustrated by Figure (11).

![Figure (11) P-6 EFFECT](image)

The area under this secondary moment curve produces an additional displacement which we will refer to as a tertiary effect and which if smaller than the secondary displacement will converge in subsequent cycles to a final value.

However, if $P$ is sufficiently high, or the wall sufficiently slender, the flexural stiffness of the wall will be insufficient to limit the displacements which will increase until material failure occurs.

The lowest value of $P$ that causes this type of failure is known as $P_{cr}$, the Euler critical buckling load.

4.1.1 GENERAL EXPRESSIONS

For a masonry wall subjected to the usual types of loads and effects (shown in Fig.(12)) the ultimate limit state expressions are:

![Figure (12) LOADS AND EFFECTS](image)
\[ M_1(h-x) + M_2 x + W(Hx-x^2) + P_6 x - Mr = 0 \text{ at any point } x \]  \( \frac{h}{2} \)  \( (10) \)

at mid height:

\[ \frac{M_1 + M_2}{2} + \frac{Wh^2}{8} + P_6 \text{max} - Mr = 0 \]  \( (11) \)

at end 1: \( M_1 - Mr = 0 \)  \( (12) \)

at end 2: \( M_2 - Mr = 0 \)  \( (13) \)

where \( Mr \) is obtained from the resistance diagram at ordinate \( P \).

The slenderness effect is contained in the term \( P-\xi \) which occurs at points along the span that have been displaced from the centroidal plane through the wall ends.

Note that once the \( P-\xi \) effect has been evaluated, the section strength governs capacity and if either \( P \) or \( \xi \) are zero there is no slenderness effect.

Walls that are not supporting axial loads have no tendency to buckle and therefore have no slenderness limitations other than those required to limit deflection under lateral loading. The displacement \( \xi \) is zero at the wall ends (in most cases) so short section strength governs at these points.

It has been proposed that slenderness effects can be ignored when:

\[ \frac{h}{r} < 40 - 20 e_1/e_2 \text{ for } 0 < e_1/e_2 \leq 1 \text{ or} \]  \( (14) \)

\[ \frac{h}{r} < 40 - 40 e_1/e_1 \text{ for } -1 \leq e_1/e_2 < 0 \]  \( (15) \)

in which \( r \) is the radius of gyration of the wall section.

There has been some controversy as to the best way of including the slenderness effect, to calculate it directly (the \( P-\xi \) method), or to use the moment magnifier method. A main objection to the moment magnifier method was some uncertainty as to whether it applied to beam-columns with lateral loading. The following derivations confirm that it is applicable.

4.2 MOMENT MAGNIFIER METHOD, BACKGROUND THEORY

Timoshenko has derived the following expressions for the behaviour of beam-columns:

For a beam-column subjected to an axial load \( S \) and a transverse load \( P \) (Fig. (13))
Figure (13) BEAM-COLUMN WITH TRANSVERSE LOAD

the differential equations for the deflection curves for the two portions of the span are

\[ E I \frac{d^2 y}{dx^2} = -S y - P c x \quad \text{and} \quad \frac{L}{L} \]  

\[ E I \frac{d^2 y}{dx^2} = -S y - P(l - c)(l - x) \quad \text{(b)} \]

letting \( P^2 = \frac{S}{E I} \) and obtaining the constants of integration from the boundary conditions at the ends of the span and the slope continuity at \( x = 0 \), we obtain for the left portion of the span

\[ y = P \sin pc \sin px - \frac{P c x}{S \sin pl} \frac{L}{L} \quad \text{(c)} \]

and by differentiating this we get

\[ \frac{dy}{dx} = P \sin pc \cos px - \frac{P c}{S \sin pl} \frac{L}{L} \]

\[ \frac{d^2 y}{dx^2} = -P \sin pc \sin px \quad \text{(e)} \]

Similarly for the right portion we can derive

\[ y = P \sin p(l - c) \sin p(l - x) - P(l - c)(l - x) \frac{S \sin pl}{S \sin pl} \frac{L}{L} \]

\[ \frac{dy}{dx} = -P \sin p(l - c) \cos p(l - x) + \frac{P(l - c)}{S \sin pl} \frac{L}{L} \]

\[ \frac{d^2 y}{dx^2} = -P \sin p(l - c) \sin p(l - x) \quad \text{(h)} \]

If \( c \) is made very small, \( M_0 \) is substituted for \( P c \) and \( p c \) substituted for \( \sin pc \) in these expressions, we obtain solutions for a beam subjected to an end moment \( M_0 \) and axial load \( S \) as follows: (Fig. (14))
Figure (14) BEAM-COLUMN WITH END MOMENT

\[ y = \frac{M_0 (\sin px - x)}{S \sin pl L} \quad (i) \]

from which

\[ \frac{dy}{dx} = \frac{M_0 (P \cos px - 1)}{S \sin pl L} \quad (j) \]

An examination of expressions (c) to (j) reveals the following concepts:

(a) The deflections, slopes and curvatures are directly proportional to the transverse loads \( P \) and applied end moments \( M_0 \).

(b) The axial load \( S \) has a non-linear effect since it appears directly and indirectly (in \( p \)) in the expressions.

From this it follows that if \( S \) is held constant, superposition can be used to determine the effects of multiple transverse loads and end moments.

Using this principle we can derive for a beam-column with uniform transverse load \( q \) (many small concentrated loads) and axial load \( S \):

\[ M_{\max } = -EI\frac{d^2y}{dx^2} = EI q (1 - \cos pl/2) = q L^2/2 \left( 1 - \cos u \right) = \frac{qL^2}{8} \left( 1 - \frac{u}{L} \right)^2 \quad (k) \]

in which

\[ x = \frac{L}{2} \quad \text{and} \quad u^2 = \frac{P^2 L^2}{4} = \frac{S L^2}{4E^2I} \]

The deflection curve of a beam can be represented by a trigonometric series as follows:

\[ y = a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + \ldots \quad (l) \]

\[ = \frac{A_1}{L} \sin x + \frac{A_2}{L} \sin 2x + \frac{A_3}{L} \sin 3x + \ldots \]
The coefficients $a_1, a_2$ etc. are obtained by considering strain energy of the beam and are for a beam with a single transverse load $P$:

$$a_n = \frac{2PL^3}{EI\pi^4} \frac{1}{n^4} \frac{\sin n\pi c}{L}$$

from which

$$y = \frac{2PL^3}{EI\pi^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \frac{\sin n\pi c \sin n\pi x}{L}$$

(m)

for a beam with a transverse load $P$ and axial load $S$ we can derive

$$a_n = \frac{2PL^3}{EI\pi^4} \frac{1}{n^2(n^2SL^2)} \frac{\sin n\pi c}{L}$$

(n)

in which the term $SL^2$ is the ratio of the axial load $S$ to the critical buckling load $Scr = \frac{2EI}{L^2}$

letting $\alpha = S/Scr = \frac{SL^2}{EI\pi^2}$ we obtain

$$a_n = \frac{2PL^3}{EI\pi^4} \frac{1}{n^2(n^2-\alpha)} \frac{\sin n\pi c}{L}$$

(o)

Substituting this into equation (a) the deflection curve is:

$$y = \frac{2PL^3}{EI\pi^4} \sum_{n=1}^{\infty} \frac{1}{n^2(n^2-\alpha)} \frac{\sin n\pi c \sin n\pi x}{L}$$

(p)

It can be shown that the first term of the series provides a good approximation of the deflection, letting $n=1$ we obtain

$$y = \frac{2PL^3}{EI\pi^4} \frac{1}{(1-\alpha)} \frac{\sin \pi c \sin \pi x}{L}$$

(q)

If we compare this with equation (m), the deflection of a beam with transverse loading but no axial load, we can see that the effect of the axial load is contained in the expression $1/(1-\alpha)$ which is a deflection "magnifier". Thus we have obtained the approximate relationship

$$\delta = \delta_0/(1-\alpha)$$

(r)
in which:

$\delta$ is the total deflection at a point due to axial and transverse loads, end moments and secondary effects.

$\delta_0$ is the primary deflection at the point due to the transverse loads and end moments.

Note that the "magnifier" is applied to the deflection, not the moment, that is:

$$\frac{M_{\text{max}} = S(e + \delta)}{1 - \alpha} = S(e + \delta_0)$$

$$M_{\text{max}} = S_e \left( \frac{1}{1 - \alpha^2} \right) \tag{18}$$

as it is commonly used.

Using and expanding upon the examples given by Timoshenko, we can see the differences.

For a strut with axial load and equal end moments $S_e$ (Fig. (15))

$$p^2 = \frac{S}{EI}$$

$$pL = \sqrt{\frac{S}{EI}} L$$

$$L = H/2$$

$$S_{cr} = \frac{\pi^2 EI}{4L^2}$$

$$\delta = S/S_{cr} = 4S_1 L^2 = \frac{4}{\pi^2} (\frac{pL}{\pi})^2 = 0.405 (pL)^2$$

Figure (15) BEAM-COLUMN
\[ \phi \quad \begin{array}{cccccc}
.1 & .5 & 1.0 & 1.5 & \pi/2 \\
\alpha & .004 & .101 & .405 & .911 & 1.0 \\
\frac{1}{1-\alpha} & 1.004 & 1.112 & 1.681 & 11.235 & \infty \\
\end{array} \]

Exact $\xi$ 
\[ .005e \quad .139e \quad .851e \quad 13.136e \quad \infty \]

Approx $\xi = \frac{SeL^2}{2El} \cdot \frac{1}{1-\alpha}$ 
\[ .005e \quad .139e \quad .841e \quad 12.639e \quad \infty \]

$H = S(e+\xi)$ 
\[ 1.005Se \quad 1.139Se \quad 1.851Se \quad 14.13Se \quad \infty \]

Approx $H = Se \left( \frac{1}{1-\alpha} \right)$ 
\[ 1.004Se \quad 1.112Se \quad 1.681Se \quad 11.235Se \quad \infty \]

| Diff. $H$ & Approx. $H$ |
|---|---|
| $2.5\%$ | $10\%$ | $25.5\%$ |

Table (1) Accuracy of Moment Magnifier Approximations

From all this we can conclude that the moment magnifier method is applicable to beam-columns with lateral loads.

4.2.1 Concept and Expressions for Proposed Method

Hatzinikolas, Longworth and Warwaruk have proposed the following expressions and procedures for a moment magnifier method of design for masonry walls:

\[ M = \frac{M_p \cdot CM}{1-P/P_{cr}} \quad (19) \]

where $M_p$ is the maximum primary moment along the span.

If $w = 0$, $CH = 0.6 + 0.4H_1/H_2 \geq 0.4$ and $M_1 < M_2 \quad (20)$

If $w > 0$ then $CH = 1$

For a plain or reinforced wall bent in single curvature

\[ P_{cr} = \pi^2 E11/H_2^2 \quad (21) \]

where:

(a) For a plain wall with no tensile strength

\[ I_1 = 8(1.5 - e/t)3I_0 \quad (22) \]

where $e = \frac{(e_1 + e_2)}{2}$
(b) For a plain wall with tensile strength \( f' t \):

\[
11 = 810 (0.5 - e/t) (d/2t)^3
\]  

(23)

in which \( d = Ef' t/F_{max} \)

\( f' t \) = Tensile strength (taken as 0.25 Mpa)

\( F_{max} = 9 \left( \frac{P}{A(1-2e/t)} \right) \) and \( E = \frac{2P}{A F_{max}} \)

(c) For a reinforced wall:

\[
11 = 8 (0.5 - e/t)^3 t_0
\]

\[
11 = (0.5 - e/t) t_0 + t_0
\]

in which \( e = (e_1 + e_2)/8 \)

For a plain or reinforced wall bent in double curvature:

\[
Pcr = \frac{To E t_0}{h^2}
\]

(24)

in which \( To \) is a buckling coefficient for a stepped column. This is presented in the references as a table of values. The following algorithm was developed to provide approximate values for the computer program written to perform the calculations:

\[
To = 10 (B + A1^3 (1-B)) + 2A1^2 + B^2 + 10
\]

(25)

in which \( B = 11/t_0 \) and \( A1 = e1/(e1+e2) \)

The ultimate limit state design requirements are illustrated by the following Loads and Effects diagram (Fig. 16) and the Section Resistance diagram (Fig. 17).

---

**Figure (16) PRIMARY LOADS AND EFFECTS**
4.2.2 PROPOSED DESIGN PROCEDURE:

The method proposed for finding the point of intersection of the magnified e line and the short section resistance interaction curve was the following:

(a) Start with initial values of $P_i$ and $M_i$ obtained from the intersection of an unmagnified e line and the section resistance curve (Fig. 18)

(b) Using this value of $P$, calculate $P_{cr}$ using expressions (21) to (24)

(c) Substituting these values of $P_{cr}$ and $M$ into the expression $M = PeCM/1-P/P_{cr}$, calculate a new value of $P$.

(d) Determine the corresponding value of $M$ from the resistance curve and without changing $e$ repeat from (b) until $P_{new} = P_{old}$ with sufficient accuracy.

This method was studied and tried but finally abandoned for several reasons:

(i) The rationale was obscure.

(ii) There was no provision to include lateral loading $W$.

(iii) Cases were tried that did not converge quickly and some that did not converge at all.

(iv) The following procedure was developed that eliminated both disadvantages and could be...
2.3 MODIFIED DESIGN PROCEDURE

For given values of e1, e2 and W to find P_{max}:
(a) Obtain initial value of P at intersection of
e1 or e2 line and resistance curve.
(b) Calculate Pcr and M_{max}=M_pCM/(1-P/Pcr) where
M_p is the maximum primary moment along span.
(c) If M_{max}>M_r @ P, reduce P.
(d) Go to (b) and repeat until M_{max}≤M_r @ P.
This is illustrated by Figure (19).

![Diagram of resistance diagram and magnified e line with initial P and P reduction increments.]

**Figure (19)** RESISTANCE DIAGRAM & MAGNIFIED e LINE MODIFIED PROCEDURE

This was particularly interesting where walls exhibited instability and no intersection occurred.
Subroutine "H&WMM" in Appendix B was written to perform these calculations. Subroutine "H&WMI" also in Appendix B is a variation of the foregoing subroutine "H&WMM" that was modified to carry out the analysis of a wall with constant P, e1 and e2 and variable lateral load W. This analysis is accomplished in the following steps:

1. Calculate section moment resistance M_r at P
2. Calculate initial W=8M_r/H_e
3. Find maximum primary moment along span (M_p).
4. Calculate Pcr and M_{max}=M_pCM/(1-P/Pcr).
5. If M_{max}>M_r at P, reduce W.
6. Go to 3 and repeat until M_{max}≤M_r @ P
4.3 THE P-S METHOD

4.3.1 CONCEPT AND EXPRESSIONS

Turkstra, Ojinaga and others have developed behavioural theory and expressions for a P-S method for masonry design. This method has been proposed in various forms but the basic concepts are:

(a) The flexural stiffness of the wall is estimated, taking into account the effects of curvature, cracking and reinforcement.

(b) Displacements (\(\delta_s\)) due to primary moments using a trial value for \(P\) (or \(W\)) are calculated.

(c) The secondary (P-S) moments are added to the primary moments \(M=M_p+P\).

(d) If \(M>M_r\) at \(P\), reduce \(P\) (or \(W\)) and repeat from (b).

It has been proposed that tertiary effects - the displacements due to the P-S moment itself - can be neglected within the range of practical slenderness ratios. However, in order to assess the importance of these effects, an iterative routine has been included in the program that cycles through the deflection analysis until the tertiary displacements have converged to a very small value. (Fig. (20))

![Figure (20) CONVERGENCE OF TERTIARY DISPLACEMENTS](image)

Ojinaga and Turkstra (1977) have recommended the following expressions for estimating the effective inertia of a wall. These are based on the assumptions shown by the diagram in Appendix C.

(a) For plain masonry walls and columns

\[
I_{eff} = \frac{(I_1 + I_2)}{4} \text{ for } 0 < e_1/e_2 \leq 1
\]

(27)

\[
I_{eff} = \text{lesser of } \frac{(I_1 + I_0)}{4} \text{ or } \frac{(I_2 + I_0)}{4} \text{ for } -1 \leq e_1/e_2 \leq 0
\]

(28)

where \(I_1\) and \(I_2\) are cracked or uncracked moments of inertia (depending on \(e_1\) and \(e_2\)) at ends 1 and 2 and \(I_0\) is the uncracked moment of inertia of the wall section.
(b) For reinforced walls and columns:

\[
\ell_{eff} = \frac{(11+21c_r+12)}{4} \text{ for } 0 \leq \ell/e_2 \leq 1
\]  
(29)

\[
\ell_{eff} = \text{lesser of } \frac{(11+21c_r+10)}{4} \text{ or } \frac{(12+21c_r+10)}{4} \text{ for } -1 \leq \ell/e_2 \leq 0
\]  
(30)

in which \(c_r\) is the cracked moment of inertia of the transformed section of the wall subjected to a pure bending moment equal to the pure moment resistance \(M_0\). 11 and 12 are also calculated using a transformed section if cracked.

4.3.2 DERIVATIONS AND SEQUENCE FOR EXACT P-\(S\) METHOD

Figure (21) illustrates the method used to calculate the wall deflection curve produced by the externally induced moments.

![Diagram of a reinforced wall showing deflection analysis](image)

**Figure (21) 10-SEGMENT DEFLECTION ANALYSIS**

Subroutine ITP-\(S\) in Appendix B was written to do this analysis. The sequence of the analysis was the following:

(a) Initialize \(P\) at intersection of maximum e line and resistance curve (Fig. (22)).

![Diagram showing initialize P](image)

**Figure (22) INITIALIZE P**
(b) Calculate primary deflections $Dp(2-10)$ due to areas under primary moment curves at nine stations (10 segments).

(c) Check mid-height stability of plain walls ($A1 & A2=0$) is $Dp(\phi)+ae2)/2 \geq t/2$ ?
If so, reduce $P$ and repeat from (b).

(d) Calculate tertiary effects (secondary deflections) $Ds(2-10)$ due to area under $P\phi$ moment curve (Fig. (23)).

![Figure (23) TERTIARY EFFECTS](image)

(e) If max $Ds(2-10)$ greater than $0.5(\max Dp(2-10))$ then poor convergence, so reduce $P$ and repeat from (b).

(f) Sum displacements: First time through set total displacement $D(2-10)=Dp(2-10)+Ds(2-10)$. On additional iterations, set $D(2-10)=Dp(2-10)+Ds(2-10)$ and set $Dp(2-10)=Ds(2-10)$.

(g) If max $Ds(2-10)$ greater than limit (2mm) then repeat from (d) (if previous cycle error less then 0.1Po).

(h) Calculate final $P\phi$ moments and add to primary moments for total moments $H(2-10)$.

(i) Select absolute maximum $H$ along section and calculate $E=M/P$.

(j) If $E < \text{Kern eccentricity }K1$ then $P=PoK1/(E+K1)$ and $M=P\times E$ and go to (l).

(k) If $E > K1$ then iterate neutral axis across section to find $P$ at $E$: (Fig. (24))

$$P2=C-Ts$$
$$H=Ts(d-t/2)+C(t/2-c)$$
$$E4=M/P2$$

Iterate until $E4 = E$
Figure (24) P-E ITERATION

(L) P3=P-P2 cycle error

(m) If ABS(P3)>1Po set flag to prevent tertiary displacement calculation on next cycle.

(n) If .ABS(P3)<.03Po go to (p)

(o) If P3<0, P=P+Po/50 and go to (b)
    If P3>.15Po, P=P-Po/10 and go to (b)
    If P3<.15Po, P=P-Po/50 and go to (b)

(p) P(4)=P/1000

(q) Finished.

Subroutine "ITP-61" in Appendix B is a variation of the foregoing subroutine "ITP-6" which has been modified to accommodate the analysis of a wall with P, e1 and e2 constant and lateral load W variable. This is accomplished in the following steps:

(a) Calculate section moment resistance Mr at P=Ptest.

(b) Initialize W using W=8Mr/H^2

(c) Calculate deflections and P-σ's at stations along span.

(d) Sum primary moments and P-σ's at stations and select maximum moment M4.

(e) Cycle error M3=Mr-M4.

(f) If M3<.1Mr then finished.

(g) If M4>Mr then reduce W and repeat from (c).

(h) If M4<Mr then increase W and repeat from (c).
4.4 APPROXIMATE P-δ METHODS

As previously discussed, these methods are proposed for walls with sufficient stiffness that tertiary effects can be safely ignored. It has been suggested that this limitation could be \( H/t > 25 \).

The following two methods have been proposed in the references and expanded into design methods for this project:

4.4.1 MODIFIED P-δ METHOD

This is similar to that described for the exact analysis (subroutine "ITP-δ") but only the primary displacements are calculated. The subroutine "ITP-δ" can be inhibited from calculating tertiary effects by assigning flag 22=1 in the argument list. The station with maximum primary + secondary moment along the span governs the design.

4.4.2 QUADRATIC P-δ METHOD

This method solves a quadratic equation \( P^2 A + P B + C = 0 \) for \( P \) at mid height of the wall. \( A, B \) and \( C \) are derived to include for primary moments, secondary \( P-δ \) moment and section resistance. The wall is checked at each end and at mid height as follows:

4.4.2.1 DERIVATIONS AND EXPRESSIONS - QUADRATIC METHOD

(a) At the ends, using equations (12) and (13):

\[
\begin{align*}
\text{Pe1} - \text{Mr} &= 0 \quad (12) \\
\text{Pe2} - \text{Mr} &= 0 \quad (13)
\end{align*}
\]

(b) At mid height calculate the primary moment

Figure (25):

\[
\frac{e_1 + e_2}{2} = 0.05
\]

Figure (25) PRIMARY MOMENTS AT MID-HEIGHT
Substituting into equation (13) we can derive the general expression:

\[
\frac{P(e_1+e_2+e_1 e_2+e_1 e_2+e_1 e_2+e_1 e_2)}{2} \cdot \frac{WH^2}{8} - Mr = 0
\]  

(31)

in which the displacements \(e_1\) and \(e_2\) can be evaluated using virtual work as follows:

\[
E_16 e_1 = \frac{P e_1 H}{2} \cdot \frac{H}{12} + \frac{P e_1 H}{2} \cdot \frac{H}{8} + \frac{P e_1 H}{2} \cdot \frac{H}{6} = \frac{P e_1 H^2}{16} \quad \text{(From Fig. (26))}
\]

\[
A_1 = \frac{P e_1 H}{2} \cdot \frac{H}{4}
\]

\[
A_2 = \frac{P e_1 H}{2} \cdot \frac{H}{2}
\]

Similarly \(E_16 e_2 = \frac{P e_2 H^2}{16}\)

**Figure (26) DERIVATION OF \(e_1\) AND \(e_2\)**

Letting \(I = I_{eff}\) from equations (27) or (28) and substituting into equation (31):

\[
P \left( \frac{e_1+e_2+P(e_1+e_2)H^2+5WH^4}{2} \cdot \frac{16I_{eff}}{384E_{Ieff}} - \frac{WH^2}{8} - Mr = 0 \right)
\]

\[
P^2 \left( \frac{(e_1+e_2)H^2+P(e_1+e_2+5WH^4)}{16I_{eff}} \cdot \frac{16E_{Ieff}}{384E_{Ieff}} - \frac{WH^2}{8} - Mr = 0 \right)
\]

(32)

In order to be able to solve this equation for \(P\), \(Mr\) must be evaluated. Using previously derived expressions for:

(a) Plain and reinforced masonry, uncracked (equation (4))

\[
P \geq \frac{P_0}{2} \text{ and } E \leq E_k
\]

\[
Mr = (P_0 - P) E_k \text{ where } E_k = 2 I / A_t \text{ (Kern's eccentricity)}
\]

Substituting into (32):

\[
P^2 \left( \frac{(e_1+e_2)H^2+P(e_1+e_2+5WH^4+E_k)}{16I_{eff}} \cdot \frac{16E_{Ieff}}{384E_{Ieff}} - \frac{WH^2}{8} - PoE_k = 0 \right)
\]

(33)
(b) Plain masonry, cracked (Equation (5))

\[ P < P_0 / 2 \text{ and } E > E_k \]

\[ M_r = \frac{P t - P^2 t + 4 P^2 l_0}{2 \cdot P_0 \cdot P o A t} = \frac{P t - P^2 t + 2 P^2 E_k}{2 \cdot P_0 \cdot P_0} \]

Substituting into equation (32)

\[ P^2 \left( \frac{(e_1 + e_2) H^2 + t - 2 E_k}{16 E_{\text{eff}} P_0} \right) + P \left( \frac{5 W H^4 - t}{384 E_{\text{eff}}^2} \right) + W^2 = 0 \]  

(34)

(c) Reinforced masonry, cracked transformed section

\[ P < P_0 / 2 \text{ E > E}_k \]

Figure (27) INTERPOLATION FOR M_r, CRACKED TRANSFORMED REINFORCED SECTION

In Fig. (27) interpolate M_r between M_o and \( P o E_k / 2 \) as P goes from \( P_0 / 2 \) to Zero.

\[ M_r = M_o + P \left( \frac{5 P_0 E_k - M_o}{5 P_0} \right) \]

\[ = M_o + P \left( \frac{E_k - 2 M_o}{P_0} \right) \]  

(35)

In this case, we must also evaluate M_o.

Referring to equations (7) and (8) we can evaluate Asbal.
If As > Asbal the section is underreinforced.
If As > Asbal the section is overreinforced.

For an underreinforced section using equation (6)

\[ M_o = F_s A s j d \text{ where } F_s = 0.5 F_y \text{ and } j d \text{ is evaluated as in 3.3.2.1} \]

For an overreinforced solid section we can use equation (11)

\[ M_o = A s E_s E_s d \left( 1 - \frac{E_m}{3(E_m + E_s)} \right) \]  

(11)

28
where $E_s$ is obtained from equation (10)

As previously discussed, $M_o$ can be evaluated for a hollow or flanged section by iteration (see Fig. (24)). The neutral axis can be stepped across the section, starting at the tension face (where $E=E_k$), evaluating $T_s$ and $C$ at each step until $C=T_s$ at which point $P=0$ and the section pure moment capacity $M_o$ will have been reached. Subroutine "P-E" in Appendix B performs this procedure when flag $Z$ (in the argument list) is set $= 3$.

Substituting $M_r$ from equation (35) into equation (32) we obtain:

$$\frac{P^2(e_1+e_2)H^2 + P(e_1+e_2 + \frac{5WH^4}{384EmEff} + 2Mo-Ek)}{16EmEff} + \frac{WH^2 - Mo = 0}{8} \quad (36)$$

Equations (33), (34) and (36) provide approximate mid-height capacities for plain and reinforced masonry walls, with axial loads, end moments and applied lateral pressures.

4.4.2.2 PROPOSED DESIGN SEQUENCE

Subroutine "APP-6" in Appendix B calculates wall capacities by this method in the following steps:

(a) Calculate short section capacities $P_1$ and $P_2$ at wall ends (at eccentricities $e_1$ and $e_2$).

(b) Substitute $e_1$, $e_2$, $W$ and $H$ into equation (33) and solve for $P$.

(c) If $P > 0.5 P_o$, finished, go to (g).

(d) If $P < 0.5 P_o$, section cracked, go to (e) or (f).

(e) If plain ($A_s1=0$ & $A_s2=0$) use equation (36) and solve for $P$.

(f) If reinforced ($A_s1>0$ or $A_s2>0$) use equation (36) and solve for $P$.

(g) Wall capacity $P(5)$ is the smallest of $P_1$, $P_2$ or $P$.

Subroutine "APP-51" in Appendix B is a variation of the foregoing subroutine "APP-6" which has been modified to accommodate the analysis of a wall with $P$, $e_1$ and $e_2$ constant and $W$ variable. This is accomplished in the following steps:
(a) Calculate section moment resistance $M_r$ at $P = P_{\text{test}}$.

(b) Using the general load resistance expression equation (32), which has been rearranged as:

$$W = \frac{(M_r - P^2(e_1 + e_2)H^2 + P(e_1 + e_2))}{16E_{\text{meff}}^2}$$

$$\left( \frac{5PH^4}{384E_{\text{meff}}^2} + H^2 \right)$$

Calculate $W$
5. APPLICATION TO TEST DATA

As it is assumed that loads, material strengths and tolerance were accurately controlled in the tested walls, no load or resistance factors have been applied to the applications of the methods to test data. Tests with zero eccentricities were run with .05t single curvature eccentricity to assess the adequacy of the proposed minimum code eccentricity and to avoid problems with some of the expressions which cease to function at zero eccentricity.

5.1 SUMMARY OF METHODS

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Variable</th>
<th>P</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact P-6 method</td>
<td>ITP-5</td>
<td>ITP-41 (Z2=0)</td>
<td></td>
</tr>
<tr>
<td>Modified P-6 method</td>
<td>ITP-62</td>
<td>ITP-63 (Z2=1)</td>
<td></td>
</tr>
<tr>
<td>Quadratic P-6 method</td>
<td>APP-6</td>
<td>APP-81</td>
<td></td>
</tr>
<tr>
<td>Moment Magnifier method</td>
<td>H&amp;WMM</td>
<td>H&amp;WMM1</td>
<td></td>
</tr>
</tbody>
</table>

5.2 TEST SERIES ANALYSED.

The wall test series analysed are as follows:

- (a) S.C.P.I. Plain brick wall test series 14
- (b) Yokel Mathey & Dikkers test series #2 16
- (c) Hatzinikolas Longworth and Warwaruk test series 67
- (d) Slender walls research program test series California Structural Engineers 14

The first three series (a), (b) and (c) were wall elements loaded axially and eccentrically. The analytical routines calculated the maximum axial load \( P \) that each wall is theoretically capable of carrying and compared this with the ultimate test load \( P_{test} \). The output tables give values for \( P_{test}/P \)

The last series were slender wall elements with a small constant axial load applied eccentrically at one end and a variable transverse pressure applied uniformly to one face. The analysis routines were modified to calculate the theoretical maximum transverse pressure \( W \) that each wall is capable of resisting and compare this value with the ultimate test load \( W_{test} \).
5.3 GRAPHIC TABULATION OF RESULTS

The following six pages are the graphically tabulated results of the test series analysis. The values have been classified according to individual test parameters H/t, e1/e2 and e/t in an attempt to expose any limits of applicability for the methods. The numerical output tables which contain the wall geometry and material parameters, Ptest (or Wtest), Ptest/Ptheory (or Wtest/Wtheory) and calculated values of H/t, e1/e2 and e/t are contained in Appendix B.

5.4 DISCUSSION OF RESULTS

All methods gave fairly consistent predictions for the strength of plain and reinforced concrete block, values of Ptest/Ptheory ranging from 0.8 to 4.1 for the P-6 methods and up to 5.5 for the moment magnifier method. However, if the cases for plain walls with e/t greater than 1/3 are eliminated, the spread for the moment magnifier method reduces to .8 to 2.5 and for the P-6 methods .8 to 4.0.

The methods all gave safe prediction of strength for plain brick walls, but the scatter was large, ranging from:

For the P-6 methods
1.8 to 5.6 for h/t ≤ 23 and e/t ≤ .33
1.8 to 21.4 for h/t up to 46 and e/t up to .34

For the Moment Magnifier Method
3.8 to 7.6 for h/t ≤ 23 and e/t ≤ .33
3.8 to 19.8 and higher (no intersection) for h/t up to 46 and e/t up to .34
GRAPHIC TABULATION

OF RESULTS
METHOD SYMBOLS

- Exact P-S Method
- Modified P-S Method (Primary Reflections)
- Quadratic P-S Method
- Moment Magnifier Method

TEST SERIES: S.C.P.I. PLAIN BRICK

Figure (28)
METHOD SYMBOLS

- Exact P-S Method
- Modified P-S Method (Primary Deflections)
- Quadratic P-S Method
- Moment Magnifier Method

TEST SERIES: YOKEL MATHEY & DIKKERS SERIES 2

Figure (29)
METHOD SYMBOLS
- Exact P-6 Method
- Modified P-6 Method (Primary Deflections)
- Quadratic P-6 Method
- Moment Magnifier Method

TEST SERIES: HATZINIKOLAS et al. L5 to K1

Figure (31)
METHOD SYMBOLS

- Exact P-S Method
- Modified P-S Method (Primary Deflections)
- Quadratic P-S Method
- Moment Magnifier Method

TEST SERIES: HATZINIKOLAS et al. S1 to N1

Figure (32)
METHOD SYMBOLS

- Exact P-S Method
- Modified P-S Method (Primary Deflections)
- Quadratic P-S Method
- Moment Magnifier Method

TEST SERIES: SLENADER WALL PROGRAM

Figure (33)
DESIGN EXAMPLES
6 DESIGN EXAMPLES

6.1 DESIGN EXAMPLE 1

Warehouse bearing sidewalk

Specified gravity loads:
Roof snow load 2.0 kPa x 6 m = 12 kN/m
Roof dead load
tar & gravel 0.6
steel 0.3
dead load 0.9 x 6 5.4
Total load 2.9 kPa 17.4 kN/m

Specified wind load .7 kPa

Eccentricities:
At top zero - use hinged bearing
At bottom - partially fixed
Assume 0.05% top & bottom
(single curvature)

Trial section:
240 concrete block f'm = 15 MPa
2 - 15M vert. at 1200 AS1 = 200

Fig. (36) MODEL SECTION

Material Strength Factors:
Masonry flexure and shear $\phi_m = 0.4$
Reinforcement $\phi_s = 0.5$

Fig. (38) ACTUAL SECTION
Section Properties:

Area $A_n = (1200)(70) + (189)(170) = 116130 \text{mm}^2$

$F_{m} = \Phi f'_{m} = (0.4)(15) = 6 \text{MPa}$ \quad $\Phi f_y = (0.5)(400) = 200$

$E_m = 1000f'_{m} = 15000 \text{MPa}$

$I_g = \frac{(189)(240^3) + (2)(1011)(35^3) + (2)(1011)(35)(102.5^2)}{12} = (968.5)(10^6 \text{mm}^4)$

**Figure (39)** TRIAL SECTION RESISTANCE DATA
For wall detailed in Figures (36), (37) and (38) and 2-15M at 1200.

**Factored Load Cases (from N.B.C. Part 4, Limit States Design)**

(a) $1.25D + 1.5L$

(b) $1.25D + 0.7(1.5L + 0.5W)$

(c) $1.25D + 1.5W$

(d) $0.85D + 1.5W$
Primary Moments:

![Diagram of primary moments]

Figure (40) PRIMARY MOMENTS

At mid-height $M_p = \frac{P(e_1 + e_2) + WH^2}{2}$

Unfactored loads (per 1.2m length)

(a) L.L. $(1.2)(12) = 14.4\text{kN}/1.2\text{m}$

(b) D.L. at top of wall = $(1.2)(5.4) = 6.48\text{kN}/1.2\text{m}$

(c) D.L. at mid ht. wall = $6.48 + (1.2)(3)(7.3)/2 = 19.62\text{kN}/1.2\text{m}$

(d) D.L. at bottom wall = $6.48 + (1.2)(3)(7.3) = 32.76\text{kN}/1.2\text{m}$

(e) Wind $(.7)(1.2) = .84\text{kN/m} (.84\text{N/mm})$

(1) P-δ METHOD:

Load Case (a) $1.25D + 1.5L$ governs at wall bottom (by inspection)

Axial load = $(1.25)(32.76) + (1.5)(14.4) = 62.55$

Moment ($e = .05t = 12\text{mm}$) = $(62.55)(.012) = .75\text{kNm}$

From Fig.(39) $M_r$ at $P=62$ is 13.5 O.K.

Load Case (b) $1.25D + 1.05L + 1.05W$ governs at mid height (by inspection)

Wind load = $(1.05)(.84) = .882\text{kN/m}$

Mid Ht. axial load = $(1.25)(19.62) + (1.05)(14.4) = 39.65\text{kN}/1.2\text{m}$
Primary Moment \( M_p = \frac{(39.65)(.012+.012)+(.882)(7.3)^2}{8} \)
\[ = 6.35 \text{kN-m} \]

Mid-Height deflection:
\[ \delta_{\text{max}} = \frac{P(1+e^2)H^2 + \frac{5WH^4}{16Eml_{\text{eff}}}}{384Eml_{\text{eff}}} \]
\[ l_{\text{eff}} = \frac{11 + 21cr + 12}{4} = (2)(968.5)(10^6) + (2)(73.5)(10^6) \]
\[ = (521)(10^6) \text{mm}^4 \]
\[ \delta(e_1+e_2) = \frac{P(12+12)(7300)^2}{(16)(15000)(521)(10^6)} = 0.000152P(N) \text{ or} \frac{0.012P(\text{kN})}{(10^6)} \]
\[ \delta_W = \frac{SW(7300)^4}{(384)(15000)(521)(10^6)} = 4.73W_{\Delta} (N/\text{mm}) \]
\[ \delta_{\text{max}} = (.0102)(39.65) + (4.73)(.882) = 4.57 \text{mm} \]
\[ P-\delta = (39.65)(.00457) = 1.81 \text{kN-m} \]
\[ M_p = 6.35 \]
\[ M_{\text{total}} = 6.53 \text{kN-m} \]

From Fig.(39) \( M_r \) at \( P=39.65 \) is 11.7 > 6.53 O.K.

Load Case (c) governs at mid height

Factored wind load = \( (1.5)(.84) = 1.26 \text{kN/m} \)
Factored gravity load = \( (1.25)(19.62) = 24.5 \)

Mid Ht. deflections:
\[ \delta_W = (1.26)(4.73) = 5.96 \text{mm} \]
\[ \delta(e_1+e_2) = (.0102)(24.5) = .25 \text{mm} \]
\[ \delta_{\text{total}} = 6.2 \text{mm} \]
\[ P-\delta = (24.5)(.0026) = .15 \text{kN-m} \]

Primary Moment
\[ M_p = \frac{(24.5)(.012) + (1.26)(7.3)^2}{8} = 8.68 \text{kN-m} \]
\[ M_{\text{tot}} = M_p + P-\delta = 8.68 + 1.15 = 8.83 \]

From Fig.(39) \( M_r \) at \( P=24.5 \) > 9.5 > 8.83 O.K.
Load Case (d)

Factored gravity load \( = (0.85)(19.62) = 16.67 \) kN/m
Factored wind \( W = (1.5)(0.84) = 1.26 \) kN/m
Mid Ht deflections: \( \delta = 5.96\text{mm} \)
\[ \delta(e1+e2) = (0.0102)(16.7) = 0.17 \]
\[ M_p - \delta = (16.7)(0.00613) = 1\text{mm} \]
\[ M_p = (16.67)(0.012) + (1.26)(7.3^2) = 8.5\text{ kN.m} \]
Mtots. = \( M_p - \delta = 8.59 + 1 = 8.69\text{kN.m} \)

From Fig. (39) Hr at P = 16.7 is 8.8 > 8.69 o.k.

(2) MOMENT MAGNIFIER METHOD.

For reinforced masonry:
If mid ht. \( e < e_k \) use \( I_{eff} = (0.5 - e/t)I_o < 0.1I_o \)
If mid ht. \( e < 2e_k \) use \( I_{eff} = 8I_o(0.5 - e/t)^3 \)

Load Case (a)
Axial load = 62.55 kN/m
Mid Ht. \( M_p = 0.75\text{kN.m and e = 12mm} \)
\( e < e_k \), so use \( I_{eff} = 8(0.5 - 12/240)^3(968.5)(10^6) = 706(10^6) \)
\[ P_{cr} = \frac{\pi^2}{7300^2}(15000)(706)(10^6) = 1961\text{kN} \]
\[ M_{tot} = (0.75)(\frac{1}{1 - \frac{62}{1961}}) = 0.77 \]

From Fig. (39) Hr at P = 62 is 13.5 > 7.77 o.k.

Load Case (b)
Mid Ht. axial load 39.65
Mid Ht. \( M_p = 6.35 \) kN/m, \( e = M/P = 160\text{mm} \)
\( e < e_k \) use \( I_{eff} = (0.5 - 160)(968.5)(10^6) \) Negative Use 0.1Io

\[ \]
\[ \text{I}_{\text{eff}} = (0.1)(968.5)(10^6) = 96.85(10^6) \]

\[ \text{Pcr} = \frac{\pi^2}{7300^2}(15000)(96.85)(10^6) = 269 \]

\[ \text{M}_{\text{tot}} = 6.35 \left( \frac{1}{1-39.6} \right) = 7.44 \]

\[ \left( \frac{269}{1447} \right) \]

From Fig. (39) Mr at P=39.6 is 11.7 > 7.44 O.K.

Try Turkstra's \text{I}_{\text{eff}}:

\[ \text{Pcr} = \frac{\pi^2}{7300^2}(15000)(521)(10^6) = 1447 \text{ kN} \]

\[ \text{M}_{\text{tot}} = 6.35 \left( \frac{1}{1-39.6} \right) = 6.52 \text{ close to } 6.53 \text{ from P-\varepsilon \ method} \]

Load Case (c)

Mid ht. axial load 24.5

Mid ht. \( M_p = 8.68 \) e=354mm

\[ \text{I}_{\text{eff}} = \frac{(0.5-354)l_0}{240} \]

\[ \text{Pcr} = 269 \]

\[ \text{M}_{\text{tot}} = 8.68 \left( \frac{1}{1-24.5} \right) = 9.54 \]

\[ \left( \frac{269}{1447} \right) \]

From Fig. (39) Mr at P=24.5 is 9.5 = 9.54 Close but O.K.

Try Turkstra's \text{I}_{\text{eff}}:

\[ \text{M}_{\text{tot}} = 8.68 \left( \frac{1}{1-24.5} \right) = 8.82 \text{ Close to } 8.83 \text{ from P-\varepsilon \ method} \]

Load Case (d)

Mid ht. axial load 16.67

Mid ht. \( M_p = 8.59 \)

\[ \text{Pcr} = 269 \text{ as before} \]

\[ \text{M}_{\text{tot}} = 8.59 \left( \frac{1}{1-16.7} \right) = 9.158 \]

\[ \left( \frac{269}{1447} \right) \]
From Fig. (39) $M_r$ at $P=16.67$ is $8.8 < 9.15$

Need more steel

Try Turkstra's $I_{ef}$:

$$M_{tot} = 8.59 \left( \frac{1}{1-16.67} \right) = 8.69 = 8.69 \text{ from the}$$

$$\left( \frac{1}{1447} \right) \quad \text{P-S method}$$

(3) S304 S.C.P.I. METHOD

$F_m = 0.33 \text{f'}m = 5 \text{MPa}$

$F_s = 165 \text{ MPa max}$

Using $e_1/e_2 = 0$ and $H/t = 30 \text{ C}_s = 0.4$

Load combinations from N.B.C. Part 4 Working Stress

Load Case (a) D+L

Axial load at mid ht. = $14.4 + 32.76 = 47.2 \text{kN/1.2m}$

**Figure (41) RESISTANCE DATA FOR SECTION - Fig. (36), (37), (38), 2-15M at 1200 and $F_m = 5$**

From Fig. (41) $P_o = 581$

$$e < t/3 : P = C_e C_s P_o$$
\( \frac{e}{t} \leq 0.05 \) \( \Rightarrow \) \( e = 0.1 \)

\[ P = (0.4)(1)(581) = 232 > 47.2 \text{ O.K.} \]

**Load Case (b) \( 0.75(D+L+W) \)**

- Grav. load at mid ht. = \( 0.75(14.4+19.62) = 25.5 \text{ kN} \)
- Wind moment \( M_w = \frac{1}{8}(0.75)(0.84)(7.3^2) = 4.2 \text{ kN-m} \)

\[ e = 4.2 / 25.5 = 0.164 \text{ m} \]

- **Plot e line:** \( M = 0.164P \) (Figure (42))

At \( M = 0 \) \( P = 0 \)

At \( M = 20 \) \( P = 20 / 0.164 = 122 \)

- **From Figure (42) intersection at** \( M = 15.5 \), \( P = 100 \)
- \( PC_s = (0.4)(100) = 40 > 25.5 \text{ O.K.} \)
- \( MC_s = (0.4)(15.5) = 6.2 > 4.2 \text{ O.K.} \)
Load Case (c) D+W

Gravity load at mid ht. = 19.62 kN

\[ M_w = \left( \frac{.84}{8} \right)(7.32) = 5.6 \text{ kN} \cdot \text{m} \]

\[ e = \frac{5.6}{19.62} \approx 0.285 \text{ m} \]

Plot e line: \( M = 0.285P \) (Figure (43))

At \( M=0 \), \( P=0 \)

At \( M=20 \), \( P=70.1 \)

From Fig. (43) intersection at \( M=9 \), \( P=25 \)

Allow \( P = (0.4)(25) = 10 < 19.62 \text{ N.G.} \)

Corresponding \( M = (0.4)(9) = 3.6 < 5.6 \text{ N.G.} \)

Figure (43)
6.2 DESIGN EXAMPLE 2

Same wall as example 1 but increased specified loads.

Roof snow load 150 kN/m

Roof D.L. 50 kN/m

Wind 1 kPa

Specified loads:

L.L. (1.2)(150) = 180 kN/1.2 m

D.L. at top of wall (1.2)(50) = 60

D.L. at mid ht. wall 60 + (3)(7.3)(1.2) = 73.14

D.L. at bottom 60 + (3)(7.3)(1.2) = 86

Wind (1.2)(1) = 1.2 kN/m

(1) P-S METHOD

Load Case (a) 1.25 D+1.5L

At wall bottom axial load = (1.25)(86) + (1.5)(180)

= 377.5

Mp = (377.5)(0.012) = 4.53 kN-m

Try same section (1-15 M ea. face at 1200)

From resistance diagram, Fig. (39), Mr at P=377 is 22 > 4.53  O.K.

Load Case (b) 1.25D+1.05L+1.05W

Factored load at mid ht.:

Wind (1.05)(1.2) = 1.26 kN/m

Axial Load (1.25)(73.14) + (1.05)(180) = 280 kN/1.2 m

From previous example:

ε = (ε1 + ε2) = 0.0102 P = (0.0102)(280) = 2.86 mm

δw = 4.73 W = (4.73)(1.26) = 5.96 mm

δtot = 8.82 mm
\[ P - \delta = (280)(0.00882) = 2.47 \text{ kN} - \text{m} \]
\[ M_p = (1.26)(7.3^3) + (280)(0.012) = 11.75 \]
\[ M_{tot.} = 2.47 + 11.75 = 14.22 \]

From Fig. (39), Mr at \( P = 250 = 25.8 > 14.22 \) O.K.

\textbf{MOMENT MAGNIFIER METHOD using Turkstra's Ieff:}
\[
\frac{M_{tot.}}{1 - 280/1447} = 14.56 \text{ Close} \]

Load Case (c) 1.25D+1.5W

Factored loads at mid ht.

Wind \((1.5)(1.2) = 1.8 \text{kN/m}\)

Axial load \((1.25)(73.14) = 91.4 \text{kN/1.2m}\)

Mid Ht. deflections:
\[ \delta_{(e1+e2)} = (0.0102)(91.4) = 0.93 \text{ mm} \]
\[ \delta_w = (4.73)(1.8) = 8.5 \text{ mm} \]
\[ \delta_{tot.} = 0.93 + 8.5 = 9.4 \text{ mm} \]
\[ P - \delta = (91.4)(0.0094) = 0.86 \]
\[ M_p = (91.4)(0.012) + (1.8)(7.3^3) \times \frac{1}{8} = 13.08 \]
\[ M_{tot.} = 0.86 + 13.08 = 13.95 \text{ kN-m} \]

From Fig. (39), Mr at \( P = 91.4 \) is 16.4 > 13.95 O.K.

\textbf{MOMENT MAGNIFIER METHOD using Turkstra's Ieff:}
\[
\frac{M_{tot.}}{1 - 91.4/1447} = 13.96 \text{ Close} \]

Load Case (d) 0.85D+1.5W

Factored loads at mid Ht.: 

\( W = 1.8 \text{kN/m} \)

Axial load = \((0.85)(73.14) = 62 \text{kN/1.2m}\)

Deflections: \[ \delta_{(e1+e2)} = (0.0102)(62) = 0.63 \]
$w = 8.5$
$\delta_{tot.} = 0.63 + 8.5 = 9.1 \text{mm}$
$P - \delta = (62)(0.0091) = 0.56 \text{kN} \cdot \text{m}$
$P_{\delta} = (62)(0.012) + (1.8)(7.3^3) = 12.73$

$M_{tot.} = 0.56 + 12.73 = 13.29$

From Fig. (39), Mr at $P = 62$ is 13.5 Close but O.K.

MOMENT MAGNIFIER METHOD, using Turkstra's $I_{eff}$:

$M_{tot.} = \frac{12.7}{1-62/1447} = 13.26$ Close

(3) S304 METHOD

Load Case (a) D+L

Axial load at mid ht. $= 73 + 180 = 232$

From Fig. (41) $P_0 = 581$ (as before)

$e/t = 0.05$, $C_e = 1$

$P = C_e C_s P_0 = (0.4)(1)(581) = 232 = 232$ O.K.

Load Case (b) $75(D+L+W)$

Axial load $(0.75)(253) = 189$

$M_w = (0.75)(1.2)(7.3^3) = 5.99 \text{kN} \cdot \text{m}$

$e = 5.99/189 = 0.0317 \text{m}$

$e/t = 0.132$, $C_e = 0.77$

$P = (0.77)(0.4)(581) = 179 < 189$ N.G.

Load Case (c) D+W

Axial load 73.14 kN

$M_w = (1.2)(7.3^3) = 7.99 \text{kN} \cdot \text{m}$

$e = 7.99/73.14 = 0.109$, $e/t = 0.109/0.24 = 0.455 > 0.33$

Plot e line $M = 0.109P$ (Fig. (44))
From Fig. (44) Intersection at \( M = 20 \) and \( P = 183 \)

\[
PCs = (.4)(183) = 73.2 \approx 73.14 \quad \text{O.K.}
\]

\[
MCs = (.4)(20) = 8 = 7.99 \quad \text{O.K.}
\]
6.3 DESIGN EXAMPLE 3

Same wall as example 1 but increase wind W to 1.5 kPa
Unfactored load (per 1.2 m length, see page 43)
L.L. 14.4
D.L. at top 6.48
D.L. at mid ht. 19/62
D.L. at bottom 32.76

Wind W (1.5)(1.2)=1.8 kN/m/1.2 m width

Load Case (a) No wind, therefore same as Example 1

Load Case (b) 1.25D+1.05L+1.05W

Factored loads at mid ht.
W=(1.05)(1.8)=1.89 kN/m
Axial load (1.25)(19.62)+(1.05)(14.4)=39.64
\( \delta(e_1+e_2)=(0.0102)(39.64)=.4 \)
\( \delta w=(1.89)(4.73)=8.93 \text{mm} \)
\( \delta \text{tot.}=.4+8.93=9.33 \text{mm} \)
\[ M_p=\frac{(1.89)(7.33)+(39.64)(.012)}{8}=14.25 \]
\[ P-\delta=\frac{(39.64)(9.33)}{1000} = .37 \]
Mtot.=.37+14.25=14.62 kN-m

From Fig(39), Mr at P=39.64 is 11.7 < 14.62 N.m
Increase steel.
Try 2-20M's As1=300
INPUT L, L1, T, T1, e1, e2
1260, 189, 240, 35, 12, 12
INPUT fFm, fFv, As1, As2, Ds1
6, 200, 300, 0.120
INPUT Em, Max. Max. STRAIN
J5800, 002

F = 6.97E+005
Mn = 115130
I1 = 9.68E+008
Ieff = 5.39E+008
Icr = 1.085E+008
k = 69

\( P = \frac{6.97 \times 10^5}{Mn} \)
\( I_{eff} = \frac{(2)(968.5)(10^6)}{4} + \frac{(2)(108.5)(10^6)}{4} = \frac{(538.5)(10^6)}{4} \)

Small change won't affect P. S much this case
Check from Fig. (45) new Mr at P = 39.6 is 14.8 ≈ 14.62
Q.K.

**MOMENT MAGNIFIER:**

New \( P_{cr} = \frac{\pi^2}{7300^2} \times \frac{(15000)(538.5)(10^6)}{1496} \)

\( M_{tot.} = \frac{14.75}{1 - 39.6} = 14.63 \) Very close

**Load Case (c) 1.25D + 1.5W**

Wind \( (1.5)(1.8) = 2.7 \text{ kN m} \)

Mid ht. grav. load = \( (1.25)(19.62) = 24.5 \text{ kN} \)

\( \delta(e1+e2) = P(e1+e2)H^2 = \frac{P(12+12)(7300^2)}{16EmIeff} \times \frac{(16)(15000)(538.5)(10^6)}{1496} = 0.0099 \text{ Pmm} \)

\( \delta_w = \frac{(5)(W)(7300^4)}{(384)(15000)(538.5)(10^6)} = 4.58 \text{ Wmm} \)

\( \delta(e1+e2) = (0.0099)(24.5) = 0.24 \text{ mm} \)

\( \delta_W = (2.7)(4.58) = 12.3 \)
\[ M_p = (24.5)(0.012) + 2.7(7.3^2) = 18.27 \]

\[ M_{tot} = 18.27 + 18.57 = 36.84 \]

From Fig. (45), Mr at P=24.5 is 12.5 < 18.57 N.G.

**NEED MORE STEEL**

Try 2-20M at 800

**Figure (46)**

Resistence Data for section detailed in Figures (47) and (48), 2-20M at 800

Using new resistance data: (Fig. (46))

\[ I_{eff} = \frac{(2)(671)(10^6) + (2)(109.4)(10^6)}{4} = (390.2)(10^6) \]

\[ \delta = \frac{p(12+12)(23002)}{(16)(15000)(390.2)(10^6)(1000)} = 0.0136 \]

\[ \delta_w = \frac{5W(7300^4)}{(384)(15000)(390.2)(10^6)} = 6.317 \text{Wmm} \]
Load Case (c) (cont'd) 1.25D+1.5W

Grav. load = 5.4 + \left( 3.5 \right) \left( 7.3 \right) \left( 0.8 \right) \left( 1.25 \right) = 18.1 \text{kN/m}\cdot 0.8 \text{m}

\text{Wind} = (1.5) \left( 0.8 \right) (1.5) = 1.8 \text{kN/m}

\delta(e1 + e2) = (0.0136) (18.1) = 0.247 \text{mm}

\delta W = (6.317) (1.8) = 11.37 \text{mm}

\delta_{\text{tot.}} = 0.247 + 11.37 = 11.61 \text{mm}

P - \delta = \left( 18.1 \right) \left( 11.61 \right) = 0.21 \text{kN}\cdot \text{m}

\frac{M_p}{1000} = \left( 18.1 \right) \left( 0.012 \right) + \left( \frac{1.8}{8} \right) (7.3^2) = 12.20

M_{\text{tot.}} = 0.21 + 12.2 = 12.41 \text{kN}\cdot \text{m}

From Fig. (46), M_r at \ P = 18.1 is 11.6 < 12.41

NEED MORE STEEL

Try 2-20M's at 600

INPUT L, L1, T, T1, e1, e2

600, 189, 240, 35, 12, 12
INPUT f_{\text{Fy}}, f_{\text{fy}}, A_s1, A_s2, D_s1

6, 200, 300, 0.120
INPUT E_m, Max. Mas. STRAIN

15000, 0.002

P_o = 4.45E+085
A_n = 74138
I_1 = 5.23E+088
I_{\text{eff}} = 3.16E+088
I_{\text{cr}} = 1.691E+088

N_o = 9.969E+086
N_n = 5.23E+088
E_{\text{eff}} = 4.74E+012
E_k = 59

\text{Figure} (49)

From new resistance data: (fig. (49))

\text{I}_{\text{eff}} = (2)(523)(10^6) + (2)(109)(10^6) = (316)(10^6)
\[ \delta_w = \frac{5W(7300^4)}{(384)(15000)(316)(106)} = 7.8 \text{ Wmm} \]

**Load Case (c) (cont'd) 1.25D+1.5W**

Grav. load = \( (0.6)(5.4) + (3.72)(7.3)(0.6)(1.25) = 14.23 \text{ kN/m} \)

Wind = \( (0.6)(1.5)(1/5) = 1.35 \text{ kN/m} \)

\[ \delta(e1+e2) = (0.0168)(14.23) = 0.24 \text{ mm} \]

\[ \delta_w = (1.35)(7.8) = 10.53 \text{ mm} \]

\[ \delta_{tot} = 0.24 + 10.53 = 10.77 \]

\[ P - \delta = (14.23)(10.77) = 0.153 \]

\[ \frac{1000}{M_p} = 14.23 \]

\[ M_p = (14.23)(0.012) + (1.35)(7.3^2) = 9.16 \]

\[ M_{tot} = 0.153 + 9.16 = 9.31 \text{ kN.m} \]

From Fig. (49), Mr at P=14.23 is 11 > 9.31 O.K.

**MOMENT MAGNIFIER:**

\[ P_{cr} = \frac{\pi^2}{7300^2}(15000)(316)(106) = 877.9 \]

\[ M_{tot} = \frac{9.16}{1-14.23} = 9.31 \text{ (Right On!)} \]

\[ \frac{877.9}{14.23} \]

**Load Case (d) 0.85D+1.5W**

Grav. Load = \( (0.85)(0.6)(18.9) = 9.67 \text{ kN/m} \)

Wind = 1.35

\[ \delta(e1+e2) = (0.0168)(9.67) = 0.162 \text{ mm} \]

\[ \delta_w = 7.8 \text{ mm} \]

\[ \delta_{tot} = 0.162 + 7.8 = 7.96 \text{ mm} \]

\[ P - \delta = (9.67)(7.96) = 0.076 \]

\[ \frac{1000}{M_p} = 9.67 \]

\[ M_p = (9.67)(0.012) + (1.35)(7.3^2) = 9.1 \]
Mtq = 0.076 + 9.1 = 9.17

From Fig. (49), Mr at P=9.67 is 10.7 > 9.17 O.K.

\[
M_{\text{tot}} = \frac{9.1}{1 - 9.67} = 9.2 \text{ (Very close)}.
\]

**S304 METHOD:**

\[
F_m = 5 \quad F_s = 165(\text{max}) \quad C_s = 0.4
\]

**INPUT L, L1, T, T1, e1, e2**

- 600, 189, 240, 35, 12, 12
- INPUT \( F_m, F_s, A_{s1}, A_{s2}, D_s \)
- 5, 165, 300, 0, 1.20

**INPUT E, M, Max. Mas. STRAIN**

- 15000, 0.012

- \( P_0 = 3.71E + 0.05 \)
- \( M_0 = 8.277E + 0.06 \)
- \( A_n = 74130 \)
- \( I_1 = 5.23E + 0.08 \)
- \( I_{\text{eff}} = 3.16E + 0.08 \)
- \( I_{\text{cr}} = 1.091E + 0.09 \)
- \( e_k = 59 \)

**Figure (50)**

**Load Case (a) D+L**

**Grav. Load**

\[
\frac{(.6)(5.4)(3.72)(7.3)}{2} + 12 = 18.6 \text{ kN/m}
\]

\[
P = C_e C_s P_0 = (0.4)(371) = 148.4 > 18.6 \text{ O.K.}
\]

**Load Case (b) .75(D+L+W)**

**Mid ht. grav. load**

\[
.75(18.6) = 13.95
\]

\[
M_w = \frac{(.75)(.6)(1.5)(7.3^2)}{8} = 4.49 \text{ kN-m}
\]

\[
4.49 = \text{.32}
\]

\[
13.95
\]

Plot e line. \( M = .32P \) (Fig. (51))
From Fig. (51) intersection at $M=10.5$, $P=32.8$

$PCs=(32.8)(.4)=13.1 \approx 13.9$ Close

Load Case (c) $D+iW$

Mid Ht. grav. load = $(.6)(5.4+(3.72)(7.3)) \approx 11.38 kN$

$M_w=(.6)(1.5)(7.3^2) = 5.99 kN \cdot m$

$e = 5.99 \cdot .526m \approx \frac{11.38}{4}$

Plot e line $M=.526P$ (Fig. (51))

From Fig. (51), intersection at $M=9.7$, $P=18.4$

$PCs=(.4)(18.4)=7.37 < 11.38$ N.G.
SUMMARY

7. COMMENTS ON METHODS

7.1 GENERAL

When load factors 1.5L and 1.25D and resistance factor .4 are applied, the predicted resistances will increase by approximately 1.3/4 = 3.25 which should raise the least conservative results (.8) to around 2.5 which would seem adequately conservative to deal with most strength and loading variations.

7.2 THE EXACT P-6 METHOD

This method as would be expected gave slightly more conservative prediction of strength than the other methods, but it is questionable whether the extra computation time is justified.

7.3 THE MODIFIED P-6 METHOD

This method seems sufficiently accurate for most cases and has the capability to find the largest moment along the span for any type of curvature and loading and so should be able to deal with unusual or special cases more competently than the simpler methods.

7.4 THE QUADRATIC P-6 METHOD

This method uses considerably less computation time than either of the other P-6 methods, but has the disadvantage that it may not detect the maximum moment along a double curvature span with applied lateral loading since it checks only at ends and mid-height of span. It is also necessary to prevent the mid-height eccentricity from being zero in order to obtain reasonable results.
7.5 THE MOMENT MAGNIFIER METHOD

This method has the advantage that a simple mathematical expression provides a multiplier or "Magnifier" that is applied to the maximum moment along the span to obtain the critical design moment. However, it is very sensitive to the value of Pcr used in the calculations.

It is suspected that the conservative scatter obtained for the plain brick test series is due to an underestimate of the tensile strength f't (taken as 0.025MPa) which affects the value of Ieff used in calculating Pcr.

In the design examples, this method gave identical answers to those given by the P-S method when the same Ieff was used for both methods.

It is not clear, at this stage, how this method will cope with cases of double (or triple) curvature since the magnifier is applied to the maximum primary moment along the span, which may not be at the point where the maximum M_p+P-S moment occurs. This problem deserves more study.

8. POTENTIAL FOR DESIGN AIDS

The design problem resolves into the following steps:

(a) Define the factored loading and a trial geometry and material strengths.
   - e1, e2, w and P
   - f'm, Fy, Em and Es
   - H, L, L1, As, t and t1

(b) Calculate the section properties, the effective moment of inertia of the span Ieff and generate the resistance (interaction) curve using factored resistance parameters.

(c) Calculate the primary moments.

(d) For the P-S method, calculate the displacements (s's) and add the P-S's to the primary moments, determining the maximum total moment M_{max} along the span.
(e) For the moment magnifier method, multiply the maximum primary moment along the span by the "multiplier" CM/(1-P/Pcr) to determine Mmax.

(f) Using the resistance (interaction) curve, check if:
   (i) Mmax ≤ Mr at P then finished.
   (ii) Mmax > Mr at P then increase strength and repeat from (b)

8.1 DESIGN AIDS

Step (b) can be easily tabulated in variations of the resistance data output diagrams produced by program "Resist" for the design examples.

Step (c) is best executed by a small computer or programmable calculator that steps along the span calculating and printing the primary moments as it goes.

Step (d) could be tabulated - station displacements for unit values of M1, M2 and Mw which could be calculated by hand and the P-δ's added at each station. This could also be easily done by a small computer or programmable calculator.

Step (e) values of Pcr could be tabulated - perhaps in the same table as the resistance data (see above).

9 CONCLUSIONS

(a) The methods all seem applicable to the range of design problems that will be permitted by the proposed code.

(b) The P-δ methods have the advantage that they encourage the designer to visualize the structural action and to approach the design in a simple direct manner.

(c) The Moment Magnifier Method has the advantages of simple application and speed. However its use in special cases such as double curvature with lateral loading is not straightforward. This method gives identical results to those of the P-δ method for simple cases when the same leff is used for both methods.
(d) The methods have not been adequately tested against walls with large axial and transverse loads.

10 RECOMMENDATIONS

(a) That both the P-S and Moment Magnifier Methods be permitted by the Code.

(b) That the same leff be used for both methods.

(c) That the Moment Magnifier Method be limited to single curvature where transverse loads are applied.

(d) That the minimum design eccentricity .05t be specified single curvature for all walls including those with equal and opposite end eccentricities.

(e) That a test series be run for walls with large axial and transverse loads and single and double curvature end eccentricities.


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SHRIWE, N.G. 1981. The failure mechanism of face shell bedded (ungrouted and unreinforced) masonry subjected to compressive loading. Research Report No. CE81-3, Department of Civil Engineering, University of Calgary.


TURKSTRA, C.J. 1981. Limit states design in masonry, a status report. Printed Notes, A Short Course on Masonry for Structural Engineers, University of Calgary.


APPENDIX A

Test series output tables
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APPENDIX B

Computer Program Listings
20: INITIALIZE
30 OPTION BASE 1
40 DIM J(1), P(B, T16(200)), N(10, 16(200))
50 I0BUFFER 14, 15**
60 $1 = 2 GOSUB 330
70 BY $1 = 'ITP-6 'ITP-52 APP-6 HWWMM'
80 PRINT B$ & PRINT ' 1, 2 : B$'
90 B$ = 'NK $YPHE T H L L I 1 1 F'AS e1 e2 = PTEST P/P P/P P/P P/P P/P T16 'NK $YPHE T H L L I 1 1 F'AS e1 e2 = PTEST P/P P/P P/P P/P P/P P/P P/P P/P P/P'
100 PRINT B$ & PRINT ' 1, J : B$'
120 FOR J = 4 TO M1+3
130 FOR J2 = 1 TO 8 P(J2) = 0 & NEIT J2
140 GOSUB 330
150 CALL "COST1" (A, D4, E1, E2, E5, E6, E7, F1, F2, F3, I1, I1, I1, I1, I1, L1, L1, L2, M, P0, P1, S1, S2, T1, T1)
160 CALL "ITP-6" (D4, E1, E2, E6, F2, F3, H1, L1, L2, M, P0, P1, S1, S2, T1, T1)
180 ! RE-RUN WITHOUT TERTIARY DEFL. 'CALC'MS
190 CALL "ITP-6" (D4, E1, E2, E6, F2, F3, H1, L1, L2, M, P0, P1, S1, S2, T1, T1)
200 CALL "NWWMM" (A, D4, E1, E2, E5, E6, E7, F2, F3, F4, H1, K1, L1, L2, M, P0, P1, S1, S2, T1, T1)
230 IF E2 < .1 THEN E2 = 0
240 FOR J2 = 4 TO 8
250 IF P(J2) = 0 THEN P(J2) = 0 ELSE P(J2) = P(P(J2))
260 NEIT J2
270 PRINT USING '300 ; M4, T4, H, L1, T1, T1, F1, S2, E1, E2, M, P0, P1, P2, P3, P4, P5, P6'
280 OUTPUT 1 USING '300 ; M4, T4, H, L1, T1, T1, F1, S2, E1, E2, M, P0, P1, P2, P3, P4, P5, P6'
290 PRINT '1, J ; T1, J**'
300 IMAGE 2(AAA, 11), 3(DDDD, 11), DDDD, 11, DD, 11, DD, 1, DDDD, 21, 4(DDDD, D1)
310 NEIT J
320 GOTO 590
330 ! SBR. DATA
340 M1 = 7 : EQUIV. FACT
350 S1 = 0 & D4 = 0 & F4 = 0.025
360 IF J = 1 THEN READ T14, M4, M14 PRINT T14, M4 & ASSIGN 0 TO M4 & PRINT '1, J4, M4 & RETURN
370 DATA "S.C.F. 1. PAINT BRICK", "PDATB", 14
380 READ M4, T4, H, L1, T1, T1, M14, E3, E4, M2, P9
390 L = 609
400 DATA B2, BRK, 2093, 609, 92, 38, 28.9, 0.15, -15, 0, 1126
410 DATA B3, BRK, 4242, 609, 92, 38, 28.7, 0.15, -15, 0, 516
420 DATA B4, BRK, 2, 609, 2093, 38, 28.4, 0, 0.61, -61, 0, 1334
430 DATA B5, BRK, 2093, 609, 92, 38, 28.8, 0, 0, 31, -31, 0, 748
440 DATA P11, BRK, 4242, 609, 92, 38, 28.9, 0, 31, -31, 0, 536
450 DATA B26, BRK, 2222, 609, 92, 38, 28.6, 0, 0, 15, 0, 919
460 DATA B27, BRK, 4242, 609, 92, 38, 28.6, 0, 0, 15, 0, 380
470 DATA B30, BRK, 2093, 609, 92, 38, 29.1, 0, 0, 31, 0, 483
480 DATA B31, BRK, 4242, 609, 92, 38, 27.6, 0, 0, 31, 0, 127
490 DATA B35, BRK, 2751, 609, 197, 38, 33.1, 0, 33, 33, 0, 2235
500 DATA B36, BRK, 2093, 609, 92, 38, 28.5, 0, 15, 0, 615
510 DATA B38, BRK, 4242, 609, 92, 38, 28.9, 0, 15, 0, 282
520 DATA B43, BRK, 2093, 609, 92, 38, 29.0, 0, 31, 0, 192
530 DATA B46, BRK, 4242, 609, 92, 38, 30.0, 31, 0, 52
540 IF ABS(E3) > ABS(E4) THEN E1 = E3 & E2 = E4 & GOTO 560
550 IF ABS(E3) > ABS(E4) THEN E1 = E4 & E2 = E3
560 IF E1 = 0 AND E2 = 0 THEN E2 = .1
570 M = M241/1000
580 RETURN
590 ASSIGN 1 TO J
600 END
10 50  "CONST" (A, D4, E1, E2, E3, E5, E6, E7, F1, F2, F3, I1, I2, I4, I5, I6, I7, I8, I9, L2, M, F0, P11, P12, S1, S2, T, T1, T8)
15  OPTION BASE 1
20 IF T1="RCK" THEN ES=F2S22O, SA(+1)=1066 ELSE ES=MIN(144F1+5641, 20700)
25  ES=ES-.003 & F6=.25 & F7=.460
30 : CALC, CONSTANTS
35  L2=L1-L1 & F3=ES4ES & IF F3=F1 THEN F3=F1
40  IN=2009000/ES
50 : CALC UNECRACKED SECT PROP
60  A=1(I1+2I111L2
70  $I=1(I111+2I111112/12+2I11111+I(I111-1111/2)
80  $S=2I111/T + P0=A1F3 & K=S/K
90 : PRINT USING 100 ; P0/1000
100  IMAGE /, "Po", .P0000
110 : CALC 1st
120 : CALC 1cr 148
130 : FOR I1 = 1 TO 2
140  IF I1 THEN E=ARS(E11) ELSE E=ARS(E2)
150  IF E<1 THEN (I11=1) & E2=2I111(I11T1) & P1=P0(ES/E/E2) & GOTO 200
160  I=0 : FIND 1, P FOR GIVEN E
170  CALL "P-E" (D1, D2, D3, D4, E.F2, F2, L1, L2, M, N, F, P1, S1, S2, T, T1, T5, T6, X, X, Y, Y)
180  CALL "ICR" (D1, D2, D3, D4, E.F2, F2, L1, L2, M, N, F, P1, S1, S2, T, T1, T5, T6, X, X, Y, Y)
190  P1=P & I1=12
200  NEXT I
210  IF ABS(E21) > ARS(E11) THEN E7=E1/E2 ELSE E7=E2/E1
220  IF E1 > OR S200 THEN GOTO 260
230 : PLAIN MASONRY
240  IF E7 > AND E7<1 THEN I6=(I11+I12)/4 & GOTO 340
250  I6=(I11+MIN(I11, I12))/4 & GOTO 340
260 : REINF'D MASONRY
270 : CALC lc (P=0)
280  I=3 : FIND (X, A) FOR C=1s
290  CALL "P-E" (D1, D2, D3, D4, E.F2, F2, L1, L2, M, N, F, P1, S1, S2, T, T1, T5, T6, X, X, Y, Y)
300 : CALC lc (P=0)
310 - CALL "ICR" (D1, D2, D3, D4, E.F2, F2, L1, L2, M, N, F, P1, S1, S2, T, T1, T5, T6, X, X, Y, Y)
320  IF E70 > AND E7<1 THEN I6=(I11+I12+I12)/4 & GOTO 340
330  I6=(MIN(I11, I12))/4 & GOTO 340
340  ES=I1I6E5
350 : PRINT USING 360 ; A, I1, S, P0, K1
370 : PRINT USING 360 ; E1, E2, E7
380  IMAGE "E1=", DDD, "E2=", DDD, "E7=", DDD
390 : PRINT USING 400 ; I111, I122, I16, E5
400  IMAGE "I111=", D.DDE, "I122=", D.DDE, "I16=", D.DDE, "E5=", D.DDE
410 : SUBEND
20 IF 0 FINDS 1 FOR E; IF 1 FINDS 1 FOR P; IF 2 DRAWS INT CURVE; 2=3 FINDS 1 FOR E:15 = P
30 L1 = 1/100 6 16/12000 6 16/1000 6 61/2 6 64/2 6 62=1/2 6 63=1/2 6 64/2
40 FOR K=1 TO 100
45 IF 1=1 THEN 420
47 IF 1=1, 115 THEN I = I-15 ELSE 1=1-16
50 11=13, 14=0
52 IF I=0 THEN 1=1
56 IF I=0 THEN 160
70 IF S=0 THEN 160
60 F=NIFS (D-1)/3/2 6 IF F>F2 THEN F=F2
90 T2=151
160 IF I=120 THEN 160
120 IF S=0 THEN 140
120 F=NIFS (D-1)/3/2 6 IF F>F2 THEN F=F2
130 T3=152
140 IF I=3 THEN 160
150 IF S=0 THEN 160
160 F=NIFS (D-1)/3/2 6 IF F>F2 THEN F=F2
170 T4=151
180 IF I=1 THEN 160 ELSE 16=1
190 IF X=X-1 THEN 55=0 ELSE 55=X-1+1
200 F=F+51 (X-1)/2
210 C1=51 (F3-F4)*161/2 6 C2=F41612
220 C3=F31112
230 C4=F3112 (1/2)
240 P=C1+C2+C3+C4-13-14
250 IF I=3 THEN GOTO 260
260 IF PK0 AND 15>16 THEN I=I+15 @ 15=16 & GOTO 390
270 IF F>0 THEN 390
280 N=C1 (1/22-16/2)+C2 (1/2-16/2)+C3 (1/2-15/2)+C4 (1/2-14/2)+C5 (1/2-13/2)
290 IF P>0 THEN 390
295 IF 15=16 THEN 7=15 6 15=16 6 GOTO 350
299 P=1 6 GOTO 420
300 E4=W/P
310 PRINT USING 320: ;1.F(1000,E4
320 IMAGE "F", DGG, "F", DGG, "E4", DGG
330 IF Z=1 THEN 360
340 IF ABS(E1)=E4 AND 15 16 THEN I=I+15 @ 15=16 6 GOTO 350
350 IF ABS(E1)=E4 THEN 420 ELSE 390
360 IF Z=2 THEN 410
370 IF F<P1 AND 15 16 THEN 2=2+15 6 15=16 6 GOTO 350
366 IF F<P1 THEN 420
390 NEXT k
410 GOTO 420
410 GOTO 390
420 GOTO 390
10 SUB "GRAPH5" (A, d4, E1, E2, E5, E7, F2, F3, H1, H2, M, N, P0, R2, S1, S2, T1, W)
15 OPTION BASE 1
20 R2=2111/(u11) & U1=1000 & U2=1000000
30 Y1=1.2Z8/60 & Z2=15X/60 & Z3=122/4 & Y2=Y1/4
40 PEN 1 & ECLEAR
50 SCALE =12, Y2, Y1
60 R1110 90, 90, 0, 0, 12
70 YAJIS 0, 100, 100, Y1
80 MOVE 313/4, Y1/2 & LDIR 90 & LABEL "P1M1"
90 MOVE IX/2, Y2 & LDIR 0 & LABEL "M1H-M1"
100 FOR I=1 TO Y1/560
110 X=5001
120 MOVE -313/8, A & LDIR 0 & LABEL VA1(K1) & DRAW 1,K
130 HEIT 1
140 FOR I=1 TO 12/50
150 X=501
160 MOVE K, -313/4 & LDIR 90 & LABEL VA1(K1) & DRAW K, 10
170 HEIT 1
180 MOVE 0, P0/U1
190 P0=P0/2
200 MOV R21 (PO-PS)
210 DRAW M/U2, PS/U1
220 Z2 & P1=PS & CALL "P-E" (D1, d2, d3, E4, E2, F2, F3, L1, L2, M, N, P0, P1, S1, S2, T1, T5, T6, Z, 2)
230 SUBEND
16 SUB "IIP-S" (D4,E1,E2,EE,F3,F5,H1,L1,L2,M,F,PO,PIE,SL,1,1,1,1,1)
17 : I2=1 INHIBIT TERTIARY VACancies
20 OPTION BASE 1
30 DIM M5(2),C12,DI11,DI11,AL11,B22(1),M11,P2(10)
35 : PRINT "OLT P-S METHOD"
40 : PRIMARY REFLECTIONS
50 IF P1(1)=P1(2) THEN P4,P2=P+F+1 ELSE P4,P2=P+F(2)
60 : PRINT USING "64 ; P2/1600"
70 IMAGE *P2=PDDD
80 I4=M10 & VI=1000 & u2=100000 & v17,19,1,2,2=15=P0/15 & 16=15/5 & M11 P2=ZERO
90 DB=0
100 FOR I=1 TO 11
110 DI11,B211,M11,DI1=0
120 NEXT I
130 M1=P21E1 & M2=F21E2
140 FOR J=1 TO 10
150 I=(I-1)/114
160 FOR K=1 TO 10
170 C(J)=(J-1)/114 & C(J)=J14
180 NEXT K=1 TO 2
190 MS(K)=W4(C(J)+H-C(J)/2)+M21(C(J)+H-K111H-C(J11))/11
200 NEXT J
210 GOSUR B36 ; M4
220 D11=DI11*A71K6
230 NEXT J
240 IF ABS(D11))DB THEN DB=ABS(D11)
245 IF J2=1 THEN D11=DI11)
250 NEXT J
260 IF I2-O AND ABS((E1+E2)/2)+D61/2 AND S1=0 AND S2=0 THEN P4,P2=.54F2 & GDP B3
265 J7=1
270 : PRINT "STAY PRINT.S"s"
280 FOR I=1 TO 11
290 : PRINT USING "sD3X,DDD" ; 1.DI11)
300 NEXT I
305 IF 22=1 THEN S60
310 : SECONDARY DEFLECTIONS
320 K=0
330 D9=0 & K=K+1
340 FOR J=2 TO 10
350 I=(I-1)/114
360 FOR J=1 TO 10
370 C(J)=(J-1)/114
380 MS(J)=DI111P2
390 MS(J)=DI11111P2
400 GOSUR B36 ; M4
410 D22111=DI1111A71K6
420 NEXT J
430 IF ABS(D211))DB THEN DB=ABS(D211)
440 NEXT J
450 : PRINT "STAY 5"S, SECY 5"S"
460 FOR I=1 TO 11
470 IF K=1 THEN D11=D111+D2111
480 IF K=2 THEN D11=D111+D211
490 : PRINT USING "sD3X,25D3DP3,31)" ; 1.DI111.B2111
500 DI111=B211
510 D211=0
570 IF DF<DE OR A<6 THEN P4, P2 = .74P2 & GOTO 830 - UNSTABLE => WILL REDUCE P2
575 IF 15=0 THEN 560 - BYPASS TERTIARY DEFLECT Cycles
580 DO-9
585 IF DO=2 THEN 330
590 CALL P-65
595 PRINT "STN Pn P C M NT
580 FOR P+1 TO 11
590 I=1 TO 11
600 IF K2=21/2+K11*(H-1)/H*K12/2
605 IF K3=0 THEN K3=.001
610 P1=P21(D)
620 M1=M3+P1 * R(1)=M11/M
625 PRINT USING "DD,2X,3(SDDD,51),D.DD" ; 1, M5/23, P1/23, M11/23, R1
700 NEXT I
705 SELECT M AT
710 R4=M=0
720 FOR I=1 TO 11
730 IF M(ABS(R(1))) THEN M=M(ABS(R(1)))
740 IF R4=R(1) THEN R4=R(1)
750 NEXT I
760 E=M4/F2 & IF E=X1 THEN F=P041E/((E+1))=E=51E & GOTO 760
770 IF 0=0 THEN 700 FOR GIVEN E
780 CALL "F.E" (D1,D2,D3,D4,E,F2,F3,F1,L1,L2,M,N,P,1,S1,S2,T1,T5,T6,1,2)
790 PRINT "E(m) R11=23 P1(P1=) P(1=) M/P=1953/P0"
800 PRINT USING "DD, D, 23(DDDD, D, 21),DD, 2X, D.DD" ; E,M/23,P/23, R, K
810 IF P3=1.2F4 THEN P2=P4 & GOTO 720
820 P3=P2-P
830 IF ABS(P3<.15P0) THEN I=1 ELSE I=0
840 IF AO+P3(1,631P0) THEN 910
890 IF I=1 AND P3<0 THEN 910
B10 IF P3>0 THEN P2=P2+16 & GOTO B15
B15 IF F3+515 THEN P2=P2+15 ELSE P2=P2-16
B15 K2=K+1 & FOR 1=4 TO D; P21(I)=P21(I+1) & NEXT I & P21(I)=P2
B16 IF K2=10 THEN B30
B17 FOR I=1 TO 4 & IF P2=P21(I) THEN 620 - OSCILLATION DETECTOR
816 NEXT I
820 GOTO B30
820 P2D I=1 TO 10 & IF P2=P21(I) THEN P2=P21(I)
821 NEXT I
822 GOTO 920
830 PRINT USING 840 ; P2/UI
840 I/NASE /* "TRY P=", DDDD
850 GOTO 90
860 ! SUBR M6
870 A7=(M511+M521)/11/2(116)
880 C6=M11+M512(2)M117(311M511+M521)
890 IF K<6 THEN M6=M611-H ELSE M6=M6111/H
900 RETURN
910 IF SI=0 OR SK2=0 THEN 920
915 IF E2=1/2 THEN F2,P4=.1P2 & GOTO B30
920 IF 2Z= THEN P1=10 & P2=UI ELSE P1=8 & P2=UI
930 SUBEND
10 10 SUB 'APP-S' (A, D4, E1, E2, E6, F2, F3, H11, K1, L1, L2, M1, N, P0, P1, S1, S2, T, T1, T1) 
12 OPTION BASE 1 
15 !PRINT "TLS DIR (APPROX) P-S METHOD" 
20 U1=1000 & U2=1000000 
25 !CALC MID HI E 
30 E=(E1+E2)/2 & IF ABS(E)-.0517 THEN E=.0517 
40 GOSUB 120 'P', S1Po 
50 IF P>-.51Po THEN 300 
60 IF S1X0 OR S2X0 THEN GOSUB 170 & GOTO 300 
70 GOSUB 250 'Pc', S1Po, PLAIN 
75 GOTO 300 
80 !SUB GUARD: 
85 D=SQR((E2-4)*T1)1C 
85 ON ERROR GOTO 320 
100 P=-(R-D)/(.2141) 
102 IF P>MIN(P+1,F+2) THEN P=MIN(P+1,F+2) 
105 RETURN 
120 !SUB P, S1Po 
130 A1=H-2E/(1816) 
140 E=E+SINH^4/(384E6)*K1 
150 E=SINH^2/5-F01X1 
155 GOSUB 80 
160 RETURN 
170 !SUB P, S1Po, REINF'D 
180 I=3 ! FINDS I,M ETC FOR P=0 
190 CALL "P-E" (D1, D2, D3, D4, E, F2, F3, L1, L2, M, P, P1, S1, S2, T, T1, T5, T6, 1, 1) 
200 M=H 
210 A1=H-2E/(1816) 
220 E=E+SINH^4/(384E6)*2+M0/P0-K1 
230 C=SINH^2/5-M0 
235 GOSUB 80 
240 RETURN 
250 !SUB P, S1Po, PLAIN 
260 A1=H-2E/(1816)+T/P0-3I1/P0 
270 E=E+SINH^4/(384E6)-1/2 
280 C=SINH^2/5B 
290 =-GOSUB 80 
295 RETURN 
300 !PRINT USING 310 ; P/U1,F/F0 
310 IMAGE "P=", "D0", "P/P0=", "DON" 
315 GOTO 335 
320 OFF ERROR & PRINT "NO SOL'A PADDABLY P&E:N/MR" & P=0 
335 P(1)=P/1000 
340 GOTO 440 
350 CALL "DLOGCS" (A, D4, E1, E2, E5, E7, F2, F3, H11, K1, L1, L2, M, P0, R2, S1, S2, T, T1, T1) 
360 !CALC P(e+SF+SM)+44.2/6 
370 Q1=P6/2O & P3=P0/2 
380 FOR J=1 TO 16 
390 E3=E+F3SH^24E/(9817)+SINH^4/(384E6) 
400 M3=P3E3+SINH^2/6 
420 P3=P3+Q1 
430 NEXT I 
435 COPY 
440 SUSREM
SUB "MAIN" (A,B,C,E1,E2,..,F1,F2,F3,F6,H,H1,K1,L1,L2,M,P,R,S1,S2,T,T1,T2)
20 OPTION BASE 1
30 PRINT "MODIFIED HAW MAM.MAGNIFIER METHOD"
40 PRINT USING 26 ; P
50 IMAGE /, "F4", 1000
60 PLOT SHORT SEC'N INT. CURVE
70 CALL "GAPACE" (A,D1,E1,E2,E5,E7,F2,F3,H1,K1,L1,L2,M,P,R,S1,S2,T,T1,T2)
80 PRINT USING "P*/USE P'S FROM SUB'CONST"
90 IF ABS(W)<10 THEN GOSUB 140, 2 GOTO 100 "SUAR Max" 
100 IF E7=0 THEN E=E+1(E1+E2)/2 ELSE E=MN(MAX(ABS(E1),ABS(E2)) 
110 IF E<0 THEN E=E 
120 IF F3>P3/2 THEN N=N1+1(P0-P3) GOTO 150 
130 J=J+1 GOTO P1+P3 
140 CALL "P" (D1,D2,D3,D4,E2,F3,L1,L2,M,M,P,P1,S1,S2,T,T1,T2,T3) 
150 N-1 
160 'CALC Pcr (PS)
170 GOSUB 440 'Pcr 
180 IF F3>P3 THEN PDC=P*DPS 
190 PDC=PDC .41E1 
200 IF ABS(W)<10 THEN GOSUB 140, 2 GOTO 60 
210 GOSUB 440 ' Pcr 
220 D5=C5/I3-P3/PS 
230 M5=P3D5 
240 IF J=0 THEN GOSUB 140, 2 GOTO 70 "SUAR Min" 
250 D5=C5/I3-P3/PS 
260 PRINT "P" 
270 PRINT USING "(PDC, .41E1, P0)" ; P3/PS, PS/I3 
280 IF N3=N3 AND F3=0 THEN GOTO 60 
290 IF N3<N3 AND F3=0 THEN GOTO 60 
295 IF N3<N3 AND F3=0 THEN GOTO 60 
300 P3=P3/PS 
310 IF P3=0 THEN GOSUB 140, 2 SUBEX 
320 IF P3=0 THEN GOSUB 140, 2 SUBEX 
330 GOTO 600 
340 GOTO 70 "SUAR Min" 
350 IF N3=N3 AND F3=0 THEN N=PS/I3 
360 IF N3=N3 AND F3=0 THEN N=PS/I3 
370 FOR I=1 TO 10 
380 IF I=N THEN GOTO 70 "SUAR Min" 
390 W=W+1(I=1) 
400 IF N=ABS(N) THEN GOTO 70 "SUAR Min" 
410 NEXT I 
420 E=E/P3 
430 RETURN 
440 'SUB Pcr
450 IF E(X1) AND S1>0 OR S2>0 THEN G010 S60
460 F9=44(F5/(S1+114-21E/111)
470 E9=211F3/(AIFV)
480 T9=E91F6/F9
490 B9=0.5-E/T+19/(2T))**3
500 ! E7=E1/E2
510 IF E7>0 THEN P5=9.871E512111/H**2 @ G010 S90
520 ! E7 NEG.
530 A9=ABS(E1)/(ABS(E1)*ABS(E2))
540 TB=104(B9+ABS(1-B9)+2169*2+B9*3) @ IF 1610 THEN 16=10
550 PS=9.871E5111/H**2 @ G010 S90
560 ! REINF'D WALL e1/T
570 R3=.5-E/T @ IF R3<.1 THEN R3=.1
580 P5=9.871R31E5111/H**2
590 RETURN

10 SUB "APP-S1" (A,D4,E1,E2,E6,F2,F3,H,11,K1,L1,L2,N,P0,P1,P2,P3,P4,P5,P6,P7,P8,P9)
20 ! CALCULATES VARIABLE H (P=Ptest)
30 OPTION BASE 1
40 P=P11
50 ! CALC MID HT E
60 E=(E1+E2)/2 @ IF ABS(E)<.0541 THEN E=.0541
70 ! CALC N&P
80 IF P<.510 THEN 100
90 N=(P0-P1)*K1 @ G010 S150
100 IF S1>0 OR S2>0 THEN 140
110 ! PLAIN MASONRY, P<P0/2
120 H=P11/2-P*211/P0+21P*21K1/P0 @ G010 S150
130 ! REINF'D, P<P0/2
140 P1=P 1=1 @ CALL "P-E" (D1,D2,D3,D4,E,F2,F3,L1,L2,M,N,P,F2,F1,S1,S2,T,T1,T5,T6,X,Z)
150 M=(P-H)*P211E412111/G1E61+P1E/211/(51P1K4/(5841E61+H**2/G))
160 SUBEIT
10 SUB "ITP-51" (D4,E1,E2,E6,F2,F3,H,K1,L1,L2,N,P,P0,P1,S1,S2,T1,T1,UZ2,W1)
12 ! CALCULATES VARIABLE W (IP CONSTANT)
15 ! IZ=1 INHIBITS TERTIARY DEFLECTIONS
20 OPTION BASE 1
30 DIM M$(2), C$(2), D(11), R(11), D2(11), H(11)
35 * PRINT "DLT P-C METHOD"
40 ! PRIMARY DEFLECTIONS
50 IF P(1)<P(2) THEN P4,P2=P(1) ELSE P4,P2=P(2)
60 ! PRINT USING 70, P2/1000
70 IMAGE "P2=""DDDD"
80 L4=H/10 @ U1=1000 & U2=100000 & I7,19,U,K2=0 & I5=P0/10 & I6=15/5
81 ! CALC M&P=P(test4W2init.
84 IF P2>P0/2 THEN M=K1*(P0-P2) @ GOTO 86
85 CALL "P=E"(D1,D2,D3,D4,E,F2,F3,L1,L2,N,P,P0,P2,S1,S2,T1,T5,T6,T1)
86 M2=81/M/H^2
90 DB=0 ! PRIMARY DEFLECTIONS
100 FOR I=1 TO 11
110 D1(I),Q2(I),M(I),D1(I)=O
120 NEXT I
130 M1=P2E1 @ M2=P2E2
140 FOR I=2 TO 10
150 J1=I-1
160 FOR J=1 TO 10
170 C(I)=I-J+11 @ C(2)=J14
180 FOR K=1 TO 2
190 M5(K)=M2*(C(I)+11)/2+M2*(C(K)+11)/H+M1*H-C(I)/H
200 NEXT K
210 GOSUB 840 ! M
220 D1(I)=Q1(I)+A7*M6
230 NEXT J
240 IF ABS(D1(I))>DB THEN DB=ABS(D1(I))
245 IF Z2=1 THEN D(I)=D(I)
250 NEXT I
305 IF Z2=1 THEN 560
310 ! SECONDARY DEFLECTIONS
320 K=0
330 D9=0,2 K=K+1
340 FOR I=2 TO 10
350 J1=I-1
360 FOR J=1 TO 10
370 C(I)=I-J+11
380 M5(I)=D1(I)+P2
390 M5(2)=D1(I)+P2
400 GOSUB 860 ! M
410 D2(I)=D2(I)+A7*M6
420 NEXT J
430 IF ABS(D2(I))>D9 THEN D9=ABS(D2(I))
440 NEXT I
450 ! PRINT "STA $S Sec'y $S"
460 FOR I=1 TO 11
470 IF K=1 THEN D(I)=D(I)+D2(I)
480 IF K=2 THEN D(I)=D(I)+D2(I)
490 ! PRINT USING "DD,3X,2(SDDD,3X)" ; I,D(I),D2(I)
500 D(I)=D2(I)
510 D2(I)=0
520 NEXT I
530 IF I9=0 THEN S60 ! Bypass tertiary def. cycles
540 DB=D9'
550 IF D92=3 THEN 330
560 ! CALC P-'S'
570 ! PRINT "STN PM P-6 Mt Mt/Np"
580 FOR I=1 TO 11
590 K=(I-1)/11
600 M3=M211*(H1-X^2)/2*K111/(H^2)*M211/H
610 IF M3=0 THEN M3=.001
620 P1=P211D(I)
630 ! PRINT USING "DD,3X,2(SDDD,3X),.DD" ; I,M3/U2,P1/U2,M(I)/U2,R(I)
640 NEXT I
650 ! SELECT MAI MDH
660 R4,M4=0
670 FOR I=1 TO 11
680 IF M4(ABS(M(I))) THEN M4=ABS(M(I))
690 IF M4(R(I)) THEN M4=R(I)
700 NEXT I
710 M3=M-M4
720 IF ABS(M3)<1*M THEN 610
730 IF M4>M THEN W2=1.11W2 & GOTO 90
740 IF M4<M THEN W2=.99W2 & GOTO 90
750 IF X=0 THEN W(1)=W2ELSE W(1)=W2
760 GOTO 930
770 ! SUBR Mm
780 D7=M5(1)+M5(2)+I4/(2*E6)
790 C6=C(1)+M5(1)+2*M5(2)I4/((I31(M5(1)+M5(2))
800 IF X=C6 THEN M6=(X-C6)I4/H ELSE M6=(X-I)C6/H
810 RETURN
820 SUBEND
10  SUB "H6WNI" (A,D4,E,E1,E2,E5,E7,F2,F3,F6,H11,K1,L1,L2,M6,P0,P1,S1,S2,T,T1,W1)
15 OPTION BASE 1
30  \* PLOT SHORT SEC'N INT. CURVE
40  \* CALL "ERPMCS" (A,D4,E,E1,E2,E5,E7,F2,F3,M11,K1,L1,L2,M6,P0,P2,S1,S2,T,T1)
50  \* INITIALIZE P-USE P'S FROM SUB'CONST'
60  P3=P1(1)
70  J=0  @  UI=1000  @  U2=10000000
90  IF E7>0 THEN E=(E1+E2)/2 ELSE E=MAX(Abs(E1),Abs(E2))
100 IF E=0 THEN E=-E
110 IF P3=P0/2 THEN M=K11(P0-P3) \* GOTO 150
120 I=1  @  P1=P3
140 CALL "P-E" (D1,D2,D3,D4,E,F2,F3,L1,L2,M6,P0,P1,S1,S2,T,T1,T5,T6,T7)
150 W=841/H*2
155 GOSUB 340 \* Surr Max
160 \* CALC Pcr (P5)
170 GOSUB 440 \* L Pcr
179 C5=.44+.10E7
200 IF ABS(W)>0 THEN C5=1
210 GOSUB 440 \* Pcr
220 D5=C5/(1-P3/P5)
230 HS=H4DS
240 IF J=0 THEN MOVE M5/U2,P3/U1  @  J=1
250 DRAW M5/U2,P3/U1
260 PRINT "P  P  P  E  M  Pcr"
270 PRINT USING "DDDD,3(DDD,D,11),DDDDDDDD" ; P3/U1,D5,M5/U2,M/U2,P5/U1
275 K3=K-N5
276 IF ABS(K3)<.11M THEN W1=W \* SBEIXI
280 IF K5>M THEN W=.91W  @  GOTO 155
285 IF K5M THEN W=1.11W  @  GOTO 155
295 DRAW M5/U2,0  @  MOVE M5/U2,P3/U1  @  DRAW 0,P3/U1
296 MOVE 10,100+P3/U1  @  LDIR 0  @  LABEL VAL((IP(P3/U1))
300 P7=P3/U1
310 PRINT "E(na)-H1(W-n) P(kW) PIP0"
320 PRINT USING "SDDD,25SDDD,S2)DDD,21,L,DD" ; E,M5/U2,P3/U1,P3/P0,D5
330 GOTO 600
340 \* Surr Max
350 14=M/10  @  N1=P3E1  @  M2=P3E2
360 M4=0
370 FOR I=1 TO 10
380 I=1114  @  I=N-I
390 M4=M111/M4+M111/M4+M111(N-1)/2
400 IF M4<ABS(H6) THEN M4=ABS(H6)
10 MAIN PGN RESIST
20 OPTION BASE 1
30 DIM I(3),F(2)
40 PRINT ALL
50 Disp "INPUT L,L1,T,T1,e1,e2"
60 INPUT L,L1,T,T1,e1,e2
70 Disp "INPUT F,G,F,Y,As1,As2,Ds1"
80 INPUT F1,F2,S1,S2,D4
90 Disp "INPUT Ec,Max.Mas,Strain"
100 INPUT ES,ES
110 NORM
120 CALL "CONST2" (A,D4,E1,E2,E3,E5,E6,E7,F1,F2,F3,11,11,16,16,I1,I1,L1,L2,M,P0,P1,S1,S2,T,T1,T6)
130 CALL "GRAPHC2" (A,D4,E1,E2,E3,E5,F2,F3,H1,K1,L1,L2,N,P0,R2,S1,S2,T,T1,T6)
150 COPY
160 END

10 SUR "CONST2" (A,D4,E1,E2,E3,E5,E6,E7,F1,F2,F3,11,11,16,K1,L1,L2,N,P0,P1,S1,S2,T,T1,T6)
15 OPTION BASE 1
20 'CALC CONSTANTS
30 L2=L-L1 & F3=ES*ES & IF F3>F1 THEN F3=F1
40 N=200000/ES
50 !CALC UNCRACKED SECT PROP
60 A=F1+2F1*L2
70 T1=L*(3/12+2T1)*3L2/12+2T1*L2*(T-T1)/2*2
80 S=2T1*T2 & PO=A0F3 & K1=5/A
110 !CALC lcr T&B
120 !CALC lcr & T&B
130 FOR I=1 TO 2
140 IF I=1 THEN E=ABS(E1) ELSE E=ABS(E2)
150 IF E<=K1 THEN 111=(111)/(A11) & P1=P1R2/(E+R2) & GOTO 200
160 E=0: FIND X,P FOR GIVEN E
170 CALL "P-E" (D1,D2,D3,D4,E,F2,F3,L1,L2,M,M,P1,S1,S2,T,T1,T5,T6,I,1)
180 CALL "ICR" (D1,D2,D3,L1,L2,M,S1,S2,T,T1,T5,T6,I)
190 P(1)=P(1)+1(1)=1(1)
200 NEXT I
210 IF ABS(E2)>ABS(E1) THEN E7=E1/E2 ELSE E7=E2/E1
220 IF S1>0 OR S2>0 THEN GOTO 260
230 !PLAIN MASONRY
240 IF E7>0 AND E7<1 THEN 16=(111+2112)/(121)/4 & GOTO 340
250 16=(111+MIN(111,212))/4 & GOTO 340
260 !REINF'D MASONRY
270 !CALC lcr (P=O)
280 E=5: FIND X,H,AI FOR C=Ts.
290 CALL "P-E" (D1,D2,D3,D4,E,F2,F3,L1,L2,M,M,P1,S1,S2,T,T1,T5,T6,I)
300 !CALC lcr (P=O)
310 CALL "ICR" (D1,D2,D3,D4,L1,L2,M,S1,S2,T,T1,T5,T6,I)
320 1(1)=12
330 PRINT USING 316; P0,M
331 IMAGE X, "P0=",D.DDE,4X, "M=",D.DDE
340 IF E7>0 AND E7<1 THEN 16=(111+2112+2112)/4 & GOTO 340
350 16=(MIN(111,212)+2112)/4
10 SUB "6RPHC2" (A, B4, E, E1, E2, ES, E7, F2, F3, H, I1, K1, L1, L2, N, P0, P2, S1, S2, T, U1, W)
15 OPTION BASE 1
20 R2=21111/11111  \ U1=1000  \ U2=1000000
30 T1=1.21PO/UL  \ T2=1.6021PO/UL  \ T3=12/4  \ T4=1/4
40 PO=1  \ & CLEAR
50 SCALE -I3, I2, -Y2, Y1
60 TAXIS 0, 1, 0, I2
70 YAXIS 0, 20, 0, Y1
80 MOVE -31I3/4, Y1/2  \ LDIR 90  \ LABEL "P(kH)"
90 MOVE, I2/2, -Y2  \ LDIR 0  \ LABEL "H(kH-m)"
100 FOR I=1 TO Y1/200
110 K=20011
120 MOVE -5I3/I8, K  \ LDIR 0  \ LABEL VALS(K)  \ DRAW I2, K
130 NEXT I
140 FOR I=1 TO I2/10
150 K=1041
160 MOVE I, -31Y2/4  \ LDIR 90  \ LABEL VALS(K)  \ DRAW K, Y1
170 NEXT I
175 MOVE 0, 0  \ DRAW PO+ABS(E1)/U2, PO/U1  \ MOVE 0, 0  \ DRAW PO+ABS(E2)/U2, PO/U1
180 MOVE 0, PO/U1
190 P0=P0/2
200 M=R21(P0-P5)
210 DRAW M/U1, P8/U1
220 CALL "P-E1" (D1, D2, D3, D4, E, F2, F3, L1, L2, M, N, F, P1, S1, S2, T, U1, T5, T6, X)
250 SUBEND
FROM DESIGN OF PLAIN MASONRY WALLS  OJINAGA & TURKSTRA 1977

\[ I_{\text{eff}} = \frac{I_1 + I_2}{4} = \frac{I_1}{2} = \frac{I_2}{2} \]

**a) single curvature**

\[ I_{\text{net section uncracked}} \]

\[ I_{\text{eff}} = \frac{I_1 \cdot I_2}{A} = \frac{I_2 \cdot I_1}{4} \]

**b) double curvature**

**FIG. 3** REASONING BEHIND THE EFFECTIVE INERTIA ASSUMPTION.
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